INDEX NUMBER THEORY AND MEASUREMENT ECONOMICS

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CHAPTER 15: **The Treatment of Seasonal Products**

1. The Problem of Seasonal Commodities

The existence of seasonal commodities poses some significant challenges for price statisticians. *Seasonal commodities* are commodities which are either: (a) not available in the marketplace during certain seasons of the year or (b) are available throughout the year but there are regular fluctuations in prices or quantities that are synchronized with the season or the time of the year.[1] A commodity that satisfies (a) is termed a *strongly seasonal commodity* whereas a commodity which satisfies (b) will be called a *weakly seasonal commodity*. It is strongly seasonal commodities that create the biggest problems for price statisticians in the context of producing a monthly or quarterly Consumer Price Index because if a commodity price is available in only one of the two months (or quarters) being compared, then obviously it is not possible to calculate a relative price for the commodity and traditional bilateral index number theory breaks down. In other words, if a commodity is present in one month but not the next, how can the month to month amount of price change for that commodity be computed?[2] In this Chapter, a solution to this problem will be presented which "works" even if the commodities consumed are entirely different for each month of the year.[3]

There are two main sources of seasonal fluctuations in prices and quantities: (a) climate and (b) custom.[4] In the first category, fluctuations in temperature, precipitation and hours of daylight cause fluctuations in the demand or supply for many commodities; e.g., think of summer versus winter clothing, the demand for light and heat, vacations, etc. With respect to custom and convention as a cause of seasonal fluctuations consider the following quotation:

> "Conventional seasons have many origins—ancient religious observances, folk customs, fashions, business practices, statute law… Many of the conventional seasons have considerable effects on economic behaviour. We can count on active retail buying before Christmas, on the Thanksgiving demand for turkeys, on the first of July demand for fireworks, on the preparations for June weddings, on heavy dividend and interest payments at the beginning of each quarter, on an increase in bankruptcies in January, and so on." Wesley C. Mitchell (1927; 237).

[1] This classification of seasonal commodities corresponds to Balk's narrow and wide sense seasonal commodities; see Balk (1980a; 7) (1980b; 110) (1980c; 68). Diewert (1998b; 457) used the terms type 1 and type 2 seasonality.

[2] Zarnowitz (1961; 238) was perhaps the first to note the importance of this problem: "But the main problem introduced by the seasonal change is precisely that the market basket is different in the consecutive months (seasons), not only in weights but presumably often also in its very composition by commodities. This is a general and complex problem which will have to be dealt with separately at later stages of our analysis."

[3] However, the same commodities must reappear each year for each separate month!

[4] This classification dates back to Mitchell (1927; 236) at least: "Two types of seasons produce annually recurring variations in economic activity--those which are due to climates and those which are due to conventions."

Examples of important seasonal commodities are: many food items; alcoholic beverages; many clothing and footwear items; water; heating oil; electricity; flowers and garden supplies; vehicle purchases; vehicle operation; many entertainment and recreation expenditures; books, insurance expenditures; wedding expenditures; recreational equipment; toys and games; software; air travel and tourism expenditures. For a "typical" country, seasonal expenditures will often amount to one fifth to one third of all consumer expenditures.[5]

In the context of producing a monthly or quarterly Consumer Price Index, it must be recognized that there is no completely satisfactory way for dealing with strongly seasonal commodities. If a commodity is present in one month but missing from the market place in the next month, then none of the index number theories that were considered in earlier chapters can be applied because all of these theories assumed that the dimensionality of the commodity space was constant for the two periods being compared. However, if seasonal commodities are present in the market during each season, then, in theory, traditional index number theory can be applied in order to construct month to month or quarter to quarter price indices. This "traditional" approach to the treatment of seasonal commodities will be followed in sections 8, 9 and 10 below. The reason why this straightforward approach is deferred to the end of the chapter is twofold:

- The approach that restricts the index to commodities that are present in every period often does not work well in the sense that systematic *biases* can occur.
- The approach is not fully *representative*; i.e., it does not make use of information on commodities that are not present in every month or quarter.

In section 2, a modified version of Turvey's (1979) artificial data set is introduced. This data set will be used in order to numerically evaluate all of the index number formula that are suggested in this chapter. It will be seen in section 7 that very large seasonal fluctuations in quantities combined with systematic seasonal changes in price can make month to month or quarter to quarter price indexes behave rather poorly.

Even though existing index number theory cannot deal satisfactorily with seasonal commodities in the context of constructing month to month indexes of consumer prices, it can deal satisfactorily with seasonal commodities if the focus is changed from month to month indexes to indexes that compare the prices of one month with the prices of the *same* month in a previous year. Thus in section 3 below, *year over year monthly Consumer Price Indexes* are studied.[6] Turvey's seasonal data set is used to evaluate the performance of these indexes and they are found to perform quite well.

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[5] Alterman, Diewert and Feenstra (1999; 151) found that over the 40 months between September 1993 and December 1996, somewhere between 23 and 40 percent of U.S. imports and exports exhibited seasonal variations in quantities whereas only about 5 percent of U.S. export and import prices exhibited seasonal fluctuations.

[6] The same theory applies to producer price indexes.

In section 4, the year over year monthly indexes defined in section 3 are aggregated into an *annual index* that compares all of the monthly prices in a given calendar year with the corresponding monthly prices in a base year. In section 5, this idea of comparing the prices of a current calendar year with the corresponding prices in a base year is extended to annual indexes that compare the prices of the last 12 months with the corresponding prices in the 12 months of a base year. The resulting *rolling year indexes* can be regarded as seasonally adjusted price indexes. The modified Turvey data set is used to test out these year over year indexes and they are found to work very well on this data set.

The rolling year indexes can provide an accurate gauge of the movement of prices in the current rolling year compared to the base year. However, this measure of price inflation can be regarded as a measure of inflation for a year that is centered around a month that is six months prior to the last month in the current rolling year. Hence for some policy purposes, this type of index is not as useful as an index that compares the prices of the current month to the previous month so that more up to date information on the movement of prices can be obtained. However, in section 6, it will be shown that under certain conditions, the current month year over year monthly index, along with last month's year over year monthly index, can successfully *predict* or *forecast* a rolling year index that is centered around the current month.

The year over year indexes defined in section 3 and their annual averages studied in sections 4 and 5 offer a theoretically satisfactory method for dealing with *strongly seasonal commodities*; i.e., commodities that are available only during certain seasons of the year. However, these methods rely on the year over year comparison of prices and hence these methods cannot be used in the month to month or quarter to quarter type of index, which is typically the main focus of a consumer price program. Thus there is a need for another type of index, which may not have very strong theoretical foundations, but which can deal with seasonal commodities in the context of producing a *month to month index*. In section 7, such an index is introduced and it is implemented using the artificial data set for the commodities that are available during each month of the year. Unfortunately, due to the seasonality in both prices and quantities in the always available commodities, this type of index can be systematically biased and for the modified Turvey data set, this bias shows up.

Since many Consumer Price Indexes are month to month indexes that use *annual basket quantity weights*, this type of index is studied in section 8. For months when the commodity is not available in the marketplace, the last available price is carried forward and used in the index. In section 9, an annual quantity basket is again used but instead of carrying forward the prices of seasonally unavailable items, an imputation method is used to fill in the missing prices. The annual basket type indices defined in sections 8 and 9 are implemented using the artificial data set. Unfortunately, the empirical results are not satisfactory in that the indexes show tremendous seasonal fluctuations in prices so that they would not be suitable for users who wanted up to date information on *trends* in general inflation.

In section 10, the artificial data set is used in order to evaluate another type of month to month index that is frequently suggested in the literature on how to deal with seasonal commodities; namely the *Bean and Stine Type C* (1924) or *Rothwell* (1958) index. Again, this index does not get rid of the tremendous seasonal fluctuations that are present in the modified Turvey data set.

Sections 8 and 9 will show that the annual basket type indexes with carry forward of missing prices (section 8) or imputation of missing prices (section 9) do not get rid of seasonal fluctuations in prices. However, in section 11, it is shown how seasonally adjusted versions of these annual basket indexes can be used to successfully *forecast* rolling year indexes that are centered in the current month. In addition, the results in section 11 show how these annual basket type indexes can be seasonally adjusted (using information obtained from rolling year indexes from prior periods or by using traditional seasonal adjustment procedures) and hence these seasonally adjusted annual basket indexes could be used as successful indicators of general inflation on a timely basis.

Section 12 concludes.

2. A Seasonal Commodity Data Set

It will prove to be useful to illustrate the index number formulae that will be defined in subsequent sections by computing them for an actual data set. Turvey (1979) constructed an artificial data set for 5 seasonal commodities (apples, peaches, grapes, strawberries and oranges) for 4 years by month so that there are 5 times 4 times 12 observations, equal to 240 observations in all. At certain times of the year, peaches and strawberries (commodities 2 and 4) are unavailable so in Tables 1 and 2, the prices and quantities for these two commodities are entered as zeros.[7] The data in Tables 1 and 2 are essentially equal to that constructed by Turvey except that a number of adjustments were made to it in order to illustrate various points. The two most important adjustments were:

- The data for commodity 3 (grapes) were adjusted so that the annual Laspeyres and Paasche indexes (which will be defined in section 4 below) would differ more than in the original data set.[8]
- After the above adjustments were made, each price in the last year of data was escalated by the monthly inflation factor 1.008 so that month to month inflation for the last year of data would be at an approximate monthly rate of 1.6% per month compared to about 0.8% per month for the first three years of data.[9]

[7] The corresponding prices are not zeros but they are entered as zeros for convenience in programming the various indexes.

[8] After the first year, the price data for grapes was adjusted downward by 30% each year and the corresponding volume was adjusted upward by 40% each year. In addition, the quantity of oranges (commodity 5) for November 1971 was changed from 3548 to 8548 so that the seasonal pattern of change for this commodity would be similar to that of other years. For similar reasons, the price of oranges in December 1970 was changed from 1.31 to 1.41 and in January 1971 from 1.35 to 1.45.

[9] Pierre Duguay of the Bank of Canada, while commenting on a preliminary version of this chapter, observed that rolling year indexes would not be able to detect the *magnitude* of systematic changes in the month to month inflation rate. The original Turvey data set was roughly consistent with a month to month

Table 1: An Artificial Seasonal Data Set: Prices

Year t	Month m	$p_1^{t,m}$	$p_2^{t,m}$	$p_3^{t,m}$	$p_4^{t,m}$	$p_5^{t,m}$
1970	1	1.14	0	2.48	0	1.30
	2	1.17	0	2.75	0	1.25
	3	1.17	0	5.07	0	1.21
	4	1.40	0	5.00	0	1.22
	5	1.64	0	4.98	5.13	1.28
	6	1.75	3.15	4.78	3.48	1.33
	7	1.83	2.53	3.48	3.27	1.45
	8	1.92	1.76	2.01	0	1.54
	9	1.38	1.73	1.42	0	1.57
	10	1.10	1.94	1.39	0	1.61
	11	1.09	0	1.75	0	1.59
	12	1.10	0	2.02	0	1.41
1971	1	1.25	0	2.15	0	1.45
	2	1.36	0	2.55	0	1.36
	3	1.38	0	4.22	0	1.37
	4	1.57	0	4.36	0	1.44
	5	1.77	0	4.18	5.68	1.51
	6	1.86	3.77	4.08	3.72	1.56
	7	1.94	2.85	2.61	3.78	1.66
	8	2.02	1.98	1.79	0	1.74
	9	1.55	1.80	1.28	0	1.76
	10	1.34	1.95	1.26	0	1.77
	11	1.33	0	1.62	0	1.76
	12	1.30	0	1.81	0	1.50
1972	1	1.43	0	1.89	0	1.56
	2	1.53	0	2.38	0	1.53
	3	1.59	0	3.59	0	1.55
	4	1.73	0	3.90	0	1.62
	5	1.89	0	3.56	6.21	1.70
	6	1.98	4.69	3.51	3.98	1.78
	7	2.07	3.32	2.73	4.30	1.89
	8	2.12	2.29	1.65	0	1.91
	9	1.73	1.90	1.15	0	1.92
	10	1.56	1.97	1.15	0	1.95
	11	1.56	0	1.46	0	1.94
	12	1.49	0	1.73	0	1.64
1973	1	1.68	0	1.62	0	1.69
	2	1.82	0	2.16	0	1.69
	3	1.89	0	3.02	0	1.74
	4	2.00	0	3.45	0	1.91
	5	2.14	0	3.08	7.17	2.03

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inflation rate of 0.8 % per month; i.e., prices grew roughly at the rate 1.008 each month over the 4 years of data. Hence this second major adjustment of the Turvey data was introduced to illustrate Duguay's observation, which is quite correct: the centered rolling year indexes pick up the correct magnitude of the new inflation rate only after a lag of half a year or so. However, they do quickly pick up the direction of change in the inflation rate.

6	2.23	6.40	3.07	4.53	2.13
7	2.35	4.31	2.41	5.19	2.22
8	2.40	2.98	1.49	0	2.26
9	2.09	2.21	1.08	0	2.22
10	2.03	2.18	1.08	0	2.31
11	2.05	0	1.36	0	2.34
12	1.90	0	1.57	0	1.97

Table 2: An Artificial Seasonal Data Set: Quantities

Year t	Month m	$q_1^{t,m}$	$q_2^{t,m}$	$q_3^{t,m}$	$q_4^{t,m}$	$q_5^{t,m}$
1970	1	3086	0	82	0	10266
	2	3765	0	35	0	9656
	3	4363	0	9	0	7940
	4	4842	0	8	0	5110
	5	4439	0	26	700	4089
	6	5323	91	75	2709	3362
	7	4165	498	82	1970	3396
	8	3224	6504	1490	0	2406
	9	4025	4923	2937	0	2486
	10	5784	865	2826	0	3222
	11	6949	0	1290	0	6958
	12	3924	0	338	0	9762
1971	1	3415	0	119	0	10888
	2	4127	0	45	0	10314
	3	4771	0	14	0	8797
	4	5290	0	11	0	5590
	5	4986	0	74	806	4377
	6	5869	98	112	3166	3681
	7	4671	548	132	2153	3748
	8	3534	6964	2216	0	2649
	9	4509	5370	4229	0	2726
	10	6299	932	4178	0	3477
	11	7753	0	1831	0	8548
	12	4285	0	496	0	10727
1972	1	3742	0	172	0	11569
	2	4518	0	67	0	10993
	3	5134	0	22	0	9621
	4	5738	0	16	0	6063
	5	5498	0	137	931	4625
	6	6420	104	171	3642	3970
	7	5157	604	202	2533	4078
	8	3881	7378	3269	0	2883
	9	4917	5839	6111	0	2957
	10	6872	1006	5964	0	3759
	11	8490	0	2824	0	8238
	12	5211	0	731	0	11827
1973	1	4051	0	250	0	12206
	2	4909	0	102	0	11698
	3	5567	0	30	0	10438

4	6253	0	25	0	6593
5	6101	0	220	1033	4926
6	7023	111	252	4085	4307
7	5671	653	266	2877	4418
8	4187	7856	4813	0	3165
9	5446	6291	8803	0	3211
10	7377	1073	8778	0	4007
11	9283	0	4517	0	8833
12	4955	0	1073	0	12558

Turvey sent his artificial data set to statistical agencies around the world, asking them to use their normal techniques to construct monthly and annual average price indexes. About 20 countries replied and Turvey summarized the responses as follows:

"It will be seen that the monthly indices display very large differences, e.g., a range of 129.12 -169.50 in June, while the range of simple annual means is much smaller. It will also be seen that the indices vary as to the peak month or year." Ralph Turvey (1979; 13).

The above (modified) data will be used to test out various index number formulae in subsequent sections.

3. Year over Year Monthly Indexes

It can be seen that the existence of seasonal commodities that are present in the marketplace in one month but not the next causes the accuracy of a month to month index to fall.[10] A way of dealing with these strongly seasonal commodities is to change the focus from short term month to month price indexes and instead focus on making year over year price comparisons for each month of the year. In the latter type of comparison, there is a good chance that seasonal commodities that appear say in February will also appear in subsequent Februarys so that the overlap of commodities will be maximized in these year over year monthly indexes.

For over a century, it has been recognized that making year over year comparisons[11] provides the simplest method for making comparisons that are free from the contaminating effects of seasonal fluctuations:

"In the daily market reports, and other statistical publications, we continually find comparisons between numbers referring to the week, month, or other parts of the year, and those for the corresponding parts of a previous year. The comparison is given in this way in order to avoid any variation due to the time of the year. And it is obvious to everyone that this precaution is necessary. Every branch of industry and commerce must be affected more or less by the revolution of the seasons, and we must allow for what is due to this cause before we can learn what is due to other causes." W. Stanley Jevons (1884;3).

[10] In the limit, if each commodity appeared in only one month of the year, then a month to month index would break down completely.

[11] In the seasonal price index context, this type of index corresponds to Bean and Stine's (1924; 31) Type D index.

The economist Flux and the statistician Yule also endorsed the idea of making year over year comparisons to minimize the effects of seasonal fluctuations:

"Each month the average price change compared with the corresponding month of the previous year is to be computed. … The determination of the proper seasonal variations of weights, especially in view of the liability of seasons to vary from year to year, is a task from which, I imagine, most of us would be tempted to recoil." A. W. Flux (1921; 184-185).

"My own inclination would be to form the index number for any month by taking ratios to the corresponding month of the year being used for reference, the year before presumably, as this would avoid any difficulties with seasonal commodities. I should then form the annual average by the geometric mean of the monthly figures." G. Udny Yule (1921; 199).

In more recent times, Zarnowitz also endorsed the use of year over year monthly indexes:

"There is of course no difficulty in measuring the average price change between the same months of successive years, if a month is our unit 'season', and if a constant seasonal market basket can be used, for traditional methods of price index construction can be applied in such comparisons." Victor Zarnowitz (1961; 266).

In the remainder of this section, it is shown how year over year Fisher indexes and approximations to them can be constructed.[12] For each month m = 1,2,...,12, let S(m) denote the set of commodities that are available in the marketplace for each year t = 0,1,...,T. For t = 0,1,...,T and m = 1,2,...,12, let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of commodity n that is in the marketplace in month m of year t for n belongs to S(m). Let $p^{t,m}$ and $q^{t,m}$ denote the month m and year t price and quantity vectors respectively. Then *the year over year monthly Laspeyres, Paasche and Fisher indexes* going from month m of year t to month m of year t+1 can be defined as follows:

$$(1)\ P_L(p^{t,m},p^{t+1,m},q^{t,m}) \equiv \sum_{n\in S(m)} p_n^{t+1,m}q_n^{t,m} / \sum_{n\in S(m)} p_n^{t,m}q_n^{t,m} ; \qquad m = 1,2,...12;$$

$$(2)\ P_P(p^{t,m},p^{t+1,m},q^{t+1,m}) \equiv \sum_{n\in S(m)} p_n^{t+1,m}q_n^{t+1,m} / \sum_{n\in S(m)} p_n^{t,m}q_n^{t+1,m} ; \qquad m = 1,2,...12;$$

$$(3)\ P_F(p^{t,m},p^{t+1,m},q^{t,m},q^{t+1,m}) \equiv [P_L(p^{t,m},p^{t+1,m},q^{t,m})\, P_P(p^{t,m},p^{t+1,m},q^{t+1,m})]^{1/2} ; \qquad m = 1,2,...,12.$$

The above formulae can be rewritten in price relative and monthly expenditure share form as follows:

$$(4)\ P_L(p^{t,m},p^{t+1,m},s^{t,m}) \equiv \sum_{n\in S(m)} s_n^{t,m}\,(p_n^{t+1,m}/p_n^{t,m}) ; \qquad m = 1,2,...12;$$

$$(5)\ P_P(p^{t,m},p^{t+1,m},s^{t+1,m}) \equiv [\sum_{n\in S(m)} s_n^{t+1,m}\,(p_n^{t+1,m}/p_n^{t,m})^{-1}]^{-1} ; \qquad m = 1,2,...12;$$

$$(6)\ P_F(p^{t,m},p^{t+1,m},s^{t,m},s^{t+1,m}) \equiv [P_L(p^{t,m},p^{t+1,m},s^{t,m})P_P(p^{t,m},p^{t+1,m},s^{t+1,m})]^{1/2} ; \qquad m = 1,2,...,12$$
$$= [\sum_{n\in S(m)} s_n^{t,m}\,(p_n^{t+1,m}/p_n^{t,m})]^{1/2}\,[\sum_{n\in S(m)} s_n^{t+1,m}\,(p_n^{t+1,m}/p_n^{t,m})^{-1}]^{-1/2}$$

[12] Diewert (1996b; 17-19) (1999a; 50) noted various separability restrictions on consumer preferences that would justify these year over year monthly indexes from the viewpoint of the economic approach to index number theory.

where the monthly expenditure share for commodity $n \in S(m)$ for month m in year t is defined as:

(7) $s_n^{t,m} \equiv p_n^{t,m} q_n^{t,m} / \sum_{i \in S(m)} p_i^{t,m} q_i^{t,m}$; $\quad m = 1,2,...,12$; $n \in S(m)$; $t = 0,1,...,T$

and $s^{t,m}$ denotes the vector of month m expenditure shares in year t, $[s_n^{t,m}]$ for $n \in S(m)$.

Current period expenditure shares $s_n^{t,m}$ are not likely to be available. Hence it will be necessary to approximate these shares using the corresponding expenditure shares from a base year 0.

Use the base period monthly expenditure share vectors $s^{0,m}$ in place of the vector of month m and year t expenditure shares $s^{t,m}$ in (4) and use the base period monthly expenditure share vectors $s^{0,m}$ in place of the vector of month m and year t+1 expenditure shares $s^{t+1,m}$ in (5). Similarly, replace the share vectors $s^{t,m}$ and $s^{t+1,m}$ in (6) by the base period expenditure share vector for month m, $s^{0,m}$. The resulting *approximate year over year monthly Laspeyres, Paasche and Fisher indexes* are defined by (8) to (10) below:[13]

(8) $P_{AL}(p^{t,m},p^{t+1,m},s^{0,m}) \equiv \sum_{n \in S(m)} s_n^{0,m} (p_n^{t+1,m}/p_n^{t,m})$; $\quad m = 1,2,...12;$

(9) $P_{AP}(p^{t,m},p^{t+1,m},s^{0,m}) \equiv [\sum_{n \in S(m)} s_n^{0,m} (p_n^{t+1,m}/p_n^{t,m})^{-1}]^{-1}$; $\quad m = 1,2,...12;$

(10) $P_{AF}(p^{t,m},p^{t+1,m},s^{0,m},s^{0,m}) \equiv [P_{AL}(p^{t,m},p^{t+1,m},s^{0,m})P_{AP}(p^{t,m},p^{t+1,m},s^{0,m})]^{1/2}$; $\quad m = 1,2,...,12$
$$= [\sum_{n \in S(m)} s_n^{0,m} (p_n^{t+1,m}/p_n^{t,m})]^{1/2} [\sum_{n \in S(m)} s_n^{0,m} (p_n^{t+1,m}/p_n^{t,m})^{-1}]^{-1/2} .$$

The approximate Fisher year over year monthly indexes defined by (10) will provide adequate approximations to their true Fisher counterparts defined by (6) only if the monthly expenditure shares for the base year 0 are not too different from their current year t and t+1 counterparts. Hence, it will be useful to construct the true Fisher indexes on a delayed basis in order to check the adequacy of the approximate Fisher indexes defined by (10).

The year over year monthly approximate Fisher indexes defined by (10) will normally have a certain amount of upward bias, since these indexes cannot reflect long term substitution of consumers towards commodities that are becoming relatively cheaper over time. This reinforces the case for computing true year over year monthly Fisher indexes defined by (6) on a delayed basis so that this substitution bias can be estimated.

[13] If the monthly expenditure shares for the base year, $s_n^{0,m}$, are all equal, then the approximate Fisher index defined by (10) reduces to Fisher's (1922; 472) formula 101. Fisher (1922; 211) observed that this index was empirically very close to the unweighted geometric mean of the price relatives, while Dalén (1992; 143) and Diewert (1995a; 29) showed analytically that these two indexes approximated each other to the second order. The equally weighted version of (10) was recommended as an elementary index by Carruthers, Sellwood and Ward (1980; 25) and Dalén (1992; 140).

Note that the approximate year over year monthly Laspeyres and Paasche indexes, P_{AL} and P_{AP} defined by (8) and (9) above, satisfy the following inequalities:

(11) $P_{AL}(p^{t,m},p^{t+1,m},s^{0,m})P_{AL}(p^{t+1,m},p^{t,m},s^{0,m}) \geq 1$; $m = 1,2,...,12$;
(12) $P_{AP}(p^{t,m},p^{t+1,m},s^{0,m})P_{AP}(p^{t+1,m},p^{t,m},s^{0,m}) \leq 1$; $m = 1,2,...,12$

with strict inequalities if the monthly price vectors $p^{t,m}$ and $p^{t+1,m}$ are not proportional to each other.[14] The inequality (11) says that the approximate year over year monthly Laspeyres index *fails the time reversal test* with an upward bias while the inequality (12) says that the approximate year over year monthly Paasche index *fails the time reversal test* with a downward bias. Hence the fixed weight approximate Laspeyres index P_{AL} has a built in upward bias and the fixed weight approximate Paasche index P_{AP} has a built in downward bias. *Statistical agencies should avoid the use of these formulae*. However, they can be combined as in the approximate Fisher formula (10) and the resulting index should be free from any systematic formula bias (but there still could be some substitution bias).

The year over year monthly indexes defined in this section are illustrated using the artificial data set tabled in section 2 above. Although fixed base indexes were not formally defined in this section, these indexes have similar formulae to the year over year indexes that were defined in this section except that the variable base year t is replaced by the fixed base year 0. The resulting 12 year over year monthly fixed base Laspeyres, Paasche and Fisher indexes, are listed in Tables 3 to 5.

Table 3: Year over Year Monthly Fixed Base Laspeyres Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2060	1.2442	1.3062	1.2783	1.2184	1.1734	1.2364	1.1827	1.1049	1.1809	1.2550	1.1960
1973	1.3281	1.4028	1.4968	1.4917	1.4105	1.3461	1.4559	1.4290	1.2636	1.4060	1.5449	1.4505

Table 4: Year over Year Monthly Fixed Base Paasche Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1.1218	1.0824
1972	1.2023	1.2436	1.3038	1.2773	1.2024	1.1657	1.2307	1.1455	1.0695	1.1274	1.2218	1.1901
1973	1.3190	1.4009	1.4912	1.4882	1.3715	1.3266	1.4433	1.3122	1.1664	1.2496	1.4296	1.4152

Table 5: Year over Year Monthly Fixed Base Fisher Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1.1251	1.0837
1972	1.2041	1.2439	1.3050	1.2778	1.2104	1.1695	1.2336	1.1640	1.0870	1.1538	1.2383	1.1930

[14] See Hardy, Littlewood and Pólya (1934; 26).

1973	1.3235	1.4019	1.4940	1.4900	1.3909	1.3363	1.4496	1.3694	1.2140	1.3255	1.4861	1.4327

Comparing the entries in Tables 3 and 4, it can be seen that the year over year monthly fixed base Laspeyres and Paasche price indexes do not differ substantially for the early months of the year but that there are substantial differences between the indexes for the last 5 months of the year by the time the year 1973 is reached. The largest percentage difference between the Laspeyres and Paasche indexes is 12.5% for month 10 in 1973 (1.4060/1.2496 = 1.125). However, all of the year over year monthly series show a nice smooth year over year trend.

Approximate fixed base year over year Laspeyres, Paasche and Fisher indexes can be constructed by replacing current month expenditure shares for the 5 commodities by the corresponding base year monthly expenditure shares on the 5 commodities. The resulting approximate Laspeyres indexes are equal to the original fixed base Laspeyres indexes so there is no need to table the approximate Laspeyres indexes. However the approximate year over year Paasche and Fisher indexes do differ from the fixed base Paasche and Fisher indexes found in Tables 4 and 5 above so these new approximate indexes are listed in Tables 6 and 7.

Table 6: Year over Year Approximate Monthly Fixed Base Paasche Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1.1209	1.0813
1972	1.2025	1.2421	1.3036	1.2757	1.2110	1.1640	1.2267	1.1567	1.0788	1.1309	1.2244	1.1862
1973	1.3165	1.3947	1.4880	1.4858	1.3926	1.3223	1.4297	1.3315	1.1920	1.2604	1.4461	1.4184

Table 7: Year over Year Approximate Monthly Fixed Base Fisher Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1.1247	1.0831
1972	1.2043	1.2432	1.3049	1.2770	1.2147	1.1687	1.2316	1.1696	1.0918	1.1557	1.2396	1.1911
1973	1.3223	1.3987	1.4924	1.4888	1.4015	1.3341	1.4428	1.3794	1.2273	1.3312	1.4947	1.4344

Comparing the entries in Table 4 with the corresponding entries in Table 6, it can be seen that with a few exceptions, the entries correspond fairly closely. One of the bigger differences is the 1973 entry for the fixed base Paasche index for month 9, which is 1.1664, while the corresponding entry for the approximate fixed base Paasche index is 1.1920 for a 2.2% difference (1.1920 /1.1664 = 1.022). In general, the approximate fixed base Paasche indexes are a bit bigger than the true fixed base Paasche indexes, as could be expected, since the approximate indexes have some substitution bias built into them as their expenditure shares are held fixed at the 1970 levels.

Turning now to the chained year over year monthly indexes using the artificial data set, the resulting 12 year over year monthly chained Laspeyres, Paasche and Fisher indexes, P_L, P_P and P_F, where the month to month links are defined by (4) to (6), are listed in Tables 8 to 10.

Table 8: Year over Year Monthly Chained Laspeyres Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2058	1.2440	1.3058	1.2782	1.2154	1.1720	1.2357	1.1753	1.0975	1.1690	1.2491	1.1943
1973	1.3274	1.4030	1.4951	1.4911	1.4002	1.3410	1.4522	1.3927	1.2347	1.3593	1.5177	1.4432

Table 9: Year over Year Monthly Chained Paasche Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1074	1.1070	1.1471	1.1486	1.1115	1.0827	1.1075	1.0699	1.0414	1.0762	1.1218	1.0824
1972	1.2039	1.2437	1.3047	1.2777	1.2074	1.1682	1.2328	1.1569	1.0798	1.1421	1.2321	1.1908
1973	1.3243	1.4024	1.4934	1.4901	1.3872	1.3346	1.4478	1.3531	1.2018	1.3059	1.4781	1.4305

Table 10: Year over Year Monthly Chained Fisher Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1080	1.1069	1.1474	1.1487	1.1137	1.0835	1.1089	1.0741	1.0453	1.0831	1.1251	1.0837
1972	1.2048	1.2438	1.3052	1.2780	1.2114	1.1701	1.2343	1.1660	1.0886	1.1555	1.2405	1.1926
1973	1.3258	1.4027	1.4942	1.4906	1.3937	1.3378	1.4500	1.3728	1.2181	1.3323	1.4978	1.4368

Comparing the entries in Tables 8 and 9, it can be seen that the year over year monthly chained Laspeyres and Paasche price indexes have smaller differences than the corresponding fixed base Laspeyres and Paasche price indexes in Tables 3 and 4. This is a typical pattern: *the use of chained indexes tends to reduce the spread between Paasche and Laspeyres indexes compared to their fixed base counterparts*. The largest percentage difference between corresponding entries for the chained Laspeyres and Paasche indexes in Tables 8 and 9 is 4.1% for month 10 in 1973 (1.3593/1.3059 = 1.041). Recall that the fixed base Laspeyres and Paasche indexes differed by 12.5% for the same month so that *chaining does tend to reduce the spread between these two equally plausible indexes*.

The chained year over year Fisher indexes listed in Table 10 are regarded as the "best" estimates of year over year inflation using the artificial data set.

The year over year chained Laspeyres, Paasche and Fisher indexes listed in Tables 8 to 10 above can be approximated by replacing current period commodity expenditure shares for each month by the corresponding base year monthly commodity expenditure shares. The resulting 12 year over year monthly approximate chained Laspeyres, Paasche and Fisher indexes, P_{AL}, P_{AP} and P_{AF}, where the monthly links are defined by (8) to (10), are listed in Tables 11 to 13.

Table 11: Year over Year Monthly Approximate Chained Laspeyres Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1085	1.1068	1.1476	1.1488	1.1159	1.0844	1.1103	1.0783	1.0492	1.0901	1.1284	1.0849
1972	1.2056	1.2440	1.3057	1.2778	1.2168	1.1712	1.2346	1.1770	1.0989	1.1692	1.2482	1.1939
1973	1.3255	1.4007	1.4945	1.4902	1.4054	1.3390	1.4491	1.4021	1.2429	1.3611	1.5173	1.4417

Table 12: Year over Year Monthly Approximate Chained Paasche Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1077	1.1057	1.1468	1.1478	1.1135	1.0818	1.1062	1.0721	1.0426	1.0760	1.1209	1.0813
1972	1.2033	1.2424	1.3043	1.2764	1.2130	1.1664	1.2287	1.1638	1.0858	1.1438	1.2328	1.1886
1973	1.3206	1.3971	1.4914	1.4880	1.3993	1.3309	1.4386	1.3674	1.2183	1.3111	1.4839	1.4300

Table 13: Year over Year Monthly Approximate Chained Fisher Indexes

Month	1	2	3	4	5	6	7	8	9	10	11	12
1970	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1.1081	1.1063	1.1472	1.1483	1.1147	1.0831	1.1082	1.0752	1.0459	1.0830	1.1247	1.0831
1972	1.2044	1.2432	1.3050	1.2771	1.2149	1.1688	1.2317	1.1704	1.0923	1.1565	1.2405	1.1912
1973	1.3231	1.3989	1.4929	1.4891	1.4024	1.3349	1.4438	1.3847	1.2305	1.3358	1.5005	1.4358

The year over year chained indexes listed in Tables 11 to 13 approximate their true chained counterparts listed in Tables 8 to 10 very closely. For the year 1973, the largest discrepancies are for the Paasche and Fisher indexes for month 9: the chained Paasche is 1.2018 while the corresponding approximate chained Paasche is 1.2183 for a difference of 1.4% and the chained Fisher is 1.2181 while the corresponding approximate chained Fisher is 1.2305 for a difference of 1.0%. It can be seen that for the modified Turvey data set, the approximate year over year monthly approximate Fisher indexes listed in Table 13 approximate the theoretically preferred (but practically infeasible in a timely fashion) Fisher chained indexes listed in Table 10 quite satisfactorily. Since the approximate Fisher indexes are just as easy to compute as the approximate Laspeyres and Paasche indexes, it may be useful to ask that statistical agencies make available to the public these approximate Fisher indexes along with the approximate Laspeyres and Paasche indexes.

4. Year over Year Annual Indexes

Assuming that each commodity in each season of the year is a separate "annual" commodity is the simplest and theoretically most satisfactory method for dealing with seasonal commodities when the goal is to construct annual price and quantity indexes. This idea can be traced back to Mudgett in the consumer price context and to Stone in the producer price context:

"The basic index is a yearly index and as a price or quantity index is of the same sort as those about which books and pamphlets have been written in quantity over the years." Bruce D. Mudgett (1955; 97).

"The existence of a regular seasonal pattern in prices which more or less repeats itself year after year suggests very strongly that the varieties of a commodity available at different seasons cannot be transformed into one another without cost and that, accordingly, in all cases where seasonal variations in price are significant, the varieties available at different times of the year should be treated, in principle, as separate commodities." Richard Stone (1956; 74-75).

Using the notation introduced in the previous section, the *Laspeyres, Paasche and Fisher annual (chain link) indexes* comparing the prices of year t with those of year t+1 can be defined as follows:

$$(13)\ P_L(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t,1},...,q^{t,12}) \equiv \sum_{m=1}^{12} \sum_{n\in S(m)} p_n^{t+1,m}q_n^{t,m} / \sum_{m=1}^{M} \sum_{n\in S(m)} p_n^{t,m}q_n^{t,m} ;$$

$$(14)\ P_P(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t+1,1},...,q^{t+1,12}) \equiv \sum_{m=1}^{12} \sum_{n\in S(m)} p_n^{t+1,m}q_n^{t+1,m} / \sum_{m=1}^{M} \sum_{n\in S(m)} p_n^{t,m}q_n^{t+1,m} ;$$

$$(15)\ P_F(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t,1},...,q^{t,12};q^{t+1,1},...,q^{t+1,12}) \equiv [P_L(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t,1},...,q^{t,12})P_P(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};q^{t+1,1},...,q^{t+1,12})]^{1/2}.$$

The above formulae can be rewritten in price relative and monthly expenditure share form as follows:

$$(16)\ P_L(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};\sigma_1{}^t s^{t,1},...,\sigma_{12}{}^t s^{t,12}) \equiv \sum_{m=1}^{12} \sum_{n\in S(m)} \sigma_m{}^t s_n{}^{t,m} (p_n{}^{t+1,m}/p_n{}^{t,m}) = \sum_{m=1}^{12} \sigma_m{}^t P_L(p^{t,m},p^{t+1,m},s^{t,m}) ;$$

$$(17)\ P_P(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};\sigma_1{}^{t+1} s^{t+1,1},...,\sigma_{12}{}^{t+1} s^{t+1,12}) \equiv [\sum_{m=1}^{12} \sum_{n\in S(m)} \sigma_m{}^{t+1} s_n{}^{t+1,m} (p_n{}^{t+1,m}/p_n{}^{t,m})^{-1}]^{-1} = [\sum_{m=1}^{12} \sigma_m{}^{t+1} \sum_{n\in S(m)} s_n{}^{t+1,m} (p_n{}^{t+1,m}/p_n{}^{t,m})^{-1}]^{-1} = [\sum_{m=1}^{12} \sigma_m{}^{t+1} [P_P(p^{t,m},p^{t+1,m},s^{t+1,m})]^{-1}]^{-1} ;$$

$$(18)\ P_F(p^{t,1},...,p^{t,12};p^{t+1,1},...,p^{t+1,12};\sigma_1{}^t s^{t,1},...,\sigma_{12}{}^t s^{t,12};\sigma_1{}^{t+1} s^{t+1,1},...,\sigma_{12}{}^{t+1} s^{t+1,12}) \equiv [\{\sum_{m=1}^{12} \sum_{n\in S(m)} \sigma_m{}^t s_n{}^{t,m} (p_n{}^{t+1,m}/p_n{}^{t,m})\}\{\sum_{m=1}^{12} \sum_{n\in S(m)} \sigma_m{}^{t+1} s_n{}^{t+1,m} (p_n{}^{t+1,m}/p_n{}^{t,m})^{-1}]^{-1}\}]^{1/2} = [\sum_{m=1}^{12} \sigma_m{}^t P_L(p^{t,m},p^{t+1,m},s^{t,m})]^{1/2}[\sum_{m=1}^{12} \sigma_m{}^{t+1} [P_P(p^{t,m},p^{t+1,m},s^{t+1,m})]^{-1}]^{-1/2}$$

where *the expenditure share for month m in year t* is defined as:

$$(19)\ \sigma_m{}^t \equiv \sum_{n\in S(m)} p_n{}^{t,m}q_n{}^{t,m} / \sum_{i=1}^{12} \sum_{j\in S(i)} p_j{}^{t,i}q_j{}^{t,i} ; \qquad m = 1,2,...,12 ; t = 0,1,...,T$$

and the year over year monthly Laspeyres and Paasche (chain link) price indexes $P_L(p^{t,m},p^{t+1,m},s^{t,m})$ and $P_P(p^{t,m},p^{t+1,m},s^{t+1,m})$ were defined in the previous section by (4) and (5) respectively. As usual, the annual chain link Fisher index P_F defined by (18), which compares the prices in every month of year t with the corresponding prices in year t+1, is the geometric mean of the annual chain link Laspeyres and Paasche indexes, P_L and P_P, defined by (16) and (17). The last equation in (16), (17) and (18) shows that these annual indexes can be defined as (monthly) share weighted averages of the year over year monthly chain link Laspeyres and Paasche indexes, $P_L(p^{t,m},p^{t+1,m},s^{t,m})$ and $P_P(p^{t,m},p^{t+1,m},s^{t+1,m})$, defined earlier by (4) and (5). Hence once the year over year monthly indexes defined in the previous section have been numerically calculated, it is easy to calculate the corresponding annual indexes.

Fixed base counterparts to the formulae defined by (16) to (18) can readily be defined: simply replace the data pertaining to period t by the corresponding data pertaining to the base period 0.

Using the data from the artificial data set tabled in section 2 above, the annual fixed base Laspeyres, Paasche and Fisher indexes are listed in Table 14.

Table 14: Annual Fixed Base Laspeyres, Paasche and Fisher Price Indexes

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2091	1.1884	1.1987
1973	1.4144	1.3536	1.3837

Viewing Table 14, it can be seen that by 1973, the annual fixed base Laspeyres index exceeds its Paasche counterpart by 4.5%. Note that each series increases steadily.

The annual fixed base Laspeyres, Paasche and Fisher indexes can be approximated by replacing any current shares by the corresponding base year shares. The resulting annual approximate fixed base Laspeyres, Paasche and Fisher indexes are listed in Table 15. Also listed in the last column of Table 15 is the fixed base Geometric Laspeyres annual index, P_{GL}. It is the weighted geometric mean counterpart to the fixed base Laspeyres index, which is equal to a base period weighted arithmetic average of the long term price relatives. It can be shown that P_{GL} approximates the approximate fixed base Fisher index P_{AF} to the second order around a point where all of the long term price relatives are equal to unity.[15]

Table 15: Annual Approximate Fixed Base Laspeyres, Paasche, Fisher and Geometric Laspeyres Indexes

Year	P_{AL}	P_{AP}	P_{AF}	P_{GL}
1970	1.0000	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982	1.0983
1972	1.2091	1.1903	1.1996	1.2003
1973	1.4144	1.3596	1.3867	1.3898

It can be seen that the entries for the Laspeyres price indexes are exactly the same in Tables 14 and 15. This is as it should be because the fixed base Laspeyres price index uses only expenditure shares from the base year 1970 and hence the approximate fixed base Laspeyres index is equal to the true fixed base Laspeyres index. Comparing the columns labelled P_P and P_F in Table 14 and P_{AP} and P_{AF} in Table 15 shows that the approximate Paasche and approximate Fisher indexes are quite close to the corresponding

[15] See footnote 13 above.

annual Paasche and Fisher indexes. Hence for the artificial data set, *the true annual fixed base Fisher can be very closely approximated by the corresponding approximate Fisher index* P_{AF} (or the Geometric Laspeyres index P_{GL}), *which, of course, can be computed using the same information set that is normally available to statistical agencies*.

Using the data from the artificial data set tabled in section 2 above, the annual chained Laspeyres, Paasche and Fisher indexes can readily be calculated, using the formulae (16) to (18) for the chain links. The resulting indexes are listed in Table 16.

Table 16: Annual Chained Laspeyres, Paasche and Fisher Price Indexes

Year	P_L	P_P	P_F
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0961	1.0984
1972	1.2052	1.1949	1.2001
1973	1.3994	1.3791	1.3892

Viewing Table 16, it can be seen that the use of chained indexes has substantially narrowed the gap between the Paasche and Laspeyres indexes. The difference between the chained annual Laspeyres and Paasche indexes in 1973 is only 1.5% (1.3994 versus 1.3791) whereas from Table 14, the difference between the fixed base annual Laspeyres and Paasche indexes in 1973 is 4.5% (1.4144 versus 1.3536). *Thus the use of chained annual indexes has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indexes*. Comparing Tables 14 and 16, it can be seen that for this particular artificial data set, the annual fixed base Fisher indexes are very close to their annual chained Fisher counterparts. However, the annual chained Fisher indexes should normally be regarded as the more desirable target index to approximate, since this index will normally give better results if prices and expenditure shares are changing substantially over time.[16]

Obviously, the current year weights, $s_n^{t,m}$ and σ_m^{t} and $s_n^{t+1,m}$ and σ_m^{t+1}, which appear in the chain link formulae (16) to (18) can be approximated by the corresponding base year weights, $s_n^{0,m}$ and σ_m^{0}. This leads to the annual approximate chained Laspeyres, Paasche and Fisher indexes listed in Table 17.

Table 17: Annual Approximate Chained Laspeyres, Paasche and Fisher Price Indexes

Year	P_{AL}	P_{AP}	P_{AF}
1970	1.0000	1.0000	1.0000
1971	1.1008	1.0956	1.0982
1972	1.2051	1.1952	1.2002
1973	1.3995	1.3794	1.3894

[16] "Better" in the sense that the gap between the Laspeyres and Paasche indices will be normally be reduced using chained indices under these circumstances. Of course, if there are no substantial trends in prices so that prices are just randomly changing, then it will generally be preferable to use the fixed base Fisher index.

Comparing the entries in Tables 16 and 17 shows that the approximate chained annual Laspeyres, Paasche and Fisher indexes are extremely close to the corresponding true chained annual Laspeyres, Paasche and Fisher indexes. Hence for the artificial data set, the true annual chained Fisher can be very closely approximated by the corresponding approximate Fisher index, which can be computed using the same information set that is normally available to statistical agencies.

The approach to computing annual indexes outlined in this section, which essentially involves taking monthly expenditure share weighted averages of the 12 year over year monthly indexes, should be contrasted with the approach that simply takes the arithmetic mean of the 12 monthly indexes. The problem with the latter approach is that months where expenditures are below the average (e.g., February) are given the same weight in the unweighted annual average as months where expenditures are above the average (e.g., December).

5. Rolling Year Annual Indices

In the previous section, the price and quantity data pertaining to the 12 months of a calendar year were compared to the 12 months of a base calendar year. However, there is no need to restrict attention to calendar year comparisons: any 12 consecutive months of price and quantity data could be compared to the price and quantity data of the base year, provided that the January data in the noncalendar year is compared to the January data of the base year, the February data of the noncalendar year is compared to the February data of the base year, …, and the December data of the noncalendar year is compared to the December data of the base year.[17] Alterman, Diewert and Feenstra (1999; 70) called the resulting indices *rolling year* or *moving year* indexes.[18]

In order to theoretically justify the rolling year indexes from the viewpoint of the economic approach to index number theory, some restrictions on preferences are required. The details of these assumptions can be found in Diewert (1996b; 32-34) (1999a; 56-61).

The problems involved in constructing rolling year indexes for the artificial data set that was introduced in section 2 are now considered. For both fixed base and chained rolling year indexes, the first 13 index number calculations are the same. For the year that ends with the data for December of 1970, the index is set equal to 1 for the Laspeyres, Paasche and Fisher moving year indexes. The base year data are the 44 nonzero price and quantity observations for the calendar year 1970. When the data for January of 1971 become available, the 3 nonzero price and quantity entries for January of calendar year 1970 are dropped and replaced with the corresponding entries for January of 1971. The

[17] Diewert (1983c) suggested this type of comparison and termed the resulting index a "split year" comparison.
[18] Crump (1924; 185) and Mendershausen (1937; 245) respectively used these terms in the context of various seasonal adjustment procedures. The term "rolling year" seems to be well established in the business literature in the UK.

data for the remaining months of the comparison year remain the same; i.e., for February through December of the comparison year, the data for the rolling year are set equal to the corresponding entries for February through December of 1970. Thus the Laspeyres, Paasche or Fisher rolling year index value for January of 1971 compares the prices and quantities of January 1971 with the corresponding prices and quantities of January 1970 and for the remaining months of this first moving year, the prices and quantities of February through December of 1970 are simply compared with the exact same prices and quantities of February through December of 1970. When the data for February of 1971 become available, the 3 nonzero price and quantity entries for February for the last rolling year (which are equal to the 3 nonzero price and quantity entries for February of 1970) are dropped and replaced with the corresponding entries for February of 1971 and the resulting data become the price and quantity data for the second rolling year. The Laspeyres, Paasche or Fisher rolling year index value for February of 1971 compares the prices and quantities of January and February of 1971 with the corresponding prices and quantities of January and February of 1970 and for the remaining months of this first moving year, the prices and quantities of March through December of 1970 are compared with the exact same prices and quantities of March through December of 1970. This process of exchanging the price and quantity data of the current month in 1971 with the corresponding data of the same month in the base year 1970 in order to form the price and quantity data for the latest rolling year continues until December of 1971 is reached when the current rolling year becomes the calendar year 1971. Thus the Laspeyres, Paasche and Fisher rolling year indexes for December of 1971 are equal to the corresponding fixed base (or chained) annual Laspeyres, Paasche and Fisher indexes for 1971 listed in Tables 14 or 16 above.

Once the first 13 entries for the rolling year indexes have been defined as indicated above, the remaining fixed base rolling year Laspeyres, Paasche and Fisher indexes are constructed by taking the price and quantity data of the last 12 months and rearranging the data so that the January data in the rolling year is compared to the January data in the base year, the February data in the rolling year is compared to the February data in the base year,..., and the December data in the rolling year is compared to the December data in the base year. The resulting fixed base rolling year Laspeyres, Paasche and Fisher indexes for the artificial data set are listed in Table 18.

Once the first 13 entries for the fixed base rolling year indexes have been defined as indicated above, the remaining *chained* rolling year Laspeyres, Paasche and Fisher indexes are constructed by taking the price and quantity data of the last 12 months and comparing these data to the corresponding data of the rolling year of the 12 months preceding the current rolling year. The resulting chained rolling year Laspeyres, Paasche and Fisher indexes for the artificial data set are listed in the last 3 columns of Table 18. Note that the first 13 entries of the fixed base Laspeyres, Paasche and Fisher indexes are equal to the corresponding entries for the chained Laspeyres, Paasche and Fisher indexes. It will also be noted that the entries for December (month 12) of 1970, 1971, 1972 and 1973 for the fixed base rolling year Laspeyres, Paasche and Fisher indexes are equal to the corresponding fixed base annual Laspeyres, Paasche and Fisher indexes listed in Table 14 above. Similarly, the entries in Table 18 for December (month 12) of 1970, 1971, 1972 and 1973 for the chained rolling year Laspeyres, Paasche and Fisher indexes are equal to the corresponding chained annual Laspeyres, Paasche and Fisher indexes listed in Table 16 above.

Table 18: Rolling Year Laspeyres, Paasche and Fisher Price Indexes

Year	Month	P_L (fixed)	P_P (fixed)	P_F (fixed)	P_L (chain)	P_P (chain)	P_F (chain)
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0087	1.0085	1.0082	1.0087	1.0085
	2	1.0161	1.0170	1.0165	1.0161	1.0170	1.0165
	3	1.0257	1.0274	1.0265	1.0257	1.0274	1.0265
	4	1.0344	1.0364	1.0354	1.0344	1.0364	1.0354
	5	1.0427	1.0448	1.0438	1.0427	1.0448	1.0438
	6	1.0516	1.0537	1.0527	1.0516	1.0537	1.0527
	7	1.0617	1.0635	1.0626	1.0617	1.0635	1.0626
	8	1.0701	1.0706	1.0704	1.0701	1.0706	1.0704
	9	1.0750	1.0740	1.0745	1.0750	1.0740	1.0745
	10	1.0818	1.0792	1.0805	1.0818	1.0792	1.0805
	11	1.0937	1.0901	1.0919	1.0937	1.0901	1.0919
	12	1.1008	1.0961	1.0984	1.1008	1.0961	1.0984
1972	1	1.1082	1.1035	1.1058	1.1081	1.1040	1.1061
	2	1.1183	1.1137	1.1160	1.1183	1.1147	1.1165
	3	1.1287	1.1246	1.1266	1.1290	1.1260	1.1275
	4	1.1362	1.1324	1.1343	1.1366	1.1342	1.1354
	5	1.1436	1.1393	1.1414	1.1437	1.1415	1.1426
	6	1.1530	1.1481	1.1505	1.1528	1.1505	1.1517
	7	1.1645	1.1595	1.1620	1.1644	1.1622	1.1633
	8	1.1757	1.1670	1.1713	1.1747	1.1709	1.1728
	9	1.1812	1.1680	1.1746	1.1787	1.1730	1.1758
	10	1.1881	1.1712	1.1796	1.1845	1.1771	1.1808
	11	1.1999	1.1805	1.1901	1.1962	1.1869	1.1915
	12	1.2091	1.1884	1.1987	1.2052	1.1949	1.2001
1973	1	1.2184	1.1971	1.2077	1.2143	1.2047	1.2095
	2	1.2300	1.2086	1.2193	1.2263	1.2172	1.2218
	3	1.2425	1.2216	1.2320	1.2393	1.2310	1.2352
	4	1.2549	1.2341	1.2444	1.2520	1.2442	1.2481
	5	1.2687	1.2469	1.2578	1.2656	1.2579	1.2617
	6	1.2870	1.2643	1.2756	1.2835	1.2758	1.2797
	7	1.3070	1.2843	1.2956	1.3038	1.2961	1.3000
	8	1.3336	1.3020	1.3177	1.3273	1.3169	1.3221
	9	1.3492	1.3089	1.3289	1.3395	1.3268	1.3331
	10	1.3663	1.3172	1.3415	1.3537	1.3384	1.3460
	11	1.3932	1.3366	1.3646	1.3793	1.3609	1.3700
	12	1.4144	1.3536	1.3837	1.3994	1.3791	1.3892

Viewing Table 18, it can be seen that the rolling year indices are very smooth and free from seasonal fluctuations. For the fixed base indexes, each entry can be viewed as a *seasonally adjusted annual consumer price index* that compares the data of the 12 consecutive months that end with the year and month indicated with the corresponding price and quantity data of the 12 months in the base year, 1970. Thus rolling year indexes offer statistical agencies an *objective* and *reproducible* method of seasonal adjustment that can compete with existing time series methods of seasonal adjustment.[19]

Viewing Table 18, it can be seen that the use of chained indexes has substantially narrowed the gap between the fixed base moving year Paasche and Laspeyres indexes. The difference between the rolling year chained Laspeyres and Paasche indexes in December of 1973 is only 1.5% (1.3994 versus 1.3791) whereas the difference between the rolling year fixed base Laspeyres and Paasche indexes in December of 1973 is 4.5% (1.4144 versus 1.3536). *Thus the use of chained indexes has substantially reduced the substitution (or representativity) bias of the Laspeyres and Paasche indexes*. As in the previous section, the chained Fisher rolling year index is regarded as the *target seasonally adjusted annual index* when seasonal commodities are in the scope of the CPI. This type of index is also a suitable index for central banks to use for inflation targeting purposes.[20] The six series in Table 18 are charted in Figure 1. The fixed base Laspeyres index is the highest one, followed by the chained Laspeyres, the two Fisher indexes (which are virtually indistinguishable), the chained Paasche and finally, the fixed base Paasche is the lowest index. An increase in the slope of each graph can clearly be seen for the last 8 months, reflecting the increase in the month to month inflation rates that was built into the data for the last 12 months of the data set.[21]

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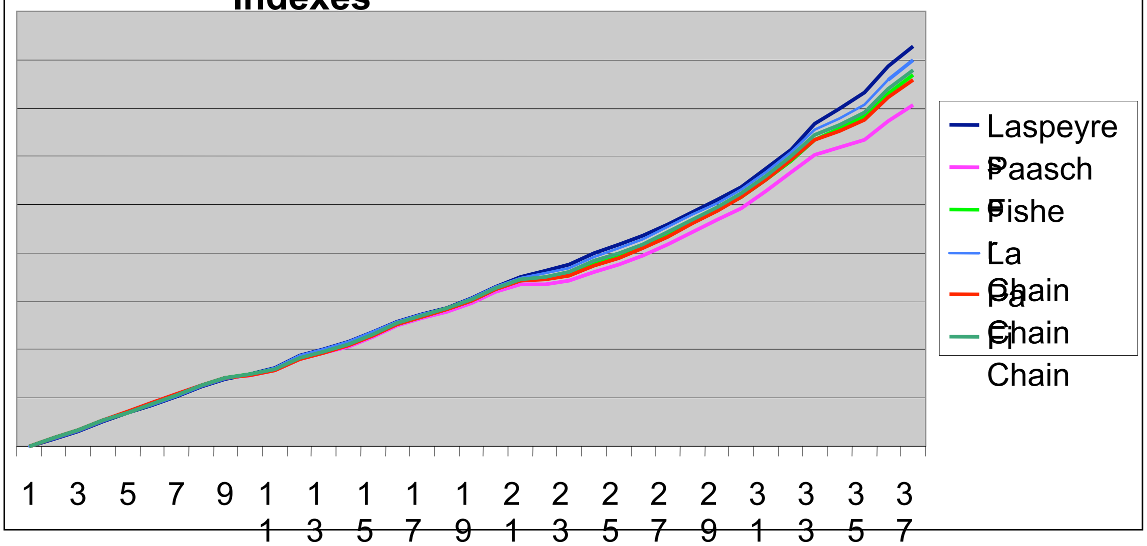
[19] For discussions on the merits of econometric or time series methods versus index number methods of seasonal adjustment, see Diewert (1999a; 61-68) and Alterman, Diewert and Feenstra (1999; 78-110). The basic problem with time series methods of seasonal adjustment is that the target seasonally adjusted index is very difficult to specify in an unambiguous way; i.e., there are an infinite number of possible target indexes. For example, it is impossible to identify a temporary increase in inflation within a year from a changing seasonal factor. Hence different econometricians will tend to generate different seasonally adjusted series, leading to a lack of reproducibility.

[20] See Diewert (2002c) for a discussion of the measurement issues involved in choosing an index for inflation targeting purposes.

[21] The arithmetic average of the 36 month over month inflation rates for the rolling year fixed base Fisher indexes is 1.0091; the average of these rates for the first 24 months is 1.0076, for the last 12 months is 1.0120 and for the last 2 months is 1.0156. Hence the increased month to month inflation rates for the last year are not *fully* reflected in the rolling year indices until a full 12 months have passed. However, the fact that inflation has *increased* for the last 12 months of data compared to the earlier months is picked up almost immediately.

Figure 1: Rolling Year Fixed Base and Chained Laspeyres Paasche and Fisher Indexes



As in the previous section, the current year weights, $s_n^{t,m}$ and σ_m^{t} and $s_n^{t+1,m}$ and σ_m^{t+1}, which appear in the chain link formulae (16) to (18) or in the corresponding fixed base formulae can be approximated by the corresponding base year weights, $s_n^{0,m}$ and σ_m^{0}. This leads to the annual approximate fixed base and chained rolling year Laspeyres, Paasche and Fisher indexes listed in Table 19.

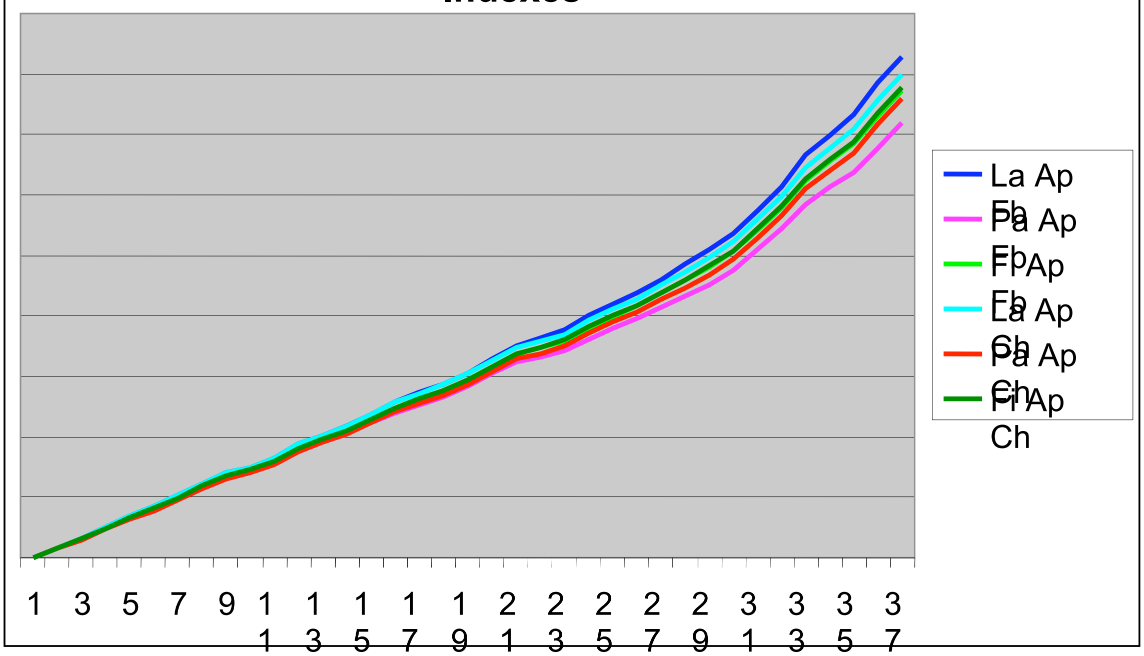
Table 19: Rolling Year Approximate Laspeyres, Paasche and Fisher Price Indexes

Year	Month	P_{AL} (fixed)	P_{AP} (fixed)	P_{AF} (fixed)	P_{AL} (chain)	P_{AP} (chain)	P_{AF} (chain)
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0082	1.0074	1.0078	1.0082	1.0074	1.0078
	2	1.0161	1.0146	1.0153	1.0161	1.0146	1.0153
	3	1.0257	1.0233	1.0245	1.0257	1.0233	1.0245
	4	1.0344	1.0312	1.0328	1.0344	1.0312	1.0328
	5	1.0427	1.0390	1.0409	1.0427	1.0390	1.0409
	6	1.0516	1.0478	1.0497	1.0516	1.0478	1.0497
	7	1.0617	1.0574	1.0596	1.0617	1.0574	1.0596
	8	1.0701	1.0656	1.0679	1.0701	1.0656	1.0679
	9	1.0750	1.0702	1.0726	1.0750	1.0702	1.0726
	10	1.0818	1.0764	1.0791	1.0818	1.0764	1.0791
	11	1.0937	1.0881	1.0909	1.0937	1.0881	1.0909
	12	1.1008	1.0956	1.0982	1.1008	1.0956	1.0982
1972	1	1.1082	1.1021	1.1051	1.1083	1.1021	1.1052
	2	1.1183	1.1110	1.1147	1.1182	1.1112	1.1147
	3	1.1287	1.1196	1.1241	1.1281	1.1202	1.1241
	4	1.1362	1.1260	1.1310	1.1354	1.1268	1.1311
	5	1.1436	1.1326	1.1381	1.1427	1.1336	1.1381
	6	1.1530	1.1415	1.1472	1.1520	1.1427	1.1473

	7	1.1645	1.1522	1.1583	1.1632	1.1537	1.1584
	8	1.1757	1.1620	1.1689	1.1739	1.1642	1.1691
	9	1.1812	1.1663	1.1737	1.1791	1.1691	1.1741
	10	1.1881	1.1710	1.1795	1.1851	1.1747	1.1799
	11	1.1999	1.1807	1.1902	1.1959	1.1855	1.1907
	12	1.2091	1.1903	1.1996	1.2051	1.1952	1.2002
1973	1	1.2184	1.1980	1.2082	1.2142	1.2033	1.2087
	2	1.2300	1.2074	1.2187	1.2253	1.2133	1.2193
	3	1.2425	1.2165	1.2295	1.2367	1.2235	1.2301
	4	1.2549	1.2261	1.2404	1.2482	1.2340	1.2411
	5	1.2687	1.2379	1.2532	1.2615	1.2464	1.2540
	6	1.2870	1.2548	1.2708	1.2795	1.2640	1.2717
	7	1.3070	1.2716	1.2892	1.2985	1.2821	1.2903
	8	1.3336	1.2918	1.3125	1.3232	1.3048	1.3139
	9	1.3492	1.3063	1.3276	1.3386	1.3203	1.3294
	10	1.3663	1.3182	1.3421	1.3538	1.3345	1.3441
	11	1.3932	1.3387	1.3657	1.3782	1.3579	1.3680
	12	1.4144	1.3596	1.3867	1.3995	1.3794	1.3894

Comparing the indexes in Tables 18 and 19, it can be seen that the approximate rolling year fixed base and chained Laspeyres, Paasche and Fisher indexes listed in Table 19 are very close to their true rolling year counterparts listed in Table 18. In particular, the approximate chain rolling year Fisher index (which can be computed using just base year expenditure share information along with current information on prices) is very close to the preferred target index, the rolling year chained Fisher index. In December of 1973, these two indexes differ by only 0.014 % (1.3894/1.3892 = 1.00014). The indexes in Table 19 are charted in Figure 2. It can be seen that Figures 1 and 2 are very similar; in particular, the Fisher fixed base and chained indexes are virtually identical in both figures.

Figure 2: Rolling Year Approximate Fixed Base and Chained Laspeyres Paasche and Fisher Indexes



From the above tables, it can be seen that year over year monthly indexes and their generalizations to rolling year indexes perform very well using the modified Turvey data set; i.e., like is compared to like and the existence of seasonal commodities does *not* lead to erratic fluctuations in the indexes. The only drawback to the use of these indexes is that it seems that they cannot give any information on *short term month to month fluctuations in prices*. *This is most evident if seasonal baskets are totally different for each month since in this case, there is no possibility of comparing prices on a month to month basis*. However, in the following section, it is shown how a current period year over year monthly index *can* be used to predict a rolling year index that is centered at the current month.

6. Predicting a Rolling Year Index using a Current Period Year over Year Monthly Index

It might be conjectured that under a regime where the long run trend in prices is smooth, changes in the year over year inflation rate for this month compared to last month could give valuable information about the long run trend in price inflation. For the modified Turvey data set, this conjecture turns out to be true as will be seen below.

The basic idea will be illustrated using the fixed base Laspeyres rolling year indexes that are listed in Table 18 and the year over year monthly fixed base Laspeyres indexes listed in Table 3. In Table 18, the fixed base Laspeyres rolling year entry for December of 1971 compares the 12 months of price and quantity data pertaining to 1971 with the corresponding prices and quantities pertaining to 1970. This index number is the first entry in the first column of Table 20 and is labelled as P_L. Thus in the first column of Table 20, the fixed base rolling year Laspeyres index, P_{LRY} taken from Table 18, is tabled starting at December of 1971 and carrying through to December of 1973, which is 24 observations in all. Looking at the first entry of this column, it can be seen that the index is a weighted average of year over year price relatives over all 12 months in 1970 and 1971. Thus this index is an average of year over year monthly price changes, centered between June and July of the two years whose prices are being compared. Hence, an *approximation* to this annual index could be obtained by taking the arithmetic average of the June and July year over year monthly indexes pertaining to the years 1970 and 1971 (see the entries for months 6 and 7 for the year 1971 in Table 3, 1.0844 and 1.1103).[22] For the next rolling year fixed base Laspeyres index corresponding to the January of 1972 entry in Table 18, *an approximation to this rolling year index*, P_{ARY}, could be obtained by taking the arithmetic average of the July and August year over year monthly indexes pertaining to the years 1970 and 1971 (see the entries for months 7 and 8 for the year 1971 in Table 3, 1.1103 and 1.0783). These arithmetic averages of the two year over year monthly indexes that are in the middle of the corresponding rolling year are listed in the third column of Table 20. From Table 20, it can be seen that column 3, P_{ARY}, does not approximate column 1 particularly well, since the approximate indexes in column 3 are seen to have some pronounced seasonal fluctuations whereas the rolling year indexes in column 1, P_{LRY}, are free from seasonal fluctuations.

In the fourth column of Table 20, some *seasonal adjustment factors* are listed. For the first 12 observations, the entries in column 4 are simply the ratios of the entries in column 1 divided by the corresponding entries in column 3; i.e., for the first 12 observations, the seasonal adjustment factors, SAF, are simply the ratio of the rolling year indexes starting at December of 1971 divided by the arithmetic average of the two year over year monthly indexes that are in the middle of the corresponding rolling year.[23] The initial 12 seasonal adjustment factors are then just repeated for the remaining entries for column 4.

Once the seasonal adjustment factors have been defined, then the approximate rolling year index P_{ARY} can be multiplied by the corresponding seasonal adjustment factor, SAF, in order to form a *seasonally adjusted approximate rolling year index*, P_{SAARY}, which is listed in column 2 of Table 20.

Table 20: Rolling Year Fixed Base Laspeyres and Seasonally Adjusted Approximate Rolling Year Price Indexes

[22] Obviously, if an average of the year over year monthly indexes for May, June, July and August were taken, a better approximation to the annual index could be obtained and if an average of the year over year monthly indexes for April, May, June, July, August and September were taken, an even better approximation could be obtained to the annual index and so on.

[23] Thus if SAF is *greater than one*, this means that the two months in the middle of the corresponding rolling year have year over year rates of price increase that average out to a number *below* the overall average of the year over year rates of price increase for the entire rolling year and conversely if SAF is less than one.

Year	Month	P_{LRY}	P_{SAARY}	P_{ARY}	SAF
1971	12	1.1008	1.1008	1.0973	1.0032
1972	1	1.1082	1.1082	1.0943	1.0127
	2	1.1183	1.1183	1.0638	1.0512
	3	1.1287	1.1287	1.0696	1.0552
	4	1.1362	1.1362	1.1092	1.0243
	5	1.1436	1.1436	1.1066	1.0334
	6	1.1530	1.1530	1.1454	1.0066
	7	1.1645	1.1645	1.2251	0.9505
	8	1.1757	1.1757	1.2752	0.9220
	9	1.1812	1.1812	1.2923	0.9141
	10	1.1881	1.1881	1.2484	0.9517
	11	1.1999	1.1999	1.1959	1.0033
	12	1.2091	1.2087	1.2049	1.0032
1973	1	1.2184	1.2249	1.2096	1.0127
	2	1.2300	1.2024	1.1438	1.0512
	3	1.2425	1.2060	1.1429	1.0552
	4	1.2549	1.2475	1.2179	1.0243
	5	1.2687	1.2664	1.2255	1.0334
	6	1.2870	1.2704	1.2620	1.0066
	7	1.3070	1.2979	1.3655	0.9505
	8	1.3336	1.3367	1.4498	0.9220
	9	1.3492	1.3658	1.4943	0.9141
	10	1.3663	1.3811	1.4511	0.9517
	11	1.3932	1.3827	1.3783	1.0032
	12	1.4144	1.4188	1.4010	1.0127

Comparing columns 1 and 2 in Table 20, the rolling year fixed base Laspeyres index P_{LRY} and the seasonally adjusted approximate rolling year index P_{SAARY} are identical for the first 12 observations, which follows by construction since P_{SAARY} equals the approximate rolling year index P_{ARY} multiplied by the seasonal adjustment factor SAR which in turn is equal to the rolling year Laspeyres index P_{LRY} divided by P_{ARY}. However, starting at December of 1972, the rolling year index P_{LRY} differs from the corresponding seasonally adjusted approximate rolling year index P_{SAARY}. It can be seen that for these last 13 months, P_{SAARY} is surprisingly close to P_{LRY}.[24]

[24] The means for the last 13 observations in columns 1 and 2 of Table 20 are 1.2980 and 1.2930. A regression of P_L on P_{SAARY} leads to an R^2 of 0.9662 with an estimated variance of the residual of .000214.

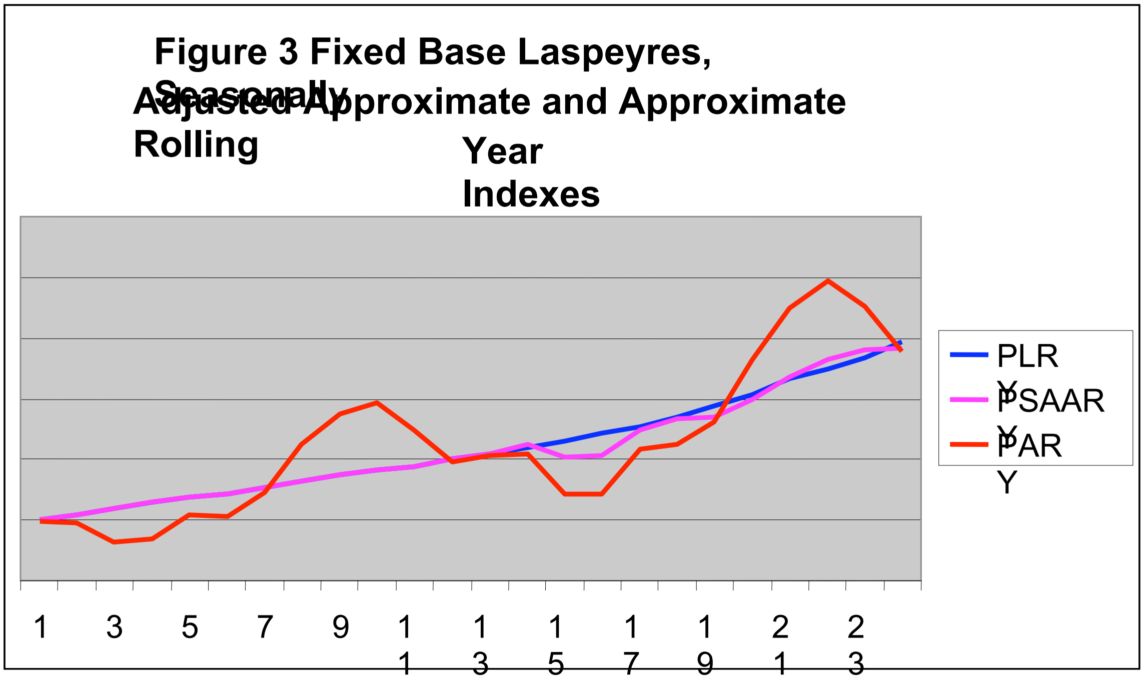


Figure 3 Fixed Base Laspeyres, Seasonally Adjusted Approximate and Approximate Rolling Year Indexes

P_{LRY}, P_{SAARY} and P_{ARY} are graphed in Figure 3. Due to the acceleration in the monthly inflation rate for the last year of data, it can be seen that the seasonally adjusted approximate rolling year series, P_{SAARY}, does not pick up this accelerated inflation rate for the first few months of the last year (it lies well below P_{LRY} for February and March of 1973) but in general, it predicts the corresponding centered year quite well.

The above results for the modified Turvey data set are quite encouraging. If these results can be replicated for other data sets, then it means that *statistical agencies can use the latest information on year over year monthly inflation to predict reasonably well the (seasonally adjusted) rolling year inflation rate for a rolling year that is centered around the last two months*. Thus policy makers and other interested users of the Consumer Price Index can obtain a reasonably accurate forecast of trend inflation (centered around the current month) some 6 months in advance before the final estimates are calculated.

The method of seasonal adjustment used in this section is rather crude compared to some of the sophisticated econometric or statistical methods that are available. Thus these more sophisticated methods could be used in order to improve the forecasts of trend inflation. However, it should be noted that if improved forecasting methods are used, it will be useful to use the rolling year indexes as *targets* for the forecasts rather than using a statistical package that simultaneously seasonally adjusts current data and calculates a trend rate of inflation. What is being suggested here is that the rolling year concept can be used in order to eliminate the lack of reproducibility in the estimates of trend inflation that existing statistical methods of seasonal generate.[25]

[25] The operator of a statistical seasonal adjustment package has to make somewhat arbitrary decisions on many factors; e.g., are the seasonal factors additive or multiplicative? How long should the moving

In this section and the previous sections, all of the suggested indexes have been based on year over year monthly indexes and their averages. In the subsequent sections of this chapter, attention will be turned to more traditional price indexes that attempt to compare the prices in the current month with the prices in a previous month.

7. Maximum Overlap Month to Month Price Indexes

A reasonable method for dealing with seasonal commodities in the context of picking a target index for a month to month CPI is the following one:[26]

- Determine the set of commodities that are present in the marketplace in both months of the comparison.
- For this maximum overlap set of commodities, calculate one of the three indexes recommended in previous chapters; i.e., calculate the Fisher, Walsh or Törnqvist Theil index.[27]

Thus the bilateral index number formula is applied only to the subset of commodities that are present in both periods.[28]

The question now arises: should the comparison month and the base month be adjacent months (thus leading to chained indexes) or should the base month be fixed (leading to fixed base indexes)? It seems reasonable to prefer chained indexes over fixed base indexes for two reasons:

- The set of seasonal commodities which overlaps during two consecutive months is likely to be much larger than the set obtained by comparing the prices of any given month with a fixed base month (like January of a base year). Hence the comparisons made using chained indexes will be more comprehensive and accurate than those made using a fixed base.
- In many economies, on average 2 or 3 percent of price quotes disappear each month due to the introduction of new commodities and the disappearance of older ones. This rapid sample attrition means that fixed base indexes rapidly become unrepresentative and hence it seems preferable to use chained indexes which can more closely follow marketplace developments.[29]

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average be and what type? Thus different operators of the seasonal adjustment package will tend to produce different estimates of the trend and the seasonal factors.

[26] For more on the economic approach and the assumptions on consumer preferences that can justify month to month maximum overlap indices, see Diewert (1999a; 51-56).

[27] In order to reduce the number of equations, definitions and tables, only the Fisher index will be considered in detail in this chapter.

[28] Keynes (1930; 95) called this the highest common factor method for making bilateral index number comparisons. Of course, this target index drops those strongly seasonal commodities that are not present in the marketplace during one of the two months being compared. Thus the index number comparison is not completely comprehensive. Mudgett (1951; 46) called the "error" in an index number comparison that is introduced by the highest common factor method (or maximum overlap method) the "homogeneity error".

[29] This rapid sample degradation essentially forces some form of chaining at the elementary level in any case.

It will be useful to review the notation at this point and define some new notation. Let there be N commodities that are available in some month of some year and let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of commodity n that is in the marketplace[30] in month m of year t (if the commodity is unavailable, define $p_n^{t,m}$ and $q_n^{t,m}$ to be 0). Let $p^{t,m} \equiv [p_1^{t,m},p_2^{t,m},...,p_N^{t,m}]$ and $q^{t,m} \equiv [q_1^{t,m},q_2^{t,m},...,q_N^{t,m}]$ be the month m and year t price and quantity vectors respectively. Let S(t,m) be the set of commodities that is present in month m of year t *and the following month*. Then the maximum overlap Laspeyres, Paasche and Fisher indexes going from month m of year t to the following month can be defined as follows:[31]

$$(20)\ P_L(p^{t,m},p^{t,m+1},q^{t,m},S(t,m)) \equiv \sum_{n\in S(t,m)} p_n^{t,m+1}q_n^{t,m} / \sum_{n\in S(t,m)} p_n^{t,m}q_n^{t,m}\ ; \qquad m = 1,2,...11;$$

$$(21)\ P_P(p^{t,m},p^{t,m+1},q^{t,m+1},S(t,m)) \equiv \sum_{n\in S(t,m)} p_n^{t,m+1}q_n^{t,m+1} / \sum_{n\in S(t,m)} p_n^{t,m}q_n^{t,m+1};\ m = 1,2,...11;$$

$$(22)\ P_F(p^{t,m},p^{t,m+1},q^{t,m},q^{t,m+1},S(t,m)) \equiv [P_L(p^{t,m},p^{t,m+1},q^{t,m},S(t,m))P_P(p^{t,m},p^{t,m+1},q^{t,m+1},S(t,m))]^{1/2}\ ;\ m = 1,2,...,11.$$

Note that P_L, P_P and P_F depend on the two (complete) price and quantity vectors pertaining to months m and m+1 of year t, $p^{t,m},p^{t,m+1},q^{t,m},q^{t,m+1}$, but they also depend on the set S(t,m), which is the set of commodities that are present in both months. Thus the commodity indices n that are in the summations on the right hand sides of (20) to (22) include indices n that correspond to commodities that are present in *both* months, which is the meaning of $n\in S(t,m)$; i.e., n belongs to the set S(t,m).

In order to rewrite definitions (20) to (22) in expenditure share and price relative form, some additional notation is required. Define the expenditure shares of commodity n in month m and m+1 of year t, using the set of commodities that are present in month m of year t and the subsequent month, as follows:

$$(23)\ s_n^{t,m}(t,m) \equiv p_n^{t,m}q_n^{t,m} / \sum_{i\in S(t,m)} p_i^{t,m}q_i^{t,m}\ ; \qquad n\in S(t,m)\ ;\ m = 1,2,...,11;$$
$$(24)\ s_n^{t,m+1}(t,m) \equiv p_n^{t,m+1}q_n^{t,m+1} / \sum_{i\in S(t,m)} p_i^{t,m+1}q_i^{t,m+1}\ ; \qquad n\in S(t,m)\ ;\ m = 1,2,...,11.$$

The notation in (23) and (24) is rather messy because $s_n^{t,m+1}(t,m)$ has to be distinguished from $s_n^{t,m+1}(t,m+1)$. The expenditure share $s_n^{t,m+1}(t,m)$ is the share of commodity n in month m+1 of year t but where n is restricted to the set of commodities that are present in month m of year t and the subsequent month whereas $s_n^{t,m+1}(t,m+1)$ is the share of commodity n in month m+1 of year t but where n is restricted to the set of commodities that are present in month m+1 of year t and the subsequent month. Thus the set of superscripts, t,m+1 in $s_n^{t,m+1}(t,m)$, indicates that the expenditure share is calculated using the price and quantity data of month m+1 of year t and (t,m) indicates that the set of

[30] It is necessary to have a target concept for the individual prices and quantities $p_n^{t,m}$ and $q_n^{t,m}$ at the finest level of aggregation. Under most circumstances, these target concepts can be taken to be unit values for prices and total quantities consumed for the quantities.

[31] The formulae are slightly different for the indices that go from December to January of the following year. In order to simplify the exposition, these formulae are left for the reader.

admissible commodities is restricted to the set of commodities that are present in both month m of year and the subsequent month.

Now define vectors of expenditure shares. If commodity n is present in month m of year t and the following month, define $s_n^{t,m}(t,m)$ using (23); if this is not the case, define $s_n^{t,m}(t,m) = 0$. Similarly, if commodity n is present in month m of year t and the following month, define $s_n^{t,m+1}(t,m)$ using (24); if this is not the case, define $s_n^{t,m+1}(t,m) = 0$. Now define the N dimensional vectors $s^{t,m}(t,m) \equiv [s_1^{t,m}(t,m),s_2^{t,m}(t,m),...,s_N^{t,m}(t,m)]$ and $s^{t,m+1}(t,m) \equiv [s_1^{t,m+1}(t,m),s_2^{t,m+1}(t,m),...,s_N^{t,m+1}(t,m)]$. Using these share definitions, the month to month Laspeyres, Paasche and Fisher formulae (20) to (22) can also be rewritten in expenditure share and price form as follows:

$$(25)\quad P_L(p^{t,m},p^{t,m+1},s^{t,m}(t,m)) \equiv \sum_{n\in S(t,m)} s_n^{t,m}(t,m)\,(p_n^{t,m+1}/p_n^{t,m})\,; \qquad m = 1,2,...11;$$

$$(26)\quad P_P(p^{t,m},p^{t,m+1},s^{t,m+1}(t,m)) \equiv [\sum_{n\in S(t,m)} s_n^{t,m+1}(t,m)\,(p_n^{t,m+1}/p_n^{t,m})^{-1}]^{-1}\,; \qquad m = 1,2,...11;$$

$$(27)\quad P_F(p^{t,m},p^{t,m+1},s^{t,m}(t,m),s^{t,m+1}(t,m)) \equiv [\sum_{n\in S(t,m)} s_n^{t,m}(t,m)\,(p_n^{t,m+1}/p_n^{t,m})]^{1/2}\,[\sum_{n\in S(t,m)} s_n^{t,m+1}(t,m)\,(p_n^{t,m+1}/p_n^{t,m})^{-1}]^{-1/2} \qquad m = 1,2,...,11.$$

It is important to recognize that the expenditure shares $s_n^{t,m}(t,m)$ that appear in the maximum overlap month to month Laspeyres index defined by (25) are *not* the expenditure shares that could be taken from a consumer expenditure survey for month m of year t: instead, they are the shares that result after expenditures on seasonal commodities that are present in month m of year t but are not present in the following month are dropped. Similarly, the expenditure shares $s_n^{t,m+1}(t,m)$ that appear in the maximum overlap month to month Paasche index defined by (26) are *not* the expenditure shares that could be taken from a consumer expenditure survey for month m+1 of year t: instead, they are the shares that result after expenditures on seasonal commodities that are present in month m+1 of year t but are not present in the preceding month are dropped.[32] The maximum overlap month to month Fisher index defined by (27) is the geometric mean of the Laspeyres and Paasche indexes defined by (25) and (26).

Table 21 lists the maximum overlap chained month to month Laspeyres, Paasche and Fisher price indexes for the data listed in section 2 above. These indexes are defined by equations (25), (26) and (27) above.

Table 21: Month to Month Maximum Overlap Chained Laspeyres, Paasche and Fisher Price Indexes

Year	Month	P_L	P_P	P_F
1970	1	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777
	3	0.9587	0.9594	0.9590

[32] It is important that the expenditure shares that are used in an index number formula add up to unity. The use of unadjusted expenditure shares from a household expenditure survey would lead to a systematic bias in the index number formula.

	4	1.0290	1.0534	1.0411
	5	1.1447	1.1752	1.1598
	6	1.1118	1.0146	1.0621
	7	1.1167	1.0102	1.0621
	8	1.1307	0.7924	0.9465
	9	1.0033	0.6717	0.8209
	10	0.9996	0.6212	0.7880
	11	1.0574	0.6289	0.8155
	12	1.0151	0.5787	0.7665
1971	1	1.0705	0.6075	0.8064
	2	1.0412	0.5938	0.7863
	3	1.0549	0.6005	0.7959
	4	1.1409	0.6564	0.8654
	5	1.2416	0.7150	0.9422
	6	1.1854	0.6006	0.8438
	7	1.2167	0.6049	0.8579
	8	1.2230	0.4838	0.7692
	9	1.0575	0.4055	0.6548
	10	1.0497	0.3837	0.6346
	11	1.1240	0.3905	0.6626
	12	1.0404	0.3471	0.6009
1972	1	1.0976	0.3655	0.6334
	2	1.1027	0.3679	0.6369
	3	1.1291	0.3765	0.6520
	4	1.1974	0.4014	0.6933
	5	1.2818	0.4290	0.7415
	6	1.2182	0.3553	0.6579
	7	1.2838	0.3637	0.6833
	8	1.2531	0.2794	0.5916
	9	1.0445	0.2283	0.4883
	10	1.0335	0.2203	0.4771
	11	1.1087	0.2256	0.5001
	12	1.0321	0.1995	0.4538
1973	1	1.0866	0.2097	0.4774
	2	1.1140	0.2152	0.4897
	3	1.1532	0.2225	0.5065
	4	1.2493	0.2398	0.5474
	5	1.3315	0.2544	0.5821
	6	1.2594	0.2085	0.5124
	7	1.3585	0.2160	0.5416
	8	1.3251	0.1656	0.4684
	9	1.0632	0.1330	0.3760
	10	1.0574	0.1326	0.3744
	11	1.1429	0.1377	0.3967
	12	1.0504	0.1204	0.3556

The chained maximum overlap Laspeyres, Paasche and Fisher indexes for December of 1973 are 1.0504, 0.1204 and 0.3556 respectively. Comparing these results to the year over year results listed in Tables 3, 4 and 5 indicates that the results in Table 21 are not at all realistic! These hugely different direct indexes compared with the last row of Table

21 indicate that *the maximum overlap indices suffer from a serious downward bias for the artificial data set.*

What are the factors that can explain this downward bias? It is evident that part of the problem has to do with the seasonal pattern of prices for peaches and strawberries (commodities 2 and 4). These are the commodities that are not present in the marketplace for each month of the year. For the first month of the year when these commodities become available, they come into the marketplace at relatively high prices and then in subsequent months, their prices drop substantially. The effects of these initially high prices (compared to the relatively low prices that prevailed in the last month that the commodities were available in the previous year) are not captured by the maximum overlap month to month indexes and so the resulting indexes build up a tremendous downward bias. The downward bias is most pronounced in the Paasche indexes, which use the quantities of the current month, which are relatively large compared to the quantities in the initial month when the commodities become available, reflecting the effects of lower prices as the quantity dumped in the market increases.

Table 22 lists the results using chained Laspeyres, Paasche and Fisher indexes for the artificial data set where the strongly seasonal commodities 2 and 4 are dropped from each comparison of prices. Thus the indexes in Table 22 are the usual chained Laspeyres, Paasche and Fisher indexes restricted to commodities 1,3 and 5, which are available in each season. These indexes are labelled as $P_L(3)$, $P_P(3)$ and $P_F(3)$.

Table 22: Month to Month Chained Laspeyres, Paasche and Fisher Price Indexes

Year	Month	$P_L(3)$	$P_P(3)$	$P_F(3)$	$P_L(2)$	$P_P(2)$	$P_F(2)$
1970	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	0.9766	0.9787	0.9777	0.9751	0.9780	0.9765
	3	0.9587	0.9594	0.9590	0.9522	0.9574	0.9548
	4	1.0290	1.0534	1.0411	1.0223	1.0515	1.0368
	5	1.1447	1.1752	1.1598	1.1377	1.1745	1.1559
	6	1.2070	1.2399	1.2233	1.2006	1.2424	1.2214
	7	1.2694	1.3044	1.2868	1.2729	1.3204	1.2964
	8	1.3248	1.1537	1.2363	1.3419	1.3916	1.3665
	9	1.0630	0.9005	0.9784	1.1156	1.1389	1.1272
	10	0.9759	0.8173	0.8931	0.9944	1.0087	1.0015
	11	1.0324	0.8274	0.9242	0.9839	0.9975	0.9907
	12	0.9911	0.7614	0.8687	0.9214	0.9110	0.9162
1971	1	1.0452	0.7993	0.9140	0.9713	0.9562	0.9637
	2	1.0165	0.7813	0.8912	0.9420	0.9336	0.9378
	3	1.0300	0.7900	0.9020	0.9509	0.9429	0.9469
	4	1.1139	0.8636	0.9808	1.0286	1.0309	1.0298
	5	1.2122	0.9407	1.0679	1.1198	1.1260	1.1229
	6	1.2631	0.9809	1.1131	1.1682	1.1763	1.1723
	7	1.3127	1.0170	1.1554	1.2269	1.2369	1.2319
	8	1.3602	0.9380	1.1296	1.2810	1.2913	1.2861
	9	1.1232	0.7532	0.9198	1.1057	1.0988	1.1022
	10	1.0576	0.7045	0.8632	1.0194	1.0097	1.0145
	11	1.1325	0.7171	0.9012	1.0126	1.0032	1.0079

	12	1.0482	0.6373	0.8174	0.9145	0.8841	0.8992
1972	1	1.1059	0.6711	0.8615	0.9652	0.9311	0.9480
	2	1.1111	0.6755	0.8663	0.9664	0.9359	0.9510
	3	1.1377	0.6912	0.8868	0.9863	0.9567	0.9714
	4	1.2064	0.7371	0.9430	1.0459	1.0201	1.0329
	5	1.2915	0.7876	1.0086	1.1202	1.0951	1.1075
	6	1.3507	0.8235	1.0546	1.1732	1.1470	1.1600
	7	1.4091	0.8577	1.0993	1.2334	1.2069	1.2201
	8	1.4181	0.7322	1.0190	1.2562	1.2294	1.2427
	9	1.1868	0.5938	0.8395	1.1204	1.0850	1.1026
	10	1.1450	0.5696	0.8076	1.0614	1.0251	1.0431
	11	1.2283	0.5835	0.8466	1.0592	1.0222	1.0405
	12	1.1435	0.5161	0.7682	0.9480	0.8935	0.9204
1973	1	1.2038	0.5424	0.8081	1.0033	0.9408	0.9715
	2	1.2342	0.5567	0.8289	1.0240	0.9639	0.9935
	3	1.2776	0.5755	0.8574	1.0571	0.9955	1.0259
	4	1.3841	0.6203	0.9266	1.1451	1.0728	1.1084
	5	1.4752	0.6581	0.9853	1.2211	1.1446	1.1822
	6	1.5398	0.6865	1.0281	1.2763	1.1957	1.2354
	7	1.6038	0.7136	1.0698	1.3395	1.2542	1.2962
	8	1.6183	0.6110	0.9944	1.3662	1.2792	1.3220
	9	1.3927	0.5119	0.8443	1.2530	1.1649	1.2081
	10	1.3908	0.5106	0.8427	1.2505	1.1609	1.2049
	11	1.5033	0.5305	0.8930	1.2643	1.1743	1.2184
	12	1.3816	0.4637	0.8004	1.1159	1.0142	1.0638

The chained Laspeyres, Paasche and Fisher indexes (using only the 3 always present commodities) for January of 1973 are 1.2038, 0.5424 and 0.8081 respectively. From Tables 8, 9 and 10, the chained year over year Laspeyres, Paasche and Fisher indexes for January of 1973 are 1.3274, 1.3243 and 1.3258 respectively. Thus the chained indexes using the always present commodities which are listed in Table 22 evidently *suffer from substantial downward biases*.

If the data in Tables 1 and 2 are examined, it can be seen that the quantities of grapes (commodity 3) on the marketplace varies tremendously over the course of a year with substantial increases in price for the months when grapes are almost out of season. Thus the price of grapes decreases substantially as the quantity in the marketplace increases during the last half of each year but the annual substantial increase in the price of grapes takes place in the first half of the year when quantities in the market are small. This pattern of seasonal price and quantity changes will cause the overall index to take on a downward bias.[33] To verify that this conjecture is true, see the last 3 columns of Table 22 where chained Laspeyres, Paasche and Fisher indexes are calculated using only commodities 1 and 5. These indexes are labelled as $P_L(2)$, $P_P(2)$ and $P_F(2)$ respectively and for January of 1973, they are equal to 1.0033, 0.9408 and 0.9715 respectively. These

[33] Baldwin (1990) used the Turvey data to illustrate various treatments of seasonal commodities and has a very good discussion of what causes various month to month indexes to behave badly. "It is a sad fact that for some seasonal commodity groups, monthly price changes are not meaningful, whatever the choice of formula." Andrew Baldwin (1990; 264).

estimates based on two always present commodities are much closer to the chained year over year Laspeyres, Paasche and Fisher indexes for January of 1973, which were 1.3274, 1.3243 and 1.3258 respectively, than the estimates based on the three always present commodities but it can be seen that *the chained Laspeyres, Paasche and Fisher indexes restricted to commodities 1 and 5 still have very substantial downward biases for the artificial data set.* Basically, the problems are caused by the high volumes associated with low or declining prices and the low volumes caused by high or rising prices. These weight effects make the seasonal price declines bigger than the seasonal price increases using month to month index number formulae with variable weights.[34]

In addition to the downward biases that show up in Tables 21 and 22, all of these month to month chained indexes show substantial seasonal fluctuations in prices over the course of a year. Hence these month to month indexes are of little use to policy makers who are interested in short term inflationary trends. Thus *if the purpose of the month to month consumer price index is to indicate changes in general inflation, then statistical agencies should be cautious about including commodities that show strong seasonal fluctuations in prices in the month to month index.*[35] If seasonal commodities are included in a month to month index that is meant to indicate general inflation, then a seasonal adjustment procedure should be used to remove these strong seasonal fluctuations. Some simple types of seasonal adjustment procedures will be considered in section 11 below.

The rather poor performance of the month to month indexes listed in the last two tables does not always occur in the context of seasonal commodities. In the context of calculating import and export price indexes using quarterly data for the U.S., Alterman, Diewert and Feenstra (1999) found that maximum overlap month to month indexes worked reasonably well.[36] However, statistical agencies should check that their month to month indexes are at least approximately consistent with the corresponding year over year indexes.

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[34] This remark has an application to the chapter on elementary indices where irregular sales during the course of a year could induce a similar downward bias in a month to month index that used monthly weights. Another problem with month to month chained indexes is that purchases and sales of individual commodities can become quite irregular as the time period becomes shorter and shorter and the problem of zero purchases and sales becomes more pronounced. Feenstra and Shapiro (2003; 125) find an *upward* bias for their chained *weekly* indexes for canned tuna compared to a fixed base index; their bias was caused by variable weight effects due to the timing of advertising expenditures. In general, these drift effects of chained indexes can be reduced by lengthening the time period, so that the *trends* in the data become more prominent than the *high frequency fluctuations*.

[35] However, if the purpose of the index is to compare the prices that consumers *actually face* in two consecutive months, ignoring the possibility that the consumer may regard a seasonal good as being qualitatively different in the two months, then the production of a month to month Consumer Price Index that has large seasonal fluctuations can be justified.

[36] They checked the validity of their month to month indexes by cumulating them for 4 quarters and comparing them to the corresponding year over year indices and found only relatively small differences. However, note that irregular high frequency fluctuations will tend to be smaller for quarters than for months and hence chained quarterly indexes can be expected to perform better than chained monthly or weekly indexes.

Obviously the various Paasche and Fisher indexes computed in this section could be approximated by indexes that replaced all current period expenditure shares by the corresponding expenditure shares from the base year. These approximate Paasche and Fisher indexes will not be reproduced here since they resemble their "true" counterparts and hence are also subject to tremendous downward bias.

8. Annual Basket Indexes with Carry Forward of Unavailable Prices

Recall that the Lowe (1823) index defined in earlier chapters had two reference periods:[37]

- A reference period for the vector of quantity weights and
- A reference period for the base period prices.

The *Lowe index* for month m say was defined by the following formula:

$$(28)\ P_{LO}(p^0,p^m,q) \equiv \sum_{n=1}^{N} p_n^m q_n / \sum_{n=1}^{N} p_n^0 q_n$$

where $p^0 \equiv [p_1^0,\ldots,p_N^0]$ is the base month price vector, $p^m \equiv [p_1^m,\ldots,p_N^m]$ is the current month m price vector and $q \equiv [q_1,\ldots,q_N]$ is the base year reference quantity vector. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the base year will be taken to be 1970 and the resulting base year quantity vector turns out to be:

$$(29)\ q \equiv [q_1,\ldots,q_5] = [53889, 12881, 9198, 5379, 68653].$$

The base period for the prices will be taken to be December of 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Lowe index with carry forward of missing prices using the modified Turvey data set can be found in column 1 of Table 23.

Baldwin's comments on this type of Annual Basket (AB) index are worth quoting at length:

"For seasonal goods, the AB index is best considered an index partially adjusted for seasonal variation. It is based on annual quantities, which do not reflect the seasonal fluctuations in the volume of purchases, and on raw monthly prices, which do incorporate seasonal price fluctuations. Zarnowitz (1961; 256-257) calls it an index of 'a hybrid sort'. Being neither of sea nor land, it does not provide an appropriate measure either of monthly or 12 month price change. The question that an AB index answers with respect to price change from January to February say, or January of one year to January of the next, is 'What would have the change in consumer prices have been if there were no seasonality in purchases in the months in question, but prices nonetheless retained their own seasonal behaviour?' It is hard to believe that this is a question that anyone would be interested in asking. On the other hand, the 12 month ratio of an AB index based on seasonally adjusted prices would be conceptually valid, if one were interested in eliminating seasonal influences." Andrew Baldwin (1990; 258).

[37] In the context of seasonal price indexes, this type of index corresponds to Bean and Stine's (1924; 31) Type A index.

In spite of Baldwin's somewhat negative comments on the Lowe index, it is the index that is preferred by many statistical agencies so it is necessary to study its properties in the context of strongly seasonal data.

Recall that the *Young* (1812) *index* was defined in earlier chapters as follows:

$$(30)\ P_Y(p^0,p^m,s) \equiv \sum_{n=1}^{N} s_n\,(p_n^{\,m}/p_n^{\,0})$$

where $s \equiv [s_1,\ldots,s_N]$ is the base year reference vector of expenditure shares. For the purposes of this section, where the modified Turvey data set is used to numerically illustrate the index, the base year will be taken to be 1970 and the resulting base year expenditure share vector turns out to be:

$$(31)\ s \equiv [s_1,\ldots,s_5] = [.3284, .1029, .0674, .0863, .4149].$$

Again, the base period for the prices will be taken to be December of 1970. For prices that are not available in the current month, the last available price is carried forward. The resulting Young index with carry forward of missing prices using the modified Turvey data set can be found in column 2 of Table 23.

The *geometric Laspeyres index* was defined in earlier chapters as follows:

$$(32)\ P_{GL}(p^0,p^m,s) \equiv \prod_{n=1}^{N} (p_n^{\,m}/p_n^{\,0})^{s_n}\,.$$

Thus the geometric Laspeyres index makes use of the same information as the Young index except that a geometric average of the price relatives is taken instead of an arithmetic one. Again, the base year is taken to be 1970 and the base period for prices is taken to be December of 1970 and the index is illustrated using the modified Turvey data set with carry forward of missing prices; see column 3 of Table 23.

It is of interest to compare the above three indexes that use annual baskets to the fixed base Laspeyres rolling year indexes computed earlier. However, the rolling year index that ends in the current month is centered five and a half months backwards. Hence the above 3 annual basket type indexes will be compared with an arithmetic average of two rolling year indexes that have their last month moved 5 and 6 months forward. This latter centered rolling year index is labelled P_{CRY} and is listed in the last column of Table 23.[38] Note that 0's are entered for the last six rows of this column since the data set does not extend 6 months into 1975 and so the centered rolling year indexes cannot be calculated for these last 6 months.

Table 23: Lowe, Young, Geometric Laspeyres and Centered Rolling Year Indexes with Carry Forward Prices

Year	Month	P_{LO}	P_Y	P_{GL}	P_{CRY}

[38] This series was normalized to equal 1 in December of 1970 so that it would be comparable to the other month to month indexes.

1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0554	1.0609	1.0595	1.0091
	2	1.0711	1.0806	1.0730	1.0179
	3	1.1500	1.1452	1.1187	1.0242
	4	1.2251	1.2273	1.1942	1.0298
	5	1.3489	1.3652	1.3249	1.0388
	6	1.4428	1.4487	1.4068	1.0478
	7	1.3789	1.4058	1.3819	1.0547
	8	1.3378	1.3797	1.3409	1.0631
	9	1.1952	1.2187	1.1956	1.0729
	10	1.1543	1.1662	1.1507	1.0814
	11	1.1639	1.1723	1.1648	1.0885
	12	1.0824	1.0932	1.0900	1.0965
1972	1	1.1370	1.1523	1.1465	1.1065
	2	1.1731	1.1897	1.1810	1.1174
	3	1.2455	1.2539	1.2363	1.1254
	4	1.3155	1.3266	1.3018	1.1313
	5	1.4262	1.4508	1.4183	1.1402
	6	1.5790	1.5860	1.5446	1.1502
	7	1.5297	1.5550	1.5349	1.1591
	8	1.4416	1.4851	1.4456	1.1690
	9	1.3038	1.3342	1.2974	1.1806
	10	1.2752	1.2960	1.2668	1.1924
	11	1.2852	1.3034	1.2846	1.2049
	12	1.1844	1.2032	1.1938	1.2203
1973	1	1.2427	1.2710	1.2518	1.2386
	2	1.3003	1.3308	1.3103	1.2608
	3	1.3699	1.3951	1.3735	1.2809
	4	1.4691	1.4924	1.4675	1.2966
	5	1.5972	1.6329	1.5962	1.3176
	6	1.8480	1.8541	1.7904	1.3406
	7	1.7706	1.8010	1.7711	0.0000
	8	1.6779	1.7265	1.6745	0.0000
	9	1.5253	1.5676	1.5072	0.0000
	10	1.5371	1.5746	1.5155	0.0000
	11	1.5634	1.5987	1.5525	0.0000
	12	1.4181	1.4521	1.4236	0.0000

It can be seen that the Lowe, Young and Geometric Laspeyres indexes have a considerable amount of seasonality in them and do not at all approximate their rolling year counterparts listed in the last column of Table 23.[39] Hence, without seasonal adjustment, the Lowe, Young and Geometric Laspeyres indexes are not suitable predictors for their seasonally adjusted rolling year counterparts.[40] The four series, P_{LO}, P_Y, P_{GL} and P_{CRY} listed in Table 23 are also plotted in Figure 4. It can be seen that the

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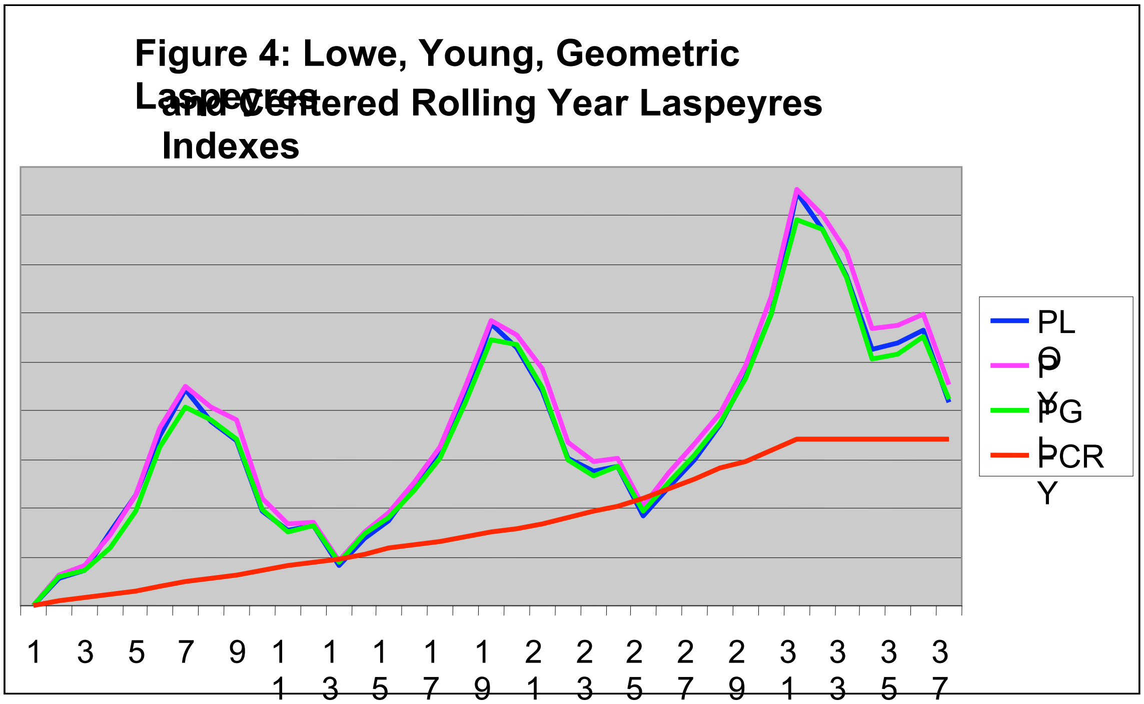
[39] The sample means of the four indexes are 1.2935 (Lowe), 1.3110 (Young), 1.2877 (Geometric Laspeyres) and 1.1282 (rolling year). Of course, the geometric Laspeyres indexes will always be equal to or less than their Young counterparts since a weighted geometric mean is always equal to or less than the corresponding weighted arithmetic mean.

[40] In section 11 below, the Lowe, Young and Geometric Laspeyres indexes will be seasonally adjusted.

Young price index is generally the highest, followed by the Lowe index and the Geometric Laspeyres is the lowest of the three month to month indexes. The centered rolling year Laspeyres counterpart index, P_{CRY}, is generally below the other three indexes (and of course does not have the strong seasonal movements of the other three series) but it moves in a roughly parallel fashion to the other three indexes.[41] Note that the seasonal movements of P_{LO}, P_Y, and P_{GL} are quite regular and this regularity will be exploited in section 11 below in order to use these month to month indexes to predict their rolling year counterparts.



Part of the problem may be the fact that the prices of strongly seasonal goods have been carried forward for the months when the commodities are not available. This will tend to add to the amount of seasonal movements in the indexes, particularly when there is high general inflation. Thus in the following section, the Lowe, Young and Geometric Laspeyres indexes will be recomputed using an imputation method for the missing prices rather than simply carrying forward the last available price.

9. Annual Basket Indexes with Imputation of Unavailable Prices

Instead of simply carrying forward the last available price of a seasonal commodity that is not sold during a particular month, it is possible to use an *imputation method* to fill in the missing prices. Alternative imputation methods are discussed by Armknecht and Maitland-Smith (1999) and Feenstra and Diewert (2001) but the basic idea is to take the last available price and *impute* prices for the missing periods that trend with another index. This other index could be an index of available prices for the general category of

[41] In Figure 4, P_{CRY} is artificially set equal to the June 1973 value for the index, which is the last month that the centered index can be constructed from the available data.

commodity or higher level components of the CPI. For the purposes of this section, the imputation index is taken to be a price index that grows at the multiplicative rate of 1.008 since the fixed base rolling year Laspeyres indexes for the modified Turvey data set grow at approximately .8% per month.[42] Using this imputation method to fill in the missing prices, the Lowe, Young and Geometric Laspeyres indexes defined in the previous section can be recomputed. The resulting indexes are listed in Table 24 along with the centered rolling year index P_{CRY} for comparison purposes.

Table 24: Lowe, Young, Geometric Laspeyres and Centered Rolling Year Indexes with Imputed Prices

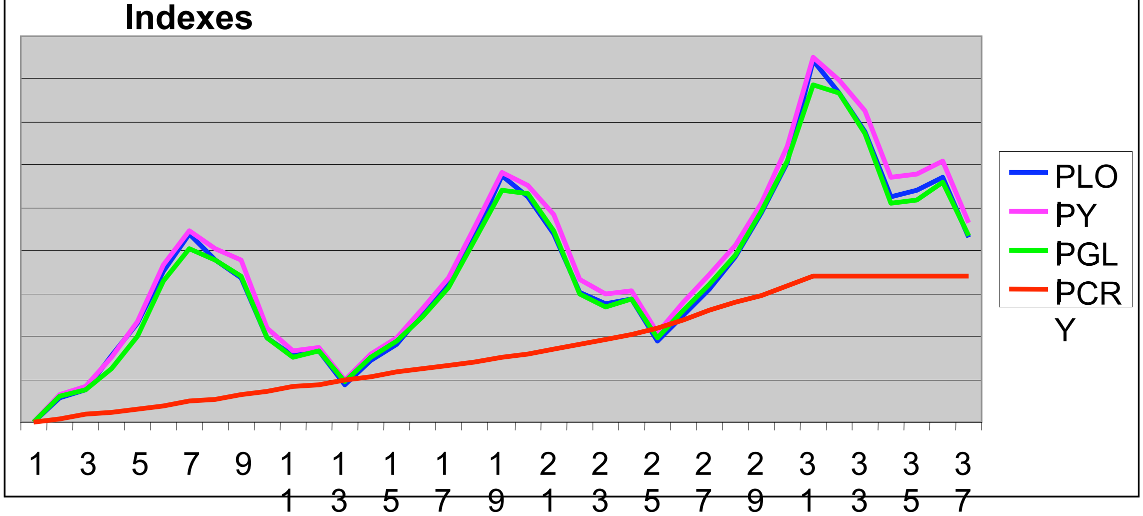
Year	Month	P_{LOI}	P_{YI}	P_{GLI}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0568	1.0624	1.0611	1.0091
	2	1.0742	1.0836	1.0762	1.0179
	3	1.1545	1.1498	1.1238	1.0242
	4	1.2312	1.2334	1.2014	1.0298
	5	1.3524	1.3682	1.3295	1.0388
	6	1.4405	1.4464	1.4047	1.0478
	7	1.3768	1.4038	1.3798	1.0547
	8	1.3364	1.3789	1.3398	1.0631
	9	1.1949	1.2187	1.1955	1.0729
	10	1.1548	1.1670	1.1514	1.0814
	11	1.1661	1.1747	1.1672	1.0885
	12	1.0863	1.0972	1.0939	1.0965
1972	1	1.1426	1.1580	1.1523	1.1065
	2	1.1803	1.1971	1.1888	1.1174
	3	1.2544	1.2630	1.2463	1.1254
	4	1.3260	1.3374	1.3143	1.1313
	5	1.4306	1.4545	1.4244	1.1402
	6	1.5765	1.5831	1.5423	1.1502
	7	1.5273	1.5527	1.5326	1.1591
	8	1.4402	1.4841	1.4444	1.1690
	9	1.3034	1.3343	1.2972	1.1806
	10	1.2758	1.2970	1.2675	1.1924
	11	1.2875	1.3062	1.2873	1.2049
	12	1.1888	1.2078	1.1981	1.2203
1973	1	1.2506	1.2791	1.2601	1.2386
	2	1.3119	1.3426	1.3230	1.2608
	3	1.3852	1.4106	1.3909	1.2809
	4	1.4881	1.5115	1.4907	1.2966
	5	1.6064	1.6410	1.6095	1.3176
	6	1.8451	1.8505	1.7877	1.3406
	7	1.7679	1.7981	1.7684	0.0000
	8	1.6773	1.7263	1.6743	0.0000
	9	1.5271	1.5700	1.5090	0.0000
	10	1.5410	1.5792	1.5195	0.0000
	11	1.5715	1.6075	1.5613	0.0000

[42] For the last year of data, the imputation index is escalated by an additional monthly growth rate of 1.008.

12	1.4307	1.4651	1.4359	0.0000

As could be expected, on average the Lowe, Young and Geometric Laspeyres indexes that used imputed prices are on average a bit *higher* than their counterparts that used carry forward prices but the variability of the imputed indexes is generally a bit *lower*.[43] The series that are listed in Table 24 are also plotted in Figure 5. It can be seen that the Lowe, Young and Geometric Laspeyres indexes that use imputed prices still have a huge amount of seasonality in them and do not closely approximate their rolling year counterparts listed in the last column of Table 24.[44] Hence, without seasonal adjustment, the Lowe, Young and Geometric Laspeyres indexes using imputed prices are not suitable predictors for their seasonally adjusted rolling year counterparts.[45] As these indexes stand, they are not suitable as measures of general inflation going from month to month.

Figure 5: Lowe, Young and Geometric Laspeyres with Imputed Prices and Centered Rolling Year Indexes



10. Bean and Stine Type C or Rothwell Indexes

The final month to month index[46] that will be considered in this chapter is the *Bean and Stine Type C* (1924; 31) or *Rothwell* (1958; 72) index.[47] This index makes use of

[43] For the Lowe indexes, the mean for the first 31 observations increases (with imputed prices) from 1.3009 to 1.3047 but the standard deviation decreases from .18356 to .18319; for the Young indexes, the mean for the first 31 observations increases from 1.3186 to 1.3224 but the standard deviation decreases from .18781 to .18730 and for the Geometric Laspeyres indexes, the mean for the first 31 observations increases from 1.2949 to 1.2994 and the standard deviation also increases slightly from .17582 to .17599. The imputed indexes are preferred to the carry forward indexes on general methodological grounds: in high inflation environments, the carry forward indexes will be subject to sudden jumps as the previously unavailable commodities become available.

[44] Note also that Figures 4 and 5 are very similar.

[45] In section 11 below, the Lowe, Young and Geometric Laspeyres indexes using imputed prices will be seasonally adjusted.

[46] For other suggested month to month indices in the seasonal context, see Balk (1980a) (1980b) (1980c) (1981).

seasonal baskets in the base year, denoted as the vectors $q^{0,m}$ for the months m = 1,2,…,12. The index also makes use of a vector of *base year unit value prices*, $p^0 \equiv [p_1^0,\ldots,p_5^0]$ where the nth price in this vector is defined as:

$$(33)\ p_n^0 \equiv \sum_{m=1}^{12} p_n^{0,m} q_n^{0,m} / \sum_{m=1}^{12} q_n^{0,m}\ ; \qquad n = 1,2,\ldots,5.$$

The *Rothwell price index for month m in year t* can now be defined as follows:

$$(34)\ P_R(p^0,p^{t,m},q^{0,m}) \equiv \sum_{n=1}^{5} p_n^{t,m} q_n^{0,m} / \sum_{n=1}^{5} p_n^0 q_n^{0,m}\ ; \qquad m = 1,\ldots,12.$$

Thus as the month changes, the quantity weights for the index change and hence the month to month movements in this index are a mixture of price and quantity changes.[48]

Using the modified Turvey data set, the base year is chosen to be 1970 as usual and the index is started off at December of 1970. The Rothwell index P_R is compared to the Lowe index with carry forward of missing prices P_{LO} in Table 25. To make the series a bit more comparable, the *normalized Rothwell index* P_{NR} is also listed in Table 25; this index is simply equal to the original Rothwell index divided by its first observation.

Table 25: The Lowe with Carry Forward Prices, Rothwell and Normalized Rothwell Indexes

Year	Month	P_{LO}	P_{NR}	P_R
1970	12	1.0000	1.0000	0.9750
1971	1	1.0554	1.0571	1.0306
	2	1.0711	1.0234	0.9978
	3	1.1500	1.0326	1.0068
	4	1.2251	1.1288	1.1006
	5	1.3489	1.3046	1.2720
	6	1.4428	1.2073	1.1771
	7	1.3789	1.2635	1.2319
	8	1.3378	1.2305	1.1997
	9	1.1952	1.0531	1.0268
	10	1.1543	1.0335	1.0077
	11	1.1639	1.1432	1.1146
	12	1.0824	1.0849	1.0577
1972	1	1.1370	1.1500	1.1212
	2	1.1731	1.1504	1.1216
	3	1.2455	1.1752	1.1459
	4	1.3155	1.2561	1.2247
	5	1.4262	1.4245	1.3889
	6	1.5790	1.3064	1.2737
	7	1.5297	1.4071	1.3719
	8	1.4416	1.3495	1.3158

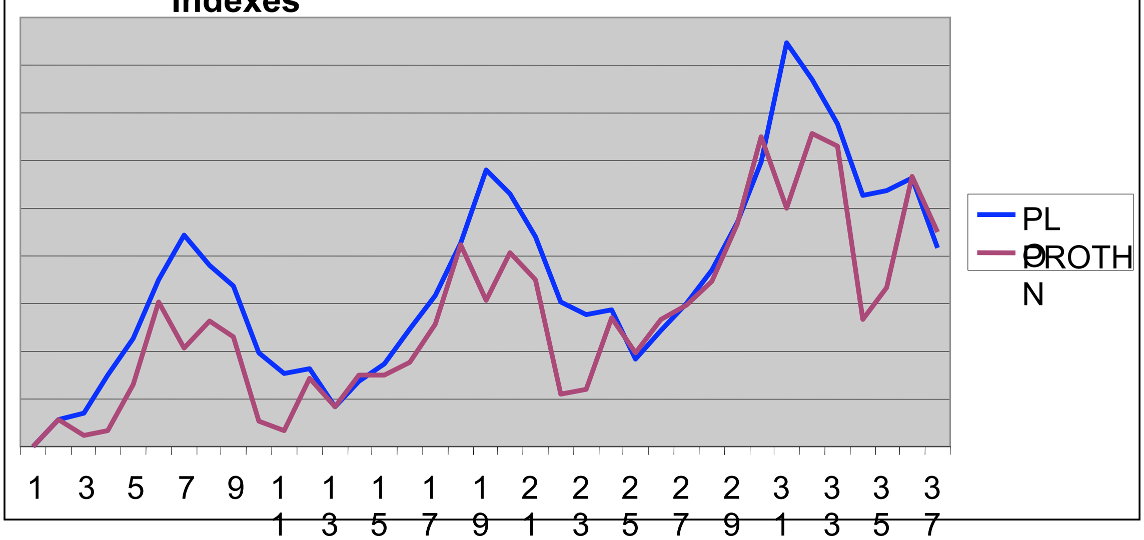
[47] This is the index favored by Baldwin (1990; 271) and many other price statisticians in the context of seasonal commodities.

[48] Rothwell (1958; 72) showed that the month to month movements in the index have the form of an expenditure ratio divided by a quantity index.

	9	1.3038	1.1090	1.0813
	10	1.2752	1.1197	1.0917
	11	1.2852	1.2714	1.2396
	12	1.1844	1.1960	1.1661
1973	1	1.2427	1.2664	1.2348
	2	1.3003	1.2971	1.2647
	3	1.3699	1.3467	1.3130
	4	1.4691	1.4658	1.4292
	5	1.5972	1.6491	1.6078
	6	1.8480	1.4987	1.4612
	7	1.7706	1.6569	1.6155
	8	1.6779	1.6306	1.5898
	9	1.5253	1.2683	1.2366
	10	1.5371	1.3331	1.2998
	11	1.5634	1.5652	1.5261
	12	1.4181	1.4505	1.4143

Viewing Figure 6, which plots the Lowe index with the carry forward of the last price and the normalized Rothwell index, it can be seen that the Rothwell index has smaller seasonal movements than the Lowe index and it is less volatile in general.[49] However, it is evident that there still are large seasonal movements in the Rothwell index and it may not be a suitable index for measuring general inflation without some sort of seasonal adjustment.

Figure 6: The Lowe and Rothwell Price Indexes



In the following section, the annual basket type indexes (with and without imputation) defined earlier in sections 8 and 9 will be seasonally adjusted using essentially the same method that was used in section 6.

[49] For all 37 observations in Table 22.25, the Lowe index has a mean of 1.3465 and a standard deviation of .20313 while the normalized Rothwell has a mean of 1.2677 and a standard deviation of .18271.

11. Forecasting Rolling Year Indexes using Month to Month Annual Basket Indexes

Recall Table 23 in section 8 which tabled the Lowe, Young, Geometric Laspeyres (using carry forward prices) and the centered rolling year indexes for the 37 observations running from December 1970 to December 1973: P_{LO}, P_Y, P_{GL} and P_{CRY} respectively. For each of the first three series, define a seasonal adjustment factor, SAF, as the centered rolling year index P_{CRY} divided by P_{LO}, P_Y and P_{GL} respectively for the first 12 observations. Now for each of the three series, repeat these 12 seasonal adjustment factors for observations 13 to 24 and then repeat them again for the remaining observations. These operations will create 3 SAF series for all 37 observations (label them SAF_{LO}, SAF_Y and SAF_{GL} respectively) but of course, only the first 12 observations in the P_{LO}, P_Y, P_{GL} and P_{CRY} series are used to create the 3 SAF series. Finally, define *seasonally adjusted Lowe, Young and Geometric Laspeyres indexes* by multiplying each unadjusted index by the appropriate seasonal adjustment factor:

(35) $P_{LOSA} \equiv P_{LO}\, SAF_{LO}$; $P_{YSA} \equiv P_Y\, SAF_Y$; $P_{GLSA} \equiv P_{GL}\, SAF_{GL}$.

These 3 seasonally adjusted annual basket type indexes are listed in Table 26 along with the target index, the centered rolling year index, P_{CRY}.

Table 26: Seasonally Adjusted Lowe, Young and Geometric Laspeyres Indexes with Carry Forward Prices and the Centered Rolling Year Index

Year	Month	P_{LOSA}	P_{YSA}	P_{GLSA}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091
	2	1.0179	1.0179	1.0179	1.0179
	3	1.0242	1.0242	1.0242	1.0242
	4	1.0298	1.0298	1.0298	1.0298
	5	1.0388	1.0388	1.0388	1.0388
	6	1.0478	1.0478	1.0478	1.0478
	7	1.0547	1.0547	1.0547	1.0547
	8	1.0631	1.0631	1.0631	1.0631
	9	1.0729	1.0729	1.0729	1.0729
	10	1.0814	1.0814	1.0814	1.0814
	11	1.0885	1.0885	1.0885	1.0885
	12	1.0824	1.0932	1.0900	1.0965
1972	1	1.0871	1.0960	1.0919	1.1065
	2	1.1148	1.1207	1.1204	1.1174
	3	1.1093	1.1214	1.1318	1.1254
	4	1.1057	1.1132	1.1226	1.1313
	5	1.0983	1.1039	1.1120	1.1402
	6	1.1467	1.1471	1.1505	1.1502
	7	1.1701	1.1667	1.1715	1.1591
	8	1.1456	1.1443	1.1461	1.1690
	9	1.1703	1.1746	1.1642	1.1806
	10	1.1946	1.2017	1.1905	1.1924

	11	1.2019	1.2102	1.2005	1.2049
	12	1.1844	1.2032	1.1938	1.2203
1973	1	1.1882	1.2089	1.1922	1.2386
	2	1.2357	1.2536	1.2431	1.2608
	3	1.2201	1.2477	1.2575	1.2809
	4	1.2349	1.2523	1.2656	1.2966
	5	1.2299	1.2425	1.2514	1.3176
	6	1.3421	1.3410	1.3335	1.3406
	7	1.3543	1.3512	1.3518	0.0000
	8	1.3334	1.3302	1.3276	0.0000
	9	1.3692	1.3800	1.3524	0.0000
	10	1.4400	1.4601	1.4242	0.0000
	11	1.4621	1.4844	1.4508	0.0000
	12	1.4181	1.4521	1.4236	0.0000

The 4 series in Table 26 coincide for their first 12 observations, which follows from the way the seasonally adjusted series were defined. Also, the last 6 observations are missing for the centered rolling year series, P_{CRY}, since data for the first 6 months of 1974 would be required in order to calculate all of these index values. Note that from December 1971 to December 1973, the three seasonally adjusted annual basket type indexes can be used to *predict* the corresponding centered rolling year entries; see Figure 7 for plots of these predictions. What is remarkable in Table 26 and Figure 7 is that *the predicted values of these seasonally adjusted series are fairly close to the corresponding target index values*.[50] This result is somewhat unexpected since the annual basket indexes use price information for only two consecutive months whereas the corresponding centered rolling year index uses price information for some 25 months![51] It should also be noted that the seasonally adjusted Geometric Laspeyres index is generally the best predictor of the corresponding rolling year index for this data set. It can be seen viewing Figure 7 that for the first few months of 1973, the 3 month to month indexes underestimate the centered rolling year inflation rate but by the middle of 1973, the month to month indexes are right on target.[52]

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[50] For observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling year series. For the seasonally adjusted Lowe index, an R^2 of .8816 is obtained; for the seasonally adjusted Young index, an R^2 of .9212 is obtained and for the seasonally adjusted Geometric Laspeyres index, an R^2 of .9423 is obtained. These fits are not as good as the fit obtained in section 6 above where the seasonally adjusted approximate rolling year index was used to predict the fixed base Laspeyres rolling year index. This R^2 was .9662; recall the discussion around Table 20.

[51] However, for seasonal data sets that are not as regular as the modified Turvey data set, the predictive power of the seasonally adjusted annual basket type indexes may be considerably less; i.e., if there are abrupt changes in the seasonal pattern of prices, one could not expect these month to month indexes to accurately predict a rolling year index.

[52] Recall that the last 6 months of P_{CRY} has been artificially held constant; six months of data for 1974 would be required to evaluate these centered rolling year index values and these data are not available.

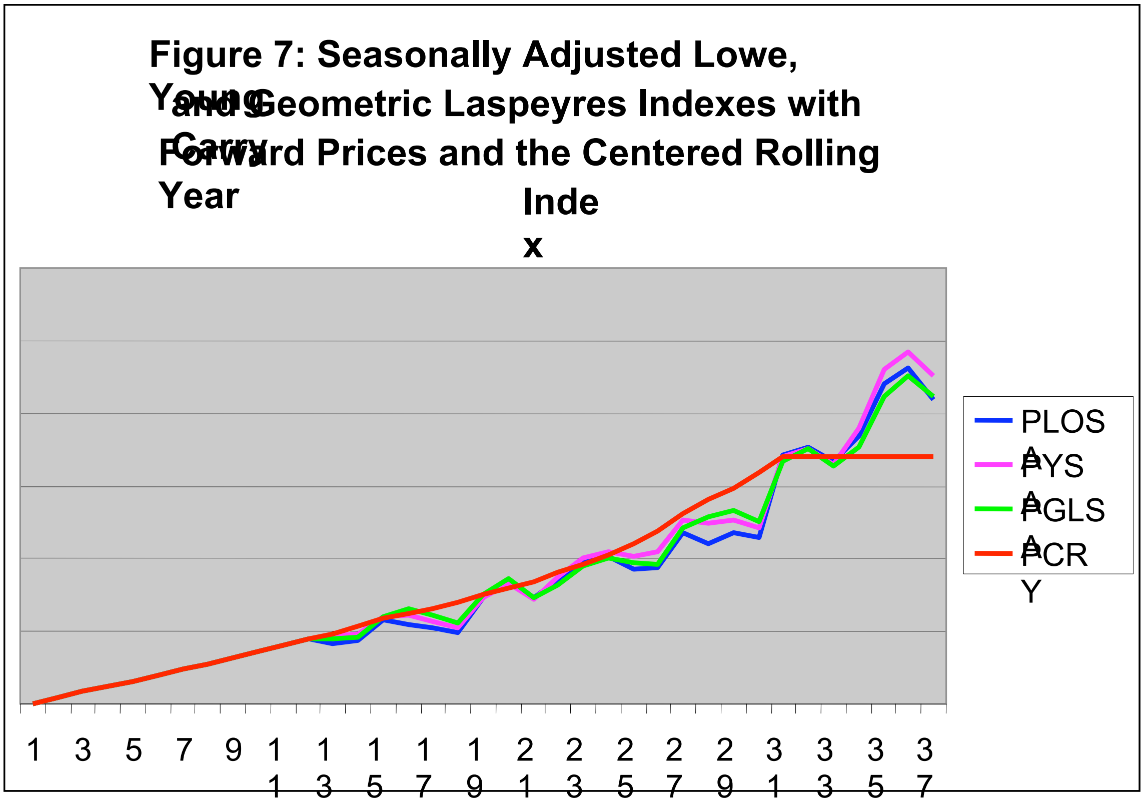


Figure 7: Seasonally Adjusted Lowe, Young and Geometric Laspeyres Indexes with Carry Forward Prices and the Centered Rolling Year Index

The above manipulations can be repeated, replacing the *carry forward* annual basket indexes by their *imputed* counterparts; i.e., use the information in Table 24 in section 9 above (instead of Table 23 in section 8) and Table 27 replaces Table 26. A seasonally adjusted version of the Rothwell index presented in the previous section may also be found in Table 27.[53] The five series in Table 27 are also graphed in Figure 8.

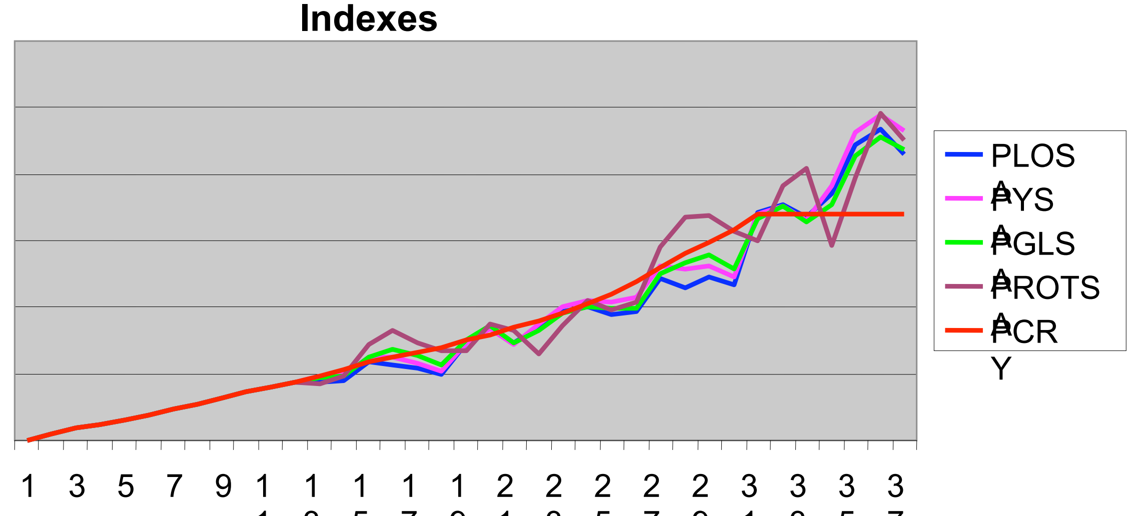
Table 27: Seasonally Adjusted Lowe, Young and Geometric Laspeyres Indexes with Imputed Prices, Seasonally Adjusted Rothwell and Centered Rolling Year Indexes

Year	Month	P_{LOSA}	P_{YSA}	P_{GLSA}	P_{ROTHSA}	P_{CRY}
1970	12	1.0000	1.0000	1.0000	1.0000	1.0000
1971	1	1.0091	1.0091	1.0091	1.0091	1.0091
	2	1.0179	1.0179	1.0179	1.0179	1.0179
	3	1.0242	1.0242	1.0242	1.0242	1.0242
	4	1.0298	1.0298	1.0298	1.0298	1.0298
	5	1.0388	1.0388	1.0388	1.0388	1.0388
	6	1.0478	1.0478	1.0478	1.0478	1.0478
	7	1.0547	1.0547	1.0547	1.0547	1.0547
	8	1.0631	1.0631	1.0631	1.0631	1.0631
	9	1.0729	1.0729	1.0729	1.0729	1.0729
	10	1.0814	1.0814	1.0814	1.0814	1.0814
	11	1.0885	1.0885	1.0885	1.0885	1.0885
	12	1.0863	1.0972	1.0939	1.0849	1.0965

[53] The same seasonal adjustment technique as was defined by equations (35) was used.

1972	1	1.0909	1.0999	1.0958	1.0978	1.1065
	2	1.1185	1.1245	1.1244	1.1442	1.1174
	3	1.1129	1.1250	1.1359	1.1657	1.1254
	4	1.1091	1.1167	1.1266	1.1460	1.1313
	5	1.0988	1.1043	1.1129	1.1342	1.1402
	6	1.1467	1.1469	1.1505	1.1339	1.1502
	7	1.1701	1.1666	1.1715	1.1746	1.1591
	8	1.1457	1.1442	1.1461	1.1659	1.1690
	9	1.1703	1.1746	1.1642	1.1298	1.1806
	10	1.1947	1.2019	1.1905	1.1715	1.1924
	11	1.2019	1.2103	1.2005	1.2106	1.2049
	12	1.1888	1.2078	1.1981	1.1960	1.2203
1973	1	1.1941	1.2149	1.1983	1.2089	1.2386
	2	1.2431	1.2611	1.2513	1.2901	1.2608
	3	1.2289	1.2565	1.2677	1.3358	1.2809
	4	1.2447	1.2621	1.2778	1.3373	1.2966
	5	1.2338	1.2459	1.2576	1.3131	1.3176
	6	1.3421	1.3406	1.3335	1.3007	1.3406
	7	1.3543	1.3510	1.3518	1.3831	0.0000
	8	1.3343	1.3309	1.3285	1.4087	0.0000
	9	1.3712	1.3821	1.3543	1.2921	0.0000
	10	1.4430	1.4634	1.4271	1.3949	0.0000
	11	1.4669	1.4895	1.4560	1.4903	0.0000
	12	1.4307	1.4651	1.4359	1.4505	0.0000

Figure 8: Seasonally Adjusted Lowe, Young and Geometric Laspeyres Indexes with Imputed Prices, Seasonally Adjusted Rothwell and Centered Rolling Year Indexes



Again, the seasonally adjusted annual basket type indexes listed in the first 3 columns of Table 27 (using imputations for the missing prices) are reasonably close to the corresponding centered rolling year index listed in the last column of Table 27.[54] The seasonally adjusted Geometric Laspeyres index is the closest to the centered rolling year index and the seasonally adjusted Rothwell index is the furthest away. The three seasonally adjusted month to month indexes that use annual weights, P_{LOSA}, P_{YSA} and P_{GLSA}, dip below the corresponding centered rolling year index, P_{CRY}, for the first few months of 1973 when the rate of month to month inflation suddenly increases but by the middle of 1973, all four indexes are fairly close to each other. The seasonally adjusted Rothwell does not do a very good job of approximating P_{CRY} for this particular data set although this could be a function of the rather simple method of seasonal adjustment that was used.

Comparing the results in Tables 26 and 27, it can be seen that it did not make a great deal of difference for the modified Turvey data set whether missing prices are carried forward or imputed; the seasonal adjustment factors picked up the lumpiness in the unadjusted indices that occurs if the carry forward method is used. However, the three month to month indexes that used annual weights and imputed prices did predict the corresponding centered rolling year indexes somewhat better than the three indexes that used carry forward prices. Hence, the use of imputed prices over carry forward prices is recommended.

The conclusions that emerge from this section are rather encouraging for statistical agencies that wish to use an annual basket type index as their flagship index.[55] It appears that for commodity groups that have strong seasonality, an annual basket type index for this group can be seasonally adjusted[56] and the resulting seasonally adjusted index value can be used as a price relative for the group at higher stages of aggregation. The preferred type of annual basket type index appears to be the Geometric Laspeyres index rather than the Lowe index but the differences between the two were not large for this data set.

12. Conclusion

A number of tentative conclusions can be drawn from the results of the previous sections in this chapter:

- The inclusion of seasonal commodities in maximum overlap month to month indexes will frequently lead to substantial biases. Hence unless the maximum overlap month

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[54] Again for observations 13 through 31, one can regress the seasonally adjusted series on the centered rolling year series. For the seasonally adjusted Lowe index, an R^2 of .8994 is obtained; for the seasonally adjusted Young index, an R^2 of .9294 is obtained and for the seasonally adjusted Geometric Laspeyres index, an R^2 of .9495 is obtained. For the seasonally adjusted Rothwell index, an R^2 of .8704 is obtained, which is lower than the other three fits. For the Lowe, Young and Geometric Laspeyres indexes using imputed prices, these R^2 are higher than those obtained using carry forward prices.

[55] Using the results of previous chapters, the use of the annual basket Young index is not encouraged due to its failure of the time reversal test and the resulting upward bias.

[56] It is not necessary to use rolling year indexes in the seasonal adjustment process but the use of rolling year indexes is recommended since they will increase the objectivity and reproducibility of the seasonally adjusted indexes.

to month indexes using seasonal commodities cumulated for a year are close to their year over year counterparts, the seasonal commodities should be excluded from the month to month index or the seasonal adjustment procedures suggested in section 11 should be used

- Year over year monthly indexes can always be constructed even if there are strongly seasonal commodities.[57] Many users will be interested in these indiexes and moreover, these indexes are the building blocks for annual indexes and for rolling year indexes. Hence, statistical agencies should compute these indexes. They can be labelled as "analytic series" in order to prevent user confusion with the primary month to month CPI.
- Rolling year indexes should also be made available as analytic series. These indexes will give the most reliable indicator of annual inflation at a monthly frequency. This type of index can be regarded as a seasonally adjusted CPI and this type of index is the most natural to use as a central bank inflation target. It has the disadvantage of measuring year over year inflation with a lag of 6 months; hence it cannot be used as a short run indicator of month to month inflation. However, the techniques suggested in sections 6 and 11 could be used so that timely forecasts of these rolling year indexes can be made using current price information.
- Annual basket indexes can also be successfully used in the context of seasonal commodities. However, most users of the CPI will want to use seasonally adjusted versions of these annual basket type indexes. The seasonal adjustment can be done using the index number methods explained in section 11 or traditional statistical agency seasonal adjustment procedures could be used.[58]
- From an a priori point of view, when making a price comparison between any two periods, the Paasche and Laspeyres indexes are of equal importance. Under normal circumstances, the spread between the Laspeyres and Paasche indexes will be reduced by using chained indexes rather than fixed base indexes. Hence, it is suggested that when constructing year over year monthly or annual indexes, the chained Fisher index (or the chained Törnqvist Theil index, which closely approximates the chained Fisher) be chosen as the target index that a statistical agency should aim to approximate. However, when constructing month to month indexes, chained indexes should always be checked against their year over year counterparts to check for chain drift. If substantial drift is found, the chained month to month indexes must be replaced by fixed base indexes or seasonally adjusted annual basket type indices.[59]
- If current period expenditure shares are not all that different from base year expenditure shares, approximate chained Fisher indexes will normally provide a very

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[57] There can be problems with the year over year indices if shifting holidays or abnormal weather changes "normal" seasonal patterns. In general, choosing a longer time period will mitigate these types of problems; i.e., quarterly seasonal patterns will be more stable than monthly patterns which in turn will be more stable than weekly patterns.

[58] However, there is a problem with using traditional X-11 type seasonal adjustment procedures for seasonally adjusting the flagship CPI due to the fact that "final" seasonal adjustment factors are generally not available until an additional 2 or 3 years data has been collected. Since the flagship CPI cannot be revised, this may preclude using X-11 type seasonal adjustment procedures on it. Note that the index number method of seasonal adjustment explained in this chapter does not suffer from this problem.

[59] Alternatively, some sort of multilateral index number formula could be used; e.g., see Caves, Christensen and Diewert (1982) or Feenstra and Shapiro (2003).

close practical approximation to the chained Fisher target indexes. Approximate Laspeyres, Paasche and Fisher indexes use base period expenditure shares whenever they occur in the index number formula in place of current period (or lagged current period) expenditure shares. Approximate Laspeyres, Paasche and Fisher indexes can be computed by statistical agencies using their normal information sets.
- The Geometric Laspeyres index is an alternative to the approximate Fisher index that uses the same information and it will normally be close to the approximate Fisher index.

It is evident that more research needs to be done on the problems associated with the index number treatment of seasonal commodities. A consensus on what is best practice in this area has not yet formed.

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