Explaining World Savings*

Colin Caines and Amartya Lahiri†

May 2014

Abstract

Data on the world saving distribution reveals two key features: (i) saving rates are significantly different across countries and they remain different for long periods of time; and (ii) some countries and regions have shown very sharp changes in their average saving rates over short periods of time. This paper provides an explanation for the observed saving patterns. We formalize a model of the world economy comprised of open economies inhabited by heterogeneous agents endowed with recursive preferences. Our assumed preferences imply increasing marginal impatience of agents as their consumption rises relative to average world consumption. Using only measured productivity shocks as an exogenous driver, we show that the model can generate the time series behavior of saving observed in the data. The model can also generate the sudden and long-lived increase in East Asian savings by incorporating an increase in aspirations represented by an exogenous increase in the reference consumption basket of East Asia toward G7 levels.

JEL Classification: E2, F3, F4

Keywords: World savings, recursive preferences

1 Introduction

Data on the world saving distribution reveals two key features: (i) cross-country differences in saving rates are significantly and persistent; and (ii) some countries and regions have shown very sharp changes in their average saving rates over short periods of time. These facts are problematic for the

---

*We would like to thank Igor Livshits, seminar participants at UBC and the 2013 SED meetings in Seoul for helpful comments and discussions. Thanks also to Roger Farmer whose previous work with Lahiri provided the basis for this paper.

†Department of Economics, University of British Columbia, 997 - 1873 East Mall, Vancouver, BC V6T 1Z1, Canada.
standard model with time-additive preferences. Without equal rates of time preference, the asymptotic distribution of world wealth is typically degenerate under additively separable preferences. While models with equal rates of time preference but cross-country differences in demographics and productivity have had some success in accounting for part of the cross-country dispersion in saving rates, a substantial amount of variation still remains unexplained in these models.

This paper provides an alternative explanation for the observed saving patterns. We formalize a model of the world economy that is comprised of open economies inhabited by infinitely-lived agents. Our main point of departure from the standard exogenous growth neoclassical model is that we endow agents with recursive preferences. Specifically, we follow Farmer and Lahiri (2005) and use a modified version of recursive preferences. The key implication of the Farmer-Lahiri specification is that it generates a determinate steady state wealth distribution within a growing world economy, a feature that typical models with recursive preferences cannot generate.

The problem with the standard recursive preference specification is that it is inconsistent with balanced growth. The existence of balanced growth requires homothetic preferences. However, under homothetic preferences, the asymptotic wealth distribution in multi-agent environments is either degenerate or reflects the initial wealth distribution.

One might of course consider the issue of balanced growth to be irrelevant to understanding savings behavior. We however believe that the inconsistency of standard recursive preferences with balanced growth is problematic if one’s goal is to explain saving rates. In particular, given the constancy of both long run average growth rates and saving rates for most groups of countries (regions or continents), understanding long run patterns of savings would appear to be intrinsically linked to long run steady state dynamics. As is well known, the Kaldor growth facts are quite stark in suggesting balanced growth to be a robust feature of long run growth. Hence, we feel that any model attempting an explanation of dispersions in long run saving rates across countries should be consistent with balanced growth.

To overcome the problems with the standard recursive preference specification, we follow Farmer and Lahiri (2005) and use a modified version of recursive preferences.
and Lahiri (2005) and construct a model of recursive utility in which agents care about relative consumption. We assume that preferences are described by an aggregator that contains current consumption, future utility, and a time-varying factor that is external to the agent but grows at the common growth rate in a balanced growth equilibrium. This time dependence allows for preferences to exhibit increasing marginal impatience, which is a necessary condition for a non-degenerate asymptotic wealth distribution. A positive productivity shock in our model induces a rise in saving which ultimately reverts back to its prior level due to the increasing marginal impatience of agents as their wealth rises relative to world wealth, thereby preserving a determinate asymptotic wealth distribution. Equally importantly, this specification implies that different preferences induce different consumption to wealth ratios of different agents which, in turn, map into different steady state saving rates of different countries.

We start by demonstrating that the modified preferences of Farmer and Lahiri (2005) can account for not only the observed differences in average long run saving rates between regions for the period 1970-2010 but also the time series behavior of region-specific saving rates. Strikingly, the model achieves these two goals without appealing to either different externality factors or frictions in factor and goods flows. We establish this feature of the model by examining a two-region, heterogeneous agent world economy without any impediments in factor or goods mobility across regions. Importantly, this version of the model assumes that the external factor in preferences is indexed to the common world steady state per capita consumption level. Hence, all agents have the same external factor. We show that a calibrated version of this two-region model can quantitatively match both the average region-specific levels of savings between 1970-2010 as well as the time series movements in saving rates in the two regions during this period. Encouragingly, we are able to match these facts inspite of allowing only two factors to vary across countries (one preference parameter and labor productivity) and using a common world productivity process as the only exogenous driver of time series fluctuations around the balanced growth path. We then establish the same results for a richer five-region world economy.

Lastly, we turn to saving miracles. In order to generate sudden switches in saving rates, we propose an additional modification. Specifically, we divide the world into three regions: the G7, the Asian tigers, and Emerging countries. Crucially, we now allow the external factor to be different in levels for the three groups even though we continue to constrain it to have the same growth rate. The basic idea behind this is that all societies have role models/peer groups that they want to keep up with or imitate. This approach to explaining miracles amounts to a hypothesis that these sudden
transformations of economies occur due to changes in their aspirations. More particularly, under our formalization steady state saving rates are functions of the external factor in preferences which describes the benchmark relative to whom the country evaluates itself. The model can generate the observed saving miracle of the Asian Tigers if their benchmark external factor is switched in 1970 from the average consumption level of the developing world to the G7 consumption level. The model predicts that saving rates rise towards the observed levels in the data as the economy starts building its consumption towards its new desired level. As consumption rises however, increasing marginal impatience starts to become stronger over time which eventually induces the saving rates to come back down. We show that the model predictions match the facts quantitatively as well as qualitatively for the Asian Tigers.

Our work is related to two different strands of literature. The first is the relatively large body of research focused on explaining the dispersion of saving rates across countries. Explanations for the observed variation in cross-country savings have typically focused on variations in per capita incomes, productivity growth, fertility rates or the age distribution of the population. Contributions along these lines can be found in Mankiw, Romer, and Weil (1992), Christiano (1989), Chen, Imrohoroglu, and Imrohoroglu (2006), Horioka and Terada-Hagiwara (2012), Loayza, Schmidt-Hebbel, and Serven (2000) and Tobing (2012). These papers typically find significant explanatory power for demographics and some explanatory power for per capita income (though the direction of causality there is somewhat unclear). However, a significant part of the saving variability in the data continues to remain unaccounted for.

This paper is also related to the work on recursive preferences and stationary wealth distributions that goes back in its modern form to Lucas and Stokey (1984) and Epstein and Hynes (1983). Of particular relevance to our work are the contributions of Boyd (1990), Dolmas (1996) and Ben-Gad (1998) who focused on characterizing the stationary wealth distribution in growing economies. A second line of research has examined the implications of recursive preferences for stationary wealth distribution in growing economies. Also relevant to our work are the papers by Uzawa (1969), Mendoza (1991) and others who examined the effects of endogenously varying discount rates on the equilibrium dynamics of the neoclassical growth model.

In the next section we describe some of the key data features that motivate our study. Section 3 quickly reviews the key issues associated with recursive preferences under balanced growth as well as the "fix" to the problem suggested by Farmer and Lahiri (2005). Section 4 presents and develops the model. In section 5 we calibrate the model and examine its quantitative fit to average
saving rates in a two-region economy. Section 6 discusses miracles while the last section concludes.

2 Two Facts on Cross-Country Saving

There are two features of the data that we want to draw attention to. First, we highlight the sustained differences in saving rates across groups of countries. To do this we collect countries into three groups: the G7, Emerging, and Sub-Saharan Africa. Figure 1 plots the savings rates of these three groups of countries between 1970 and 2010. The figure illustrates another key feature of the data. Savings rates are different for different countries for long periods of time. Further, they show little or no evidence of convergence.

While the overall pattern suggests that saving rates are persistent, the data does have another important aspect: in some countries saving rates show sudden and sharp swings over relatively short periods of time. Figure 2 highlights this by plotting the saving rates in the Asian Tigers between 1960 and 2010. Clearly, saving rates in the Asian economies increased very sharply from 1960 onward. In a short time period of 20 years starting in 1970, their saving rates rose by almost 15 percentage points. Since the mid-1980s, the average saving rate in the Asian tigers has exceeded the average saving rate in the G7 countries by over 10 percentage points.

\(^2\) The list of countries in each group is in the Appendix.
We believe this data can be explained by allowing the rate of time-preference to vary across countries using a modified version of recursive preferences. In the standard model of recursive preferences studied by Lucas and Stokey (1984) and Epstein and Hynes (1983), agents become less patient as they become richer in an absolute sense. We adapt this idea to the case of a growing economy with the assumption that agents become less patient as they become richer in a relative sense.

3 Recursive preferences and balanced growth

As discussed in Section 1, a key goal of the paper is to examine the ability of a modified version of recursive preferences to rationalize the cross-country saving facts. Before presenting the model it is worthwhile to review why a modification is needed at all. In a nutshell, the need to modify the standard recursive specification arises because we are interested in analyzing environments with steady state balanced growth. Balanced growth is not only one of the celebrated Kaldor facts but also happens to characterize the modern data aggregated by regions. This can be seen from Figure 3 below which shows the time series behavior of the consumption-to-output ratio for G7 and developing countries separately. The figure makes clear that on average, consumption and output tend to grow at similar rates over long periods of time. Hence, we would like to retain balanced growth as a key feature in any model that we develop.

The baseline recursive preference structure however is not consistent with balanced growth.

3 We should note that the celebrated Lucas and Stokey (1984) paper that studied recursive preferences in a
To see this, consider the following recursive aggregator of preferences

$$u_t = W(c_t, u_{t+1})$$

With heterogenous agents, there exists an asymptotic stationary wealth distribution if along a steady state balanced growth path all agents equate

$$W^i_u(c^i, u^i) = W^j_u(c^j, u^j) = 1/R$$ for all $i, j$ (3.1)

where $c$ is consumption, $u$ is utility and $R$ is the steady state interest factor common to all agents. Dolmas (1996) showed that this can only occur if the aggregator $W$ is homogenous of the form

$$W(\lambda x, \lambda^\gamma y) = \lambda^\gamma W(x, y)$$ (3.2)

When this homogeneity condition is satisfied $W_u$ becomes a constant along a balanced growth path thereby making it possible for the endogenous rate of time preference to remain constant and equal to the constant interest rate along a balanced growth path. More fundamentally, $W_u$ becomes a number that is independent of $c$ and $u$ in steady state.

While the condition above is intuitively obvious, Farmer and Lahiri (2005) showed that this homogeneity condition also implies that the stationary wealth distribution in a heterogenous agent heterogenous agent economy did not have long run growth. Hence, this inconsistency was not germane to their work.
economy is generically degenerate. It leads to a non-degenerate steady state wealth distribution only in the knife-edged case of $W^i = W^j$. But those are exactly the implications in the case of additively separable preferences. In other words, the key Farmer-Lahiri result is that recursive preferences do not add anything to our understanding of stationary wealth distributions beyond what we already know from additively separable preferences.

In the context of recursive preferences in environments without steady state growth, Lucas and Stokey (1984) proved the existence of a stationary wealth distribution as long as preferences exhibited increasing marginal impatience, i.e., agents became more impatient as they grew wealthier. Intuitively, rising impatience bounds the desire to accumulate assets by raising the desire to consume. The problem of the specification in equation (3.2) is that there is no force akin to the increasing marginal impatience of Lucas-Stokey that can endogenously equate it across agents. Consequently, the equilibrium has a knife-edge property to it.

Farmer and Lahiri (2005) showed however that the introduction of an externality into preferences could fix this problem. In particular, they considered preferences of the form

$$u^i = W^i (c^i_t, u^i_{t+1}, a_t)$$

where $a_t$ is a factor that is external to the individual. This could stand in for habits, the average consumption level of the economy, or any other factor provided it grows at the rate of steady state growth and, as formalized here, is external to the individual household. Farmer-Lahiri showed that as long as the aggregator $W$ was homogenous in all three arguments so that $W(\lambda x, \lambda^y y, \lambda z) = \lambda W^i$, an economy with heterogenous agents would give rise to an endogenously determined stationary wealth distribution with different agents choosing different saving levels in order to equate $W_u^i \left( \tilde{c}^i, \gamma g \tilde{u}^i, 1 \right) = W_u^j \left( \tilde{c}^j, \gamma g \tilde{u}^j, 1 \right)$ where $\tilde{x} = \frac{x}{a}$ and $g$ denotes the steady state growth. Moreover, the homogeneity property also ensures that $W_u$ would be constant in steady state. Hence, this specification can generate steady state differences in saving rates across agents facing a common vector of prices. In the rest of the paper we shall examine the potential of these preferences to account for the disparity in saving rates across the world.

## 4 The Model

We consider a world economy consisting of $N$ small economies. Each country $i$ is populated by $l_i$ agents and this measure remains constant over time. Introducing population growth into the
model is a straightforward extension that does not change any fundamental result. With no loss of generality we normalize the world population to unity so that $\sum_i l_i = 1$. We assume that the world economy has an integrated market for goods and factors. Hence, there are no impediments to goods or factor flows throughout the world. This assumption makes the role of our formulation of preferences in accounting for saving disparities particularly stark.

Agents globally are endowed with one unit of labor time which they supply inelastically to the market. We assume that all agents within a country have identical preferences but preferences of agents across countries maybe different. The preference of the representative agent in country $i$ are described by the recursive representation

$$u_{it} = \frac{c_{it}^{\theta_i} \xi_i^{1-\theta_i}}{\theta_i} + \mathbb{E}_t \left[ \beta_i \delta_i \xi_{it+1}^{1-\delta_i} \right]$$

where $c$ denotes consumption and $u$ denotes utility. This recursive preference specification is standard except for the argument $\xi_i$ which stands for an externality in preferences. It is external to the individual but is indexed by $i$ since we allow this externality parameter to vary across countries. This externality could represent a number of different things including external habits, relative consumption ("keeping up with the Jones’s") etc. Allowing it to vary across agents implies, for example, that the relative consumption targets could vary across countries. Note that these preferences reduce to the standard additively separable across time specification in the special case where when $\delta = 1$. *Ceteris paribus*, a higher $\delta$ makes agents more patient by raising the discount factor. It is easy to check that this aggregator is linearly homogenous, thereby satisfying the homogeneity and regularity conditions needed for the existence of a Balanced Growth Path (BGP) as shown in Farmer and Lahiri (2005).

Agents have two sources of income: wage income from working and capital income earned by renting out their capital to firms. Households save by accumulating capital. Income can be used for either consumption or saving. The budget constraint for households is thus given by

$$c_{it} + \iota_{it} = r_{it} k_{it} + w_{it}$$

where $k$ is the capital stock of household $i$ at the beginning of period $t$ while $\iota$ denotes saving. $r$ is the rental rate on capital from country $i$ while $w$ is the wage rate of labor from country $i$. The
capital stock of the household evolves according to the accumulation equation

\[ k_{it+1} = (1 - d) k_{it} + \nu_{it}, \quad k_{i0} \text{ given for } i = 1, ..., N \]  

(4.5)

where \( d \) is the depreciation rate.

Agents maximize utility subject to equations (4.4) and (4.5). The first order condition describing the optimal consumption-saving plan is

\[
\left( \frac{c_{it}}{\zeta_{it}} \right)^{\theta_{i}^{-1}} = \beta \E_t \left[ \left( \frac{c_{it+1}}{\zeta_{it+1}} \right)^{\theta_{i}^{-1}} \left( \frac{u_{it+1}}{\zeta_{it+1}} \right)^{\delta_{i}^{-1}} (1 + r_{it+1} - d) \right]
\]

There is a common world production technology which produces output according to

\[ Y_t = A_t K_t^\alpha \left[ \sum_{i=1}^{N} (\gamma_i l_i)^\eta \right]^{\frac{1-\alpha}{\eta}} \]

where \( K = \sum_{i=1}^{N} k_i l_i \) is capital and \( l_i \) denotes labor from country \( i \). \( \gamma_i \) is the the labor productivity of workers from country \( i \). We allow this to vary across countries but not across time. \( \eta \) controls the elasticity of substitution across the different types of labor.

\( A \) is the productivity of the technology that is given by

\[ A_t = e^{z_t} a_t^{1-\alpha} \]

where \( a \) and \( z \) are productivity processes described by

\[ a_t = ga_{t-1} \]

(4.6)

\[ z_t = \rho z_{t-1} + \sigma \varepsilon_t \]

(4.7)

Thus, \( a \) is the long run trend in TFP with \( g \) being the trend growth of productivity while \( z \) represents TFP fluctuations around the trend.

Optimality in factor markets dictates that factor prices must be given by:

\[ r_t = \alpha \frac{y_t}{k_t} \]

(4.8)
\[ w_{it} = (1 - \alpha) \frac{y_t}{l_t} \left( \frac{(\gamma_i l_t)^\eta}{\sum_{i=1}^{N} (\gamma_i l_t)^\eta} \right) \]  

(4.9)

where \( y \) and \( k \) denote per capita world output and capital, respectively. Note that since we have normalized the world population to unity, aggregate world output and capital are also per capita world output and capital: \( Y = y, K = k \). The wage rate is country specific as labor productivity is different and different types of labor are not perfect substitutes. On the other hand, the rental rate is identical for the different types of capital since they are perfect substitutes in production and they are all equally productive.

Any world equilibrium in this economy must clear goods and factor markets:

\[ \sum_{i=1}^{N} l_i (c_{it} + \iota_{it}) = y_t \]  

(4.10)

\[ \sum_{i=1}^{N} l_i k_{it} = k_t \]  

(4.11)

\[ \sum_{i=1}^{N} l_i = 1 \]  

(4.12)

Equation (4.10) is the goods market clearing condition which dictates that the total demand for consumption and investment by the world must equal the world GDP. Equation (4.11) says that total world capital supply must equal capital demand while equation (4.12) is the corresponding world labor market clearing condition.

**Definition 4.1** An equilibrium in this economy is a set of allocations \( \{c_{it}, k_{it}, \iota_{it}, y_t\} \) and prices \( \{w_{it}, r_t\} \) such that at each \( t \) (a) all households solve their optimization problem given prices; (b) firms maximize profits given prices; and (c) the allocations clear all markets.

Before proceeding further it is worth sketching out a brief description of how the recursive specification works in steady state. Let \( \bar{x} = \frac{\bar{z}}{\bar{a}} \). In steady state, the rate of time preference for agent \( i \) is given by

\[ W^i_u = \beta_i \left( g \bar{u}^i \right)^{\delta_i - 1} \]

while steady state normalized utility is

\[ \bar{u}^i = \frac{(R \beta_i)^{1 - \delta_i}}{g} \]
where $R = 1 + r - d$. Using the definition of the aggregator, we also get a steady state expression for normalized consumption:

$$
\bar{c}_i = \left[ \theta_i \bar{u}_i - \frac{\theta_i \beta_i}{\delta_i} \left( \bar{\omega}_i \theta_i \right) \delta_i \right] \frac{1}{\delta_i}
$$

Hence, each $\bar{u}$ maps into a different $\bar{c}$. In this set-up, different $\delta_i$s and $\beta_i$s imply different steady state $\bar{u}$'s. The rate of time preference $W_u$ is equated across agents by different steady state $\bar{c}$'s and $\bar{u}$'s. Hence, different $\delta_i$s and $\beta_i$s across agents induce a dispersion in steady state saving rates across agents.

For a growing economy characterized by agents with such heterogeneous preferences, Farmer and Lahiri (2005) used the results of Lucas and Stokey (1984) to prove that there exists a unique convergent path to a unique steady state with a stationary distribution of saving rates provided $c$ and $u$ are both "non-inferior" and preferences display increasing marginal impatience, i.e., $W_u$ is decreasing in $c$. Our specification satisfies all the conditions of Farmer and Lahiri (2005). Hence, their results apply to our model as well.

5 Quantifying the model

We now take the model to the data. Before proceeding to the quantification we need to address some measurement issues. Specifically, we need measures for saving, the capital stock and the share of income accruing to capital by region. The savings rate for each region $i$ is constructed from the relation

$$
\frac{S^i}{Y^i} = 1 - \frac{C^i}{Y^i} - \frac{I^i}{Y^i}
$$

where $C^i$, $I^i$ and $Y^i$ denote aggregate consumption, investment and output of region $i$. For each region we compute their aggregate output, consumption, capital stock, workers and investment by summing across all countries in the region.

We construct capital stocks for each country in our sample using the perpetual inventory method. For the capital share numbers we tried a number of alternative approaches ranging from a constant 1/3 share of capital for all countries to the numbers computed by Caselli and Feyrer (2007) as well as those from Bernanke and Gürkaynak (2002). Since the results are robust to these alternative approaches, in the following we shall set the capital share to a common 1/3 for all countries.

---

$^4$ $c$ and $u$ are non-inferior if $c < c'$ and $u > u' \Rightarrow \frac{W_u(c, u)}{W_u(c, u')} \frac{W_u(c', u')}{W_u(c', u')}, i = 1, 2$. 

12
To compute productivity in each region we use the production function to get:

$$Productivity_t = \frac{y_t}{k_t^\alpha \left( \sum_{i=1}^{N} (\gamma_i l_{it})^\eta \right)^{\frac{1-\alpha}{\eta}}}$$

We detrend this series using a linear trend and set the linear trend equal to $a_t^{1-\alpha}$. We set the detrended series equal to $z$ and use the derived series for $z$ to estimate

$$z_t = \hat{\rho} z_{t-1} + \hat{\epsilon}_t$$

We use the computed residuals $\hat{\epsilon}_t$ as shocks to the model.

The first important choice to make is the choice of the externality parameter. For our baseline case we set

$$\bar{\zeta}_{it} = \bar{c}_t$$

where $\bar{c}_t$ denotes the average per capita world consumption level. This is the most neutral starting point where there is a common reference consumption target for all countries. Later we shall relax this assumption to explore its potential in explaining saving miracles.

We divide the world into three groups: G7 economies, Asian tigers, and Emerging economies. We deliberately separate the Emerging economies from the Asian Tigers since we analyze the Tigers separately. Details regarding the countries in each region, the data and the series construction are contained in the Data Appendix.

We start by calibrating a two-country version of the model we presented above. To align the two-country model to the data we normalize the world to only comprise the G7 and Emerging economies. We calibrate the model to match data on two regions (the G7 and Emerging economies) in 1970.

Our calibration strategy is to first set the vector $(\beta_{G7}, \beta_{Emg}, \theta_{G7}, \theta_{Emg}, \eta)$ exogenously. For our baseline parameterization we set these parameters to be identical across countries so as to retain the focus of the analysis on recursive preferences and the key aspect of heterogeneity in preferences emanating from the non-additively separable component of preferences. Accordingly, we set $\beta_{G7} = \beta_{Emg} = 0.97$ and $\theta_{G7} = \theta_{Emg} = 0.8$. Lastly, we set $\eta = 1$ which implies that efficiency

\footnote{Below we shall calibrate a richer five-country world by splitting the Emerging economies into four groups. We start with the two-country specification to build intuition.}
adjusted labor from different countries are perfect substitutes. \( \beta = 0.97 \) is relatively standard number for the discount factor in cross-country studies with additively separable preferences.

We choose the regional \( l_i's \) to match the relative labour shares in 1970 which were 0.205 and 0.795 for the G7 and the emerging economies, respectively. The vector \((\delta_{G7}, \delta_{Emg}, \gamma_{G7}, \gamma_{Emg})\) is then set so that the steady state values of the regional saving rates and relative world capital shares are equal to those observed in 1970. Note that our procedure targets the regional saving rates and world capital shares. Hence, we choose four parameters. The parameter choices and the data targets for 1970 are summarized in Table 1 below:

<table>
<thead>
<tr>
<th>Table 1: Parameterization of baseline model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>G7</strong></td>
</tr>
<tr>
<td>saving rate</td>
</tr>
<tr>
<td>world capital share</td>
</tr>
<tr>
<td>( \delta_i )</td>
</tr>
<tr>
<td>( \gamma_i )</td>
</tr>
</tbody>
</table>

\( (\beta_{G7}, \beta_{Emg}, \theta_{G7}, \theta_{Emg}, \eta) = (0.98, 0.98, 0.8, 0.8, 1) \)

This table summarizes the regional data targets and the corresponding parameter choices to enable the baseline model to match those moments.

To illustrate the mechanics of the model we start by plotting the response of the saving rates of the two regions to a one standard deviation positive shock to the common productivity process \( z \). Figure 4 shows that the two regions respond differently to the same productivity shock. Specifically, the saving rate in the G7 rises less and declines faster than in the emerging economies. The reason for this is the increasing marginal impatience that is built into these preferences. The high consumption region (the G7) is also more impatient and hence responds less and adjusts downwards faster in response to the same increase in the real interest rate relative to the emerging economies. The stationary world wealth distribution is non-degenerate precisely due to this feature of preferences.

So, how well does the model explain world saving behavior? We examine this next by generating the saving from the model in response to the measured productivity shocks in the data between 1970 and 2010. Recall that the model was parameterized to mimic data in 1970. We keep all those parameters fixed across time for these simulations. Figure 5 shows the model and data generated saving rates for the G7 and the emerging economies. Panel (a) shows the time series of savings for the two regions while panel (b) of Figure 5 plots the difference between saving rates of the two regions in the model and the data. The figures make clear that the model tracks the trend behavior of saving rates in the two regions quite well. This shows up clearly in panel (b) where the differences
in saving rates between the regions in the model closely tracks the data. The noteworthy aspect of this is that despite the fact that the model has not been calibrated to match the time series behavior of the regional saving rates, it tracks turning points in saving rates of both regions quite closely. And, it does this with just one common exogenous productivity process.

Figure 5: Saving rates: data and the model

Notes: Panel (a) shows the saving rates in the G7 and emerging economies in the model and the data between 1970 and 2010. Panel (b) shows the difference between the saving rates of the G7 and emerging economies in the model and the data during this period.

Table 2 also shows that the baseline model performs quite well in reproducing the correlation of saving rates between the two regions. Additionally, the model generated saving rates also correlate
quite strongly with the actual saving rates for each region individually. How sensitive are the simulation results to the assumed parameter values for \( \theta \), a parameter for which we do not have direct estimates? Table 2 suggests that the results of the baseline model are quite robust to changing \( \theta \). Neither the volatility nor the correlation statistics change too much in response to changing the baseline symmetric specification \( (\theta_{G7}, \theta_{Dev}) = (0.8, 0.8) \) to an asymmetric one.

| Table 2: Comparing savings in the model and the data |
|-----------------|-----------------|-----------------|-----------------|
| Data            | Model \( (\theta_{G7}, \theta_{Dev}) \) | \( (0.8, 0.8) \) | \( (0.9, 0.8) \) | \( (0.8, 0.9) \) |
| std dev(\( s_{G7} \)) | 0.981 | 2.694 | 2.732 | 2.641 |
| std dev(\( s_{Dev} \)) | 1.099 | 2.940 | 2.992 | 2.919 |
| corr(\( s_{G7} \), \( s_{Dev} \)) | 0.662 | 0.973 | 0.965 | 0.950 |
| corr(\( s_{G7} \) \( model \), \( s_{G7} \) \( data \)) | 0.655 | 0.653 | 0.665 |
| corr(\( s_{Dev} \) \( model \), \( s_{Dev} \) \( data \)) | 0.572 | 0.568 | 0.572 |

Note: The table reports moments of saving rates in the data and the model for the period 1970-2010.

Overall, we view these results as suggesting that the model does a good job of accounting for saving behavior over time both within regions as well as differences across regions of the world.

### 5.1 Adjustment costs

The feature of the data that the model does not reproduce as well is the volatility of saving. In the data saving rates are much smoother than in the model. Table 2 summarizes the data and model moments of saving rates in the two regions. The table makes clear that under our baseline parameterization, the volatility of saving rates in the model is three-fold larger than in the data. This is mainly due to the fact that investment is too volatile in the model relative to the data. As has been noted by countless authors, this problem is easy to address by introducing adjustment costs on investment. We introduce adjustment costs into the model by modifying the capital accumulation equation in each country as

\[
k_{it+1} = (1 - d) k_{it} + \tau_{it} - k_{it}\phi\left(\frac{\tau_{it}}{k_{it}}\right), k_{i0} \text{ given for } i = 1, ..., N
\]

where \( \phi' > 0, \phi'' > 0 \). In our calibration we assume that the adjustment cost function is given by the quadratic form:

\[
\phi\left(\frac{\tau}{k}\right) = \frac{b}{2} \left(\frac{\tau}{k} + 1 - d - g\right)^2
\]
This specification implies that adjustment costs are zero along any deterministic steady state growth path. Moreover, the model reduces to one with no adjustment costs when \( b = 0 \). A crucial assumption underlying this formulation is that the adjustment cost function is common across countries, i.e., there is a common \( b \). Allowing \( b \) to be country-specific would simply improve the potential fit of the model.

We calibrate \( b \) to match the standard deviation of \( \sum_i \omega_{it} s_{it} \), where \( \omega_{it} = \frac{l_{it}k_{it}}{\sum_i l_{it}k_{it}} \) is the world capital share of country \( i \). Hence, \( b \) is picked to match the volatility of world savings. In the two-country world analyzed above, the implied \( b \) is 0.47. The rest of the parameter configuration for the model under adjustment costs is summarized in Table 3 below:

<table>
<thead>
<tr>
<th>Parameters</th>
<th>G7</th>
<th>Emerging economies</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta_i )</td>
<td>0.978</td>
<td>0.969</td>
</tr>
<tr>
<td>( \gamma_i )</td>
<td>2.803</td>
<td>0.535</td>
</tr>
<tr>
<td>( b )</td>
<td>0.47</td>
<td>0.47</td>
</tr>
</tbody>
</table>

This table summarizes the parameter choices to enable the model with adjustment costs to match the targeted data moments.

How does the model’s predictions for the regional saving rates change with adjustment costs? Figure 6 shows the predicted saving rates for the two economies:

Figure 6: Saving rates under adjustment costs: data and the model

Notes: The figure shows the saving rates in the G7 and emerging economies in the model with adjustment costs and the data between 1970 and 2010.
Figure 6 shows that the introduction of adjustment costs allows the model to match the volatility of country-specific saving rates without changing the fit of the average level of saving. What is possibly somewhat surprising is that the model is able to match the individual saving volatilities even with a common adjustment cost function. This reflects the fact that the primary difference in saving behavior across countries is the level of savings rather than their volatility which tends to be relatively similar.

5.2 A Five-Region Extension

The model presented above aggregated the world economy into two broad regions – the G7 and Emerging economies. However, there is a lot of dispersion in saving behavior across the Emerging group which comprises countries from sub-Saharan Africa, south Asia as well as Latin America. Can the model capture the saving heterogeneity across such a wide spectrum of countries? We answer this by expanding the two-region model to a five-region economy. We do this breaking up the Emerging economy group into four groups of countries: sub-Saharan Africa, Latin America, Middle East and North Africa, and Developing Asia. The data appendix provides the list of countries in each group.

As before we let \( \delta \) and \( \gamma \) vary across regions. These parameters are calibrated to match the saving rates in each region in 1970. The resulting parameter configuration for the 5-country world economy is given in Table 4. The adjustment cost parameter \( b \) is set so that the standard deviation of capital share-weighted world saving in the model matches the data value. The model is solved to a 2nd order approximation.

<table>
<thead>
<tr>
<th>Region</th>
<th>Saving rate</th>
<th>Capital share</th>
<th>( \delta_i )</th>
<th>( \gamma_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>G7</td>
<td>0.247</td>
<td>0.715</td>
<td>0.978</td>
<td>2.962</td>
</tr>
<tr>
<td>Sub Saharan Africa</td>
<td>0.126</td>
<td>0.034</td>
<td>0.969</td>
<td>0.787</td>
</tr>
<tr>
<td>Latin America</td>
<td>0.147</td>
<td>0.085</td>
<td>0.975</td>
<td>2.169</td>
</tr>
<tr>
<td>Middle East and North Africa</td>
<td>0.317</td>
<td>0.043</td>
<td>0.970</td>
<td>0.790</td>
</tr>
<tr>
<td>Developing Asia</td>
<td>0.165</td>
<td>0.123</td>
<td>0.966</td>
<td>0.293</td>
</tr>
</tbody>
</table>

\((\beta, \theta, \eta, \phi) = (0.98, 0.87, 1, 0.39)\)

This table summarizes the parameter choices to enable the five region model with adjustment costs to match the targeted data moments.

How well does the model perform in matching the time series behavior of regional saving rates in the data? Our particular interest lies in examining the ability of the model to match the variation
of saving rates across regions. Table 5 compares the key moments of interest in the data and the model. In terms of the time-series variability of saving rates within each region, the model does very well in matching the G7 but tends to underpredict the variability of saving rates in the other four regions, especially the Middle east and north Africa. This though is not too surprising given the special dependence of this region on volatile oil revenues and our one-shock specification. More interestingly, the Table also shows that the model does rather well in matching the dispersion of saving rates across regions. Thus, the average standard deviation of saving between regions is 6.38 in the data and 7.98 in the model. Given our previously noted inability to match the volatility of saving in the middle east, we consider this to be a success of the model in matching the cross-sectional dispersion of world savings.

Table 5: Parameterization of 5-region model with adjustment costs

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>std dev ($S_{G7}$)</td>
<td>0.981</td>
<td>0.914</td>
</tr>
<tr>
<td>std dev ($S_{SubSahara}$)</td>
<td>1.835</td>
<td>0.753</td>
</tr>
<tr>
<td>std dev ($S_{LatinAmerica}$)</td>
<td>1.163</td>
<td>0.762</td>
</tr>
<tr>
<td>std dev ($S_{MidEast+NAfrica}$)</td>
<td>4.478</td>
<td>1.011</td>
</tr>
<tr>
<td>std dev ($S_{DevAsia}$)</td>
<td>1.693</td>
<td>0.894</td>
</tr>
</tbody>
</table>

Note: The table reports moments of saving rates in the data and the model for the period 1970-2010. $\sigma_{btw}^t$ denotes the between-region standard deviation of saving at time $t$.

6 Saving miracles

The second motivation of this paper was the observation that some countries have shown sudden and dramatic increases in their average saving rates. We showed an example of this in Figure 2 which depicted the sharp pickup in the average saving rate of the Asian Tigers. We now turn to demonstrating how our model can accommodate these dramatic saving miracles. Our principal idea is that saving behavior is dictated by our goals which, in turn, is often determined by our position relative to a comparison group. If a society begins to aspire to have the wealth levels of a much richer comparison group then its saving levels have to respond to achieve that new goal. A key feature of the recursive preference structure we have formalized here is the presence of relative consumption. In the model presented in the previous section the relative consumption level in
preferences was just the world average per capita consumption. In this section we shall examine the consequences of a country changing its relative comparison group from the average world level to a richer cohort. Could such a change generate an increase in saving rates similar in magnitude to the rise in Asian savings we saw in Figure 2? We should clarify at the outset that we are not building a theory of aspirations in this paper. Rather, we are quantitatively exploring the dynamic general equilibrium consequences of a change in aspirations.

Consider a world economy comprising three regions. Let the two regions now be the G7, Emerging economies, and the Asian Tigers. In other words, we are now expanding the world economy from the two-region world of the previous section to a three-region economy that also includes the Asian Tigers. Recall that at each date $t$ the externality in preferences for each $i$ is denoted by $\zeta_i$. Let average per capita world consumption be

$$C = l_{G7}c_{G7} + l_{Emg}c_{Emg} + l_{Tigers}c_{Tigers}$$

where $c_i$ is the per capita consumption of region $i$. Consider two regimes:

Regime 1: $\zeta_{G7} = \zeta_{Emg} = \zeta_{Tigers}$

Regime 2: $\zeta_{G7} = \zeta_{Emg} = C$; $\zeta_{Tigers} = \zeta_{G7}$

Under Regime 1 all three regions value their own consumption relative to the world per capita consumption level. Under Regime 2 however, the Asian Tigers switch their comparison group to the G7 while the other two regions continue to use the world average as the relevant consumption comparison group. We consider an environment where at some date $t^*$, the regime switches from Regime 1 to Regime 2. Given that per capita consumption is higher in the G7, such a regime switch represents a switch to a higher aspiration level for the Asian Tigers.

In the context of our model, could such a regime switch account for the almost 15 percentage point increase in the saving rate of the Asian Tigers since 1970? To answer this question we calibrate the model by choosing parameter values such that the model reproduces the steady state saving rates, world capital shares and world labor shares of the three regions in 1960 under the maintained assumption of the world being in Regime 1. We then perturb the model with a regime switch to Regime 2 in 1970 where the reference consumption level for the Tigers increases to the per capita consumption of the G7 economies. Keeping the parameters underlying the initial calibration
unchanged, we then feed the estimated productivity process from 1960 to 2010 into the model and simulate the equilibrium paths for the three economies. We then compare the model generated saving rates with their data counterparts.

To clarify the role of the regime switch in generating the saving increase, we start with Figure 7 which shows the saving rates of the G7 and the Tigers when we shut down all productivity movements and only allow for a regime switch in 1970. Since the world economy is in steady state in 1960, saving rates are constant till 1970 when the regime switch occurs. From that date onward savings of the Tigers rises while the G7 saving rate initially declines before recovering towards its original level.

Figure 7: Response of savings to regime switch

![Graph showing saving rates of G7 and Tigers](image_url)

Note: The graph depicts the response of saving rates in the two regions when the only shock is a change in the reference consumption level of the Asian Tigers to per capita consumption of the G7 in 1970.

What are the predicted saving rates of the model when we incorporate the measured productivity shocks in the data? Panel (a) of Figure 8 shows the simulated path of savings under the estimated productivity process between 1960 and 2010 with a regime switch from Regime 1 to Regime 2 in 1970. As a point of contrast, panel (b) of Figure 8 plots the saving rates under the assumption of no change in regime. We find the fit in Panel (a) quite remarkable in terms of how well the model-generated saving rates track the actual saving rates.

In summary, the switch to a higher aspiration is key for the model to reproduce the sharp increase in the saving rates of the Asian Tigers. We find this result indicative of the power of the

---

6The decline in the G7 saving on impact occurs due to the fall in the interest rate that is induced by the rise in the desired savings of the Tigers. Note that we do not plot the saving rate of the Emerging economies here in order to keep the graph uncluttered.
Figure 8: Saving miracles: data and the model

(a) saving rates under regime switch
(b) saving rates without regime switch

Notes: Panel (a) shows the saving rates in the G7 and the Asian Tiger economies in the model under a switch in the comparison group for the Tigers with the data between 1970 and 2010. Panel (b) shows the saving rates of the G7 and the Asian Tigers in the model without any switch in regime.

aspiration mechanism to explain the rapid growth of savings in Asia.

7 Conclusion

The variation in saving behavior across countries has long been a puzzle and a challenge to explain for standard neoclassical models. In this paper we have explored the explanatory potential of recursive preferences and preference heterogeneity in jointly accounting for the cross-country saving data. We have used a preference specification that displays a form of relative consumption. Specifically, agents of a country derive utility from consumption relative to the consumption of a reference group. Our specification imply that when countries are poor they display high patience and high saving rates. As their consumption gets closer to the levels of their reference group however they become more impatient, a property that Lucas and Stokey (1984) called "increasing marginal impatience". This feature of preferences keeps the wealth distribution from becoming degenerate even when preferences are heterogenous across countries.

We apply these preferences to a multi-country world economy model with free capital flows across countries and calibrate the model to match the long-run differences in saving rates across countries. Using only productivity shocks to a common world production technology as an exogenous driver, we then show that that the calibrated model can also account for the short run differences in saving rates across countries.
In addition, we have also shown that a change in the aspirations of societies, as captured by a change in the reference consumption basket they use to value their own utility, can account for sudden and sharp changes in saving rates. Thus, our model can account for the rapid increase in Asian saving rates and its overall behavior between 1960 and 2010 by allowing for a change in the reference basket being used by the Asian economies from the average world consumption level to the G7 consumption level in 1970. Intuitively, a higher reference consumption level induces greater saving as accumulating greater wealth is the only way to achieve a higher steady state consumption.

We believe this class of models has great potential in also helping us understand changes in the wealth distribution within countries over time. Wealth evolves as a function of saving. Accounting for differential saving rates is thus key to explaining wealth distributions and changes therein. We hope to address this issue in future work.

References


A  Data Appendix

A.1  Countries
### Table 6: List of Countries in 3-Region Model

<table>
<thead>
<tr>
<th>Region</th>
<th>Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian Tigers</td>
<td>Hong Kong, Indonesia, Korea, Malaysia, Philippines, Singapore, Taiwan, Thailand, Vietnam*</td>
</tr>
<tr>
<td>G7</td>
<td>Canada, France, Germany*, Italy, Japan, United Kingdom, United States</td>
</tr>
</tbody>
</table>

### A.2 Calibration and data computation

The model is calibrated to match data on two regions over the period 1970. The vector \((\beta_1, \beta_2, \theta_2, \eta)\) is set exogenously, while \(l_i\) are set to match relative labour shares in at the start of the sample. \((\delta_1, \delta_2, \gamma_1, \gamma_2)\) is then set so that the steady state values of the regional saving rates and relative capital shares are equal to those observed in the data at the start of the sample. We set the capital share of output for each country as \(\alpha_i = 0.33\).

The saving rate of country is computed as

\[
1 - \text{consumption share of } \text{rgdppercapita} - \text{investment share of } \text{rgdppercapita}
\]

The savings rates of each region are averaged across regions to obtain regional saving rates (averages are unweighted).

The capital stock of each country in the sample is constructed using the perpetual inventory method. Assuming a depreciation rate of 0.06 for each country and using the growth rate of real gdp per capita we have the steady state relationship

\[
k_{i1970} = i_{i1970}/(dep + girgdp)
\]
<table>
<thead>
<tr>
<th>Region</th>
<th>Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>G7</td>
<td>Canada, France, Germany*, Italy, Japan, United Kingdom, United States</td>
</tr>
<tr>
<td>Latin America &amp; Caribbean</td>
<td>Argentina, Bahamas*, Barbados, Belize*, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Ecuador, El Salvador, Guatemala, Guyana*, Haiti, Honduras, Jamaica, Mexico, Nicaragua, Panama, Paraguay, Peru, Suriname*, Trinidad &amp; Tobago, Uruguay, Venezuela</td>
</tr>
<tr>
<td>Middle East &amp; Northern Africa</td>
<td>Algeria, Bahrain, Egypt, Iran, Iraq, Jordan, Lebanon, Mauritania, Morocco, Oman, Sudan, Syria, Tunisia</td>
</tr>
</tbody>
</table>
In computing the capital stock numbers we assume that depreciation ($dep$) is 0.06 for all countries. We measure $g_{rgdp}$ as the average growth rate of rgdp per worker in the first ten periods of the sample.

Labour supply as measured by the number of workers, is computed as

$$lab = \frac{\text{real gdp per capita} \times \text{population}}{\text{real gdp per worker}}$$

The output and labour supply of each country are summed across regions to obtain output per worker and capital per worker for each region.

TFP shocks are measures from the data. For each region, we compute $prod_t = \frac{w_t}{k_t}$ and detrend the series using a linear trend. We set the linear trend equal to $a_t$. We then set the detrended series equal to $z_t$ and estimate $z_t = \hat{\rho}z_{t-1} + \hat{\epsilon}_t$. The residuals $\hat{\epsilon}_t$ are used as shocks in the simulated model.