

Economic Growth in an Interdependent World Economy¹

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Abstract

The Solow-Swan growth model predicts that growth should be uncorrelated with the ratio of national investment to GDP. If capital markets are open, the model predicts instantaneous convergence of GDP per capita across countries. Convergence is achieved by capital flows from rich to poor countries and a consequence of these flows is that the ratio of national savings to GDP in each country should differ substantially from the ratio of investment to GDP since there is no reason to expect that countries with high savings rates should be those with large investment opportunities. In the presence of capital market imperfections, such as the inability to borrow to finance human capital accumulation, convergence is predicted to occur more slowly. But savings and investment ratios should still differ substantially across countries. In the data, investment ratios are strongly correlated with growth across countries and investment ratios are closely correlated with savings ratios within countries. Both of these facts are difficult to explain with versions of the Solow-Swan model. We study a two-sector two-country AK model and argue that this model can account for these facts and that in this sense it provides a better description of the data than the Solow-Swan model.

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1 Introduction

In this paper we address the question: What is the best way to characterize the low-frequency movements of major macroeconomic aggregates in a group of countries? In an influential paper in 1992, Mankiw, Romer and Weil, (MRW) [21] argued that an amended form of the Solow-Swan model [27], [28] provides a good description of the facts.

Examining recently available data for a large set of countries, we find that saving and population growth affect income distribution in the directions that Solow predicted. Moreover, more than half of the cross-country variation in income per capita can be explained by these two variables alone.¹

A possible alternative to the Solow-Swan model is some variant of a model of endogenous growth either in the form of the AK model developed by Romer [25], Lucas [17] and Rebelo [24] or in the subsequent group of models by Romer [26], Grossman and Helpman [10], [11] and Aghion and Howitt [1] that focus more explicitly on the determinants of technological change. In this paper we study a two-sector two-country version of the AK model and we argue that this model is a more attractive paradigm for the study of a collection of interdependent national economies than the Mankiw-Romer-Weil variant of the Solow-Swan model.

Endogenous growth models have been criticized by Jones 1995 [13] who points to an important prediction of the AK model; periods with high investment ratios should be periods of high growth rates. Jones examines post-war data from the U.S. and from a sample of eight OECD countries: He finds that while investment to GDP ratios have risen in many countries over the postwar period, GDP growth rates have stayed roughly constant or have fallen.

In 1998, McGrattan [22] argued that Jones overstates the case in favor of the Solow-Swan model. She points to evidence from longer time series for a larger panel than that examined by Jones. McGrattan finds, using data from Maddison ([19], [20]) for 1870–1989, that

¹Mankiw-Romer Weil [21] page 407.

Jones' deviations from investment and growth trends are relatively short-lived, and periods of high investment rates roughly do coincide with periods of high growth.²

More convincing (to us) is the cross-country evidence that she cites on the relationship between average growth rates and average investment rates. This is summarized in Figure 1.³

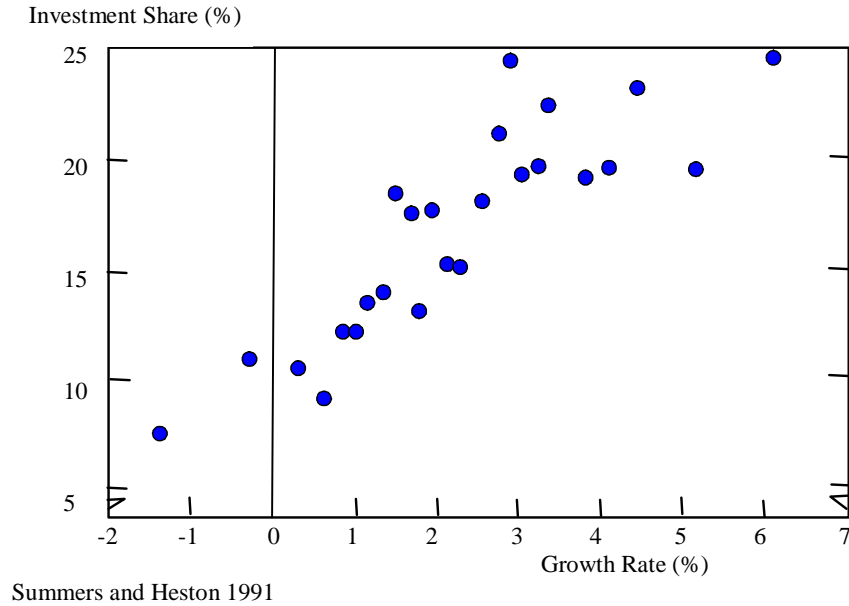


Figure 1: Growth Rates and Investment Shares

We take the view that the appropriate way to model a panel data set, such as the growth data of Heston Summers, is by assuming that each country represents a draw from an invariant probability distribution. If one takes this viewpoint then it is appropriate to compare the average growth rate and the average investment rate, across the panel, with the predictions of a theoretical model in which each country has attained a stochastic steady state. The stochastic steady state of the Solow-Swan model predicts that growth rates should be independent of investment ratios but the evidence summarized in Figure 1 shows that there is a strong correlation between these variables

²McGrattan [22] page 18.

³The figure depicts the growth rates of gross domestic product per worker and investment shares of gross domestic product for 125 countries ranked by annualized 25-year growth rates, then averaged in groups of five, 1960-85. Source, McGrattan [22] page 23.

across countries. This evidence presents a problem for models like that of Mankiw-Romer-Weil in which growth is attributed to exogenous increases in total factor productivity.⁴

We are not the only authors to express skepticism with the Mankiw-Romer-Weil orthodoxy. In a recent paper, Bernanke and Gürkaynak [6] re-estimate the MRW model using data from a pre-publication version of the most recent version of the Summers-Heston [12] data set. They are able to replicate the Mankiw-Romer-Weil findings for the period from 1960 through 1985 but for the extended data set they find much weaker support for the overidentifying restrictions implied by the Solow-Swan model. They argue that the MRW results can be divided into two parts. The first part, which is accepted by the data, is a set of restrictions that would be satisfied by any growth model with a long-run balanced growth path. The second part, a set of restrictions implied by exogenous growth models, is more contentious. Specifically Bernanke and Gürkaynak find that the implication that growth rates are independent of investment ratios is strongly rejected by the data, in line with the evidence from Figure 1 above.

Evidence on the relationship between investment ratios and growth rates is one reason to seek an alternative to the MRW model but it is not the only reason. Mankiw-Romer-Weil [21] choose to model the world as a collection of closed economies. They take this approach since, if the Solow-Swan model is opened to international borrowing and lending, it predicts instantaneous convergence of per-capita GDP. Barro et al [2] have argued that the model can be partially opened by allowing individuals to borrow against the accumulation of physical capital but by closing down the market for borrowing and lending to finance the accumulation of human capital. Although this argument goes some way towards alleviating problems inherent in the Solow-Swan model it does not go far enough since the model still predicts a much larger imbalance in the market for international borrowing and lending than we observe in practice. We examine this argument more closely in Section 6.2.

⁴We should note that this view is not uncontroversial and many authors assume instead that the typical country in the Heston-Summers data had not yet attained its steady state in 1960.

2 Related Literature

Lucas [18] drew attention to the fact that the one-sector neoclassical growth model cannot easily explain why capital does not flow from rich to poor countries and he pointed to three kinds of explanations: differences in human capital, external benefits in human capital and capital market imperfections. In this paper we elaborate on the first of these explanations by developing a two-sector model of a kind first studied by Uzawa [29] and developed more fully by Lucas [17]. Like Uzawa-Lucas, we model a world of two commodities in one which one good is interpreted as physical capital and the other as human capital. We view the “human capital” sector very broadly and, like Uzawa [29], we see human capital as a metaphor for

...various activities in the form of education health, construction and maintenance of public goods, etc., which result in an improvement in labor efficiency...⁵

Unlike the endogenous growth models of Romer [25], [26] and Lucas [17] we develop a convex model of growth that ignores the external benefits of human capital. Predecessors of our work include Rebelo [24], Jones and Manuelli [14] and Jones Manuelli and Rossi [15] each of which study convex models. Jones-Manuelli ask how the growth path would differ in response to different taxation policies and, using a result that Becker [4] calls an *equivalence principle*, they show that tax policies cause the representative agent to act as if he had a different discount factor. In our work, we allow for discount factors to differ across countries and we show that countries with high discount factors will grow faster. These differences in discount factors may be taken to be primitives of preferences or, applying Becker’s result, they may arise as a consequence of different taxation policies in a world in which countries would have the same discount rate in the absence of tax policy.⁶

⁵Uzawa [29] page 18.

⁶Chapter 6 of Becker and Boyd [5] contains a detailed discussion of similar equivalence principals in a variety of neoclassical growth models under alternative assumptions about preferences and technologies.

3 The Model

In this section we construct a two-country version of a generalized version of Lucas' model [17] of growth through human-capital accumulation. The closed economy version of our model was studied by Bond, Wang and Yip [7] who proved existence of equilibrium and characterized the transitional dynamics of non-stationary equilibria to a steady state. Our theoretical contribution is to extend their analysis to the open economy. We will show that the two-country version can account for a number of features of the data that are troublesome for the MRW model.

Our model consists of two countries each of which is inhabited by a representative consumer with rate of time preference ρ and instantaneous utility function $U(C)$, where C is the flow rate of consumption. We use the notation ρ' and C' to denote the rate of time preference and consumption in country 2.

In our model, the savings rate at low frequency will be governed by the rate of time preference. To allow for the possibility that savings rates may differ across countries we chose to allow ρ and ρ' to differ. Since we do not explicitly model the government sector, we think of cross-country differences in ρ as arising either from true differences in the rate of time preference of private agents or from differences in government policies that put a wedge between private and national savings rates.

3.1 Technology

Our economy contains three commodities, K^W , H , and H' . The term K^W represents the world stock of a unique physical commodity that can be consumed or used in production in either country and the terms H and H' refer to the stocks of human capital in countries 1 and 2. We assume that H and H' are distinct and that human capital cannot be traded internationally. There are four technologies that we describe below:

$$Y = F(K_Y, H_Y), \tag{1}$$

$$Y' = F(K'_Y, H'_Y). \tag{2}$$

Technologies (1) and (2) are identical increasing, concave, constant returns-to-scale production functions that describe the production of physical commodities. K_Y and H_Y represent the inputs of capital and human capital used to produce the physical commodity in country 1. Similarly K'_Y and H'_Y are inputs of physical and human capital in country 2.

To produce human capital we assume the following increasing, concave, constant returns-to-scale production functions:

$$\dot{H} = I = G(K_I, H_I), \quad (3)$$

$$\dot{H}' = I' = G(K'_I, H'_I). \quad (4)$$

The symbols I and I' mean investment in human capital in countries 1 and 2, and K_I and H_I (K'_I and H'_I) are inputs of physical capital and human capital to the sectors that produce human capital. Throughout the paper a dot over a variable denotes its time derivative.

Adding up constraints give the following restrictions on the alternative uses of resources,

$$K^W = K + K', \quad (5)$$

$$H = H_Y + H_I, \quad (6)$$

$$H' = H'_Y + H'_I, \quad (7)$$

$$K = K_Y + K_I, \quad (8)$$

$$K' = K'_Y + K'_I. \quad (9)$$

Equation (5) is the aggregate constraint on world capital, and K (K') denotes the stock of capital in country 1 (2). Equations (6) and (7) are the constraints on the uses of human capital in each country and equations (8) and (9) are constraints on the uses of physical capital.

The following definitions of physical to human capital ratios in each industry enable us to simplify notation by making use of the constant returns-to-scale assumption.

$$k_I = \frac{K_I}{H_I}, \quad k_Y = \frac{K_Y}{H_Y}, \quad k'_I = \frac{K'_I}{H'_I}, \quad k'_Y = \frac{K'_Y}{H'_Y}. \quad (10)$$

We assume that the world is composed of two representative agent economies that trade freely in the final good and, hence, in physical capital. However, human capital is non-traded and country specific. The structure of each of the two economies is the same as that studied by Bond-Wang and Yip [7]. Our chief interest is in determining the impact of openness on the equilibrium dynamics of the model.

4 A Social Planning Problem

Since our model has a finite number of agents with concave preferences and since technology sets are convex the welfare theorems apply and there is an equivalence between the set of Pareto Optima and the set of competitive equilibria.⁷ Since it is conceptually simpler to study Pareto optima than to study competitive equilibria we start by asking how a social planner would allocate consumption and organize production in the world economy. In Section 6 we show how to interpret the social planning optimum as a decentralized competitive equilibrium.

4.1 A Statement of the Problem

The social planner solves the problem,

$$Max \int_{t=0}^{\infty} e^{-\rho t} \left[bU(C) + e^{-(\rho' - \rho)t} (1 - b) U(C') \right], \quad (11)$$

subject to the constraints:

$$\dot{K}^W = F(K_Y, H_Y) + F(K'_Y, H'_Y) - C - C', \quad (12)$$

$$\dot{H} = G(K_I, H_I), \quad (13)$$

$$\dot{H}' = G(K'_I, H'_I). \quad (14)$$

In addition he respects the adding up constraints (5–9) and the initial conditions

$$K^W(0) = \bar{K}^W, H(0) = \bar{H}, H'(0) = \bar{H}'. \quad (15)$$

⁷For a proof of this result in the context of an exchange economy see Kehoe and Levine [16]. The extension to the growth model is straightforward.

The parameter b is the welfare weight that the social planner attributes to country 1 and ρ and ρ' are discount rates for the two countries.

4.2 Two Definitions

A solution to the social planner's problem is a set of time paths for the aggregate world capital stocks $\{K^W, H, H'\}$, a consumption plan $\{C, C'\}$, and a plan for allocating capital across countries and across industries $\{k_Y, k_I, k'_Y, k'_I\}$, that maximizes (11) subject to the dynamic constraints (12–14), the adding up constraints (5–9) and the initial conditions (15).

A balanced growth path is a set of time paths $\{K^W, H, H', C, C'\}$, with the property that the ratios $(k^W, c, c', k_Y, k_I, k'_Y, k'_I)$ are constants, where we define the terms k^w , c and c' as follows

$$k^w = \frac{K^W}{H + H'}, \quad c = \frac{C}{(H + H')}, \quad c' = \frac{C'}{H + H'}.$$

4.3 Shadow Prices and First-Order Conditions for Consumption

We let Λ , M and M' represent the costate variables associated with K^W , H and H' : We refer to these variables as *shadow prices* since in a decentralized version of the solution they would be equated to actual transaction prices. We make the following simplifying assumption that U is logarithmic:⁸

$$U(C) = \log(C).$$

The social planner chooses the controls C and C' to satisfy the following first order conditions:

$$\Lambda = \frac{b}{C} = \frac{1-b}{C'} e^{-(\rho' - \rho)t}. \quad (16)$$

Let the variables ψ and ψ' be the ratios of the shadow price of human capital production to the shadow price of physical output in the two countries;

$$\psi = \frac{M}{\Lambda}, \quad \text{and} \quad \psi' = \frac{M'}{\Lambda}. \quad (17)$$

⁸This assumption can be extended to the case of constant elasticity of substitution preferences at the cost of complicating some of the proofs in Appendix A.

In a decentralized economy these variables would be equated to the relative prices of human capital to output. We define the *human-wealth-weighted shadow price*

$$\lambda = \Lambda (H + H').$$

4.4 First-Order Conditions for an Efficient Production Plan

To describe the production side of our economy we introduce the notation $f(k_Y), g(k_Y), f(k'_Y)$, and $g(k'_I)$ to denote the production functions in intensive form:

$$\begin{aligned} f(k_Y) &= F\left(\frac{K_Y}{H_Y}, 1\right), & g(k_I) &= G\left(\frac{K_I}{H_I}, 1\right), \\ f(k'_Y) &= F\left(\frac{K'_Y}{H'_Y}, 1\right), & g(k'_I) &= G\left(\frac{K'_I}{H'_I}, 1\right), \end{aligned}$$

and we use subscripts to denote the partial derivatives of these functions with respect to human and physical capital,

$$\begin{aligned} f_K(k_Y) &= F_K(K_Y, H_Y), & f_H(k_Y) &= F_H(K_Y, H_Y), \\ f_K(k'_Y) &= F_K(K'_Y, H'_Y), & f_H(k'_Y) &= F_H(K'_Y, H'_Y), \\ g_K(k_I) &= G_K(K_I, H_I), & g_H(k_I) &= G_H(K_I, H_I), \\ g_K(k'_I) &= G_K(K'_I, H'_I), & g_H(k'_I) &= G_H(K'_I, H'_I). \end{aligned}$$

The first order conditions for the efficient allocation of resources across countries and across industries leads to the following equations:

$$f_K(k_Y) = \psi' g_K(k'_I), \tag{18}$$

$$f_K(k'_Y) = \psi' g_K(k'_I), \tag{19}$$

$$f_H(k_Y) = \psi g_H(k_I), \tag{20}$$

$$f_H(k'_Y) = \psi' g_H(k'_I), \tag{21}$$

$$\psi g_K(k_I) = \psi' g_K(k_I'), \quad (22)$$

where the variables k_Y (k_Y') and k_I (k_I'), are the sectoral capital labor ratios as defined in Equation (10). Under the relatively standard additional assumptions of, *interiority* and *no factor intensity reversals*, there is a unique value of ψ that satisfies these equations.⁹ Given these assumptions, the ratios of physical to human capital in each industry will be the same in each country and will be functions of this single variable ψ ;

$$k_Y(\psi) = k_Y'(\psi'), \quad (23)$$

$$k_I(\psi) = k_I'(\psi'), \quad (24)$$

$$\psi = \psi'. \quad (25)$$

In a decentralized equilibrium ψ represents the relative price of human to physical capital. The equality of ψ and ψ' implies that these relative prices will be equalized even if human capital cannot be traded. Since the rental rates of physical capital and human capital are functions of the capital/labor ratio this result also implies that rental rates will be equated internationally. This is a restatement of Samuelson's factor price equalization theorem which states, in the context of the two-sector two-country model, that allowing for trade in the final good is sufficient to equate the relative prices of non-traded goods and of factor prices in the two countries.

⁹ *Interiority* is the condition that the elasticities of the reduced form production functions are strictly between zero and one,

$$0 < \frac{f_K(x)x}{f(x)} < 1, \quad \text{and} \quad 0 < \frac{g_K(x)x}{g(x)} < 1.$$

No factor intensity reversals means that one of the following two conditions holds. Either

$$-\frac{f(x)f_{KK}(x)}{f_K(x)^2} > -\frac{g(x)g_{KK}(x)}{g_K(x)^2}, \quad \text{for all } x > 0,$$

or:

$$-\frac{g(x)g_{KK}(x)}{g_K(x)^2} > -\frac{f(x)f_{KK}(x)}{f_K(x)^2}, \quad \text{for all } x > 0.$$

The proof of uniqueness is a straightforward extension of the closed economy proof in Bond-Wang and Yip [7].

5 Characterizing the Planning Optima

In the closed economy version of this model Bond-Wang and Yip [7] proved the existence of a balanced growth path and showed how to characterize non-stationary optimal plans as a system of differential equations in the three variables c , k^w and ψ . This differential equation system has a saddle-path property when linearized around a balanced growth path. The saddle path property implies that, for local initial conditions, there exists a unique non-stationary equilibrium that converges to the balanced growth path.

5.1 Features of the Solution that Mirror the Closed-Economy Model

In Appendix A, we show that a solution to the planners problem is described by the following system of differential equations

$$\frac{\dot{k}^w}{k^w} = A(\psi, \lambda, k^w, s(t)), \quad (26)$$

$$\frac{\dot{\lambda}}{\lambda} = B(\psi, k^w), \quad (27)$$

$$\frac{\dot{\psi}}{\psi} = f_k(k_Y[\psi]) - g_H(k_I[\psi]), \quad (28)$$

together with the boundary conditions

$$\lim_{T \rightarrow \infty} e^{-\rho T} \lambda k^w = 0, \quad (29)$$

$$\lim_{T \rightarrow \infty} e^{-\rho T} \psi \lambda = 0, \quad (30)$$

$$k^w(0) = k_0^w. \quad (31)$$

The functions A and B are defined in the appendix. The variable $s(t)$, given by the expression

$$s(t) \equiv b + (1 - b) e^{-(\rho' - \rho)t},$$

reflects changes in the weight attached to each country over time as a consequence of differences in discount factors. The consumption of the representative agent in each country is related to the

weighted shadow price λ by the equations

$$c = \frac{b}{\lambda}, \quad c' = \frac{1-b}{\lambda} e^{-(\rho' - \rho)t}. \quad (32)$$

This part of the solution is essentially the same as that studied by Bond Wang and Yip [7] for a closed economy.

5.2 Differences from the Closed Economy Model

The complete system has two notable differences from the closed economy model. First, in the two-country model there are two types of human capital and two countries in which the production of physical capital can be located. Second, our model contains two representative consumers and we allow for their rates of time preference to differ. It is this possibility that leads to the fact that the term $s(t)$ appears in the aggregate dynamics. We will discuss this issue further below in Subsection 5.2.2. We deal first with the implications of the fact that there are two alternative production technologies.

5.2.1 The Location of Production

For the two-country model we need to add an additional variable in order to characterize the full solution. This variable, θ , represents the ratio of human capital in country 1 to world human capital and it is defined as follows,

$$\theta = \frac{H}{H + H'}.$$

We show in Appendix A that the location of production is governed by the following differential equation

$$\begin{aligned} \frac{\dot{\theta}}{\theta} &= \frac{(1-\theta)}{\theta} \left(q' - \left[\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]} \right] \right) g(k_I[\psi]), \\ \theta(0) &= \frac{H_0}{H_0 + H'_0}, \end{aligned} \quad (33)$$

where q and q' ;

$$q = \frac{H_Y}{H}, \quad q' = \frac{H'_Y}{H'},$$

are the ratios of human capital used in the production of the physical capital good in each country.

The growth rates of human capital are related to q and q' by the following expressions:

$$\frac{\dot{H}}{H} = (1 - q) g(k_I[\psi]), \quad (34)$$

$$\frac{\dot{H}'}{H'} = (1 - q') g(k'_I[\psi]). \quad (35)$$

The variables q and q' are not independent of each other. For a given q , Equations (18)–(22) determine the optimal allocation of physical and human capital across industries and for any given q there is a unique q' determined by these conditions.

The social planner cares about the time-path of aggregate consumption but he is indifferent as to the physical location of production.¹⁰ This indifference follows from the fact that although the technology is convex it is not *strictly* convex. There are many alternative production plans all of which support the same consumption allocations. Indeterminacy of the production plan shows up in the first-order conditions as the absence of a separate equation to determine q .

Suppose that the social planner begins at time 0 with human-capital levels H_0 and H'_0 . One way of allocating production would be to set $q = q'$. In this plan both countries would grow at the same rate and the differences in the initial level of human capital would be maintained through time. But this is not the only alternative. Consider the welfare-equivalent plan in which $q(s) < q'(s)$, $s < T$ and $q(s) = q'(s)$, $s \geq T$ where T is some finite but arbitrary date. In this alternative plan country 1 grows faster than country 2 until date T at which point both countries grow at the same rate. Since the date T is arbitrary, there are many equivalent production plans all of which support the same Pareto Optimal allocation of consumption. In the two-sector endogenous growth model the physical location of production is indeterminate.

¹⁰We are not the first to make this observation. Ventura [30] recognizes that the location of production is indeterminate in a model that is asymptotically an AK model.

5.2.2 Implications of Different Rates of Time Preference

In our two-person economy, the way the planner allocates consumption across households is determined by the welfare weight attached to each agent by the social planner. When agents have different rates of time preference, the optimal savings rate will be different at different points in time as the planner gives increasing weight to the more patient individual. If the planner locates production and consumption in the same physical location, a more patient country will save more, invest more and grow faster. This implies, as pointed out by Jones and Manuelli, [14], that government policies that promote savings will also increase economic growth.

Without loss of generality we may suppose that country 1 is the more patient, that is $\rho' > \rho$ and define the time-varying preference parameter $s(t)$ as;

$$s(t) \equiv b + (1 - b) e^{-(\rho' - \rho)t}.$$

We show in Appendix A that the human-capital-weighted consumption levels of the two agents will be given by the expression

$$c + c' = \frac{s(t)}{\lambda}.$$

As $t \rightarrow \infty$, the more impatient country becomes negligible as $s(t) \rightarrow b$, and $c'/(c + c')$ converges to zero. This is a strong implication but one that follows necessarily from any model in which countries have different savings rates. As we show in Section 6.1 below, savings rates have been very different in the data for long periods of time. It seems better to recognize this fact rather than to build models in which balanced growth is built into the model by giving countries the same rate of time preference.

6 Equilibrium

In the introduction to our paper we claimed that our two-sector two-country growth model provides a better vehicle, than the Solow-Swan model, for understanding growth in a collection of open

economies. In this section we explain why we take this position by studying the way that alternative planning solutions might be decentralized.

6.1 Two Open Economies: a No-Trade Result

Suppose that the world consists of two closed economies that have the same technological opportunities but representative agents in the two economies have different rates of time preference. Now suppose that the countries are unexpectedly opened to trade. We interpret the post-opening allocations as the solution to a new planning problem that gives rise to the equations (26–31) in place of two sets of closed economy equations. The welfare weight b reflects the relative wealth of the two economies at the date at which the opening occurs.

When the economies are opened to trade, their levels of GDP per capita may differ since these levels depend on relative accumulation of human wealth. But if the economies use identical technologies and if they have attained their balanced growth paths then all relative prices, including rental rates for physical and human capital, will be the same in each country. Hence there will be no incentive for capital to flow from one country to the other even if capital markets are open and GDP per capita *is different*.

The absence of an incentive to trade extends to economies that have not achieved their balanced growth paths. Bond Wang and Yip [7] have shown that if the human capital industry is more physical-capital intensive than the physical capital industry, (that is if $k_I > k_Y$), then relative prices, wages and rental rates are time invariant.¹¹ It follows in this case that there are no incentives to trade capital internationally even if the two countries have not attained their balanced growth paths. The representative agents may have different savings rates, the countries may have different levels of output per capita, they may be growing at different rates, they may not have achieved their balanced growth paths and *still* there may be no incentive to trade capital internationally.

The final case to consider is one in which the human capital industry is less physical-capital

¹¹The statement that $k_I > k_Y$ is an unambiguous statement about the properties of the technologies under our maintained assumption of no factor intensity reversals.

intensive than the physical capital industry, (that is if $k_Y > k_I$). In this case, if the two economies have not attained their balanced growth paths, then there will be a change in relative prices and also in relative factor intensities at the date that the economies are opened to trade. Before opening the economies to trade, if the event was unanticipated, the relative prices ψ and ψ' may be different: After opening to trade they must be the same. But even in this case, there still need be no flow of capital across borders since the equalization of relative prices may be attained entirely through the sectoral reallocation of human capital.

The allocation of human capital to the final goods sector in the home country is given by the expression

$$H_Y/H = q = \frac{k - k_I(\psi)}{k_Y(\psi) - k_I(\psi)},$$

where $k = K/H$ is the ratio of physical to human capital in country 1. As long as both industries operate in both countries, it is feasible to choose an optimal plan in which relative prices are equalized without the flow of capital across borders. Factor price equalization occurs instead through the reallocation of human capital across sectors, i.e., by changing q .¹² Since growth rates depend on the sectoral allocation of human capital, an allocation of human capital that leaves the distribution of world capital stocks unaltered would be accompanied by a change in growth rates.

In the neoclassical model the distribution of the world capital stock in the free trade equilibrium is determined by the assumption of diminishing-returns to accumulable factors. In contrast, in the two-sector two-country *AK* model, there are constant-returns to accumulable factors and hence the physical to human-capital ratios k and k' are indeterminate. Since per-capita GDP is a function of this ratio, the model can explain persistent difference in per-capita GDP in the face of open capital markets. This fact has important implications for the ability of the two-sector AK model to explain the data but it does not seem to be well understood in the literature.¹³

¹²This feature characterizes any two sector–two factor model where factors are mobile across sectors. A typical example is the Heckscher-Ohlin model. In the static version of the Heckscher-Ohlin model with fixed capital and labor, all changes in relative prices are accommodated through sectoral factor reallocations.

¹³For example, Barro Mankiw and Sala-i-Martin [2] assert that “Two-sector-endogenous-growth models can ex-

To summarize, in the two-sector two-country AK model, opening two closed economies to trade *may* result in a change in relative prices or it may not. The implications of openness for relative prices depends on which industry is more intensive in its use of physical capital. Whether or not there is an incentive for relative prices to change, there will be no incentive for capital to flow from one country to the other. The two countries may have different levels of per-capita GDP, they may have different savings rates, they may have different growth rates and still we may fail to observe a market for international borrowing and lending.

6.2 The Correlation of Saving with Investment

In the neoclassical model capital will flow from rich countries to poor countries to equate the ratios of the variable factor, capital, to the fixed factor, labor. But in the two-sector two-country AK model the direction of cross border capital movements is indeterminate. Our model has many feasible production plans and it is not clear which of these plans will prevail.

In practice, there are motives to trade capital internationally that we have not modeled: These arise from the benefits of diversification in a risky environment. But there are also motives *not* to locate capital abroad: These include the difficulty of monitoring disputes in a foreign legal environment and costs of international travel to monitor production. In the absence of risk there are good reasons to think that a decentralized system will favor the location of ownership and production in the same physical location. This allocation of production plans would be consistent with evidence cited by Feldstein and Horioka [9] who pointed out that savings and investment are closely correlated within countries across time.

The close correlation of saving and investment is illustrated for the case of Japan, the United States and Egypt, on Figure 2. We have chosen these countries as illustrations of high, medium and a low saving countries but the correlation illustrated on these graphs is not restricted to these three countries; it is common to every country in the world. This correlation presents problems for plain convergence based on imbalances between physical and human capital.....[But] the imbalances would vanish instantaneously across open economies." Barro et al [2] page 104.

the Solow-Swan model which predicts that we should see large capital flows from rich countries to poor countries as investors in capital rich countries like the U.S., and Japan take advantage of profit opportunities in capital poor economies like Egypt. Mankiw-Romer-Weil explain the absence of these flows by invoking capital controls or costs of adjustment. But the world capital markets are becoming increasingly open and we still observe a close correlation between national saving and investment rates.

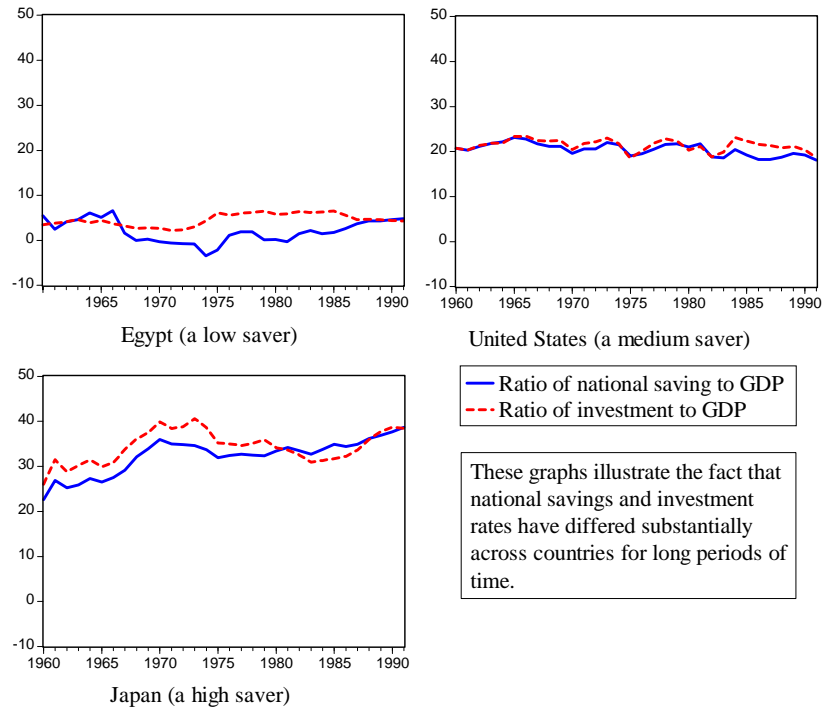


Figure 2: Ratio of National Savings and Investment to GDP in three Representative Countries

Defenders of the neoclassical position point to the work of Barro, Mankiw and Sala-i-Martin [2]. These authors recognize that

Economists have long known that capital mobility tends to raise the rate at which poor and rich countries converge.¹⁴

This statement is true but is somewhat understated. In the neoclassical model the convergence rate is finite in the presence of closed capital markets but infinite if capital markets are open. Barro

¹⁴Barro et al [2] page 114.

et al make the assumption that human capital must be financed by domestic saving and that they argue that when...

there are some types of capital ... that cannot be financed by borrowing on world markets, then open economies will converge only slightly faster than closed economies.

They go on to argue that

This prediction accords with the empirical literature, which finds that samples of open economies, such as the U.S. states, converge only slightly faster than samples of more closed economies, such as the OECD countries.¹⁵

Barro et al's defense of the neoclassical model points to the convergence properties of the Solow-Swan model. This is a relatively weak test of the model and, as we show below in Section 6.3, it is one that is also passed also by the two-sector two-country AK model. A stronger test is that trade in capital, if it occurs to any degree, will cause rich countries to accumulate claims on poor countries that build up over time. As these claims accumulate GNP and GDP will begin to diverge as a significant portion of the wealth of the rich country will be held as claims on capital in the poor country. One consequence of this divergence is that national savings, as a fraction of domestic product, should differ from the ratio of domestic investment to GDP. We do not see a difference in these ratios in the data.

Barro et al assume that countries do not borrow against human capital. But the approximate equality of national savings with domestic investment implies that countries *also fail to borrow against physical capital*. Note that we are *not saying* that trade in the international capital markets is small; this is far from the truth. But trade in the capital markets is roughly balanced in the sense that domestic holdings of foreign capital are approximately equal to foreign holdings of domestic capital. Trade of this kind could be explained by portfolio diversification in a world of uncertainty, a motive that is missing in our non-stochastic model.

¹⁵Barro et al, op. cit., page 114.

6.3 The Location of Production - Implications for Convergence

In this section we discuss evidence for convergence that has been cited by some authors in support of the Solow-Swan model. We will argue that the two-sector two-country AK model is consistent with this evidence. Our argument is based on the existence of multiple ways of implementing the social planning optimum. In a decentralized equilibrium these multiple solution paths would be implemented by differing degrees of international borrowing and lending.

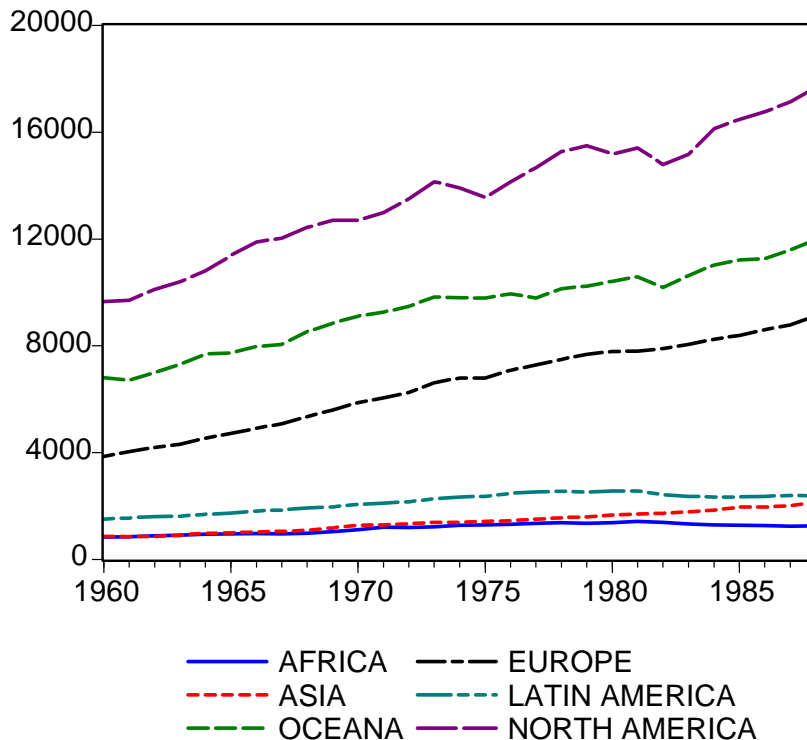


Figure 3: Continental Per-capita Real GDP

The raw data on GDP per person displays little evidence for convergence over time. Figure 3 depicts the time series plots of the continental real per capita GDP levels between 1960 and 1988. While this picture masks income mobility across countries, it nevertheless provides a rather stark picture of the absence of any clear convergence in income per person over time across the continents. The corresponding evidence at the cross-country level is similar.¹⁶

¹⁶Chari, Kehoe and McGrattan [8] provides an excellent description of the distribution of relative incomes across countries and the evolution of this distribution over time.

But although the raw data does not display convergence, there is evidence of convergence between some groups of countries, for example, the OECD economies. Barro and Sala-i-Martin [3] review the evidence from cross-country growth regressions which implies that, conditional on other factors like educational attainment and political climate, poor countries tend to grow faster than rich ones. It would be an embarrassment to the two-sector two-country model AK model if it could be shown that the model contradicts these results. But the implications of this model for the convergence regressions are ambiguous.

We have argued that, in the absence of risk, there are good reasons to think that a decentralized system will favor the location of ownership and production in the same physical location in accord with the evidence that savings and investment are closely correlated within countries across time. But the location of production in the home country is not the only feasible plan and there may be offsetting reasons for shifting production abroad. If individuals shift production abroad then growth rates will move discretely as human capital accumulation increases in one location and falls in the other. A sudden relocation of this kind will induce a period during which the two countries are away from their balanced growth paths. During this period one country will grow faster than the other.¹⁷

The prediction of our model for the correlation of growth with initial levels of the capital-output ratio depends on which of the two industries is more intensive in its use of physical capital. Suppose that the human capital industry is more intensive in its use of physical capital than the physical capital industry and suppose further that at some initial date we observe an inflow of capital from a rich country to a poor country. In this case we would expect the poor country to grow faster than the rich country for a period of time. This is what Barro and Sala-i-Martin call conditional convergence. Our model is consistent with this fact under some parameter configurations but not under others.

¹⁷Mulligan and Sala-i-Martin [23] also recognize that the two-sector closed economy model generates transitional dynamics that imply conditional convergence but they do not pursue the implications of their analysis for international trade.

6.4 Why is Investment Correlated with Growth?

Bernanke and Gürkaynak [6] have argued that one of the most robust features of international data is that growth is correlated with investment. Savings rates differ across countries for long periods of time and countries with high savings rates grow faster than those with low savings rates. This feature of the data is inconsistent with the Solow-Swan model in which growth rates are exogenous.

In the AK model, in contrast, a correlation of investment with growth is a prediction of the model. Since the model displays constant returns-to-scale in accumulable factors there is no incentive for growth to slow down and the growth rate in both the long-run and the short-run is a function of the investment ratio.

7 Conclusion

Following the work of Mankiw-Romer and Weil it has become common to use an amended version of the Solow-Swan model to understand growth by modeling the world as a collection of closed economies. We find this model unappealing for two reasons.

Our first reason to be skeptical of the Solow-Swan model is the evidence of McGrattan [22] and Bernanke and Gürkaynak [6] who find a strong positive correlation between growth rates and investment ratios. The MRW model predicts that growth rates should be independent of investment ratios but the data suggests otherwise. In contrast, our two-sector two-country AK model can account for this evidence.

Our second reason to be skeptical of the model is that it can account for persistent differences in GDP per capita only if there are strong imperfections in the international capital markets. This may have been an accurate characterization of the capital markets in the 1950's but capital controls have been considerably weakened in recent decades and yet the correlation between national savings and investment persists. The two-sector two-country AK model can explain this correlation: The Solow-Swan model cannot.

We recognize the argument of Barro et al [2], that if countries cannot borrow to purchase

human capital then convergence will be slower than would otherwise be the case: But this does not alleviate our concerns. Even in the presence of this friction, the accumulation of capital flows over time should lead to an equilibrium in which national income and investment differ by an order of magnitude greater than we observe in the data as rich countries acquire physical assets in poor countries. The fact that saving is so closely correlated with investment implies, to a first approximation, not only that countries are not borrowing to acquire human capital but also that they are not borrowing to acquire physical capital.

One reason for the popularity of the Solow-Swan model is that evidence from cross-country growth regressions, summarized by Barro and Sala-i-Martin [3], supports the proposition of conditional convergence. The one-sector AK model is inconsistent with this evidence. But the two-sector two-country AK model *is* consistent with the convergence regressions if countries depart temporarily from their balanced growth paths.

Understanding the determinants of growth remains one of the most important challenges facing economists and policymakers. Between two key competing visions of the growth process – the neoclassical and the endogenous growth models – the profession has leaned lately toward the neoclassical view. This may have been due to a combination of factors: the comfort of the profession with the traditional diminishing returns to capital specification, and accumulating evidence over the last decade in favor of conditional convergence. But evidence from investment-growth regressions and from the savings-investment correlation points to problems with the Solow-Swan model. In the conclusion to their paper, Bernanke and Gürkaynak [6] suggest that we should search for an alternative to the Mankiw-Romer-Weil model. We think that the two-sector two-country AK model may provide the alternative they are looking for.

Appendix A: Characterizing Planning Optima in the Open Economy

In this Appendix we derive the equations of motion that describe equilibria in the world economy.

1. Preliminary Definitions

Define the employment shares in each country $q \equiv \frac{H_Y}{H}$, $q' \equiv \frac{H'_Y}{H'}$. From the adding up constraints, (5)–(9) we can write these shares as follows,

$$q = \frac{k - k_I}{k_Y - k_I}, \quad q' = \frac{k' - k'_I}{k'_Y - k'_I} \quad (\text{A1})$$

$$1 - q = \frac{k_Y - k}{k_Y - k_I}, \quad 1 - q' = \frac{k'_Y - k'}{k'_Y - k'_I}. \quad (\text{A2})$$

Now let $\theta \equiv \frac{H}{H+H'}$ be the share of human capital in country 1 and let $k^w \equiv \frac{K^W}{H+H'}$ be the ratio of world capital to world human capital. Using these definitions write the adding up constraint for physical capital

$$K^W = K_Y + K_I + K'_Y + K'_I,$$

as follows:

$$k^w = \theta q k_Y + \theta (1 - q) k_I + (1 - \theta) q' k'_Y + (1 - \theta) (1 - q') k'_I. \quad (\text{A3})$$

Static efficiency requires

$$k_Y = k'_Y, \quad \text{and} \quad k_I = k'_I,$$

from which it follows that

$$k^w = [q\theta + q'(1 - \theta)] k_Y + [(1 - q)\theta + (1 - q')(1 - \theta)] k_I. \quad (\text{A4})$$

A further simplification follows by defining $\tilde{q} \equiv q\theta + q'(1 - \theta)$, to be the relative intensity with which the social planner runs the physical and human capital sectors. \tilde{q} is related to the state variables k^w and ψ by the expression

$$\tilde{q} = \frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]}, \quad 1 - \tilde{q} = \frac{k_Y[\psi] - k^w}{k_Y[\psi] - k_I[\psi]}. \quad (\text{A5})$$

One can use this definition to simplify Expression (A4) as follows

$$k^w = \tilde{q}k_Y + (1 - \tilde{q})k_I. \quad (\text{A6})$$

Next we use the definitions of q , q' , and \tilde{q} to derive the equations of motion of the international economy. We begin with the evolution of human capital in each country.

2. Human Capital Accumulation

Growth of human capital in each country is governed by Equations (13) and (14) which we write in intensive form:

$$\frac{\dot{H}}{H} = (1 - q)g(k_I[\psi]), \quad (\text{A7})$$

$$\frac{\dot{H}'}{H'} = (1 - q')g(k'_I[\psi]). \quad (\text{A8})$$

Since $k_I = k'_I$, from the static conditions for an optimum, the growth rate in each sector will depend only on the relative magnitudes of q and q' .

3. Physical Capital Accumulation

Using Definitions (36) and (36) and exploiting the linear homogeneity of the production function one can rewrite the capital accumulation equation, (12), in intensive form:

$$\frac{\dot{K}^W}{K^W} = \frac{q\theta(H + H')}{K^W}f(k_Y) + \frac{q'(1 - \theta)(H + H')}{K^W}f(k'_Y) - \frac{C}{K^W} - \frac{C'}{K^W}, \quad (\text{A9})$$

Noting that $k^w = K^W/(H + H')$ using facts that $k_I = k'_I$, $k_Y = k'_Y$ and $\tilde{q} \equiv q\theta + q'(1 - \theta)$, k^w must evolve according to

$$\frac{\dot{k}^w}{k^w} = \frac{\tilde{q}f(k_Y[\psi])}{k^w} - \left(\frac{c + c'}{k^w}\right) - (1 - \tilde{q})g(k_I[\psi]). \quad (\text{A10})$$

Equation (16) and definitions of c , c' and λ imply that

$$c + c' = \frac{s(t)}{\lambda},$$

where

$$s(t) \equiv b + (1 - b)e^{-(\rho' - \rho)t}.$$

Using this fact, plus the expression for \tilde{q} given by Equation (A5) leads to the expression we seek

$$\frac{\dot{k}^w}{k^w} = A(\psi, \lambda, k^w, s(t))$$

where:

$$A(\psi, \lambda, k^w, s(t)) = \left(\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]} \right) \frac{f(k_Y[\psi])}{k^w} - \frac{s(t)}{\lambda k^w} - \left(\frac{k_Y[\psi] - k^w}{k_Y[\psi] - k_I[\psi]} \right) g(k_I[\psi]). \quad (\text{A11})$$

This is equation (26) in the text.

4. Relative Prices

The costate variables, for an optimal plan, must satisfy the conditions:

$$\frac{\dot{\Lambda}}{\Lambda} = \rho - f_k(k_Y), \quad (\text{A12})$$

$$\frac{\dot{M}}{M} = \rho - g_H(k_I), \quad (\text{A13})$$

$$\frac{\dot{M}'}{M} = \rho - g_H(k'_I), \quad (\text{A14})$$

and transversality requires that

$$\lim_{t \rightarrow \infty} e^{-\rho t} \Lambda K^W = \lim_{t \rightarrow \infty} e^{-\rho t} M H = \lim_{t \rightarrow \infty} e^{-\rho t} M' H' = 0. \quad (\text{A15})$$

Combining Equations (A12)–(A14), using the definition of ψ :

$$\frac{\dot{\psi}}{\psi} = [f_k(k_Y[\psi]) - g_H(k_I[\psi])], \quad (\text{A16})$$

which is equation (28) in the text.

5. The Shadow Price of Consumption

Recall that $\lambda = \Lambda(H + H')$. It follows from Equations (A12), (A7) and (A8) that the equation of motion for λ is given by:

$$\frac{\dot{\lambda}}{\lambda} = \rho - f_k(k_Y[\psi]) + (1 - \tilde{q}) g(k_I[\psi]).$$

From the definition of \tilde{q} , this gives

$$\frac{\dot{\lambda}}{\lambda} = B(\psi, k^w), \quad (\text{A17})$$

where

$$B(\psi, k^w) = \rho - f_k(k_Y[\psi]) + \left(\frac{k_Y[\psi] - k^w}{k_Y[\psi] - k_I[\psi]} \right) g(k_I[\psi]).$$

which is equation (27) in the text.

6. The Evolution of θ

From the definition of θ :

$$\frac{\dot{\theta}}{\theta} = \frac{\dot{H}}{H} - \theta \frac{\dot{H}}{H} - (1 - \theta) \frac{\dot{H}'}{H'}.$$

Using equations (A7) and (A8) and the fact that $g(k_I) = g(k_I')$ we can rewrite this expression as:

$$\frac{\dot{\theta}}{\theta} = (1 - \theta) (q' - q) g(k_I[\psi]). \quad (\text{A18})$$

Using expression (A5)

$$q' - q = \frac{q' - \tilde{q}}{\theta} = \frac{1}{\theta} \left(q' - \left[\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]} \right] \right),$$

from whence it follows that

$$\frac{\dot{\theta}}{\theta} = \frac{(1 - \theta)}{\theta} \left(q' - \left[\frac{k^w - k_I[\psi]}{k_Y[\psi] - k_I[\psi]} \right] \right) g(k_I[\psi]), \quad (\text{A19})$$

which is equation (33) in the text.

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