The Exchange Rate Response Puzzle

Viktoria Hnatkovska†, Amartya Lahiri†, and Carlos A. Vegh‡

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Abstract

We present a new data fact: in response to a monetary tightening, the nominal exchange tends to appreciate in developed countries but depreciate in developing countries. A model is formalized to rationalize this puzzling pattern. It has three key channels of monetary transmission: a liquidity channel, a fiscal channel and an output channel. These have offsetting effects on the exchange rate. The paper shows that a calibrated version of the model can explain the contrast between developed and developing countries. Finally, we use the theory to identify differences in the deposit to cash ratio as being key to the contrasting responses generated by the model. The paper provides independent evidence of systematic variation in this ratio between developed and developing countries. We interpret this as evidence supporting the mechanism formalized in the model.

JEL Classification: F3, F4

Keywords: Monetary policy, interest rates, exchange rates

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1 Introduction

We examine cross-country data on 72 countries to uncover an intriguing contrast between developed and
developing countries in terms of the response of their nominal exchange rates to monetary policy shocks:
developed countries appreciate in response to a monetary tightening while developing economies depreciate.
This contrasting response is robust to controlling for the endogenous response of monetary policy to various
types of exogenous shocks. This is a new data fact. We formalize a model to explain the contrast and show
that a calibrated version of the model can explain the data patterns. The model identifies the deposit to
cash ratio to be a key factor in accounting for the different responses of the two groups of countries. We
provide independent evidence in support of this mechanism: countries that appreciate in the data tend to
have deposit-cash ratios that are significantly higher (about 3-fold greater) than in the depreciating countries.
We interpret this as support for the model and its mechanisms.

Perhaps one of the oldest issues in international economics is about the relationship between monetary
policy and the nominal exchange rate, specifically the question "what is the effect of tightening monetary policy
on the exchange rate?" In the context of modern central banking practises, this question can be rephrased
as "how does the nominal exchange rate respond when the central bank raises the interest rate?" Indeed,
all undergraduate textbooks have a treatment of some version of it, policymakers (especially in smaller, more
open economies) are extremely conscious of it, while practitioners and analysts have some priors about it when
they debate the consequences of policy interventions on the economy.

Somewhat paradoxically, the evidence on this to date has been sparse and somewhat limited. Probably, the
most well known study is due to Eichenbaum and Evans (1995) who conclude, using a vector autoregression
(VAR) analysis, that a contractionary monetary policy in the United States leads to an appreciation of the
dollar relative to all major currencies. In turn, Kim and Roubini (2000) use a structural VAR approach, which
takes care of some identification problems that had plagued this literature up to this point, to look at non-US
G-7 countries and reach the same conclusion. These results tend to provide support for the conventional
wisdom about this question which holds that exchange rates should appreciate in response to a monetary
tightening.\(^1\)

Case closed? Not in our view. In fact, we will argue in this paper that, contrary to the case of industrial
countries, in developing countries the currency depreciates in response to an increase in interest rates. We
establish this stylized fact based on a sample of 25 industrial and 47 developing countries with flexible exchange
rates.\(^2\) We start by computing simple correlations between interest rate and the exchange rate (in both levels
and growth rates) and find that the two variables are negative correlated in industrial countries, but are
positively correlated in developing countries. Moreover, the differences are highly statistically significant. To

\(^1\)The conventional wisdom probably is grounded in the predictions of the older Mundell-Fleming family of open economy
macroeconomic models based on sticky prices. However, the prediction that exchange rates should appreciate in response to a
monetary tightening is shared by a broader group of more modern but standard open economy macro models such as limited
asset market participation, fiscal theory of the price level as well as the Obstfeld-Rogoff-type microfounded versions of the
Mundell-Fleming model.

\(^2\)For several developing countries we have multiple episodes giving us a total of 55 developing country-episode pairs in our
sample.
isolate the effects of interest rate shocks on the exchange rate, we then turn to individual country VARs. In this analysis we find that for industrial countries, the domestic currency appreciates in response to an increase in interest rates in 84 percent of the cases. In sharp contrast, for developing countries we show that the nominal exchange rate increases (i.e., the domestic currency depreciates) in response to higher interest rates in 75 percent of the cases. We also confirm this finding by estimating panel VARs for industrial and developing countries separately and showing how, in response to an increase in the interest rate, the currency appreciates in industrial countries but depreciates in developing countries. We will refer to these contrasting findings in industrial versus developing countries as the “exchange rate response puzzle.”

One may argue that the “exchange rate response puzzle” is due to differences in the type and size of shocks in the two groups of countries. These could be expected inflation or output shocks which are often more pronounced in developing countries; or they could be risk premium shocks, which too tend to be more volatile in developing countries. If such shocks in developing countries depreciate their exchange rate and induce policy makers to respond to them by raising policy interest rates, we are likely to find a positive correlation between the two variables in this groups of countries. In developed countries, on the other hand, such shocks may be less important, leading to a negative correlation between interest rates and exchange rate, in line with the conventional wisdom. We investigate this conjecture in detail by considering sub-samples of countries and through various techniques. We show that even after controlling for expected inflation, output, and risk premium shocks, the divergence in the response of the exchange rate to identified interest rate shocks in developed and developing countries survives. Moreover, we also take into account one of the main criticisms of the The main conclusion we draw from this is that exchange rates in developing and developed countries respond differently to monetary policy shocks for reasons that are distinct from these channels.³

³We should note that a number of the studies that have used VARs to identify the effect of monetary policy shocks have attracted criticisms about their specific identification assumptions. An excellent summary of these criticisms can be found in Faust and Rogers (2003). Our strategy of using many different approaches including data correlations, country VARs, panel VARs, recursive identification schemes, non-recursive schemes through structural VARs etc, is a direct result of these criticisms. The multi-pronged approach is designed to alleviate concerns that can logically arise from any one approach.

We next turn to explaining the observed divergence between developed and developing countries by building a monetary model of a small open economy. The model we develop incorporates three key features which we believe are important aspects of the monetary transmission mechanism. Specifically, the model incorporates a liquidity channel, a fiscal channel and an output channel, all of which impact the transmission of monetary policy to the economy. The liquidity channel raises the demand for domestic currency liquid assets and hence has a strengthening effect on the local currency when monetary policy is tightened, i.e., when the policy controlled interest rate is raised. The other two channels tend to weaken the currency in response to a rate hike. We identify necessary and sufficient conditions for the model to give rise to appreciations or depreciations in response to interest rate changes.

With the theoretical insights in hand, we then quantify the model by calibrating it. The calibration exercise is structured around trying to determine whether differences between developed and developing economies in the strengths of the three channels described above can account for the differences in the response of their
exchange rates to monetary policy shocks. Accordingly, we undertake two calibrations: one for developed and another for developing economies. We keep all parameters identical for the two groups except for the parameters that control the liquidity, fiscal and output channels. We then examine the impulse response of exchange rates to interest rate shocks which we identify by estimating different interest rate rules for the two groups of countries. Amongst the monetary policy rules we study are exogenous interest rate rules and two different types of Taylor rules. The model impulse responses from all the different specifications yield the same result: developed country exchange rates appreciate in response to an increase in the policy interest rate while developing countries show depreciations. In other words, the impulse responses from the quantified model replicate the impulse response patterns from the data.

As a final check on the mapping between the model and the data, we look for independent evidence on the key mechanism that drives the differential response of exchange rates in developed and developing economies to monetary policy shocks. Using the theoretical model, we identify the deposit to cash ratio to be a key driver of the quantitative results. The model identifies a threshold value of 2.44 for this ratio with countries with ratios in excess of this threshold predicted to appreciate while those below it predicted to depreciate. We then revisit our empirical VAR results to isolate the appreciating and depreciating countries and examine the deposit to cash ratios in these countries. For countries for whom this data is available, we find that the ratio of deposits to cash is equal to 1.87 in depreciating countries, while it is significantly higher at 5.22 in appreciating countries. Lastly, a simple probit regression reveals that a higher deposit-cash ratio significantly increases the probability of the exchange rate appreciating in response to a rate hike. We interpret this as evidence in support of the key mechanism identified by the model.

The importance of the deposit to cash ratio suggests that the transmission of monetary policy to the economy is likely fundamentally affected by factors such as the history of expropriation of interest bearing assets, the institutional strength of the monetary regime, the presence and duration of deposit insurance schemes as well as the level of financial development. Hence, they need to be factored in explicitly when conducting monetary policy in developing countries since the outcomes may well be at odds with the established wisdom derived from developed country experiences.

We should note at the outset that our paper is not concerned with the relationship between the nominal market interest rate and the rate of currency depreciation. There is a voluminous literature which attempts to document and/or explain this relationship. This literature is concerned with the failure of the uncovered interest parity (UIP) condition (the “forward premium anomaly”). In our model interest parity holds for internationally traded bonds. Hence, we do not shed any new light on the observed deviations from UIP. Instead, our main focus is on the impact effect of policy-induced changes in interest rates on the level of the exchange rate. Crucially, this relationship in the model does not rely on the UIP condition.¹

The rest of the paper is organized as follows. The next section presents some empirical evidence from a number of developing and developed countries detailing the mixed results on the relationship between interest

¹In contrast, evidence of UIP failing in the data is problematic for models whose main result rely on the UIP condition. An example of this is the "exchange rate overshooting" result due to Dornbusch (1976).
rates and the exchange rate. Section 3 presents the model while Section 4 discusses how the model is calibrated and solved. Section 5 presents our quantitative results using the calibrated model. The last section concludes.

2 Empirical facts

We start off by empirically documenting our motivating issue through a look at the data. We use a large sample of countries during 1974:1-2010:12 period for which monthly data on exchange rates and interest rates was available. Most of the data is from International Financial Statistics (IFS) compiled by the International Monetary Fund (IMF). We use period average official exchange rates whenever available to measure exchange rates. If official rates are not available, we turn to period average market rates, otherwise we use the period average principal exchange rates. Exchange rates are in domestic currency units per U.S. dollar, so that an increase is a depreciation of local currency relative to the US dollar. Our focus is on policy-controlled interest rates, which we measured in the data as the period average T-bill rate. If T-bill rate was not available, we used the discount rate, or the money market rate for that country. We note that for majority of countries in our sample we used the T-bill rate. This rate is the closest to the overnight interbank lending rates, which would be our preferred policy rate, but is not available for most of our countries. In our analysis we focus on the interest rate differential between home and abroad computed as domestic interest rate minus U.S. Federal Funds rate.

We focus only on those countries and time periods that are characterized by a flexible exchange rate regime. To perform the selection, we rely on the Reinhart and Rogoff (2004) classification of historical exchange rate regimes. We classify a country as having a flexible exchange rate regime if, in a given year, its exchange rate was either (i) within a moving band that is narrower than or equal to +/-2% (i.e., allows for both appreciation and depreciation over time); or (ii) was classified as managed floating; or (iii) was classified as freely floating; or (iv) was classified as freely falling according to Reinhart and Rogoff (2004). These correspond to their fine classification indices of 11, 12, 13, and 14, respectively. We only focus on the post-Bretton Woods period for all countries. High income OECD countries are included in our sample, irrespective of their exchange rate classification. For the Eurozone countries, we used their national exchange rates before the introduction of the Euro as separate episodes. Since 1999:1 we included a separate episode for the Euro area, for which we used the Euro-dollar exchange rate and the ECB marginal lending facility rate as the policy rate. According to Reinhart and Rogoff (2004) regime classification, some countries had multiple episodes of flexible exchange rates. We considered each such episode separately. To be included in our sample we also require that an

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5In what follows we show that our results are robust to using only countries for which the T-bill rate is available. We also verify that our results are not driven by the fact that our measures of interest rates may potentially contain information other than the monetary policy change, i.e. changes in expected inflation or in the perceived sovereign risk.

6These categories are generally used in the literature to represent floating exchange rate regimes (see Reinhart and Rogoff (2004)). In what follows we also check for robustness of our results with respect to the regime classification (see Section 2.2).

7We also considered the coarse exchange rate classification of Reinhart and Rogoff (2004) to select countries and episodes into the sample. We found the results to be robust with respect to the classification. The coarse classification included countries that were on (i) pre announced crawling band that is wider than or equal to +/-5%; (ii) de facto crawling band that is narrower than or equal to +/-2%; (iii) moving band that is narrower than or equal to +/-2%; (iv) managed floating; (v) freely floating; (vi) freely falling. These correspond to indices 3, 4, and 5 in Reinhart and Rogoff (2004).
episode has at least 24 months of data in the flexible regime for each country. This selection gives us a sample of 25 industrial country-episode pairs and 55 developing country-episode pairs, for a total of 80 country-episode pairs. All country-episode pairs are listed in the Appendix A.1.\(^8\)

### 2.1 Bilateral interest rate-exchange rate relationship

To illustrate the relationship between interest rate and the exchange rate, we first report some simple time-series correlations between them. Panel A of Table 1 summarizes our results. We compute correlations on a country-by-country basis for both levels and first-differences of (log) exchange rate and interest rate variables, which are shown in the first column.\(^9\) Column “full sample” reports the mean of all the time-series correlations obtained for the countries in our sample. Columns labelled “developed” and “developing” computes the corresponding correlations for the two groups of countries separately. The results show that the correlation between exchange rates and interest rates is low, on average. However, when the sample is broken into developed and developing countries, the correlation is consistently negative in developed countries and consistently positive in developing economies. Recall that negative correlation occurs when an increase in interest rate is accompanied by an appreciation of the exchange rate, as in developed economies. In developing countries, higher interest rates come together with currency depreciation, resulting in positive correlation between them.

To confirm the significance of these correlations we also estimate simple regression of the log exchange rate (or its first-difference) on a constant and domestic minus US interest rate differential (or its first-difference) on a country-by-country basis. We then report the average of the slope coefficients from these regressions and its 95\% confidence interval for the full sample and separately for developed and developing countries in the Panel B of Table 1. These regressions confirm our findings from correlations: exchange rates and interest rates are negatively correlated in industrial countries; and they are positively correlated in developing countries. These results hold in both levels and first-differences and are highly statistically significant. Importantly, the confidence intervals for the slope coefficients in developed and developing countries do not overlap, indicating significant differences between them.

We next turn to an analysis of the exchange rate-interest rate relationship using vector autoregressions (VARs). We estimate VAR on a country-by-country basis for our sample using log exchange rate and interest

\(^8\)It is probably not surprising that the majority of flexible exchange rate episodes in developing countries included in our sample occur in the 1990s – the “globalization” decade.

\(^9\)Using interest rates and exchange rate series in levels has been a conventional practice in the literature (see, for instance, Kim and Roubini (2000), Faust and Rogers (2003) among others). Such approach implicitly assumes that the two variables are integrated of the same order. We confirm this result in our sample of countries. We test for the presence of a unit root in the country exchange rate and interest rate differential series using augmented Dickey-Fuller test and Phillips-Perron test. We can not reject the presence of a unit root in the levels of both interest rate and (log) exchange rate for 90 percent of all country-episode pairs in our sample. Unit root it rejected in all country-episode pairs at 10 percent significance level when both variables are in first-differences.
Table 1: Correlation between exchange rate and interest rate

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>Developed</th>
<th>Developing</th>
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<tbody>
<tr>
<td>Panel A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>corr(ln Et, it - itus)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.13</td>
<td>-0.09</td>
<td>0.24</td>
</tr>
<tr>
<td>median</td>
<td>0.10</td>
<td>-0.08</td>
<td>0.36</td>
</tr>
<tr>
<td>Panel B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ln Et = β₀ + β₁(it - itus) + εᵗ</td>
<td>1.27</td>
<td>-0.74</td>
<td>2.19</td>
</tr>
<tr>
<td>mean(β₁)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% c.i.(β₁)</td>
<td>[1.13; 1.42]</td>
<td>[-0.94; -0.54]</td>
<td>[1.99; 2.39]</td>
</tr>
<tr>
<td>Δᵗ ln Eᵗ = α₀ + α₁Δᵗ(it - itus) + ut</td>
<td>0.02</td>
<td>-0.44</td>
<td>0.24</td>
</tr>
<tr>
<td>mean(α₁)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% c.i.(α₁)</td>
<td>[-0.08; 0.13]</td>
<td>[-0.57; -0.31]</td>
<td>[0.09; 0.38]</td>
</tr>
</tbody>
</table>

Note: Panel A of the Table reports the mean and median of the cross-sectional distribution of the correlation coefficient between (log) exchange rate and interest rate (and their first-differences) for our sample of countries. Panel B presents the mean of the estimated slope coefficients from the regression ln Eᵗ = β₀ + β₁(it - itus) + εᵗ in levels and first-differences. 95% confidence intervals are in parenthesis.

rate differential between home and the U.S.. The VAR specification also includes a constant term. We use the estimated VARs to calculate the impulse response of the exchange rate to an orthogonalized one standard deviation innovation in the interest rate differential for each country. Following Eichenbaum and Evans (1995) we compute the impulse responses using the ordering: interest rate differential, exchange rate.

We start by presenting the impulse responses of the nominal exchange rate to interest rate shocks in several selected countries in our sample to illustrate the more general data fact. Figure 1 presents the impulse responses in three developed and three developing countries. The picture reveals some systematic patterns. For the developed countries – France, Sweden and the UK – there is a significant appreciation of the currency in response to an increase in the interest rate differential. This is the well-known result of Eichenbaum and Evans (1995). For the developing group the effect is the opposite. In Brazil, Mexico and Philippines, a positive innovation in the interest rate differential between home and the United States induces a significant depreciation of the currency.

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10 In our benchmark VAR specification we focus on interest rate differential and exchange rate variables only. This allows us to use and draw inference from the largest possible sample of countries. In section 2.3 we extend our benchmark VAR specification to include a broad set of other macroeconomic variables, such as output, prices, inflation, risk-premium, etc. Due to data limitations such analysis can only be conducted for a much smaller sample of countries. Nevertheless it allows us to provide robustness checks for the results in this section.

11 We also tried a VAR specification with a trend and have found that the results remained largely unchanged.

12 In each individual VAR we used the Akaike criterion to choose the lag length. The results remain unchanged when Schwarz’s Bayesian information criterion (BIC) is used for selecting the lag length as the two criteria choose the same lag length in 97 percent of all cases.

13 We conduct robustness checks with respect to the ordering of the variables in Section 2.4.

14 Notice that some of these impulse responses have a hump-shaped pattern, which came to be known as the “delayed overshooting” result (see, for instance, Sims (1992), Eichenbaum and Evans (1995), among others). While there is ongoing debate as for the reasons for such “delayed overshooting” pattern in exchange rate responses to monetary policy shocks (Faust and Rogers (2003), Bacchetta and van Wincoop (2010), Engel (2011)), our interest is on the immediate response of the exchange rate. Thus when presenting results, we focus on the immediate responses.
Figure 1: Country VARs: Impulse responses of exchange rate to interest rate shock

Note: These figures present impulse responses of exchange rate to a positive interest rate innovation from individual country VARs estimated on (log) exchange rate and interest rate differential between home and abroad. The following ordering is used: $i - i^{US}, E$.

To check the generality of this differing relationship between interest rates and the exchange rate in developed and developing countries, we ran individual country level VARs for all countries in our sample.\footnote{One may be concerned that the use of a linear VAR specification is not warranted in countries that experienced large jumps in the level of the exchange rate or crisis episodes. We check the robustness of our results with respect to crisis episodes and periods of high inflation in Section 2.2.} We adopted several approaches to classifying a country as exhibiting appreciation: (i) if the response of its exchange rate after an interest rate shock is negative on impact; (ii) if the response of its exchange rate to an interest rate shock is negative at the end of the 1st month; and (iii) if the response of its exchange rate to an interest rate shock is negative at the end of the 1st quarter (3rd month). Depreciation is defined similarly. Table 2 summarizes the results from the individual country VARs, where we estimated a bivariate specification with variables ordered as $i - i^{US}, \ln E$. Panel (a) reports the share of developed countries that have experienced appreciations and the share of developing countries that experienced depreciations of their exchange rates following a positive shock to the interest rate differential between home and abroad, based on level VARs.

The results clearly indicate that an overwhelming majority of industrial economies see their exchange rate appreciating after a positive interest rate shock both on impact (84 percent of all industrial countries), as well as one month (88 percent of all industrial countries) and three months after (84 percent of all industrial
Table 2: Individual country VARs: Summary

<table>
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<th></th>
<th>(a). Levels</th>
<th>(b). First-differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>impact 1 month 3 months</td>
<td>impact 1 month 3 months</td>
</tr>
<tr>
<td>**Bivariate VAR: ( i - i^US ), ln ( E ) **</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial countries: appreciation</td>
<td>84% 88% 84%</td>
<td>84% 88% 52%</td>
</tr>
<tr>
<td>Developing countries: depreciation</td>
<td>75% 75% 75%</td>
<td>70% 62% 60%</td>
</tr>
</tbody>
</table>

Note: The table reports the fraction of developed (developing) countries that experience an appreciation (depreciation) of their exchange rate following a positive shock to the interest rate differential. Appreciations and depreciations are defined based on the impact, 1st month and 1st quarter (3 months) impulse responses from a country-by-country VAR analysis. The ordering used to obtain the orthogonalized impulse responses to interest rate shocks is \( i - i^US, E \).

For developing countries on the other hand, 75 percent of countries show a depreciation following a positive interest rate shock on impact, after 1 month, and the proportion remains at 75 percent if the cutoff is raised to the end of the 3rd month. If we restrict our sample of countries to only those with T-bill data available, we find that our results for developing countries become even stronger. In particular, in that subsample 83 percent of all developing countries experienced an impact depreciation after an interest rate shock, and 80 percent saw their currency depreciate one month later.

To check the robustness of our findings, we also re-estimate the individual VARs with the first difference of the (log) exchange rate and the interest rate differential. The results from this estimation and associated impulse responses are summarized in panel (b) of Table 2. Our earlier results remain robust. In particular, we find that among industrial economies, 84 percent have experienced exchange rate appreciation after an interest rate shock on impact, 88 percent still saw their currency appreciate after the 1st month and 52 percent did so by the end of the first quarter. For the developing countries, the corresponding numbers were 70 percent, 62 percent and 60 percent, respectively.\(^\text{16}\)

We further confirm our empirical findings by running unrestricted, bivariate panel VARs for industrial and developing countries separately. We start with a simple specification in which both the (log) exchange rate and interest rate variables are included in levels. In the panel VAR analysis country heterogeneity is likely to be important which suggests the presence of unobservable individual country fixed effects. We eliminate country-specific fixed effects and common deterministic trends by de-meaning and linearly de-trending both variables for each country. This within-transformation wipes out fixed effects, but does not eliminate the fact that the lagged dependent variable and the error term are correlated. This could lead the within-estimators to be inconsistent, unless \( T \) – the time-series dimension of the data – is large. In our sample, the average number of periods across countries is quite high, equal to 106 months in developing countries and 324 months in developed economies. While this does not eliminate the bias in the estimates, it lends credibility to our level-based results.\(^\text{17}\)

An alternative transformation that eliminates the fixed effects is the first-difference

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\(^{16}\)We should note that when the model is estimated in first-differences, the exchange rate in the third month is the difference between the exchange rate levels in months three and four. Hence, it is not surprising that the differences in the responses of the groups to temporary shocks in the third month appear to be much smaller in first-differences than in levels since the first-difference observation reflects an additional period.

\(^{17}\)We are interested in obtaining the results from the panel VAR in levels to retain comparability with the individual VAR
transformation. We present the results from the panel VARs on the first-differenced data below.

Under either transformation of the data, the correlation between the lagged dependent variable and the remainder error term remains. The standard approach of addressing this correlation is to estimate the model coefficients by an instrumental variable (IV) method. We follow this practice and apply the system generalized method of moments (GMM) of Arellano and Bond (1991) that uses lagged regressors as instruments.

Figure 2 presents the impulse response of exchange rate to a positive interest rate innovation together with the 90 percent confidence bands separately for our sample of industrial countries and developing economies. It is easy to see that in response to an increase in the interest rate, the currency appreciates in industrial countries but depreciates in developing countries.

Figure 2: Panel VAR: Impulse responses of exchange rate to interest rate shock (levels)

Note: Figures present the impulse responses of the exchange rate to a positive interest rate differential shock from panel VARs estimated for developed and developing countries. Estimation uses (log) exchange rates and interest rates in levels. Both series are de-meaned and linearly de-trended.

Figure 3 presents the resulting impulse responses from the model estimated in first-differences. As before, the exchange rate appreciates in our sample of developed countries; and depreciates for developing countries, with the key difference being that these responses are more short-lived.

2.2 Exchange rate classification, crisis episodes and inflation

We begin by checking the robustness of our results with respect to the exchange rate classification. As we noted above, we used the definition of the floating exchange rate regime following Reinhart and Rogoff (2004) results we presented earlier. An alternative transformation that preserves the VAR estimation in levels, but does not induce serial correlation, is based on the forward mean differencing (the Helmert procedure) as in Holtz-Eakin, Newey, and Rosen (1988) and Love and Zicchino (2006). We find our results to be robust to this transformation. These results are available from the authors upon request.
Figure 3: Panel VAR: Impulse responses of exchange rate to interest rate shock (1st differences)

Note: Figures present the impulse responses of the exchange rate to a positive interest rate differential shock from panel VARs estimated for developed and developing countries. Estimation uses (log) exchange rates and interest rates in first differences.

and the existing literature. This classification included the following categories: exchange rate in a given year was (i) within a moving band that is narrower than or equal to +/-2% (i.e., allows for both appreciation and depreciation over time); or (ii) was classified as managed floating; or (iii) was classified as freely floating; or (iv) was classified as freely falling. The latter “freely falling” category included countries in the following circumstances. First, it included countries that have experienced inflation rates above 40 percent over the 12 month period. Second, it included periods during the six months immediately following a currency crisis and accompanied by a regime switch from a fixed or quasi fixed regime to a managed or independently floating regime.

To verify that our results are not driven by the high-inflation countries or crisis episodes, we exclude these “freely falling” country-episodes from our benchmark sample. This leaves us with a selection of 58 country-episode pairs in total, of which 25 are developed country-episode pairs and 33 are developing country-episode pairs. Table 3 reports correlation and regression results for this modified sample. As is easy to see, all results remain practically unchanged and highly significant.

We also verify our individual country VARs and panel VARs for this restricted sample of countries and episodes. Our results change only marginally for developing countries. For instance, in bivariate VARs estimated on the levels of (log) exchange rate and interest rate differential we find that 71 percent of developing countries experienced depreciation on impact of a positive shock to the interest rate differential, 74 percent saw their currency depreciating 1 month after the shock, and exchange rate continued to depreciate 3 months after the shock in 76 percent of developing countries.\textsuperscript{18} The panel VAR results also go through unchanged.

\textsuperscript{18}Note that no industrial country is classified as “freely falling” in Reinhart and Rogoff (2004).
Table 3: Correlation between exchange rate and interest rate: No crisis or high inflation episodes

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>Developed</th>
<th>Developing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Panel A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$corr(ln E_t, i_t - i_t^{us})$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>0.15</td>
<td>-0.09</td>
<td>0.33</td>
</tr>
<tr>
<td>median</td>
<td>0.11</td>
<td>-0.08</td>
<td>0.42</td>
</tr>
<tr>
<td>$corr(\Delta_t \ln E, \Delta_t (i - i_t^{us}))$</td>
<td>0.00</td>
<td>-0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>mean</td>
<td>-0.03</td>
<td>-0.11</td>
<td>0.05</td>
</tr>
</tbody>
</table>

|                              | Panel B     |           |            |
| $\ln E_t = \beta_0 + \beta_1(i_t - i_t^{us}) + \varepsilon_t$ | 1.21        | -0.74     | 2.69       |
| mean($\hat{\beta}_1$)       |             |           |            |
| 95% c.i.($\hat{\beta}_1$)   | [1.05; 1.37] | [-0.94; -0.54] | [2.45; 2.93] |
| $\Delta_t \ln E_t = \alpha_0 + \alpha_1 \Delta_t (i_t - i_t^{us}) + u_t$ | -0.06       | -0.44     | 0.22       |
| mean($\hat{\alpha}_1$)       |             |           |            |
| 95% c.i.($\hat{\alpha}_1$)   | [-0.16; 0.04] | [-0.57; -0.31] | [0.08; 0.37] |

Note: Panel A of the Table reports the mean and median of the cross-sectional distribution of the correlation coefficient between (log) exchange rate and interest rate (and their first-differences) for our sample of countries. Panel B presents the mean of the estimated slope coefficients from the regression $\ln E_t = \beta_0 + \beta_1(i_t - i_t^{us}) + \varepsilon_t$ in levels and first-differences. 95% confidence intervals are in parenthesis.

2.3 VAR specification

One concern that may arise in the bivariate VAR specification we estimated above is related to inflation shocks. In particular, developing countries often experience higher inflation rates and may be more susceptible to inflationary shocks. Hence, their interest rate innovations might be endogenous policy responses to inflationary shocks which also tend to depreciate the currency. To account for this possibility we amend our VAR specification to include monthly CPI. There are two specifications that are popular in the literature. We examine both.

**Specification (1): Price level shocks.** First, following Eichenbaum and Evans (1995), for every country we estimate a three-variable VAR that includes its (log) consumer price index (CPI), its interest rate differential with the U.S., and its (log) exchange rate. This is the same as our benchmark specification except that we have now also included the domestic price level. To obtain orthogonalized impulse responses we use the same ordering as in Eichenbaum and Evans (1995): Price level, interest rate differential, exchange rate. The data on all three variables, however, is available only for a subsample of our countries. Thus, our subsample with CPI consists of 59 country-episode pairs, of which 17 are for developed economies and 42 are for developing countries. We find that our results remain robust under this extended model specification. As shown in panel (1) of Table 4, among developed countries, 82 percent exhibit a currency appreciation on impact after a positive interest rate innovation. In contrast, 76 percent of developing countries saw their currency depreciating on impact after a positive interest rate shock. By the end of the first month, 82 percent of developed countries saw an appreciation of their currency, while 67 percent of developing countries experienced depreciations. By the end of the third month, the corresponding numbers were 82 percent and 74 percent. When estimated
in growth rates, our VAR analysis suggests an impact exchange rate appreciation following a positive shock
to the interest rate differential in 82 percent of developed countries and an exchange rate depreciation in 73
percent of all developing countries.

**Specification (2): Inflation shocks.** Second, we try another specification that is common in the literature which
uses the inflation rate rather than the price level (see, for instance, Grilli and Roubini (1995, 1996)). Hence,
we estimate a three-variable VAR that includes the domestic CPI inflation rate differential over the U.S.
CPI inflation rate, the interest rate differential between home and the US, and the (log) exchange rate. We
obtain orthogonalized impulse responses using the ordering: Inflation rate differential, interest rate differential,
exchange rate. As can been seen in panel (2) of Table 4, under this specification, an orthogonalized interest
rate shock led to an impact appreciation of the exchange rate in 82 percent of all developed countries but a
depreciation in 67 percent of developing countries. At the end of the first and third month, the corresponding
numbers were 82 percent for developed countries and 69 percent for developing economies.\(^\text{19}\)

<table>
<thead>
<tr>
<th>Table 4: Individual country VARs: Robustness</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) With CPI level: ( \ln P; i - \ln P; US, \ln E )</td>
</tr>
<tr>
<td>Industrial countries: appreciation</td>
</tr>
<tr>
<td>Developing countries: depreciation</td>
</tr>
</tbody>
</table>

| (2) With inflation differential: \( \pi - \pi; US, i - i; US, \ln E \) |
| Industrial countries: appreciation | 82% | 82% | 82% | 82% | 82% | 52% |
| Developing countries: depreciation | 67% | 69% | 69% | 69% | 69% | 69% |

| (3) With forward inflation differential: \( \pi_{t+1} - \pi; US, i; US, \ln E_t \) |
| Industrial countries: appreciation | 82% | 82% | 82% | 82% | 82% | 82% |
| Developing countries: depreciation | 71% | 69% | 71% | 71% | 71% | 71% |

| (4) With risk-premium: \( rp; i - i; US, \ln E \) |
| Industrial countries: appreciation | 72% | 72% | 84% | 84% | 84% | 84% |
| Developing countries: depreciation | 72% | 72% | 69% | 69% | 69% | 69% |

| (5) With output: \( \ln y; i - \ln y; US, \ln P; i - \ln P; US, \ln E \) |
| Industrial countries: appreciation | 84% | 89% | 84% | 84% | 84% | 84% |
| Developing countries: depreciation | 84% | 89% | 84% | 84% | 84% | 84% |

| (6) With output, CPI and risk-premium: \( rp; \ln y, \ln P; i - \ln P; US, \ln E \) |
| Industrial countries: appreciation | 83% | 92% | 92% | 92% | 92% | 92% |
| Developing countries: depreciation | 70% | 60% | 70% | 70% | 70% | 70% |

Note: The table reports the fraction of developed (developing) countries that experience an appreciation (de-
preciation) of their exchange rate following a positive shock to the interest rate differential. Appreciations and
depreciations are defined based on the impact, 1st month and 1st quarter (3 months) impulse responses from a
country-by-country VAR analysis.

\(^\text{19}\)Note that we do not run this specification in first-differences since CPI inflation is already the first-difference of the log price level.
\textit{Specification (3): Expected inflation shocks.} Inflation may matter for the interest rate-exchange rate relationship in another important way. It may be the case that interest changes reflect endogenous policy responses to expected inflationary shocks. To account for this possibility, we estimate another modified VAR in which we include the one month ahead inflation differential between home and the U.S. We order variables as follows: Forward CPI inflation differential, interest rate differential, exchange rate. The results are presented in Panel (3) and confirm that our earlier findings remain unchanged for developed countries and in fact become stronger for developing countries. Clearly, factors orthogonal to expected inflation shocks are important and key to understanding the different responses of the exchange rate in the two groups of countries.

\textit{Specification (4): Risk premium shocks.} Another potential concern is that the joint dynamics of exchange rates and interest rates are driven by the changes in country risk-premiums. For instance, if the country risk-premium rises, its currency may depreciate. At the same time, its Central Bank may be compelled to raise domestic interest rates to counterweight the effect of the rising risk-premium. To account for such a possibility, we control for the risk-premium in the country VARs. Unfortunately, country-specific measures of risk-premium are available only for a very small group of developing countries. Instead we proxy developing country risk premia with junk bond spreads that are known to be highly correlated with the sovereign bond spreads – a standard measure of the country risk-premium (see Uribe and Yue (2006)). More precisely, we use Moody’s Seasoned Baa Corporate Bond Yield spread over the U.S. T-bill rate as a measure of risk-premium.\footnote{Blanchard (2004) also uses Baa spread and shows that it is a good instrument for risk-premium in Brazil.} We re-run our VARs and obtain orthogonalized impulse responses using the ordering: Risk-premium, interest rate differential, exchange rate. The proportions of appreciating developed countries and depreciating developing countries are reported in panel (4) of Table 4. As is easy to see, an overwhelming majority of all industrial countries in our sample still see their exchange rate appreciating following shocks to the interest rate, even after controlling for changes in the risk-premium. In contrast, in the majority of developing countries in our sample, exchange rates depreciate in response to interest rate shocks. We also use the Merryl Lynch High Yield Master II bond yield spreads to measure the risk-premium and find that the results remain robust.\footnote{We prefer to report the results for the BAA spread because it is available for a longer period thus allowing us to estimate VARs for a larger sample of countries (79 country-episode pairs, as opposed to 44 country-episode pairs if we use high-yield spread instead).} Both series are available from the Board of Governors of the Federal Reserve System.

\textit{Specification (5): Taylor rules.} Our policy-controlled interest rates may also be driven by endogenous policy responses to changes in domestic business cycles due to a Taylor rule. To account for this possibility we include industrial production (index, 2000 base year) in our baseline VAR specification. This would be our preferred VAR specification, but industrial production data is only available for less than a half (30 country-pairs to be precise, of which 19 belong to developed countries and 11 belong to developing countries) of all country-episode pairs in our sample.\footnote{The ordering follows Eichenbaum and Evans (1995), where industrial production appears first, followed by interest rate differential and (log) exchange rate.} The results for this subsample are reported in Panel (5) of Table 4. Importantly, our results are confirmed again: on impact, 84 percent of developed countries showed appreciation after an orthogonalized shock to interest rate, while developing countries showed depreciation in 64 percent of...
all cases. One month later 89 percent industrial countries currencies continued to appreciate, while 73 percent of developing countries saw depreciation. Three months later the proportions were 84 percent and 64 percent, respectively.

*Specification (6): All shocks.* Finally, we estimate an extended VAR specification, where we include CPI level, industrial production and risk-premium into the benchmark specification. We assume the following ordering for the variables: risk-premium, industrial production, price level, interest rate differential, exchange rate. This identification strategy implies that innovations to interest rates have effects on domestic real activity, the price level and the risk-premium with a one-period lag, but, as before, can affect exchange rates contemporaneously. This identification scheme also implies that shocks to output, prices and the risk-premium can affect domestic interest rates contemporaneously. This ordering reflects the standard assumption in the literature that macroeconomic variables react to monetary policy shocks with a lag, while monetary policy can respond to macroeconomic shocks immediately. A similar structure is assumed for the relationship between the exchange rate and macroeconomic variables: exchange rate can respond immediately to all shocks, but its effect on macroeconomic variables percolates only with a lag. The ordering of the first three variables assumes that risk-premium shocks are the most exogenous.\(^{23}\) The assumption that output shocks affect prices immediately is standard in the literature (see, for instance, Bernanke and Blinder (1992)).

Due to limited data availability, this extended VAR can only be estimated for 22 country-pairs, of which 12 are industrial country-pairs and 10 are developing country-pairs. The results for this VAR specification are presented in Panel (6). A shock to interest rate that is orthogonal to domestic output, the price level, and risk-premium, leads to currency appreciation on impact in 83 percent of developed countries, the share increases to 92 percent of all developed countries after one month, and 92 percent see their currency appreciating three months following the shock. The corresponding numbers for developing countries are 70 percent, on impact, 60 percent after 1 month and 70 percent after 3 months.

### 2.4 Structural VAR

In our VAR analysis so far we obtained identification of interest rate shocks by placing zero contemporaneous restrictions on the interaction between interest rates and the exchange rate. This assumption, while standard in the literature, rules out potential simultaneity effects between interest rates and exchange rates in identifying interest rate shocks. However, contemporaneous feedback between interest rates and exchange rates may be important in developing countries, as emphasized in the “fear of floating” literature (see Calvo and Reinhart (2002)). This literature documents the tendency of monetary authorities, especially in developing countries, to respond to fluctuations in the exchange rate. Furthermore, since the exchange rate is a forward looking variable, it may contain information about the future prospects of the economy to which the monetary authority may want to react. Both these concerns raise the question of whether our results are sensitive to the identifying restrictions we used?

\(^{23}\)We also try an alternative ordering where risk-premium variable is placed after output and price level, and find that results remain unchanged.
To address this question we estimate a structural VAR (SVAR) in which we allow for a contemporaneous correlation between the interest rate and the exchange rate. Identification is obtained by imposing a long-run restriction that interest rates have no long-run effects on the real exchange rate. This is a standard neutrality assumption that holds in a number of theoretical monetary models (see Clarida and Gali (1994)) and has been recently used in several empirical studies (see Bjørnland (2009)). Thus, we estimate a structural VAR containing interest rate differential, \( i - i^{US} \), and the first difference of the log real exchange rate \( \Delta \lnrer \), imposing the long-run neutrality restriction described above. We find that our results remain largely unchanged. Based on structural impulse responses, we find that 73 percent of all developing countries in our sample experienced impact depreciations following an interest rate shock, 69 percent depreciated 1 month after the shock while 55 percent experienced depreciations 3 months after the shock. We do not interpret these results as necessarily suggesting that the contemporaneous feedback between interest rates and exchange rate is not important. Rather, we think that the exchange rate classification scheme of Reinhart and Rogoff (2004) that we used to identify flexible exchange rate countries, by being based on the de-facto exchange rate regime, allowed us to focus on the countries and episodes for which “fear of floating” was less of a concern.

Overall, based on the variety of samples and the battery of approaches, the evidence suggests that interest rates and exchange rates are negatively related in industrial countries, consistent with the existing theories. However, the relationship between the two variables is reversed for developing countries, thus challenging the existing theory. We will refer to these contrasting findings in industrial versus developing countries as the “exchange rate response puzzle.” In the next section we show that a simple modification of the existing theoretical frameworks can rationalize the puzzle.

### 3 The model

Consider a model of a small open economy that is perfectly integrated with the rest of the world in both goods and capital markets. It is populated by four types of agents: households, firms, banks and the government. The infinitely-lived representative household receives utility from consuming a (non-storable) good and disutility from supplying labor. The world price of the good in terms of foreign currency is fixed and normalized to unity. Free goods mobility across borders implies that the law of one price applies. The representative firm combines capital and labor to produce final goods, and is subject to a working capital requirement. As a result, it must borrow from the banks. The representative bank acts as an intermediary between households and firms, but also lends to the government. The latter is comprised of a fiscal and monetary authority. We describe the problem of each agent in details next.

---

24 This restriction is also satisfied by the model we develop in this paper.
25 We estimate the SVAR on the first differenced (log) real exchange rate so that, in the spirit of Blanchard and Quah (1989), the effects of the interest rate shock on the level of the exchange rate add up to zero. See Appendix A.2 for econometric details.
3.1 Households

Household’s lifetime welfare is given by

\[ V = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, x_t), \tag{3.1} \]

where \( c \) denotes consumption, \( x \) denotes labor supply, and \( \beta > 0 \) is the exogenous and constant rate of time preference. \( \mathbb{E} \) denotes expectations. We assume that the period utility function of the representative household is given by

\[ U(c, x) = \frac{1}{1-\sigma} (c - \zeta x^\nu)^{1-\sigma}, \quad \zeta > 0, \quad \nu > 1. \]

Here \( \sigma \) is the intertemporal elasticity of substitution, \( \nu - 1 \) is the inverse of the elasticity of labor supply with respect to the real wage. These preferences are well-known from the work of Greenwood, Hercowitz, and Huffman (1988), which we will refer to as GHH.\textsuperscript{26}

Households use cash, \( H \), and nominal demand deposits, \( D \), for reducing transactions costs. Specifically, the transactions costs technology is given by

\[ s_t = v \left( \frac{H_t}{P_t} \right) + \psi \left( \frac{D_t}{P_t} \right), \tag{3.2} \]

where \( P \) is the nominal price of goods in the economy, and \( s \) denotes the non-negative transactions costs incurred by the consumer. Let \( h (= H/P) \) denote cash and let \( d (= D/P) \) denote interest-bearing demand deposits in real terms. We assume that the transactions technology is strictly convex. In particular, the functions \( v(h) \) and \( \psi(d) \), defined for \( h \in [0, \bar{h}] \), \( \bar{h} > 0 \), and \( d \in [0, \bar{d}] \), \( \bar{d} > 0 \), respectively, satisfy the following properties:

\[ v \geq 0, \quad v' \leq 0, \quad v'' > 0, \quad v'(\bar{h}) = v(\bar{h}) = 0, \]
\[ \psi \geq 0, \quad \psi' \leq 0, \quad \psi'' > 0, \quad \psi'(\bar{d}) = \psi(\bar{d}) = 0. \]

Thus, additional cash and demand deposits lower transactions costs but at a decreasing rate. The assumption that \( v'(\bar{h}) = \psi'(\bar{d}) = 0 \) ensures that the consumer can be satiated with real money balances.

In addition to the two liquid assets, households also hold a real internationally-traded bond, \( b \), and physical capital, \( k \), which they can rent out to firms. The households flow budget constraint in nominal terms is

\[ P_t b_{t+1} + D_t + H_t + P_t (c_t + I_t + s_t + \kappa_t) = P_t \left( Rb_t + w_t x_t + \rho_t k_{t-1} + \tau_t + \Omega^I_t + \Omega^b_t \right) + (1 + i_t^d) D_{t-1} + H_{t-1}. \]

\textsuperscript{26}These preferences have been widely used in the real business cycle literature as they provide a better description of consumption and the trade balance for small open economies than alternative specifications (see, for instance, Correia, Neves, and Rebelo (1995)). The key analytical simplification introduced by GHH preferences is that there is no wealth effect on labor supply.
Foreign bonds are denominated in terms of the good and pay the gross interest factor \( R(= 1 + r) \), which is constant over time. \( i^d_t \) denotes the deposit rate contracted in period \( t - 1 \) and paid in period \( t \). \( w \) and \( p \) denote the wage and rental rates. \( \tau \) denotes lump-sum transfers received from the government. \( \Omega^f \) and \( \Omega^b \) represent dividends from firms and banks respectively. \( \kappa \) denotes capital adjustment costs

\[
\kappa_t = \kappa (I_t, k_{t-1}) \text{, } \kappa_I > 0, \kappa_{II} > 0, \quad (3.3)
\]

e.g., adjustment costs are convex in investment. Lastly,

\[
I_t = k_t - (1 - \delta) k_{t-1} \text{.} \quad (3.4)
\]

In real terms the flow budget constraint facing the representative household is thus given by

\[
\begin{align*}
&b_{t+1} + h_t + d_t + c_t + I_t + s_t + \kappa_t \\
&= Rb_t + w_t x_t + \rho_t k_{t-1} + \frac{h_{t-1}}{1 + \pi_t} + \left( \frac{1 + i^d_t}{1 + \pi_t} \right) d_{t-1} + \Omega^f_t + \Omega^b_t,
\end{align*}
\]

where \( \Omega^f \) and \( \Omega^b \) denote dividends received by households from firms and banks, respectively. \( 1 + \pi_t = \frac{P_t}{P_{t-1}} \) denotes the gross rate of inflation between periods \( t - 1 \) and \( t \). We define the nominal interest rate as

\[
1 + i_{t+1} = RE_t \left( 1 + \pi_{t+1} \right). \quad (3.6)
\]

Households maximize their lifetime welfare equation (3.1) subject to equations (3.2), (3.3), (3.4) and (3.5).

### 3.2 Firms

The representative firm in this economy produces the perishable good using a constant returns to scale technology over capital and labor

\[
y_t = F(k_{t-1}, A_t l_t) = A_t k_{t-1}^{\alpha} l_t^{1-\alpha}, \quad (3.7)
\]

with \( \alpha > 0 \), and \( A_t \) denoting the current state of productivity which is stochastic. \( l \) is labor demand. At the beginning of the period, firms observe shocks for the period and then make production plans. They rent capital and labor. However, a fraction \( \phi \) of the total wage bill needs to be paid upfront to workers. Since output is only realized at the end of the period, firms finance this payment through loans from banks. The loan amount along with the interest is paid back to banks next period.\(^{27}\) Formally, this constraint is given by

\[
N_t = \phi P_t w_t l_t, \quad \phi > 0, \quad (3.8)
\]

\(^{27}\)Alternatively, we could assume that bank credit is an input in the production function, in which case the derived demand for credit would be interest rate elastic. This would considerably complicate the model without adding any additional insights.
where $N$ denotes the nominal value of bank loans. The assumption that firms must use bank credit to pay the wage bill is needed to generate a demand for bank loans.

The firm’s flow constraint in nominal terms is given by

$$P_t b^f_{t+1} - N_t = P_t \left( R b^f_t + y_t - w_t l_t - \rho_t k_{t-1} - \Omega^f_t \right) - (1 + i^*_t) N_{t-1},$$

where $i^*$ is the lending rate charged by bank for their loans and $\Omega^f$ denotes dividends paid out by the firms to their shareholders. $b^f_t$ denotes foreign bonds held by firms which pay the going world interest factor $R$. In real terms the flow constraint reduces to

$$b^f_{t+1} - n_t = R b^f_t - \left( \frac{1 + i^*_t}{1 + \pi_t} \right) n_{t-1} + y_t - w_t l_t - \rho_t k_{t-1} - \Omega^f_t.$$

Define

$$a^f_{t+1} \equiv b^f_{t+1} - \frac{(1 + i^*_t)}{R(1 + \pi_t)} n_t.$$

Substituting this expression together with the credit-in-advance constraint into the firm’s flow constraint in real terms gives

$$a^f_{t+1} + \Omega^f_t = R a^f_t + y_t - \rho_t k_{t-1} - w_t l_t \left[ 1 + \phi \left\{ \frac{1 + i^*_t + R(1 + \pi_t)}{R(1 + \pi_t)} \right\} \right].$$

(3.9)

Note that $\phi \left\{ \frac{1 + i^*_t + R(1 + \pi_t)}{R(1 + \pi_t)} \right\} w_t l_t = \left\{ \frac{1 + i^*_t + R(1 + \pi_t)}{R(1 + \pi_t)} \right\} n_t$ is the additional resource cost that is incurred by firms due to the credit-in-advance constraint.$^{28}$

The firm chooses a path of $l$ and $k$ to maximize the present discounted value of dividends subject to equations (3.7), (3.8) and (3.9). Given that households own the firms, this formulation is equivalent to the firm using the household’s stochastic discount factor to optimize. The first order conditions for this problem are given by two usual conditions and an Euler equation which is identical to the household’s Euler equation. The two usual conditions are standard – the firm equates the marginal product of the factor to its marginal cost. In the case of labor the cost includes the cost of credit. This is proportional to the difference between the nominal lending rate and the nominal interest rate.

### 3.3 Banks

The banking sector is assumed to be perfectly competitive. The representative bank holds foreign real debt, $d^b$, accepts deposits from consumers and lends to both firms, $N$, and the government in the form of domestic government bonds, $Z$.^{29} It also holds required cash reserves, $\theta D$, where $\theta > 0$ is the reserve-requirement ratio

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$^{28}$We should note that the credit-in-advance constraint given by equation (3.8) holds as an equality only along paths where the lending spread $1 + i^* - R(1 + \pi)$ is strictly positive. We will assume that if the lending spread is zero, this constraint also holds with equality.

$^{29}$Commercial bank lending to governments is particularly common in developing countries. Government debt is held not only as compulsory (and renumerated) reserve requirements but also voluntarily due to the lack of profitable investment opportunities in crisis-prone countries. This phenomenon was so pervasive in some Latin American countries during the 1980’s that Rodriguez
imposed on the representative bank by the central bank. Banks face a cost $q$ (in real terms) of managing their portfolio of foreign assets. Moreover, we assume that banks also face a constant proportional cost $\phi^n$ per unit of loans to firms. This is intended to capture the fact that domestic loans to private firms are potentially special as banks need to spend additional resources in monitoring loans to private firms.\textsuperscript{30} The nominal flow constraint for the bank is

$$N_t + Z_t - (1 - \theta) D_t + P_t q_t - P_t d^b_{t+1} = (1 + i^t_l - \phi^n) N_{t-1} + (1 + i^g_t) Z_{t-1} - (1 + i^d_t) D_{t-1} + \theta D_{t-1} - P_t R d^b_t - P_t \Omega^b_t,$$

(3.10)

where $i^g$ is the interest rate on government bonds. We assume that banking costs are a convex function of the foreign debt held by the bank:

$$q_t = q (d^b_t + 1), \quad q' > 0, \quad q'' > 0.$$

The costly banking assumption is needed to break the interest parity condition between domestic and foreign bonds. Throughout the paper we assume that the banking cost technology is given by the quadratic function:

$$q_t = \frac{\gamma}{2} (d^b_{t+1} - \bar{d}^b)^2,$$

(3.11)

where $\gamma > 0$ and $\bar{d}^b$ are constant parameters.\textsuperscript{31}

Deflating the nominal flow constraint by the price level gives the bank’s flow constraint in real terms:

$$\Omega^b_t = \left[ \frac{R(1 + \pi_t) - 1}{1 + \pi_t} \right] [(1 - \theta) d_{t-1} - n_{t-1} - z_{t-1}] + \frac{i^f_t - \phi^n}{1 + \pi_t} n_{t-1} + \frac{i^g_t}{1 + \pi_t} z_{t-1} - \frac{i^d_t}{1 + \pi_t} d_{t-1} - q_t,$$

(3.12)

where we have used the bank’s balance sheet identity: $P_t d^b_{t+1} = N_t + Z_t - (1 - \theta) D_t$. Note that this is equivalent to setting the bank’s net worth to zero at all times. Also, the quadratic specification for banking costs along with the zero net worth assumption implies that these banking costs can also be reinterpreted as a cost of managing the portfolio of net domestic assets since $d^b_{t+1} = \frac{N_t + Z_t - (1 - \theta) D_t}{1 + \pi_t}$.

The representative bank chooses sequences of $N$, $Z$, and $D$ to maximize the present discounted value of profits subject to equations (3.10) taking as given the paths for interest rates $i^l$, $i^d$, $i^g$, $i$, and the value of $\theta$ and $\phi^n$. We assume that the bank uses the household’s stochastic discount factor to value its profits. Note that $i^f_{t+1}$, $i^d_{t+1}$ and $i^d_{t+1}$ are all part of the information set of the household at time $t$.

The bank optimality conditions imply that we must have

$$i^f_{t+1} = i^g_{t+1} + \phi^n,$$

(3.13)

\textsuperscript{(91)} aptly refers to such governments as “borrowers of first resort”. For evidence, see Rodriguez (1991) and Druck and Garibaldi (2000).

\textsuperscript{30}We should note that this cost $\phi^n$ is needed solely for numerical reasons since, as will become clear below, it gives us a bigger range of policy-controlled interest rates to experiment with. Qualitatively, all our results would go through with $\phi^n = 0$.

\textsuperscript{31}Similar treatment of banking costs of managing assets and liabilities can be found in Díaz-Giménez, Prescott, Fitzgerald, and Álvarez (1992) and Edwards and Vegh (1997). This approach to breaking the interest parity condition is similar in spirit to Calvo and Vegh (1995).
\[ i^d_{t+1} = (1 - \theta) i^g_{t+1}. \quad (3.14) \]

These conditions are intuitive. Loans to firms and loans to the government are perfect substitutes from the perspective of commercial banks up to the constant extra marginal cost \( \phi^n \) of monitoring loans to private firms. Hence, equation (3.13) says that the interest rate charged by banks on private loans should equal the rate on loans to the government plus \( \phi^n \). For every unit of deposits held the representative bank has to pay \( i^d \) as interest. The bank can earn \( i^g \) by lending out the deposit. However, it has to retain a fraction \( \theta \) of deposits as required reserves. Hence, equation (3.14) shows that at an optimum the deposit rate must equal the interest on government bonds net of the resource cost of holding required reserves.

It is instructive to note that as the marginal banking costs becomes larger the bank will choose to keep its holdings of foreign assets closer to \( d_b \). This can be checked from the bank first order conditions; all of them imply that \( \lim_{x \to \infty} a^d_{t+1} = d_b \). Hence, in the limit as banking costs becomes prohibitively large, the bank will choose to maintain a constant portfolio of external assets or liabilities.

### 3.4 Government

The government issues high powered money, \( M \), and domestic bonds, \( Z \), makes lump-sum transfers, \( \tau \), to the public, and sets the reserve requirement ratio, \( \theta \), on deposits. Domestic bonds are interest bearing and pay \( i^g \) per unit. Since we are focusing on flexible exchange rates, we assume with no loss of generality that the central bank’s holdings of international reserves are zero. We assume that the government’s transfers to the private sector are fixed exogenously at \( \bar{\tau} \) for all \( t \). Hence, the consolidated government’s nominal flow constraint is

\[ P_t \bar{\tau} + (1 + i^g_t) Z_{t-1} = M_t - M_{t-1} + Z_t. \]

As indicated by the left-hand-side of this expression, total expenditures consist of lump-sum transfers, debt redemption and debt service. These expenditures may be financed by issuing either high powered money or bonds. In real terms the government’s flow constraint reduces to

\[ \bar{\tau} + \frac{1 + i^g_t}{1 + \pi_t} Z_{t-1} = m_t + z_t - \frac{1}{1 + \pi_t} m_{t-1}. \quad (3.15) \]

Lastly, the rate of growth of the nominal money supply is given by:

\[ \frac{M_{t+1}}{M_t} = 1 + \mu_{t+1}, \quad M_0 \text{ given.} \quad (3.16) \]

It is worth noting that from the central bank’s balance sheet the money base in the economy is given by

\[ M_t = H_t + \theta D_t. \]

Hence, \( M \) can also be interpreted as the level of nominal domestic credit in the economy.
The consolidated government (both the fiscal and monetary authorities) has three policy instruments: (a) monetary policy which entails setting the rate of growth of nominal money supply; (b) interest rate policy which involves setting $i^g$ (or alternatively, setting the composition of $m$ and $z$ and letting $i^g$ be market determined); and (c) the level of lump sum transfers to the private sector $\tau$. Given that lump-sum transfers are exogenously given, only one of the other two instruments can be chosen freely while the second gets determined through the government’s flow constraint (equation (3.15)). Since the focus of this paper is on the effects of interest rate policy, we shall assume throughout that $i^g$ is an actively chosen policy instrument. This implies that the rate of money growth $\mu$ adjusts endogenously so that equation (3.15) is satisfied.

### 3.5 Resource constraint

By combining the flow constraints for the consumer, the firm, the bank, and the government (equations (3.5), (3.9), (3.12) and (3.15)) and using equations (3.7) and (3.8), we get the economy’s flow resource constraint:

$$a_{t+1} = Ra_t + y_t - c_t - I_t - \kappa_t - s_t - q_t,$$

where $a = b + b^d - d^d$. Note that the right hand side of equation (3.17) is simply the current account.

### 3.6 Equilibrium relations

We start by defining an equilibrium for this model economy. The three exogenous variables in the economy are the productivity process $A$ and the two policy variables $\tau$ and $i^g$. We denote the entire state history of the economy till date $t$ by $s^t = (s_0, s_1, s_2, ..., s_t)$. An equilibrium for this economy is defined as:

Given a sequence of realizations $A(s^t), i^g(s^t), \tau$ and $i^g$, an equilibrium is a sequence of state contingent allocations $\{c(s^t), x(s^t), l(s^t), h(s^t), d(s^t), k(s^t), b(s^t), b^d(s^t), d^d(s^t), n(s^t), z(s^t)\}$ and prices $\{P(s^t), \pi(s^t), i(s^t), i^d(s^t), i^l(s^t), w(s^t), \rho(s^t)\}$ such that (a) at the prices the allocations solve the problems faced by households, firms and banks; (b) factor markets clear; and (c) the government budget constraint (equation (3.15)) is satisfied.

Combining the government flow constraint with the central and commercial bank balance sheets yields the combined government flow constraint:

$$\bar{\tau} = h_t - \left(\frac{1}{1 + \pi_t}\right) h_{t-1} + \theta \left( d_t - \frac{d_{t-1}}{1 + \pi_t} \right) + z_t - \left(\frac{1 + \nu^2_t}{1 + \pi_t}\right) z_{t-1}.$$  

(3.18)

It is useful at this stage to clarify the process of nominal exchange rate determination in this model. Let $m = M/E$ be real money while nominal money is denoted by $M = H + \theta D$. Since $h_t$ and $d_t$ are functions of $i_{t+1}$ and $i^d_{t+1} - i^d_{t+1}$ respectively, the money market equilibrium condition can be written implicitly as $h_t + \theta d_t = L(i_{t+1}, i^d_{t+1})$ where $L$ denotes the implicit aggregate demand for cash and deposits. Note that in writing the implicit $L$ function we have used the fact that $i^d$ is linked one-for-one with $i^g$. At any date $t$, $M_t$ is known while its growth rate $\mu_{t+1}$ is endogenous. Money market equilibrium then dictates that at date $t$ the
nominal exchange rate is given by

\[ E_t = \frac{M_t}{L(i_{t+1}, i^g_{t+1})}. \]  

(3.19)

Recalling that \( P_t = E_t \) and \( 1 + \pi_t = \frac{P_t}{P_{t-1}} \), one can substitute equation (3.19) into (3.18) to solve out for \( 1 + \pi_t \) in terms of nominal money supply and real money demand at date \( t \). For any given policy rate \( i^g_{t+1} \), the nominal interest rate \( i_{t+1} \) (and hence the expected inflation rate between \( t \) and \( t + 1 \)) is then determined from the government budget constraint (3.18). From equation (3.19), knowledge of \( i^g_{t+1} \) and \( i_{t+1} \) are sufficient to determine the nominal exchange rate \( E_t \) at that date for a given \( M_t \). Note that the rate of nominal money growth \( \mu \) between dates \( t \) and \( t + 1 \) also gets determined at date from equation (3.15). Hence, \( M_{t+1} \) gets determined at date \( t \).

From equations (3.18) and (3.19) it is easy to see that the effect of an interest rate increase on the equilibrium nominal exchange rate depends not just on monetary conditions but also on the real side of the economy as well as the state of public finances. Interest rate changes impact these fundamentals in often opposing ways. This is likely to make their end effect on the exchange rate non-linear and possibly non-monotonic. We explore these possibilities quantitatively below.

4 Calibration

Our next point of interest is whether this model can generate the difference in exchange rate behavior between developed and developing countries that we saw in the data. In order to examine this, we conduct policy experiments on a calibrated version of the model developed above. We proceed by choosing two different sets of parameterizations for the calibrated model – one for developed and another for developing countries. We then examine whether the response of the exchange rate to domestic interest rate shocks can reproduce the documented differences between developed and developing countries.

Our basic approach is to keep the majority of the parameters of the model common to both sets of countries. The parameters that we calibrate separately for developed and developing countries are those that control the three key features that we have introduced in the model: the money demand effect, the fiscal effect and the output effect. By restricting the differences between the two groups of countries, we feel that this approach allows us to better ascertain the quantitative power of the margins we have introduced in the model. Clearly, the more parameters we calibrate separately for the two groups the greater our ability to explain differences in the data patterns since developed and developing countries differ along many more margins than the three that we have chosen to focus on here.

We calibrate the model to match the properties of the two groups of countries. The benchmark parameterization for the developed countries group utilizes data for 6 industrial economies – Australia, Canada,
Netherlands, New Zealand, Sweden and UK – during the period 1974-2010. For developing countries we use the data for Argentina, Brazil, Korea, Mexico, Philippines, and Thailand for the same 1974-2010 period. When focusing on nominal variables, i.e. nominal interest rates, we restrict the sample to the 1998-2010 period to eliminate the periods of high interest rate volatility and high inflation in developing countries before and during the East Asian crisis. Detailed data description and data sources are discussed in the Appendix A.3. The model calibration is such that one period in the model corresponds to one quarter.

4.1 Functional forms and parameters

We assume that the capital adjustment cost technology is given by

\[ \kappa(I_t, k_{t-1}) = \frac{\xi}{2} k_{t-1} \left( \frac{I_t - \delta k_{t-1}}{k_{t-1}} \right)^2, \quad \xi > 0, \]

with \( \xi \) being the level parameter.

As in Rebelo and Vegh (1995), we assume that the transactions costs functions \( u(.) \) and \( \psi(.) \) have quadratic forms given by

\[ s_{\kappa} \left( \kappa^2 - \lambda_{\kappa} \kappa + \left( \frac{\lambda_{\kappa}}{2} \right)^2 \right), \tag{4.20} \]

where \( \kappa \) represents cash or demand deposits, \( \kappa = \{ h, d \} \), while \( s_{\kappa} \) and \( \lambda_{\kappa} \) are the level parameters. This formulation implies that the demand for money components are finite and that transaction costs are zero when the nominal interest rate is zero.

The transaction technology for the banks is given by a quadratic function

\[ q_t = \frac{\gamma}{2} \left( d_{t+1}^b - \bar{d}^b \right)^2, \]

where \( d_{t+1}^b = \frac{N_t + Z_t - (1 - \rho)D_t}{P_t} \). Here \( \gamma \) is a constant and \( \bar{d}^b \) is a steady state level of banks’ debt to GDP ratio.

We begin by discussing parameters that are set to be common to both developed and developing countries. Most of these parameter values are borrowed from Neumeyer and Perri (2005) and Mendoza (1991).\(^{33}\) In particular, we set the coefficient of relative risk aversion, \( \sigma \), to 5, while the curvature of the labor, \( \psi \), is set to 1.6. This value is within the range of values used in the literature.\(^{34}\) This implies the elasticity of labor demand with respect to real wage, \( \frac{1}{\psi - 1} \), equal to 1.67, consistent with the estimates for the U.S. Labor weight parameter \( \zeta \) in the utility function is chosen to match the average working time of 1/5 of total time and is set to 2.48. Subjective discount factor, \( \beta \), is set to 0.97, as in Uribe and Yue (2006). Capital income share, \( \alpha \), is chosen to be equal to 0.38, while a depreciation rate for capital, \( \delta \), of 4.4% per quarter. Capital adjustment costs parameter \( \xi \) is calibrated to replicate the volatility of investment relative to the volatility of output in our sample. Table 5 summarizes calibration for parameters that are common across the two groups of countries.

\(^{33}\) Note that all of these common parameters are almost the same in the two papers – Neumeyer and Perri (2005) who focus on Argentina and in Mendoza (1991) who calibrates to Canada.

\(^{34}\) For example, Mendoza (1991) uses \( \psi \) equal to 1.455 for Canada, while Correia, Neves, and Rebelo (1995) set \( \psi \) to 1.7 for Portugal.
Table 5: Benchmark parameter values: Common across countries

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>discount factor</td>
<td>$\beta$</td>
<td>0.971</td>
</tr>
<tr>
<td>risk-aversion</td>
<td>$\sigma$</td>
<td>5</td>
</tr>
<tr>
<td>labor curvature</td>
<td>$\nu$</td>
<td>1.6</td>
</tr>
<tr>
<td>labor weight</td>
<td>$\zeta$</td>
<td>2.48</td>
</tr>
<tr>
<td>TECHNOLOGY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>capital income share</td>
<td>$\alpha$</td>
<td>0.38</td>
</tr>
<tr>
<td>depreciation rate</td>
<td>$\delta$</td>
<td>0.044</td>
</tr>
<tr>
<td>share of wage-in-advance</td>
<td>$\phi$</td>
<td>0.15</td>
</tr>
<tr>
<td>capital adjustment costs</td>
<td>$\xi$</td>
<td>4.5</td>
</tr>
<tr>
<td>banks cost technology</td>
<td>$\gamma$</td>
<td>100</td>
</tr>
</tbody>
</table>

The remaining parameters are calibrated to developed and developing countries separately using the sample of 6 developed and 6 developing countries discussed above. Table 6 summarizes targeted data moments and values for parameters in the two groups of countries that minimize the distance between these moments in the data and in the model.

Table 6: Benchmark targets and parameter values: Country-specific

<table>
<thead>
<tr>
<th>DATA TARGETS</th>
<th>DEVELOPED</th>
<th>DEVELOPING</th>
</tr>
</thead>
<tbody>
<tr>
<td>reserve requirement ratio</td>
<td>0.03</td>
<td>0.10</td>
</tr>
<tr>
<td>M1/GDP</td>
<td>0.20</td>
<td>0.10</td>
</tr>
<tr>
<td>Deposits/Cash</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Cash interest elasticity</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>Deposits interest elasticity</td>
<td>-0.04</td>
<td>-0.04</td>
</tr>
<tr>
<td>Net lending/borrowing by the general government/GDP</td>
<td>-0.013</td>
<td>-0.021</td>
</tr>
<tr>
<td>Spread of nominal lending rate over money market rate</td>
<td>0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>Net foreign assets/GDP</td>
<td>-0.26</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MODEL PARAMETERS</th>
<th>DEVELOPED</th>
<th>DEVELOPING</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ : reserve requirement</td>
<td>0.03</td>
<td>0.10</td>
</tr>
<tr>
<td>$s_{\kappa, \varpi} = {h, d}$ : transaction cost</td>
<td>$s_h =24.55$, $s_d =0.097$</td>
<td>$s_h =100$, $s_d =4.8$</td>
</tr>
<tr>
<td>$\lambda_{\kappa, \varpi} = {h, d}$ : transaction cost</td>
<td>$\lambda_h =0.244$, $\lambda_d =1.303$</td>
<td>$\lambda_h =0.125$, $\lambda_d =0.138$</td>
</tr>
<tr>
<td>$\tau$ : lump-sum transfers</td>
<td>1.3% of GDP</td>
<td>2.1% of GDP</td>
</tr>
<tr>
<td>$\delta^n$ : per unit loans costs</td>
<td>0.05</td>
<td>0.09</td>
</tr>
<tr>
<td>$d^b$ : debt to GDP ratio</td>
<td>-0.26</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

Parameter $\theta$ determines the reserve requirement ratio in the model and is calibrated to match the observed reserve requirements in each group of countries. We measure reserve requirements in the data following Brock (1989), who computes reserve requirements as the ratio of monetary base less currency outside banks to M2 less currency outside banks. This gives us $\theta$ equal to 0.03 in developed countries and 0.10 in developing economies over our sample period. Reserve requirement ratio $\theta$, together with $s_{\kappa}$ and $\lambda_{\kappa, \varpi} = \{h, d\}$ parameters in the transactions costs technology for banks, jointly determine the level of money demand in the model. We calibrate them to match several targets.

First, we match the average ratios of M1 to GDP in the data equal to 20% in developed countries and
10\% in developing economies. Second, we match the relative size of deposits to currency in circulation in the data equal to 1 in developing countries and 4 in industrial economies. Third, since estimates for the interest elasticities of deposits and cash are not separately available, we discipline our calibration by picking parameters such that these elasticities are equalized within each group of countries and across the two groups of countries in the steady state. Furthermore, the parameters are picked to match interest elasticity of money demand equal to \(-0.04\) annually – a value estimated by Mulligan and Sala-I-Martin (1992) for the U.S. and is in line with the estimate obtained by Ball (2001). This value is also in the midrange of estimates reported in the literature for various countries, time-periods, and methodologies (see Kumar, Chowdhury, and Rao (2010) for a recent overview and summary of the estimates of interest rate elasticities of money demand).\textsuperscript{35}

The lump-sum transfers paid by the government to the private sector, \(\tau\), are measured as the net lending/borrowing by the general government as a share of GDP. Over our sample period, this ratio is equal to \(-1.3\%\) in developed economies, and \(-2.1\%\) in developing countries.

It is important to note that the differences in the moments between developed and developing countries that we relied on so far to calibrate our key parameters are systematic. For instance, all developing countries in our calibration sample have higher reserve requirement ratios, significantly lower ratio of M1 to GDP and ratio of deposits to cash, as well as larger negative fiscal imbalances.\textsuperscript{36} Based on these evidence, we believe that our key parameters are correlated with the level of income and can be used to distinguish developed and developing countries.

The share of wage bill paid in advance, \(\phi\), is a difficult parameter to calibrate. Most of the existing studies that incorporate such working capital constraints focus on industrial economies, and typically assume that firms must borrow the entire wage bill in advance (see Christiano, Eichenbaum, and Evans (2005), Altig, Christiano, Eichenbaum, and Linde (2011)). Schmitt-Grohé and Uribe (2006) deviate from this practice and calibrate the share of the wage bill paid in advance to match the average money-to-output ratio in the post-war US data. Their calibration implies that only 51\% of wage payments must be held in money. Rabanal (2007), whose main goal is to assess the importance of the cost channel in monetary policy, estimates the wage-in-advance parameter in the U.S. equal to 0.15. For developing countries, Neumeyer and Perri (2005) assume \(\phi\) equal to 1, while Uribe and Yue (2006), find that a value of \(\phi\) greater than 1 is needed to match the empirical impulse responses of several macroeconomic aggregates with their counterparts in their model. Given the great uncertainty in the literature associated with this parameter, we proceed as follows. We use the value for \(\phi\) equal to 0.15, as estimated by Rabanal (2007), and we fix this value to be the same for both developed and developing countries. We then investigate the sensitivity of our quantitative results with respect to this parameter. As we will argue later, this parameter determines the strength of the “output” effect in the model, which works to depreciate the exchange rate following rises in \(i^g\). By requiring \(\phi\) to be the same in developed and developing countries under our benchmark parameterization, we eliminate the differential contribution of this effect to the exchange rate dynamics in the two sets of countries. If the working capital

\textsuperscript{35}Total total transaction costs are very small in our calibration: they are 0.04\% of GDP in developed countries, and 0.48\% of GDP in developing countries.

\textsuperscript{36}The only exception is Korea, whose fiscal balance to GDP ratio is positive, on average.
requirements are more pronounced in developing countries, so that the output effect is stronger for them, by setting $\phi$ to be the same in developed and developing countries, we give up an important degree of freedom in generating depreciation in developing countries in our quantitative exercises.

Lastly, we calibrate parameters $\gamma$ and $\delta$ to match the average net foreign asset position to GDP ratios equal to -26% in developed economies and -33% in developing countries over our sample period. The proportional cost parameter $\phi^b$ in the banking sector’s problem is chosen to match the average spread of nominal lending rate over money market rate equal 9% in developing countries and 5% in developed economies over our sample period.

4.2 Calibration of the shock processes

There are two sources of uncertainty in our benchmark model: exogenous productivity realizations, $A$, and the policy-controlled interest rate realizations, $i^g$. We now describe how we calibrate the total factor productivity (TFP) and the process for interest rates. We will use a “hat” over a variable to denote the deviation of that variable from its balanced growth path.

We assume that productivity, $\hat{A}_t$, in both developed and developing countries is an independent AR(1) process with autoregressive coefficient, $\rho_A$, equal to 0.95. The innovations, $\varepsilon^A$, to this process are assumed to be independent and identically normally distributed. When characterizing business cycles properties of the model we set the volatility of productivity shocks in developed and developing countries calibrations such that the simulated volatility of output in the model matches the volatility of output in the data for the two groups of countries.

The process for the policy-controlled interest rate $i^g$ is estimated separately for developed and developing countries. To proxy the policy-controlled interest rates in the data we use the period average T-bill rate. For Netherlands we used a 3-month interbank rate in the Euro area. For Argentina, Australia, Brazil, Korea, Philippines and Thailand the T-bill rate was either not available or had large gaps in coverage, so we used the money market rate for these countries. We focus on the period between 1997:Q3 and 2010:Q4 to eliminate the periods of excess volatility in interest rates before and during the East Asian crisis. During the period under study, the average (annualized) level of $i^g$ was 9% in developing countries and 4% in developed economies.

We consider three different interest rate rules which differ in their exogenoeity or endogeneity to macroeconomic conditions.

(i) **Exogenous interest rate rule**: The first rule we consider is an exogenous interest rate rule, where we estimate the first-order autoregressive process for $i^g$ as

$$i^g_{t+1} = \rho_i i^g_t + \varepsilon^g_{t+1},$$

In fact, we restrict $\gamma$ to be the same in the two groups of countries to reduce the number of the free parameters. The resulting banking costs, $q$, in the steady state are very small, equal to less than 0.00086% of GDP.

For all the experiments reported below we checked to ensure that the implied inflation tax revenues are on the upward sloping portion of the Laffer-curve.
with $\varepsilon_{t+1}^g$ – i.i.d. normal innovations.\(^{39}\) While this rule is not very realistic, it allows us to flesh out the mechanism of the model.

(ii) **Taylor rule:** The second rules we consider is the well-known Taylor rule due to Taylor (1993) where interest rate responds to current output and inflation. We also allow for inertia in the interest rate. Such rules are typically referred to as Generalized Taylor rules in the literature. More precisely, we estimate Generalized Taylor rule of the form:

$$i_{t+1}^g = \rho g i_t^g + \alpha_1 (\pi_t - \pi^*) + \alpha_2 y_{t+1}^{gap} + \varepsilon_{t+1}^g,$$

with $\varepsilon_{t+1}^g$ – i.i.d. normal innovations, as before. $y_{t+1}^{gap}$ is output gap, measured as the deviation of industrial production in a given country from its Hodrick-Prescott trend.\(^{40}\) $\pi_t$ is inflation measured as (annualized) CPI growth rate, while $\pi^*$ is the inflation target.

(iii) **Inflation-Forecast-Based (IFB) monetary rule:** Our third rule is a “forward looking” version of the Taylor rule as it assumes that policy rate responds to a forecast of future inflation rather than the current inflation. We estimate simple IFB rule of the following form:

$$i_{t+1}^g = \rho g i_t^g + \alpha_1 E_t (\pi_{t+1} - \pi^*) + \alpha_2 y_{t+1}^{gap} + \varepsilon_{t+1}^g.$$  

We conduct the estimation of the equations above separately for a panel of developed and developing countries. This approach is intended to capture the dynamics of $i_t^g$ in an average emerging market economy and an average industrial country. Coefficient estimates in the three interest rate rules are presented in Table 7.\(^{41}\)

We find a substantial degree of policy inertia in the interest rate rules in both developed and developing countries, with the persistence being slightly higher in developed countries. In our estimation the loading on inflation, either contemporaneous or expected future, tend to be somewhat higher in developing countries, while the loadings on output are comparable in the two groups of countries. These results are comparable with the findings in a number of studies: for instance, Levin, Wieland, and Williams (2003) evaluate the performance of IFB rules in various monetary models and estimate the degree of policy inertia to be quite high - around 1. Laxton and Pesenti (2003) confirm their results for emerging market economies. Our estimates are also in line with Clarida, Galí, and Gertler (2000).

We also find that interest rates in developing countries to be significantly more volatile - the average standard deviation of $i_t^g$ is 1.42% in developed countries and 5.21% in developing economies (see row labelled $\sigma(i^g)$).\(^{42}\) The same differences extend to the volatility of innovations to interest rates in the two countries (row labelled $\sigma(\varepsilon_{t+1}^g)$). For instance, we get $\sigma(\varepsilon_{t+1}^g) = 0.0048$ in developed countries and $\sigma(\varepsilon_{t+1}^g) = 0.0215$ in

\(^{39}\)We also considered a differential between a country interest rate and the U.S. Federal Funds rate in our interest rate rules and found that the results remained practically unchanged.

\(^{40}\)If industrial production at quarterly frequency was not available, we used GDP volume instead.

\(^{41}\)We also considered Taylor rules with real exchange rate as discussed in Taylor (2001), but find that the estimates remain mostly unchanged as the coefficients on the real exchange rate and its lags are insignificant.

\(^{42}\)In the estimation of the process for $i^g$ for developing countries we excluded Argentina as its interest rate turned out to be 3 times more volatile than in any of the developing countries in our sample.
Table 7: Estimated interest rate rules

<table>
<thead>
<tr>
<th></th>
<th>Developed countries</th>
<th></th>
<th>Developing countries</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exogenous</td>
<td>Generalized Taylor</td>
<td>IFB</td>
</tr>
<tr>
<td></td>
<td>(i)</td>
<td>(ii)</td>
<td>(iii)</td>
</tr>
<tr>
<td>(i^g_t)</td>
<td>0.982***</td>
<td>0.918***</td>
<td>0.897***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.018)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>(y^g_{t\text{gap}})</td>
<td>0.054***</td>
<td>0.069***</td>
<td>0.116***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.013)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>(\pi_t - \pi^*)</td>
<td>0.076***</td>
<td>0.382***</td>
<td>0.076***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.138)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>(\mathbb{E}<em>t(\pi</em>{t+1} - \pi^*))</td>
<td>0.107***</td>
<td>0.107***</td>
<td>0.107***</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.030)</td>
<td>(0.030)</td>
</tr>
</tbody>
</table>

Note: Column (i) and (iv) present estimation results for equation (4.21); columns (ii) and (v) are for equation (4.22), while columns (iii) and (vi) are for equation (4.23). We obtain \(y^g_{t\text{gap}}\) in each country as deviations of industrial production from its Hodrick-Prescott trend. Due to the presence of lagged dependent variable, all equations are estimated with instrumental variables, where we used lagged values of interest rate as instruments. Rows \(\sigma(i^g)\) and \(\sigma(\epsilon^g_{t+1})\) report standard deviations of interest rate and innovations to interest rates, respectively. Robust standard errors are in parentheses. *, **, and *** indicate significance at 10%, 5%, and 1%, respectively.

developing countries in the simple exogenous rule, on average.\(^{43}\)

Once the shock processes and other parameter values are set, we solve the model by linearizing the equations characterizing equilibrium around the steady state and solving the resulting system of linear difference equations.\(^{44}\)

5 Results

We simulate the two version of the model calibrated for developed and developing countries for a sequence of random productivity and interest rate shocks and compute the key business cycles moments. We find that the model replicates the higher volatility of output, interest rates and net exports in developing countries. It also predicts procyclical consumption, investment, and employment; and countercyclical net exports (more co in developing countries), all of which are consistent with the data for the two groups of countries. The model also produces a negative correlation between policy rates and output in developing countries, but close to zero correlation in developed economies, again consistent with the data patterns. The contrast of data moments in developed and developing countries can be found in Neumeyer and Perri (2005).

We next turn to the relationship between interest rate and the exchange rate. We compute the level of nominal exchange rate in the model as follows. First, from the money market equilibrium condition in conjunction with PPP, \(M_0/E_0 = m^d(i^g_0, i_0)\), we get the initial level of exchange rate, \(E_0\), for a given level of

\(^{43}\) We also estimated country-specific processes for \(i^g\), and found them to be along the lines of the aggregate estimates.

\(^{44}\) In our economy, international bonds follow a unit root process. To account for this non-stationarity, we impose a small quadratic bond holding cost, \(\Phi(a_t) = \frac{\vartheta}{2} \mathbb{H}\left(\frac{a_t}{y_t} - \bar{a}\right)^2\), where \(\bar{a}\) denotes the steady state ratio of bond holdings to GDP, and \(\vartheta\) is a level parameter.
Next, with \( E_0 \) in hand, we construct the sequence of \( E_t \) using the process for exchange rate depreciation, \( \pi_t \), predicted by the model. Clearly, the exchange rate is non-stationary in our model. We transform \( E_t \) into stationary terms by dividing it by the model-implied \( M_t \). This is a standard transformation used in the literature to normalize nominal non-stationary variables in monetary models. Before proceeding, it is important to note that only nominal variables are non-stationary in the model. All real variables, including real money demand, in our model are stationary in that temporary shocks leave their long run levels unchanged.

Figure 4 presents impulse responses of the exchange rate to a temporary positive one standard deviation shock to the policy-controlled interest rate \( i^g \) in the model under the three interest rate rules: exog – refer to the exogenous rule in equation (4.21); GT – is for Generalized Taylor rule in equation (4.22); and IFB – is for Inflation-Forecast-Based rule in equation (4.23). Panel (a) is based on the model parameterized for a developed country, while panel (b) is for the model calibrated to a developing country. We also report the impulse responses of the market interest rate, \( i \), to the same \( i^g \) shock in developed and developing countries on Figure 4. Note that the response of \( i \) also illustrates the response of expected inflation through equation (3.6).

For developed countries, the model predicts an impact appreciation of the exchange rate under all three policy rules.\(^{45}\) For instance, a one standard deviation increase in \( i^g \) is associated with a 0.02% appreciation of the exchange rate on impact in developed countries and a 0.06% depreciation of exchange rate on impact in emerging market economies under an exogenous interest rate rule. When endogenous interest rate rules are considered, the model predicts smaller appreciation in developed countries and smaller depreciation in developing countries. These differences in quantitative responses of the exchange rate are primarily due to the differences in estimated persistence and size of shocks in the three interest rate rules. Note that these responses not only match the signs of the empirical impulse responses, but are also in line with the quantitative estimates of those responses in Figures 1 and 2.

The impulse responses in Panels (a) and (b) of Figure 4 highlight the inherent non-monotonicities in the relationship between interest rates and the exchange rate present in our model. One might argue that the interest rates measured in the data and used in our empirical analysis might not be fully controlled by the policymakers. Our theoretical impulse responses above highlight the fact that the differing relationship between exchange rate and interest rate for developed and developing countries also holds more generally for market interest rates, not just policy-controlled interest rate. There is a positive comovements between the market interest rate and exchange rate in developing countries but negative comovement in developed countries rather than a systematic relationship for all.\(^{46}\)

\(^{45}\)The fact that exogenous and endogenous policy rules produce the same qualitative exchange rate responses is not surprising, since the impact response of exchange rate should be the same under the two types of rules.

\(^{46}\)Note that following the shock, the market interest rate remains above it’s steady state value for a prolonged period of time in both developed and developing country calibrations. Since uncovered interest parity holds in our model, this implies a depreciating path for the exchange rate in both countries immediately following the differing impact response. This is indeed the case in our model. However, while panel (a) in Figure 4 is consistent with this pattern in that the exchange rate exhibits a depreciating path after the impact appreciation, the response of developing countries shown in panel (b) appears to contradict it. The exchange rate in developing countries seem to be appreciating after an impact depreciation. This pattern is an artefact of the normalization by \( M \) that we used. Because \( M \) is also increasing after the shock, but does so at a faster rate than the exchange rate for developing countries, the ratio of \( E/M \) is falling.
Figure 4: Impulse responses following 1 std dev shock to $i^g$

Panel (a). Developed countries

Panel (b). Developing countries

Note: The left panels present the responses of nominal exchange rate to a 1 std. dev. positive shock to policy-controlled interest rate, $i^g$, under three interest rate rules: exog–exogenous rule given in equation (4.21); GT–Generalized Taylor rule given in equation (4.22); and IFB–Inflation-Forecast-Based rule given in equation (4.23). The right panels show the response of the market interest rate, $i$, to the same shock. Panel (a) presents impulse responses from the model calibrated to developed countries, while panel (b) does the same for developing countries.

It is important to note from Figure 4 that the market interest rate, $i$, increases after the shock to $i^g$ in both countries. This reflects the fact that under our calibration, the expected inflation rate responds similarly to $i^g$ shocks in both groups. Hence, the different impact responses of the level of the exchange rate are not due to differences in the response of expected inflation. This is consistent with the empirical results we presented in Section 2.3 where the opposing responses of the exchange rate to interest rate shocks arose even after controlling for shocks to the expected inflation rate.

5.1 Under the hood

So what is behind these contrasting dynamics in interest rates and the exchange rate in developed and developing countries? Since our results are robust to various interest rate rules, and to help flesh out the effects of policy interest rate in the model, in what follows we focus on the case with exogenous interest rate rule.

To build intuition for the results, consider a perfect foresight version of the model. In addition, assume
that $\gamma \approx \infty$ so that banking costs are infinitely large. In this case the banking sector will hold a constant amount of foreign assets, i.e., $d^b = \bar{d}^b$. From expression (3.19), the exchange rate at any date $t$ is determined by $E_t = \frac{M_t}{h_t + \theta d_t}$, where $M_t$ is given. Log differentiating this relationship gives

$$\hat{E}_t = \dot{M}_t - \frac{h_t}{m} \dot{h}_t - \frac{\theta d_t}{m} \dot{d}_t$$

(5.24)

where hats over a variable indicate its percentage change and $m = M/E$. Suppose $\dot{M} = 0$, i.e., nominal money supply is unchanged at date $t$. Clearly, the effect of changes in $i^g$ on $E$ depends on the effect that changes in $i^g$ have on $h$ and $d$.

The optimality conditions for cash and deposits holdings along with the definition for the nominal interest rate under perfect foresight, $1 + i_{t+1} = R_t (1 + \pi_{t+1})$, yield the equilibrium cash and deposit demands in the model as

$$h_t = \hat{h} \left( \frac{i_{t+1}}{1 + \pi_{t+1}} \right) \quad \text{and} \quad d_t = \hat{d} \left( \frac{i_{t+1} - (1 - \theta) i^g_{t+1}}{1 + \pi_{t+1}} \right),$$

(5.25)

where both functions are decreasing in their arguments.47 Clearly, a rise in $i^g$ positively affects $E$ by increasing the demand for deposits $d$. This is the direct effect of $i^g$ on $E$. However, $i^g$ also affects $E$ indirectly through its effect on the market interest rate, $i$, since $i$ affects the demand for both cash and deposits. The effect of $i^g$ on $i$ in turn is determined from the government flow budget constraint

$$\tau + \frac{1 + i^g_{t+1}}{1 + \pi_t} \pi_t z_{t-1} = h_t - \frac{\pi_{t-1}}{1 + \pi_t} \theta d_t - \frac{\theta d_{t-1}}{1 + \pi_t} + z_t,$$

(5.26)

where $z = (1 - \theta) d - n + \bar{d}^b$ (assuming $\gamma \approx \infty$). The left hand side of equation (5.26) gives total expenditures while the right hand side gives net revenues. For a given $\tau$ and $i^g$, equations (5.26) and (5.24) along with the equilibrium conditions for cash and demand deposits in (5.25) pin down both $\pi_t$ and $i_{t+1}$.484950 Finally, $i_{t+1}$ and $i^g_t$ jointly determine aggregate money demand, which in turn pins down $E_t$.

There are three indirect channels through which changes in $i^g$ end up affecting $i$. The first operates through the government budget constraint (5.26). Specifically, an increase in $i^g$ directly increases the cost of servicing government bonds $z$ tomorrow, which increases the future fiscal burden and hence the required inflation rate. Second a rise in $i^g$ raises the lending spread $i^l - i$. A higher lending spread $i^l - i$ lowers the amount of private loans, $n$, for a given level of demand deposits, and thereby raises commercial bank loans to the public sector, i.e., $z$ rises. This reduces the reliance on inflationary finance today but raises the future fiscal burden through a higher base level of debt. This effect arises as a consequence of the “output” effect.

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47 Derivations are provided in Appendix A.4.

48 Note that $i_t$ and $i^g_t$ are given at any date $t$ while $\pi_t$ and $P_t$ are determined within the period. Since $P_{t-1}$ is given, pinning down $\pi_t = \frac{P_t}{P_{t-1}}$ is equivalent to determining $P_t$.

49 It is important to note that pinning down $i_{t+1}$ is not equivalent to determining $P_{t+1}$. To see this, note that $1 + i_{t+1} = R_{t+1} (1 + \pi_{t+1}) = R_{t+1} (1 + \frac{P_{t+1}}{P_t})$. Clearly, this is not equivalent to determining $P_{t+1}$ which is free to deviate from $E_t (P_{t+1})$ in response to shocks that hit the economy after the formation of expectations about period $t + 1$.

50 Note equation (5.26) can be rewritten as a first order difference equation in $i_{t+1}$ and $i_t$. The standard condition for a unique flexible price monetary equilibrium is that the difference equation in $i$ be unstable. We impose it through out. It can be verified that this stability condition also implies government revenues are increasing in the nominal interest rate $i$. 

32
Third, a higher $i^g$ lowers the deposit spread $i - i^d$ which raises demand deposits with commercial banks. For a given level of private loans, this reduces the reliance on inflationary finance today to finance government spending. This effect arises due to the “money demand” effect. These indirect effects impact the inflation rate that is required to finance the government budget and thereby affect the market nominal interest rate $i$. The changes in $i$ impact the demand for cash as well as deposits and loans through their effect on the deposits and lending spreads (see equations (5.25), (3.14) and (3.13)). These indirect effects determine the end impact of $i^g$ on money demand, and therefore, on the level of the exchange rate.

We illustrate these effects by plotting the impulse responses of various variables to a positive shock to $i^g$. To facilitate the comparison across developed and developing countries calibrations we use the same size of the shock (equal to developed country standard deviation of $i^g$) in both calibrations. Also, since the steady states are different in the two calibrations, we present all impulse responses as deviations from the steady state (multiplied by 100). Results are in Figure 5.

Panel (a) of Figure 5 shows that deposits rise in both groups in response to an increase in $i^g$. Crucially though, deposits respond much more in developed countries.\footnote{This argument relies on the matching interest rate elasticities of deposit demand in the developed and developing countries in our calibration. A given interest rate change is therefore associated with a larger change in deposit demand in the group of countries where the steady state level of deposits is higher – developed economies.} We return to this issue below. Panel (b) shows that output falls symmetrically on impact in both groups. This is to be expected since we set the wage-in-advance parameter $\phi$ to be the same in both the developed and developing groups.\footnote{Note that the loan to output ratio is $wl_y$. Under the Cobb-Douglas specification for technology, $wl_y = 1 - \alpha$ is the same for both groups. Hence, under our maintained parameterization of a common $\phi$, the loan-output ratio is the same across countries. This is the reason for the symmetric output effect on impact for developed and developing countries in 5. We investigate the robustness of our results with respect to this parameter in section 5.2 below.} The response is more long-lived in developed countries due to a higher persistence of the process for $i^g$ in that group of countries.

Panel (c) of Figure 5 shows that government bond holdings of commercial banks rise in both groups of countries but much more sharply in developed countries. This is due to the fact that deposit demand increases more in developed than in developing countries, while the fall in loan demand is symmetric across the two countries. Hence, banks’ demand for government bonds rises by more in industrial economies, as they seek to rebalance their portfolios. Panel (d) shows that the response of inflation, $\pi$, is practically symmetric in the two groups of countries. This is due to the offsetting effects to the government budget of the higher cost of servicing existing government bonds and the additional infusion of funds to the budget through higher $d_t$ and new government bond issuances $z_t$. Notice that the magnitude of these effects are larger for developed countries but enter in offsetting ways in the government budget constraint. Hence, the net effect is very similar for the two sets of countries. The symmetric inflation effect implies that the increase in $i$ is also symmetric in developed and developing countries.\footnote{The key parameter that affects the strength of the “fiscal” effect is the lump-sum transfers paid by the government to the private sector, $\tau$. In our calibration $\tau$ captures the state of public finances and is found to be smaller (in absolute value) in developed economies. This leads to a lower steady state inflation and market interest rate in developed countries than in developed economies. We investigate the robustness of our results with respect to this parameter in section 5.2 below.}

The rise in the market interest rate $i$ reduces the demand for cash in both groups of countries, since a rise in $i$ unambiguously increases the opportunity cost of holding cash. This effect is larger for developed countries.
due to a larger steady state value of \( h \) in that group (see panel (e) of Figure 5). An increase in \( i \) also raises the opportunity cost of holding deposits which partially offsets the increase in deposits arising from the direct effect of higher \( i^g \). Finally, aggregate money demand shows opposing responses in the two countries: the indirect effect of lower cash and deposits demand from higher \( i \) swamps the direct effect on deposits from higher \( i^d \) in developing countries, but it is not strong enough to overturn the direct effect in developed countries. As a result, the nominal exchange rate appreciates in developed countries while depreciating in developing countries. This is illustrated in panel (f) of Figure 5.

From the perspective of understanding the difference between developing and developed economies in their exchange rate responses to interest rate shocks, the preceding discussion illustrates that the key lies in uncovering the reason for the differing responses of real money demand \( m \). It is instructive then to differentiate \( m = h + \theta d \) with respect to \( i^g \) to get

\[
\frac{dm}{di^g} = \left[ \tilde{h} \frac{\partial I}{\partial i} + \tilde{d} \frac{\partial I^d}{\partial i} \right] \frac{\partial i}{\partial i^g} + \theta (1 - \theta) \tilde{d} \frac{\partial I^d}{\partial i^d}.
\]

Here we denoted \( \frac{i}{1+i} = I \) and \( \frac{i-i^d}{1+i} = I^d \).

Noting that \( \frac{\partial I}{\partial m} = \frac{1}{(1+i)^2} \) and \( \frac{\partial I^d}{\partial m} = \frac{1+i^d}{(1+i)^2} \), this can be rewritten as

\[
\eta_m = \eta_i \left[ -\eta_h \frac{h}{m} \left( \frac{1}{1+i} \right) - \eta_d \frac{d}{m} \left( 1+i^d \right) \left( \frac{i}{1+i} \right) \right] + \eta_d \frac{d}{m} \left( \frac{i^d}{i-i^d} \right)
\]

where \( \eta_h = -\tilde{h} \frac{I}{h} > 0 \), \( \eta_d = -\tilde{d} \frac{I^d}{h} > 0 \), \( \eta_i = \frac{\partial i}{\partial i^g} \), \( \eta_m = \frac{dm}{di^g} \). Notice that from panel (d) of Figure 5, our calibration yields \( \eta_i > 0 \) since inflation always responds positively to an increase in \( i^g \) in both groups of countries. Hence, if the RHS is positive, i.e., the last term dominates, then \( \eta_m > 0 \) and \( m \) rises with \( i^g \) leading to \( E \) appreciation, else there is a depreciation.

Our steady state calibration sets \( \eta_h = \theta \eta_d \) and \( \eta_h(\text{emerging}) = \eta_h(\text{developed}) \), i.e., these elasticities are the same across countries. Hence, around the steady state we can write \( \eta_m \) as

\[
\eta_{ms} = \eta_i^{ss} \frac{d}{m} \int d \left\{ 1 - \left( \frac{1+i^d}{i^d} \right) \left( \frac{i}{1+i} \right) \eta_i^{ss} \right\} - \eta_i^{ss} \frac{h}{m}.
\]

Our calibration (based on the data) sets \( \frac{d}{m} = 4 \frac{h}{m} \) in developed countries and \( \frac{d}{m} = \frac{h}{m} \) in developing countries. Hence, the depreciation in developing countries is due to the fact that

\[
\frac{i^d}{I^d} \left\{ 1 - \left( \frac{1+i^d}{i^d} \right) \left( \frac{i}{1+i} \right) \eta_i^{ss} \right\} < \eta_i^{ss}
\]

while the appreciation in developed economies is because

\[
\frac{4 i^d}{I^d} \left\{ 1 - \left( \frac{1+i^d}{i^d} \right) \left( \frac{i}{1+i} \right) \eta_i^{ss} \right\} > \eta_i^{ss}
\]

\[54\] The logic here is the same as for deposit demand: higher steady state level of cash holdings in developed countries implies a larger change in their cash holdings, given identical interest rate elasticities of cash demand in the two groups.
Our quantitative results imply that $\eta_i^{ss}$ is similar for the two groups since the inflation effect is quite symmetric (see panel (d) of Figure 5). Moreover, $i$ and $i^d$ are both higher in developing countries. Since they move in opposite directions, the offsetting effects tend to keep $\left( \frac{1+i^d}{1+i} \right) \left( \frac{i}{1+i} \right)$ at similar levels for developed and developing countries. Hence, the term in the curly brackets is similar in magnitude for developed and
developing countries. The opposing results for the the two groups then arise because \( \frac{i^d}{\pi} \), while higher in developing countries, is not high enough to overcome the negative effect coming from \( \eta_i^{gs} \). In developed countries however, \( 4 \frac{i^d}{\pi} \) is high enough to overcome \( \eta_i^{gs} \) (where the multiplication factor 4 reflects the four-fold higher deposit to cash ratio in developed countries).

The preceding highlights that a key factor behind the differing exchange rate responses in developed and developing countries is the fact that steady state deposits are significantly larger in developed countries than in developing economies. Recall that the steady state level of deposits to cash is four-times higher in developed economies than in developing countries in our calibration, reflecting the same difference in the data. In combination with our calibration of matching interest rate elasticities in the two countries, this implies that the same increase in \( i^g \) will lead to a larger rise in \( d \) in developed economies. This makes the “money demand” effect much stronger in developed countries. More generally, the money demand effect appears to be key to understanding the differences between developing and developed economies.

The analysis above suggests to us that there exists a threshold level of deposits to cash ratio such that countries whose deposits to cash is above that threshold will see their currency appreciating after a positive \( i^g \) shocks, while countries below that threshold will see their exchange rate depreciate. We compute this threshold in the developed country calibration by lowering the target for deposits to cash ratio, while adjusting parameters to hit the targets for aggregate money demand to GDP and interest elasticities of cash and deposits. This exercise produces a threshold ratio of deposits to cash at 2.44, i.e. when deposits to cash falls below this level, keeping all other targets unchanged, exchange rate begins to depreciate following a positive shock to interest rate \( i^g \).

Does the data support the importance of money demand channel in determining exchange rate responses to interest rate shocks, and in particular, the role played by the deposit to cash ratio? To answer this question we return to our empirical VAR results and compare the ratio of deposits to cash in countries whose currency appreciated following interest rate shocks with the same ratio in countries that saw their currency depreciate after a rise in policy interest rate.\(^{55}\) The ratio of deposits to cash is equal to 1.87 in depreciating countries, while it is significantly higher at 5.22 in appreciating countries. We also investigate the relationship between deposits to cash ratio and the probability of currency appreciation using a probit regression, and find that the relationship is positive and significant. Namely, the marginal effect of deposits to cash ratio is 0.07, so that when the ratio rises by 100 percent, the probability of currency appreciation increases by 7 percent.

The importance of the deposit/cash ratio in rationalizing the different exchange rate responses in the two groups of countries raises a fundamental question: why is this ratio so much higher in developed countries relative to developing economies? The answer is likely related to a multitude of factors such as the the history of expropriation of interest bearing deposits, the institutional strength of the monetary regime, the presence and duration of deposit insurance schemes as well as the level of financial development. More generally, our results suggest that the transmission of monetary policy to the economy is likely fundamentally affected by

\(^{55}\)The data on deposits and cash (notes and coins) was taken from the IFS and individual country Central Banks websites. This way we were able to put together data on deposits to cash ratio for 36 countries in our dataset.
these factors. Hence, they need to be factored in explicitly when conducting monetary policy in developing countries since the outcomes may well be at odds with the established wisdom derived from developed country experiences.

5.2 Varying key parameters

Aside from contribution of the money demand effect, our model had two other effects – the fiscal effect and the output effect. How sensitive are our results to different strengths of these two effects?

The size of the fiscal effect in the model is captured by \( \bar{\tau} \) and \( \bar{i}^g \). An increase in \( \bar{\tau} \), ceteris paribus, raises the financing burden on the government. Equation (5.26) makes clear that the financing can be done by some combination of a rise in \( \pi_t \) (or equivalently, \( R_t \)) and \( i_{t+1} \). Consider equation (5.26) in a perfect foresight steady state:

\[
\bar{\tau} = \left( \frac{i - r}{1 + \bar{i}} \right) (h_{ss} + \theta d_{ss}) + \left( \frac{1 + i - R(1 + \bar{i}^g)}{1 + \bar{i}} \right) z_{ss}
\]

(5.27)

where we have used the steady state relation \( 1 + \bar{i} = R(1 + \pi) \). Keeping everything else constant, a higher \( \bar{\tau} \) necessitates a higher steady state \( \bar{i} \). Since \( 1 + i = R(1 + \pi) \) and \( 1 + \pi_t = \frac{E_t}{E_{t-1}} \), a higher \( i \) implies an immediate depreciation of the currency. Thus, the model predicts that the higher is \( \bar{\tau} \) the more depreciated the country’s currency.

Figure 6 shows the fiscal effect in play for both developed countries (left panel) and emerging economies (right panel). In each panel, the figures have two lines – a solid line and a dashed line. Both lines are derived for the baseline calibration for the country in question. The solid line plots the initial level of the exchange rate as a function of \( \bar{\tau} \) for a given and constant initial level of money supply \( M_0 \). The initial level of the exchange rate is measured on the left axis. The dashed lines in the two panels of the Figure 6 depict the impact effect of an increase in \( \bar{i}^g \) on the exchange rate (expressed as percent deviation from the steady state). This is measured on the right axis.

The solid lines in the Figure show that the initial \( E \) rises with \( \bar{\tau} \) for both developed and emerging economies, exactly as the model suggests. The dashed lines reveal an important difference between the two groups. For developed countries, a higher \( \bar{i}^g \) induces an impact currency appreciation for the entire range of \( \bar{\tau} \)’s plotted. For developing economies however, a higher \( \bar{i}^g \) induces a depreciation of the currency on impact, i.e., a rise in \( E \). Note though that the size of the impact depreciation declines as \( \tau \) becomes larger, reflecting partly the fact that the exchange rate changes are in percent deviations from the steady state and the higher \( \tau \) raises the steady state level of \( E \) itself for both groups.

We draw two main conclusions from these results. First, for any given level of nominal money supply, a higher \( \bar{\tau} \), which is partially responsible for the strength of the fiscal effect, always leads to a more depreciated currency in the steady state. Second, the difference between developed and emerging economies that we found in Figure 4 is robust to changes in the level of \( \bar{\tau} \) since changes in \( \tau \) do not appear to qualitatively change the impact effect of \( \bar{i}^g \) on the exchange rate.

The size and importance of the output effect is captured by the wage-in-advance parameter \( \phi \). The higher
Figure 6: Comparative statics for parameter $\tau$

(a). Developed countries  

(b). Developing countries

Note: The figure presents the responses of nominal exchange rate to changes in $\tau$. The developed country calibration is shown in the left panel while the right panel shows the developing country calibration. Dotted vertical line indicates the level of $\tau$ under our benchmark calibration in each country group.

As we noted earlier, there are complications in calibrating this parameter with precision. However, this makes it all the more important to examine the sensitivity of the results to variations in $\phi$. Figure 7 shows
the effects of varying \( \phi \) in our model. The figure is symmetric to the ones for \( \tau \) and \( \theta \). The upward sloping solid lines in the two panels of Figure 7 demonstrate that the higher is \( \phi \) the more depreciated is the exchange rate in the initial steady state for a constant level of initial nominal money supply \( M_0 \). In terms of the impact effect of a temporary increase in \( i^g \) on the exchange rate, in developed countries the response is an appreciation while in emerging economies it is a depreciation for a broad range of values for \( \phi \).

Given that Figure 7 consistently reveals a depreciation for developing countries and an appreciation for developed economies for a broad range of values for \( \phi \) leads us to conclude that the differences in the impulse responses for the two groups to interest rate shocks highlighted in Figure 4 are robust to variations in the wage-in-advance parameter \( \phi \).

It is important to emphasize that our calibration sets \( \phi \) to be the same in developed and developing countries. This implies that the strength of the output effect is the same across the two groups of countries, and thus does not contribute to their differing exchange rate dynamics. To the extent that the output effect may be stronger in developing countries, potentially through higher reliance of private firms on bank’s finance in those countries, there will be an even more pronounced tendency for their exchange rate to depreciate following rises in \( i^g \). This would make the differences in the transmission of interest rate shocks to the exchange rates in developed and developing countries even more pronounced.

Our broad conclusion from these results is that the channels of differential responses of exchange rate to interest rate shocks in developed and developing countries are quite robust in our model.

6 Conclusions

The effect of monetary policy on the exchange rate has long been one of the fundamental concerns of academics and practitioners alike. A large preponderance of existing models predict that monetary policy tightening should induce an exchange rate appreciation. What does the evidence suggest though? In this paper we have used a panel dataset comprising of 72 countries between 1974 and 2010 to show that while most developed countries indeed exhibit exchange rate appreciations in response to interest rate increases, in developing countries the effect is the opposite: most of them exhibit depreciating currencies in response to interest rate increases. Importantly, the differing responses in developing and developed countries is not due to simply differences in the nature of expected inflation shocks in the two groups. We call this puzzling new data fact the “exchange rate response puzzle”.

We have provided an explanation for this puzzle using a simple open economy monetary model. Our explanation rests on the contrasts in the interplay between three key effects between developed and developing countries. Our model formalized three important effects of raising interest rates – a larger fiscal burden, a negative output effect and a positive effect on real money demand. While the first two effects tend to depreciate the currency, the last tends to appreciate it. Using a calibrated version of the model, we have shown that the differences in the relative importance of these three effects between the two groups of countries can account for the contrasting responses in the two groups.
References


A Appendix

A.1 Empirical evidence

In this Appendix we describe our data sources used in the empirical sections of the paper. Our primary data sources are the International Financial Statistics (IFS) compiled by the International Monetary Fund (IMF) and the World Development Indicators (WDI) compiled by the World Bank. In our analysis we considered all countries in the IFS and WDI datasets for which monthly data on exchange rates and interest rates was available for any fraction of the 1974-2010 period.

Data description and sources are summarized in Table A1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Source</th>
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</thead>
<tbody>
<tr>
<td>Exchange rate</td>
<td>official exchange rate, period average</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td></td>
<td>market exchange rate, period average</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td></td>
<td>principal exchange rate, period average</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td></td>
<td>commercial exchange rate, period average</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td>Interest rate</td>
<td>T-bill rate, period average</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td></td>
<td>discount rate, period average</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td></td>
<td>money market rate, period average</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td>Output</td>
<td>Industrial production index</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td>CPI</td>
<td>Consumer price index</td>
<td>IFS by IMF</td>
</tr>
<tr>
<td>Risk premium</td>
<td>BofA Merrill Lynch US High Yield Master II Option-Adjusted Spread</td>
<td>BofA Merrill Lynch</td>
</tr>
<tr>
<td></td>
<td>Moody’s Seasoned Baa Corporate Bond Yield</td>
<td>Board of Governors of the Federal Reserve System</td>
</tr>
</tbody>
</table>

As we mentioned in the main text, in our empirical analysis we restrict the sample to only those countries and time periods that are characterized by a flexible exchange rate regime. To perform the selection, we rely on the Reinhart and Rogoff (2004) classification of historical exchange rate regimes. In particular, we classify a country as having a flexible exchange rate regime if, in a given year, its exchange rate was either (i) within a moving band that is narrower than or equal to +/-2% (i.e., allows for both appreciation and depreciation over time); or (ii) was classified as managed floating; or (iii) was classified as freely floating; or (iv) was classified as freely falling according to Reinhart and Rogoff (2004). These correspond to their fine classification indices of 11, 12, 13, and 14, respectively. We only focus on the post-Bretton Woods period for all countries. High income OECD countries are included in our sample, irrespective of their exchange rate classification. Table A2 contains the list of country-episode pairs that are included in our sample.
Table A2: Sample used in the empirical work

<table>
<thead>
<tr>
<th>No.</th>
<th>From to Country</th>
<th>No. from to Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1994m7  2001m12 Albania</td>
<td>41  1999m1  1995m12 Luxembourg</td>
</tr>
<tr>
<td>2</td>
<td>1988m1  1995m1 Algeria</td>
<td>42  2002m10  2007m12 Madagascar</td>
</tr>
<tr>
<td>3</td>
<td>2002m5  2005m5 Angola</td>
<td>43  1983m1  1994m12 Malawi</td>
</tr>
<tr>
<td>4</td>
<td>1974m1  2010m11 Australia</td>
<td>44  1997m8  2003m8 Malawi</td>
</tr>
<tr>
<td>5</td>
<td>1974m1  1998m12 Austria</td>
<td>45  1987m11 2000m12 Malta</td>
</tr>
<tr>
<td>6</td>
<td>1994m1  2002m10 Belarus</td>
<td>46  1982m2  1988m11 Mexico</td>
</tr>
<tr>
<td>7</td>
<td>1974m1  1998m12 Belgium</td>
<td>47  1995m1  2007m12 Mexico</td>
</tr>
<tr>
<td>8</td>
<td>1999m2  2007m12 Brazil</td>
<td>48  1991m1  1994m12 Myanmar</td>
</tr>
<tr>
<td>9</td>
<td>1992m1  1996m12 Bulgaria</td>
<td>49  1996m8  1999m12 Myanmar</td>
</tr>
<tr>
<td>10</td>
<td>1974m1  2010m11 Canada</td>
<td>50  1974m1  1990m8 Netherlands</td>
</tr>
<tr>
<td>11</td>
<td>1999m9  2001m12 Chile</td>
<td>51  1978m1  2010m11 New Zealand</td>
</tr>
<tr>
<td>12</td>
<td>1990m3  1992m7 China, P.R. (Mainland)</td>
<td>52  1991m7  2007m12 Nigeria</td>
</tr>
<tr>
<td>13</td>
<td>1994m7  2004m2 Congo, Dem. Rep. of</td>
<td>53  1974m1  2009m5 Norway</td>
</tr>
<tr>
<td>14</td>
<td>1980m10 1983m10 Costa Rica</td>
<td>54  1990m1  1993m10 Peru</td>
</tr>
<tr>
<td>15</td>
<td>1997m6  2001m12 Czech Republic</td>
<td>55  1997m7  1999m11 Philippines</td>
</tr>
<tr>
<td>16</td>
<td>1981m5  1988m12 Denmark</td>
<td>56  1982m4  1998m5 Portugal</td>
</tr>
<tr>
<td>17</td>
<td>2003m11 2007m12 Dominican Republic</td>
<td>57  1994m3  2001m3 Romania</td>
</tr>
<tr>
<td>18</td>
<td>1999m1  2010m11 Euro Area</td>
<td>58  2003m4  2007m12 Serbia, Rep. of</td>
</tr>
<tr>
<td>19</td>
<td>1974m1  1998m12 Finland</td>
<td>59  2002m1  2005m6 Sierra Leone</td>
</tr>
<tr>
<td>20</td>
<td>1974m1  1998m12 France</td>
<td>60  1974m1  2007m12 Singapore</td>
</tr>
<tr>
<td>21</td>
<td>1986m1  1991m9 Gambia, The</td>
<td>61  1974m1  1998m5 South Africa</td>
</tr>
<tr>
<td>22</td>
<td>1975m7  1998m12 Germany</td>
<td>62  1995m3  2007m12 South Africa</td>
</tr>
<tr>
<td>23</td>
<td>1985m4  2001m3 Ghana</td>
<td>63  1979m1  1998m12 Spain</td>
</tr>
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<td>24</td>
<td>1983m1  2000m12 Greece</td>
<td>64  1974m1  2010m11 Sweden</td>
</tr>
<tr>
<td>25</td>
<td>2000m5  2002m6 Guinea</td>
<td>65  1980m1  2010m11 Switzerland</td>
</tr>
<tr>
<td>27</td>
<td>1994m10 2007m12 Haiti</td>
<td>67  1998m10 2002m10 Tajikistan</td>
</tr>
<tr>
<td>28</td>
<td>1987m6  2010m11 Iceland</td>
<td>68  2001m2  2007m12 Thailand</td>
</tr>
<tr>
<td>29</td>
<td>1997m8  2007m12 Indonesia</td>
<td>69  1991m12 1996m5 Turkey</td>
</tr>
<tr>
<td>30</td>
<td>1977m1  1979m10 Iran, I.R. of</td>
<td>70  2001m2  2007m12 Turkey</td>
</tr>
<tr>
<td>31</td>
<td>1974m1  1998m12 Ireland</td>
<td>71  1980m1  1980m8 Uganda</td>
</tr>
<tr>
<td>32</td>
<td>1977m1  1998m12 Italy</td>
<td>72  1989m10 1992m12 Uganda</td>
</tr>
<tr>
<td>33</td>
<td>1990m10 1992m12 Jamaica</td>
<td>73  1992m12 1996m9 Ukraine</td>
</tr>
<tr>
<td>34</td>
<td>1974m1  2010m11 Japan</td>
<td>74  1974m1  2010m10 United Kingdom</td>
</tr>
<tr>
<td>35</td>
<td>1994m4  1996m5 Kazakhstan</td>
<td>75  1991m12 1995m9 Uruguay</td>
</tr>
<tr>
<td>36</td>
<td>1987m1  1995m12 Kenya</td>
<td>76  2002m5  2005m5 Uruguay</td>
</tr>
<tr>
<td>38</td>
<td>1994m1  1999m11 Kyrgyz Republic</td>
<td>78  1978m1  1983m7 Zambia</td>
</tr>
<tr>
<td>40</td>
<td>1984m3  1991m7 Lebanon</td>
<td>80  1991m10 1999m3 Zimbabwe</td>
</tr>
</tbody>
</table>

A.2 Structural VAR: Details

Let \( y_t \) be a \((2\times1)\) vector consisting of interest rate differential and \((log)\) exchange rate: \( y_t = [i_{t} - i_{t}^{US}, \ln E_t] \).

The VAR can be written as:
\[
y_t = (I - A_1 L - ... - A_p L^p)^{-1} u_t,
\]
where \( u_t \) is a \((2\times1)\) vector of reduced-form residuals, assumed to be i.i.d., with zero mean and \( E u_t u_t' = \Sigma \). \( L \) is a lag operator, while \( A_1, ..., A_p \) are \((2\times2)\) matrices. Following the literature we assume that orthogonal structural shocks, \( \varepsilon_t \), are linear combinations of \( u_t \): \( u_t = B \varepsilon_t \), where \( \varepsilon_t \) are i.i.d. with mean zero and the variance-covariance matrix equal to the identity matrix. The VAR can then be written in terms of structural shocks as
\[
y_t = (I - A_1 L - ... - A_p L^p)^{-1} B \varepsilon_t.
\] (A1)

Parameters in \( A_1, ..., A_p \) and \( \Sigma \) can be estimated from the data. Thus, to obtain structural interpretation from the reduced form VAR we need to identify matrix \( B \). Matrix \( B \) satisfies \( \Sigma = BB' \). To obtain identification in our structural VAR we impose a long-run neutrality restriction as discussed in the text. Specifically, we assume that interest rate shocks cannot have long-run effects on the real exchange rate. This is achieved by
setting the values of the relevant lag coefficients in equation (A1) to zero. For instance, if we redefine

\[(I - A_1 L - \ldots - A_p L^p)^{-1} B = C(L)\]

and write the long-run expression of \(C(L)\) as \(C^* = \sum_{j=0}^{\infty} C_j\), then our long-run neutrality restriction reduces to \(C_{21}^* = 0\). As we discussed in the text, this identification scheme allows for contemporaneous link between interest rates and exchange rate, while maintaining comparability with the Cholesky ordering results.

### A.3 Calibration: Data sources

In this Appendix we describe data and sources used in model calibration. We focused on a sample of 6 industrial economies – Australia, Canada, Netherlands, New Zealand, Sweden and UK and 6 developing countries – Argentina, Brazil, Korea, Mexico, Philippines, and Thailand – during 1974-2010 period. When focusing on nominal variables, i.e. nominal interest rates, we restrict the sample to 1998-2010 period to eliminate the periods of high interest rate volatility and high inflation in developing countries before and during the East Asian crisis.

**Monetary variables:** M1 (in local currency) for all countries comes from World Development Indicators (WDI) and Global Development Finance (GDF) datasets compiled by the World Bank. GDP (in local currency) was obtained from the same dataset. Reserve ratio was computed for each country following Brock (1989) as the ratio of monetary base less currency outside banks to M2 less currency outside banks. All series used in the computation were obtained from the International Financial Statistics (IFS) by the International Monetary Fund (IMF). To obtain the ratio of deposits to cash holdings we computed the level of deposits in each country as M1 minus currency outside banks. Cash holdings were measured by the currency outside banks. Consumer price (CPI) data is from the IFS database.

**Fiscal variables:** We used general government net lending/borrowing as a share of GDP to calibrate parameter \(\tau\). This data is from the World Economic Outlook (WEO) dataset of the IMF.

**Other:** We obtained average net foreign asset position (NFA) from Lane and Milesi-Ferretti (2007) dataset. To proxy the policy-controlled interest rates in the data we use the period average T-bill rate. For Netherlands we used a 3-month interbank rate in the Euro area. For Argentina, Australia, Brazil, Korea, Philippines and Thailand the T-bill rate was either not available or had large gaps in coverage, so we used the money market rate for these countries. This data is from the IFS database.

### A.4 Deriving money demand

Household’s first order conditions are

\[U_c(c_t, x_t) = \beta R E_t U_c(c_{t+1}, x_{t+1}),\]  \hspace{1cm} (A2)

\[\nu \zeta x_t^{\nu-1} = w_t,\]  \hspace{1cm} (A3)
Equation (A2) is the standard intertemporal Euler equation for optimal consumption. Under our maintained assumption of $\beta = 1/R$ it says that the household should equate the expected marginal utilities across time. Equation (A3) shows that labor supply depends only on the real wage. Moreover, the assumption $\nu > 1$ implies that labor supply $x$ is an increasing function of the real wage, $w$. Equation (A6) dictates the optimal capital accumulation decision by households. Note that by combining equations (A2) and (A6) one can derive a modified no-arbitrage condition which determines the optimal portfolio composition between bonds and physical capital.

Equations (A4) and (A5) implicitly define the demand for cash and demand deposits as a decreasing function of their respective opportunity costs. To see this apply the functional forms for transaction costs functions $v(h_t)$ and $\psi(d_t)$ specified in (4.20) and substitute in the definition of the nominal interest rate in (3.6) to the first order conditions for cash and deposit demand to obtain their implicit demand functions:

$$U_c(c_t, x_t) (1 + v'(h_t)) = \beta E_t \left[ U_c(c_{t+1}, x_{t+1}) \frac{P_t}{P_{t+1}} \right],$$

(A4)

$$U_c(c_t, x_t) (1 + \psi'(d_t)) = \beta E_t \left[ U_c(c_{t+1}, x_{t+1}) \left(1 + i_{t+1}^d\right) \frac{P_t}{P_{t+1}} \right],$$

(A5)

$$U_c(c_t, x_t) = \beta E_t \left[ \frac{\rho_{t+1} + (1 - \delta) (1 + \kappa_l (t + 1)) - \kappa_k (t + 1)}{1 + \kappa_l (t)} U_c(c_{t+1}, x_{t+1}) \right]$$

(A6)

Equations (A4) and (A5) implicitly define the demand for cash and demand deposits as a decreasing function of their respective opportunity costs. To see this apply the functional forms for transaction costs functions $v(h_t)$ and $\psi(d_t)$ specified in (4.20) and substitute in the definition of the nominal interest rate in (3.6) to the first order conditions for cash and deposit demand to obtain their implicit demand functions:

$$h_t = \tilde{h} \left( \frac{i_{t+1}}{1 + i_{t+1}} \right), \quad \tilde{h}' < 0.$$  

(A7)

$$d_t = \tilde{d} \left( \frac{i_{t+1} - i_{t+1}^d}{1 + i_{t+1}} \right), \quad \tilde{d}' < 0.$$  

(A8)

Explicit solutions for $h_t$ and $d_t$ can be derived under perfect foresight.