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The Measurement of Income

By E. Diewert

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The Measurement of Income

Erwin Diewert,¹
Department of Economics,
University of British Columbia,
Vancouver, Canada, V6T 1Z1.
Email: diewert@econ.ubc.ca

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1. Introduction

In this chapter, we will study alternative income concepts. This would seem to be a very straightforward subject but as we shall see, it is far from being simple, even when we assume that there is only a single homogeneous reproducible capital good.

Virtually all economic discussions about the economic strength of a country use Gross Domestic or Gross National Product as “the” measure of output. But gross product measures do not account for the capital that is used up during the production period; i.e., the gross measures neglect depreciation. Thus in section 2, we consider why gross measures are more popular than net measures.

Even though it may be difficult empirically to estimate depreciation and hence to estimate net output as opposed to gross output, we nevertheless conclude that for welfare purposes, the net measure is to be preferred. Net measures of output are also known as *income* measures. In section 3, we study in some detail Samuelson’s (1961) discussion on alternative income concepts and how they might be implemented empirically. In particular, Samuelson (1961; 46) gives a nice geometric interpretation of Hicks’ (1939; 174) Income Number 3.

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In section 4, we digress temporarily and generalize Samuelson's (1961; 45-46) index number method for measuring "income" change; i.e., we cover the pure theory of the output quantity index that was developed by Samuelson and Swamy (1974), Sato (1976) and Diewert (1983).

In section 5, we note that Samuelson's measures of income do not capture all of the complexities of the concept. Samuelson worked with a net investment framework but net investment is equal to capital at the end of the period less capital at the beginning of the period. Unfortunately, prices at the beginning of the period are not necessarily equal to prices at the end of the period. Thus Hicks noted that there was a "kind of index number problem" in comparing capital stocks at the beginning and end of the period:

"At once we run into the difficulty that if Net Investment is interpreted as the difference between the value of the Capital Stock at the beginning and end of the year, the transformation would not be possible. It is only in the special case when the prices of all sorts of capital instruments are the same (if their condition is the same) at the end of the year as at the beginning, that we should be able to measure the money value of Real Net Investment by the increase in the Money value of the Capital stock. In all probability these prices will have changed during the year, so that we have a kind of index number problem, parallel to the index number problem of comparing real income in different years. The characteristics of that other problem are generally appreciated; what is not so generally appreciated is the fact that before we can begin to compare real income in different years, we have to solve a similar problem within the single year—we have to reduce the Capital stock at the beginning and end of the years into comparable real terms." J.R. Hicks (1942; 175-176).

In section 5, we look at various possible alternatives for making the capital stocks at the beginning and end of the year comparable to each other in real terms.

In section 6, we return to the accounting problems associated with the profit maximization problem of a production unit, using the Hicks (1961; 23) and Edwards and Bell (1961; 71-72) Austrian production function framework studied in section 9.2 of chapter I. In this section, we show how the traditional gross rentals user cost formula can be decomposed into two terms—one reflecting the reward for "waiting" and the other reflecting depreciation—and then we show how the depreciation term can be transferred from the list of inputs and regarded as a negative output, which leads to an income concept that was studied in section 5.

In section 7 we present various approximations to the theoretical target income concept—approximations that can be implemented empirically. Sections 6 and 7 also revisit the obsolescence and depreciation controversy that was discussed earlier in section 7 of chapter I. Section 8 concludes.

2. Measuring National Product: Gross versus Net

Real Gross Domestic Product, per capita real GDP and labour productivity (real GDP divided by hours worked in the economy) are routinely used to compare "welfare" levels between countries (and between time periods in the same country). Gross Domestic Product is the familiar $C + G + I + X - M$ or in a closed economy, it is simply $C + I$, consumption plus gross investment that takes place during an accounting period.

However, economists have argued for a long time that GDP is not the “right” measure of output for welfare purposes; rather NDP (Net National Product) equal to consumption plus net investment accruing to nationals is a much better measure, where net investment equals gross investment less depreciation.² Why has GDP remained so much more popular than NDP, given that NDP seems to be the better measure for “welfare” comparison purposes?³

Samuelson (1961) had a good discussion of the arguments that have been put forth to justify the use of GDP over NDP:

Within the framework of a purely theoretical model such as this one, I believe that we should certainly prefer net national product, NNP, to gross national product, GNP, if we were forced to choose between them. This is somewhat the reverse of the position taken by many official statisticians, and so let me dispose of three arguments used to favour the gross concept.” Paul A. Samuelson (1961; 33).

The first argument that Samuelson considered was that our estimates of depreciation are so inaccurate that it is better to measure GDP or GNP rather than NDP or NNP. Samuelson was able to dispose of this argument in his context of a purely theoretical model as follows:

“Within our simple model, we know precisely what depreciation is and so for our present purpose this argument can be provisionally ruled out of order.” Paul A. Samuelson (1961; 33).

However, in our practical measurement context, we cannot dismiss this argument so easily and we have to concede that the fact that our empirical estimates of depreciation are so shaky, is indeed an argument to focus on measuring GDP rather than NDP.⁴

The second argument that Samuelson considered was the argument that GNP reflects the productive potential of the economy:

“Second, there is the argument that GNP gives a better measure than does NNP of the maximum consumption sprint that an economy could make by consuming its capital in time of future war or emergency.” Paul A. Samuelson (1961; 33).

² See Marshall (1890) and Pigou (1924; 46) (1935; 240-241) (1941; 271) for example.

³ In the present chapter, we will assume that the economy is closed so that the distinction between domestic product and national product (e.g., NDP versus NNP) vanishes. Hence our focus is on justifying either a gross product or a net product concept.

⁴ Hicks (1973; 155) conceded that this argument for GDP or GNP has some validity: “There are items, of which Depreciation and Stock Appreciation are the most important, which do not reflect actual transactions, but are estimates of the changes in the value of assets which have not yet been sold. These are estimates in a different sense from that previously mentioned. They are not estimates of a statistician’s true figure, which happens to be unavailable; there is no true figure to which they correspond. They are estimates relative to a purpose; for different purposes they may be made in different ways. This is of course the basic reason why it has become customary to express the National Accounts in terms of Gross National Product (before deduction of Depreciation) so as to clear them of contamination with the ‘arbitrary’ depreciation item; though it should be noticed that even with GNP another arbitrary element remains, in stock accumulation.”

Samuelson (1961; 34) is able to dismiss this argument by noting that NNP is not the maximum short run production that could be squeezed out of an economy; by running down capital to an extraordinary degree, we could increase present period output to a level well beyond current GNP.

The third argument that Samuelson considered had to do with the difficulties involved in determining obsolescence:

“A third argument favouring a gross rather than net product figure proceeds as follows: new capital is progressively of better quality than old, so that net product calculated by the subtraction of all depreciation and obsolescence does not yield an ideal measure ‘based on the principle of keeping intact the physical productivity of the capital goods in some kind of welfare sense’.” Paul A. Samuelson (1961; 35).

Again Samuelson dismisses this argument in the context of his theoretical model (where all is known) but in the practical measurement context, we have to concede that this argument has some validity, just as did the first argument.

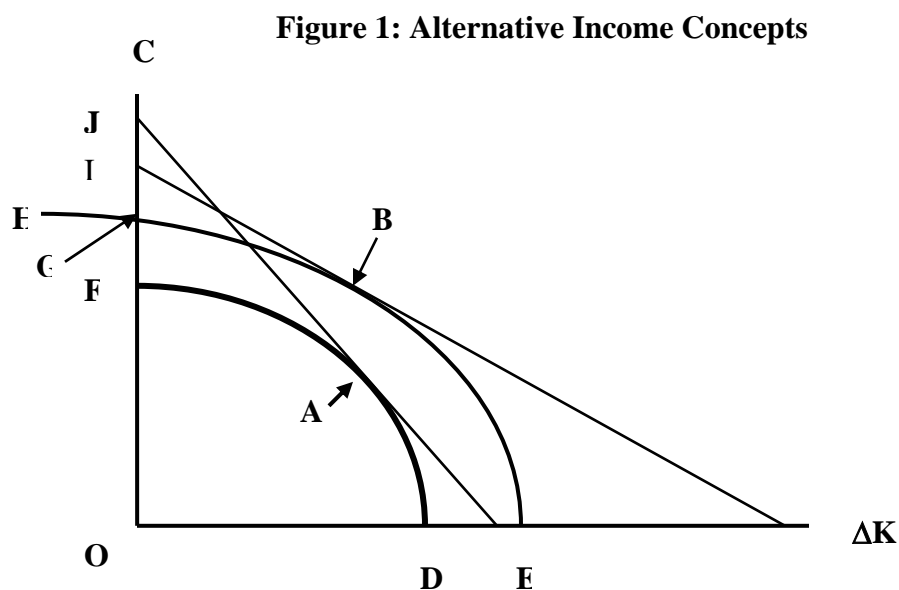
From our point of view, the problem with the gross concept is that it gives us a measure of output that is not sustainable. By deducting even an imperfect measure of depreciation (and obsolescence) from gross investment, we will come closer to a measure of output that could be consumed in the present period without impairing production possibilities in future periods. Hence, for welfare purposes, measures of net product seem to be much preferred to gross measures, even if our estimates of depreciation and obsolescence are imperfect.⁵

In the following section, we will look at some alternative definitions of net product. Given a specific definition for net product and given an accounting system that distributes the value of outputs produced to inputs utilized, each definition of net product gives rise to a corresponding definition of “income”. In the economic literature, most of the discussion of alternative measures of net output has occurred in the context of alternative “income” measures and so in the following section, we will follow the literature and discuss alternative “income” measures rather than alternative measures of “net product”.

3. Measuring Income: Hicks versus Samuelson

⁵ This point of view is also expressed in the *System of National Accounts 1993*: “As value added is intended to measure the additional value created by a process of production, it ought to be measured net, since consumption of fixed capital is a cost of production. However, as explained later, consumption of fixed capital can be difficult to measure in practice and it may not always be possible to make a satisfactory estimate of its value and hence of net value added.” Eurostat (1993; 121). “The consumption of fixed capital is one of the most important elements in the System. ... Moreover, consumption of fixed capital does not represent the aggregate value of a set of transactions. It is an imputed value whose economic significance is different from entries in the accounts based only on market transactions. For these reasons, the major balancing items in national accounts have always tended to be recorded both gross and net of consumption of fixed capital. This tradition is continued in the System where provision is also made for balancing items from value added through to saving to be recorded both ways. In general, the gross figure is obviously the easier to estimate and may, therefore, be more reliable, but the net figure is usually the one that is conceptually more appropriate and relevant for analytical purposes.” Eurostat (1993; 150).

Samuelson (1961; 45-46) constructed a nice diagram which illustrated alternative income concepts in a very simple model where the economy produces only two goods: consumption C and a durable capital input K . *Net investment* during period t is defined as $\Delta K^t \equiv K^t - K^{t-1}$, the end of the period capital stock, K^t , less the beginning of the period capital stock, K^{t-1} . In Figure 1 below, let the economy's period 2 production possibilities set for producing combinations of consumption C and net investment ΔK be represented by the curve $HGBE^6$ and let the economy's period 1 production possibilities set for producing consumption and nonnegative net investment be represented by the curve FAD . Assume that the actual period 2 production point is represented by the point B and the actual period 1 production point is represented by the point A .



Samuelson used the definition of income that was due to Marshall (1890) and Haig (1921), who (roughly speaking) defined income as consumption plus the consumption equivalent of the increase in net wealth over the period:

“The Haig-Marshall definition of income can be defended by one who admits that consumption is the ultimate end of economic activity. In our simple model, the Haig-Marshall definition measures the economy's *current power to consume* if it wishes to do so.” Paul A. Samuelson (1961; 45).

Samuelson went on to describe a number of methods by which the Haig-Marshall definition of income or net product could be implemented. Three of his suggested methods will be of particular interest to us.

⁶ The point H on the period 2 production possibilities set would represent a consumption net investment point where the end of the period capital stock is less than the beginning of the period stock so that consumption is increased at the cost of running down the capital stock. The period 1 production possibilities set could similarly be extended to the left of the point F .

Method 1: The Market Prices Method

If producers are maximizing the value of consumption plus net investment subject to available labour and initial capital resources in each period, then in each period, there will be a market revenue line that is tangent to the production possibilities set. Thus the revenue line BI is tangent to the period 2 set and the line JA is tangent to the period 1 production possibilities set. Each of these revenue lines can be used to convert the period's net investment into consumption equivalents at the market prices prevailing in each period. Thus in period 1, the consumption equivalent of the observed production point A is the point J while in period 2, the consumption equivalent of the observed production point B is the point I and so using this method, Haig-Marshall income is higher in period 1 than in 2, since J is above I.⁷

Method 2: Samuelson's Index Number Method

Let the point A be the period 1 consumption, net investment point C^1, I^1 with corresponding market prices P_C^1, P_I^1 and let the point B be the period 2 consumption, net investment point C^2, I^2 with corresponding market prices P_C^2, P_I^2 . Samuelson suggested computing the Laspeyres and Paasche quantity indexes for net output, Q_L and Q_P :

$$(1) Q_L \equiv [P_C^1 C^2 + P_I^1 I^2] / [P_C^1 C^1 + P_I^1 I^1];$$

$$(2) Q_P \equiv [P_C^2 C^2 + P_I^2 I^2] / [P_C^2 C^1 + P_I^2 I^1].$$

If Q_L and Q_P are both greater than one, then Samuelson would say that income in period 2 is greater than in period 1; if Q_L and Q_P are both less than one, then Samuelson would say that income in period 2 is less than in period 1; if Q_L and Q_P are both equal to one, then Samuelson would say that income in period 1 is equal to period 2 income. If Q_L and Q_P are such that one is less than one and the other greater than one, then Samuelson would term the situation inconclusive.⁸

We will indicate in the following section how Samuelson's analysis can be generalized to deal with the indeterminate case, at least in theory.

Method 3: Hicksian Income

⁷ Some statisticians would, I think, tend to measure incomes by the vertical intercepts of the tangent lines through A and B. On their definition, A would involve more income than B." Paul A. Samuelson (1961; 45).

⁸ "Neither Haig nor Marshall have told us exactly how they would evaluate and compare A and B in Fig. 3. Certainly some economic statisticians would interpret them as follows: Money national income is meaningless; you must deflate the money figures and reduce things to constant dollars. To deflate, apply the price ratios of B to the A situation and compare with B; alternatively, apply the price ratios of A to the B situation and compare with A. If both tests give the same answer—and in Fig. 3 they will, because B lies outside A on straight lines parallel to the tangent at either A or at B—then you can be sure that one situation has 'more income' than the other. If these Laspeyres and Paasche tests disagree, reserve judgment or split the difference depending upon your temperament." Paul A. Samuelson (1961; 45-46).

Hicks (1939) made a number of definitions of income. The one that Samuelson chose to model is Hicks' income Number 3:⁹

"Income No. 3 must be defined as the maximum amount of money which the individual can spend this week, and still expect to be able to spend the same amount *in real terms* in each ensuing week." J.R. Hicks (1939; 174).

Referring back to Figure 1 above, Samuelson (1961; 46) interpreted Hicksian income in period 1 as the point F (which is where the period 1 production frontier intersects the consumption axis so that net investment would be 0 at this point) and Hicksian income in period 2 as the point G (which is where the period 2 production frontier intersects the consumption axis so that net investment would be 0 at this point). However, Samuelson also noted that this definition of income is less useful to the economic statistician than the above two definitions because the economic statistician will not be able to determine where the production frontier will intersect the consumption axis:

"Others (e.g. Hicks of the earlier footnote) want to measure income by comparing the vertical intercepts of the curved production possibility schedules passing respectively through A and B. This is certainly one attractive interpretation of the spirit behind Haig and Marshall. The practical statistician might despair of so defining income: using market prices and quantities, he could conceivably apply any of the other definitions; but this one would be non-observable to him." Paul A. Samuelson (1961; 46).

All three of the above definitions of income have some appeal. At this stage, we will not commit to any single definition since we have not yet explored the full complexities of the income concept.¹⁰ We conclude this section with another astute observation made by Samuelson:

"Our dilemma is now well depicted. The simplest economic model involves two current variables, consumption and investment. A measure of national income is one variable. How can we fully summarize a doublet of numbers by a single number?" Paul A. Samuelson (1961; 47).

4. The Theory of the Output Index

In this section, we will have another look at Samuelson's index number method for measuring income growth; i.e., his second income or net product concept studied in the previous section. We consider a more general model where there are M consumption goods and net investment goods and N primary inputs. We also consider more general indexes than the Laspeyres and Paasche output quantity indexes considered by Samuelson.

⁹ This is the "best" Hicksian definition in my opinion but it has some ambiguity associated with it: how exactly do we interpret the word "real"?

¹⁰ Samuelson's model did not have the added complexities of the Edwards and Bell (1961; 71-72) and Hicks (1961; 23) Austrian production model that distinguished the beginning of the period and end of the period capital stocks as separate inputs and outputs. Also Samuelson had only a single consumption good and a single capital input in his model and we need to also consider the problems involved in aggregating over consumption and capital stock components.

We assume that the market sector of the economy produces quantities of M (net) outputs, $y \equiv [y_1, \dots, y_M]$, which are sold at the positive producer prices $P \equiv [P_1, \dots, P_M]$. We further assume that the market sector of the economy uses positive quantities of N primary inputs, $x \equiv [x_1, \dots, x_N]$ which are purchased at the positive primary input prices $W \equiv [W_1, \dots, W_N]$. In period t , we assume that there is a feasible set of output vectors y that can be produced by the market sector if the vector of primary inputs x is utilized by the market sector of the economy; denote this period t production possibilities set by S^t . We assume that S^t is a closed convex cone that exhibits a free disposal property.¹¹

Given a vector of output prices P and a vector of available primary inputs x , we define the period t market sector net product function, $g^t(P, x)$, as follows:¹²

$$(3) \quad g^t(P, x) \equiv \max_y \{P \cdot y : (y, x) \text{ belongs to } S^t\}; \quad t = 0, 1, 2, \dots$$

Thus market sector NDP depends on t (which represents the period t technology set S^t), on the vector of output prices P that the market sector faces and on x , the vector of primary inputs that is available to the market sector.

If P^t is the period t output price vector and x^t is the vector of inputs used by the market sector during period t and if the NDP function is differentiable with respect to the components of P at the point P^t, x^t , then the period t vector of market sector outputs y^t will be equal to the vector of first order partial derivatives of $g^t(P^t, x^t)$ with respect to the components of P ; i.e., we will have the following equations for each period t :¹³

$$(4) \quad y^t = \nabla_P g^t(P^t, x^t); \quad t = 1, 2.$$

Thus the period t market sector (net) supply vector y^t can be obtained by differentiating the period t market sector NDP function with respect to the components of the period t output price vector P^t .

If the NDP function is differentiable with respect to the components of x at the point P^t, x^t , then the period t vector of input prices W^t will be equal to the vector of first order partial

¹¹ For more explanation of the meaning of these properties, see Diewert (1973) (1974; 134) or Woodland (1982) or Kohli (1978) (1991). The assumption that S^t is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an ex post rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period. The function g^t is known as the *NDP function* or the *net national product function* in the international trade literature (see Kohli (1978)(1991), Woodland (1982) and Feenstra (2004; 76). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include: (i) the *gross profit function*; see Gorman (1968); (ii) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*; see Diewert (1973) (1974).

¹² The function $g^t(P, x)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of x ; see Diewert (1973) (1974; 136). Notation: $P \cdot y \equiv \sum_{m=1}^M P_m y_m$.

¹³ These relationships are due to Hotelling (1932; 594). Note that $\nabla_P g^t(P^t, x^t) \equiv [\partial g^t(P^t, x^t) / \partial P_1, \dots, \partial g^t(P^t, x^t) / \partial P_M]$.

derivatives of $g^t(P^t, x^t)$ with respect to the components of x ; i.e., we will have the following equations for each period t :¹⁴

$$(5) W^t = \nabla_x g^t(P^t, x^t); \quad t = 1, 2.$$

Thus the period t market sector input prices W^t paid to primary inputs can be obtained by differentiating the period t market sector NDP function with respect to the components of the period t input quantity vector x^t .

The constant returns to scale assumption on the technology sets S^t implies that the value of outputs will equal the value of inputs in period t ; i.e., we have the following relationships:

$$(6) g^t(P^t, x^t) = P^t \cdot y^t = W^t \cdot x^t; \quad t = 1, 2.$$

With the above preliminaries out of the way, we can now consider a definition of a family of output indexes which will capture the idea behind Samuelson's second definition of income or net output in the previous section. Diewert (1983; 1063) defined a *family of output indexes* between periods 1 and 2 for each reference output price vector P as follows:¹⁵

$$(7) Q(P, x^1, x^2) \equiv g^2(P, x^2) / g^1(P, x^1).$$

Note that the above definition combines the effects of technical progress and of input growth. A *family of technical progress indexes* between periods 1 and 2 can be defined as follows for each reference input vector x and each reference output price vector P :¹⁶

$$(8) \tau(P, x) \equiv g^2(P, x) / g^1(P, x).$$

Thus in definition (8), the market sector of the economy is asked to produce the maximum output possible given the same reference vector of primary inputs x and given that producers face the same reference net output price vector P but in the numerator of (8), producers have access to the technology of period 2 whereas in the denominator of (8), they only have access to the technology of period 1. Hence, if $\tau(P, x)$ is greater than 1, there has been *technical progress* going from period 1 to 2.

¹⁴ These relationships are due to Samuelson (1953) and Diewert (1974; 140). Note that $\nabla_x g^t(P^t, x^t) \equiv [\partial g^t(P^t, x^t) / \partial x_1, \dots, \partial g^t(P^t, x^t) / \partial x_N]$.

¹⁵ Diewert generalized the definitions used by Samuelson and Swamy (1974) and Sato (1976; 438). Samuelson and Swamy assumed only one input and no technical change while Sato had many inputs and outputs in his model but no technical change. These authors recognized the analogy of the output quantity index with Allen's (1949) definition of a quantity index in the consumer context.

¹⁶ Definition (8) may be found in Diewert (1983; 1063), Diewert and Morrison (1986; 662) and Kohli (1990).

A family of input growth indexes $\gamma(P, t, x^1, x^2)$ between periods 1 and 2 can be defined for each reference net output price vector P and each technology indexed by the time period t as follows:¹⁷

$$(9) \gamma(P, t, x^1, x^2) \equiv g^t(P, x^2)/g^t(P, x^1).$$

Thus using the period t technology and the reference net output price vector P , we say that there has been positive input growth going from the period 1 input quantity vector x^1 to the observed period 2 input quantity vector x^2 if $g^t(P, x^2) > g^t(P, x^1)$ or equivalently, if $\gamma(P, t, x^1, x^2) > 1$.

Problems

1. Show that the output quantity index defined by (7) has the following decompositions:

$$(a) Q(P, x^1, x^2) = \tau(P, x^2) \gamma(P, 1, x^1, x^2);$$

$$(b) Q(P, x^1, x^2) = \tau(P, x^1) \gamma(P, 2, x^1, x^2).$$

Thus the output quantity index between periods 1 and 2 does combine the effects of technical progress and input growth between periods 1 and 2.

2. We now specialize definition (7) to the case where the reference net output price vector is chosen to be the period 1 price vector P^1 , which leads to the following *Laspeyres type theoretical output quantity index*:

$$(a) Q(P^1, x^1, x^2) \equiv g^2(P^1, x^2)/g^1(P^1, x^1).$$

If we choose P to be the period 2 price vector P^2 , we obtain the following *Paasche type theoretical output quantity index*:

$$(b) Q(P^2, x^1, x^2) \equiv g^2(P^2, x^2)/g^1(P^2, x^1).$$

Under assumptions (6) above, show that the theoretical output quantity indexes defined by (a) and (b) above satisfy the following inequalities:

$$(c) Q(P^1, x^1, x^2) \geq P^1 \cdot y^2 / P^1 \cdot y^1 \equiv Q_L(P^1, P^2, y^1, y^2);$$

$$(d) Q(P^2, x^1, x^2) \leq P^2 \cdot y^2 / P^2 \cdot y^1 \equiv Q_P(P^1, P^2, y^1, y^2)$$

where $Q_L(P^1, P^2, y^1, y^2)$ and $Q_P(P^1, P^2, y^1, y^2)$ are the observable Laspeyres and Paasche net output quantity indexes.

3. Under what conditions will the inequalities (c) and (d) in problem 2 above hold as equalities?

¹⁷ Definition (9) can also be found in Diewert (1983; 1063).

4. Is the constant returns to scale assumption required to derive the results in problems 1 and 2 above?

5. Illustrate the two inequalities in problem 2 above using Figure 1; i.e., specialize M to the case $M = 2$, and then modify Figure 1 to illustrate the two inequalities in problem 2.

5. Maintaining Capital Again: the Physical versus Real Financial Perspectives

Recalling the material in section 2 of chapter I (on aggregation problems within the period; i.e., the beginning, end and middle of the period decomposition of the period) and section 9.2 in chapter I (on the Austrian production function concept), we see that Samuelson's $C + I$ framework for discussing alternative income concepts is not quite adequate to illustrate all of the problems involved in defining income concepts.

Recall that when using Samuelson's second income concept, nominal income in period 1 was defined as $P_C^1 C^1 + P_I^1 I^1$ where I^1 was defined to be net investment in period 1. Net investment can be redefined in terms of the difference between the beginning and end of period 1 capital stocks, K^0 and K^1 , so that I^1 equals $K^1 - K^0$. If we substitute this definition of net into Samuelson's definition of period 1 nominal income, we obtain the following definition for *period 1 nominal income*:

$$(10) \text{ Income 1} \equiv P_C^1 C^1 + P_I^1 I^1 = P_C^1 C^1 + P_I^1 (K^1 - K^0) = P_C^1 C^1 + P_I^1 K^1 - P_I^1 K^0.$$

Note that in the above definition, the beginning and end of period capital stocks are valued at the same price, P_I^1 . But this same price concept does not quite fit in with our Austrian one period production function framework where the beginning of the period capital stock should be valued at the beginning of the period opportunity cost of capital, P_K^0 say, and the end of the period capital stock should be valued at the end of the period expected opportunity cost of capital, P_K^1 .¹⁸ Replacing P_I^1 in (10) by P_K^1 (for K^1) and by P_K^0 (for K^0) leads to the following estimate for period 1 nominal income:

$$(11) \text{ Income 2} \equiv P_C^1 C^1 + P_K^1 K^1 - P_K^0 K^0.$$

But Income 2 is expressed in heterogeneous units: P_C^1 reflects the average level of prices of the consumption good in period 1 whereas P_K^1 reflects the price of capital at the *end* of period 1 while P_K^0 reflects the price of capital at the *beginning* of period 1. The problem is that there could be a considerable amount of price change going from the beginning to the end of period 1. In order to simplify our algebra, we will assume that it is not necessary to adjust P_C^1 into an end of period 1 price. Hence, all we need to do is to adjust the beginning of the period price of capital, P_K^0 , into a comparable end of period price that eliminates the effects of inflation over the duration of period 1. There are two possible price indexes that we could use: a (capital) *specific price index* $1+i^0$ or a *general price index* $1+\rho^0$ that is based on the movement of consumer prices; i.e., define i^0 and ρ^0 as follows:

¹⁸ For now, we will assume that expectations are realized in order to save on notational complexity. We will return to the problem of modeling expectations later in the chapter.

$$(12) 1+i^0 \equiv P_K^1/P_K^0;$$

$$(13) 1+\rho^0 \equiv P_C^1/P_C^0.$$

Now insert either $1+i^0$ or $1+\rho^0$ in front of the term $P_K^0 K^0$ in (11) and we obtain the following *two income concepts that measure income from the perspective of the level of prices prevailing at the end of period 1*:

$$(14) \text{Income 3} \equiv P_C^1 C^1 + P_K^1 K^1 - (1+\rho^0) P_K^0 K^0;$$

$$(15) \text{Income 4} \equiv P_C^1 C^1 + P_K^1 K^1 - (1+i^0) P_K^0 K^0$$

$$= \text{Income 1}$$

using (10) and (12).

It can be seen that Income 4 or 1 is a type of *specific price level adjusted income* that arose in the accounting literature surveyed in chapters II and III and Income 3 is a type of *general price level adjusted accounting income*.

We are now faced with a problem: how do we choose between Income 3 and Income 1 as the “best” income concept? We will address this question in the following section.

Problems

6. Refer back to Samuelson’s Method 1 in section 3 and construct a measure of income growth going from period 1 to 2 using Income 3 in place of Samuelson’s income concept.

7. Refer back to Samuelson’s Method 2 in section 3. Use Income 3 in place of Samuelson’s income measure and construct the Laspeyres and Paasche measures of income growth going from period 1 to 2. Are there any potential problems due to the fact that not all components of Income 3 have positive signs?

6. Measuring Business Income: the End of the Period Perspective

In order to see if one of the income concepts explained in the previous sections of this chapter can emerge as being the “right” concept, we will return to the one period profit maximization problem of the market sector of the economy using the Austrian one period production function framework explained in section 9.2 of chapter I.¹⁹

Using the notation introduced in the previous section and adding labour L as an input (with price W) and letting the market sector of the economy face the beginning of period 1 nominal interest rate r^0 , the period 1 Austrian profit maximization problem can be defined as follows:

$$(16) \max_{C^1, L^1, K^1} \{(1+r^0)^{-1}(P_C^1 C^1 - W^1 L^1 + P_K^1 K^1) - P_K^0 K^0 : (C^1, L^1, K^0, K^1) \in S^1\}$$

¹⁹ Recall that this framework is based on Hicks (1961; 23) and Edwards and Bell (1961; 71-72).

where S^1 is the period 1 Austrian production possibilities set. Note that we have treated the price P_C^1 of period 1 consumption and the price of period 1 labour W^1 as end of period 1 prices and hence the corresponding value flows are discounted to their beginning of period 1 equivalents using the beginning of period 1 nominal interest rate r^0 . From a practical measurement perspective, it is more useful to work with end of the period equivalents and so if we multiply the objective function in (16) through by $(1+r^0)$, we obtain the following *period 1 (end of period perspective) profit maximization problem*:

$$(17) \max_{C^1, L^1, K^1} \{P_C^1 C^1 - W^1 L^1 + P_K^1 K^1 - (1+r^0)P_K^0 K^0 : (C^1, L^1, K^0, K^1) \in S^1\}.$$

Recall equation (12) above, $1+i^0 \equiv P_K^1/P_K^0$, which defined the period 1 asset specific inflation rate i^0 , and equation (13) above, $1+\rho^0 \equiv P_C^1/P_C^0$, which defined the period 1 general inflation rate. The period 1 general inflation rate, ρ^0 , can be used to define the beginning of period 1 *real interest rate* r^{0*} and the period 1 *real rate of asset price inflation* i^{0*} as follows:

$$(18) 1+r^{0*} \equiv (1+r^0)/(1+\rho^0).$$

$$(19) 1+i^{0*} \equiv (1+i^0)/(1+\rho^0).$$

Now substitute (18) into the objective function in (17) and we obtain the following expression for period 1 *pure profits*:

$$\begin{aligned} (20) \quad & P_C^1 C^1 - W^1 L^1 + P_K^1 K^1 - (1+r^0)P_K^0 K^0 \\ & = P_C^1 C^1 - W^1 L^1 + P_K^1 K^1 - (1+r^{0*})(1+\rho^0)P_K^0 K^0 \\ & = P_C^1 C^1 + P_K^1 K^1 - (1+\rho^0)P_K^0 K^0 - \{W^1 L^1 + r^{0*}(1+\rho^0)P_K^0 K^0\} \\ & = \text{Income 3} - \{W^1 L^1 + U^1 K^0\} \end{aligned}$$

where Income 3 was defined by (14) in the previous section and the *period 1 waiting services user cost of the initial capital stock* is defined as

$$(21) U^1 \equiv r^{0*}(1+\rho^0)P_K^0.$$

With a constant returns to scale technology, competitive pricing on the part of market sector producers and correct expectations, pure profits will be zero and hence (20) set equal to zero will give us the following equations:

$$(22) \text{Income 3} = P_C^1 C^1 + P_K^1 K^1 - (1+\rho^0)P_K^0 K^0 \\ = W^1 L^1 + U^1 K^0$$

where $W^1 L^1$ represents period 1 payments to labour and $U^1 K^0$ represents interest payments to holders of the initial capital stock in terms of end of period 1 dollars. Note that all prices in (22) are expressed in end of period 1 equivalents.

What is the significance of equation (22)? This equation evidently suggests that Income 3 is the “right” concept of net output for period 1!²⁰

At this point, the reader may be slightly confused and may well ask: what happened to our usual user cost formula? The user cost U^1 defined by (21) does not look very familiar and so something must be wrong with the above algebra. In order to address this issue, we will specialize the Austrian model to the usual production function model, defined as follows:

$$(23) C^1 = F(I_G^1, L^1, K^0); K^1 = (1-\delta)K^0 + I_G^1$$

where I_G^1 is gross investment in period 1, C^1 is period 1 consumption output, L^1 is period 1 labour input, K^0 is the start of period 1 capital stock, K^1 is the end of period 1 finishing capital stock, $0 < \delta < 1$ is the constant (geometric) physical depreciation rate and F is the production function, which is decreasing in I_G and increasing in L and K .

If we substitute (23) into the objective function in (17) and solve the resulting period 1 profit maximization problem, we find that the optimal objective function can be written as follows:

$$\begin{aligned} (24) P_C^1 C^1 - W^1 L^1 + P_K^1 K^1 - (1+r^0)P_K^0 K^0 \\ &= P_C^1 C^1 - W^1 L^1 + P_K^1 [(1-\delta)K^0 + I_G^1] - (1+r^{0*})(1+\rho^0)P_K^0 K^0 && \text{using (18)} \\ &= P_C^1 C^1 + P_K^1 I_G^1 - W^1 L^1 - (1+r^{0*})(1+\rho^0)P_K^0 K^0 + (1-\delta)P_K^1 K^0 \\ &= P_C^1 C^1 + P_K^1 I_G^1 - W^1 L^1 - (1+r^{0*})(1+\rho^0)P_K^0 K^0 + (1-\delta)(1+i^0)P_K^0 K^0 && \text{using (12)} \\ &= P_C^1 C^1 + P_K^1 I_G^1 - W^1 L^1 - (1+r^{0*})(1+\rho^0)P_K^0 K^0 + (1-\delta)(1+\rho^0)(1+i^{0*})P_K^0 K^0 && \text{using (19)} \\ (25) &= P_C^1 C^1 + P_K^1 I_G^1 - W^1 L^1 - [(1+r^{0*})(1+\rho^0) - (1-\delta)(1+\rho^0)(1+i^{0*})]P_K^0 K^0 \\ &= P_C^1 C^1 + P_K^1 I_G^1 - (1+\rho^0)[1 - (1+i^{0*})(1-\delta)]P_K^0 K^0 - \{W^1 L^1 + r^{0*}(1+\rho^0)P_K^0 K^0\} \\ (26) &= P_C^1 C^1 + P_K^1 I_G^1 - (1+\rho^0)[1 - (1+i^{0*})(1-\delta)]P_K^0 K^0 - \{W^1 L^1 + U^1 K^0\}. \end{aligned}$$

The term in square brackets in (25) times P_K^0 represents the usual (end of period) gross rental user cost of capital u^1 ; i.e., we have²¹

$$(27) u^1 \equiv [(1+r^{0*})(1+\rho^0) - (1-\delta)(1+\rho^0)(1+i^{0*})]P_K^0 = [(1+r^0) - (1-\delta)(1+i^0)]P_K^0.$$

Thus if pure profits are zero for the period 1 data, expression (25) set equal to 0 translates into the following usual gross output equals labour payments plus gross payments to the starting stock of capital:

²⁰ Note that our Income 3 follows the adjustments to cash flows recommended by the accountant Sterling: “It follows that the appropriate procedure is to (1) adjust the present statement to current values and (2) adjust the previous statement by a price index. It is important to recognize that *both* adjustments are necessary and that neither is a substitute for the other. Confusion on this point is widespread.” Robert R. Sterling (1975; 51). Sterling (1975; 50) termed his income concept *Price Level Adjusted Current Value Income*. Unfortunately, Sterling’s income concept has not been widely endorsed in accounting circles.

²¹ See section 3 of chapter I.

$$(28) P_C^1 C^1 + P_K^1 I_G^1 = W^1 L^1 + u^1 K^0.$$

Now look at the term in square brackets in (26) times P_K^0 :

$$(29) [1 - (1+i^{0*})(1-\delta)]P_K^0 \equiv \pi^1.$$

It can be seen that π^1 is the *ex ante real time series depreciation* for a unit of capital that was defined by (38) in chapter I. Note that if the (anticipated) real asset inflation rate i^{0*} is 0, then π^1 equals normal wear and tear depreciation, δP_K^0 . We multiply π^1 by $(1+\rho^0)$ and define the resulting price as the *period 1 price of depreciation*, P_D^1 :

$$(30) P_D^1 \equiv (1+\rho^0)\pi^1 = (1+\rho^0)[1 - (1+i^{0*})(1-\delta)]P_K^0.$$

If pure profits are zero for the period 1 data, (26) set equal to zero translates into the following decomposition of net product, where we have used definition (30):

$$(31) P_C^1 C^1 + P_K^1 I_G^1 - P_D^1 K^0 = W^1 L^1 + U^1 K^0 ;$$

i.e., the value of consumption $P_C^1 C^1$ plus the value of gross investment at end of period 1 prices $P_K^1 I_G^1$ less real time series depreciation restated to end of period (anticipated) values $-(1+\rho^0)\pi^1 K^0 = -P_D^1 K^0$ equals the value of labour payments $W^1 L^1$ plus the real value of interest payments at end of period prices $U^1 K^0 = r^{0*}(1+\rho^0)P_K^0 K^0$.

All of this algebra shows that Income 4 does not contradict the usual (gross) user cost of capital approach to capital services measurement. The Income 3 concept simply decomposes the usual (gross) user cost term $u^1 K^0$ into the sum of two terms: a net user cost term $U^1 K^0$ plus a real time series depreciation term $(1+\rho^0)\pi^1 K^0$ and then takes the depreciation term over to the output side of the equation but with a negative sign in front of it so that it acts as an offset to the gross investment term $P_K^1 I_G^1$. The resulting net output can be interpreted as an income concept that maintains real financial capital. Thus at this point, we might tentatively conclude that working with the usual discounted profits model leads to *a preference for a maintenance of real financial capital income concept over a maintenance of a specific inflation adjusted income concept*, at least at the theoretical level. However, this tentative conclusion is not quite correct:²² it turns out that we can manipulate the discounted profits model in a way that will justify the maintenance of real physical capital as opposed to real financial capital. We show this in the following paragraphs.

Using the above material, it can be seen that the traditional gross user cost or rental price of capital services for period 1, u^1 defined by (27), was decomposed into the sum of two terms, $U^1 + P_D^1$, using (21) and (30), where the first term reflects “waiting” capital services and the second term combines “physical” depreciation and anticipated obsolescence (scarcity) that is due to a possible real price decline (or appreciation) of the asset. In the literature, the second term is usually called an anticipated revaluation term.

²² I owe this point to Paul Schreyer.

Instead of decomposing the gross user cost of capital into the sum of two terms, we now decompose it into the sum of three terms. The first term is U^1 as before but now P_D^1 is decomposed into the sum of two terms, $D^1 + R^1$, where D^1 is the *physical depreciation term* and R^1 is the *revaluation term*, defined as follows:

$$(32) D^1 \equiv \delta(1+\rho^0)P_K^0;$$

$$(33) R^1 \equiv -i^{0*}(1+\rho^0)P_K^0.$$

Using (21), (27), (30), (32) and (33), it can be verified that the gross rentals user cost u^1 and the real time series user cost P_D^1 have the following decompositions in terms of U^1 , D^1 and R^1 :

$$(34) u^1 = U^1 + D^1 + R^1;$$

$$(35) P_D^1 = D^1 + R^1.$$

Thus the real time series price of depreciation P_D^1 is equal to the sum of the physical depreciation price D^1 plus the revaluation price R^1 and the gross rentals user cost u^1 is equal to the sum of D^1 , R^1 and the price of waiting services U^1 . We use the definition of R^1 to establish the following identity:

$$(36) (1+\rho^0)P_K^0K^0 - (1+i^0)P_K^0K^0 = (1+\rho^0)P_K^0K^0 - (1+i^{0*})(1+\rho^0)P_K^0K^0 \quad \text{using (19)}$$

$$= -i^{0*}(1+\rho^0)P_K^0K^0$$

$$= R^1K^0 \quad \text{using (33).}$$

Now recall the two expressions for Income 3 in (22). If we add the top line in (36) to Income 3, we obtain Income 4 or 1. Thus using (22) and (36), we obtain the following expressions for Income 1:

$$(37) \text{Income 1} \equiv P_C^1C^1 + P_K^1K^1 - (1+i^0)P_K^0K^0$$

$$= W^1L^1 + U^1K^0 + R^1K^0.$$

Thus if we adopt a physical maintenance of capital point of view to measure income, the “correct” user cost for the beginning of the period stock of capital K^0 is $U^1 + R^1$, the sum of the waiting services and revaluation terms.

If we specialize our Austrian model to the usual production function model with geometric depreciation defined by (23), then (24) set equal to 0 gives us the following decomposition of the optimized objective function for the period 1 profit maximization problem (17):

$$(38) 0 = P_C^1C^1 - W^1L^1 + P_K^1K^1 - (1+r^0)P_K^0K^0$$

$$= P_C^1C^1 + P_K^1I_G^1 - (1+\rho^0)[1 - (1+i^{0*})(1-\delta)]P_K^0K^0 - \{W^1L^1 + U^1K^0\} \quad \text{using (24)}$$

$$= P_C^1C^1 + P_K^1I_G^1 - \{W^1L^1 + U^1K^0 + D^1K^0 + R^1K^0\} \quad \text{using (32) and (33)}$$

$$= P_C^1C^1 + P_K^1I_G^1 - D^1K^0 - \{W^1L^1 + U^1K^0 + R^1K^0\}.$$

Using (37) and (38), we obtain the following expression for Income 1 in the case where we have geometric depreciation:

$$(39) \text{ Income 1} = P_C^1 C^1 + P_K^1 I_G^1 - D^1 K^0.$$

Thus the Austrian production framework is consistent with both income concepts: the financial maintenance of capital concept (Income 3) and the physical maintenance of capital concept (Income 1 or 4). In fact, (24) shows that the Austrian production framework is also consistent with an “income” concept that is equal to gross product. Table 1 below summarizes the definitions of the different income concepts in the case of geometric depreciation and gives the user cost concept that matches up with the corresponding income concept.

Table 1: Alternative Income Concepts and the Corresponding User Costs of Capital

Income Concept	Corresponding Net Output Definition	Corresponding User Cost Value
Gross Output	$P_C^1 C^1 + P_K^1 I_G^1$	$U^1 K^0 + D^1 K^0 + R^1 K^0 = u^1 K^0$
Income 1	$P_C^1 C^1 + P_K^1 I_G^1 - D^1 K^0$	$U^1 K^0 + R^1 K^0$
Income 3	$P_C^1 C^1 + P_K^1 I_G^1 - D^1 K^0 - R^1 K^0$	$U^1 K^0$

Looking at Table 1, it can be seen that the usual gross output (or GDP) definition of “income” matches up with the usual gross rentals user cost of capital, u^1 . The Income 1 definition, which corresponds to a physical maintenance of capital income concept, takes the physical depreciation term $D^1 K^0$ out of the gross rentals user cost and treats it as a negative contribution to output. Finally, the Income 3 definition, which corresponds to a maintenance of real financial capital concept, takes both the physical depreciation term $D^1 K^0$ and the revaluation or obsolescence term $R^1 K^0$ out of the gross rentals user cost and treats them both as a negative contributions to output. Thus the Austrian model of production is consistent with all three income concepts.

Typically, the price of capital goods will decline relative to the price of consumption goods and services (or will increase at a lower rate) so that the real asset inflation rate, i^{0*} defined by (19), will usually be negative. Under this hypothesis, R^1 will be positive²³ and we will have the following inequalities between the three income concepts:

$$(40) \text{ Gross Income} > \text{Income 1} > \text{Income 3}.$$

We conclude this section with a brief discussion on which income concept is “best” from the perspective of describing household consumption possibilities over time. The gross income concept clearly overstates long run consumption for the consumer and so this concept can be dismissed. It is clear that Income 2 is a very defective measure of sustainable consumption prospects, since this measure can be made very large if there is a substantial amount of inflation between the beginning and end of the period. All of the

²³ Under these conditions, we can say that the capital good is experiencing a form of obsolescence.

other income measures are invariant to general inflation between the beginning and end of the accounting period.²⁴ However, choosing between the physical and real financial maintenance perspectives is more problematical: reasonable economists could differ on this choice. The merits of the two perspectives were discussed in section 7 of chapter I where we reviewed the controversy between Pigou (1941; 273-274), who favored the maintenance of physical capital approach (Income 1), and Hayek (1941; 276-277), who favored the maintenance of real financial capital approach (Income 3). Our preference is for Income 3, following in the footsteps of Hayek and Hill (2000; 6), who felt that Income 1 would generally overstate the real value of consumption in any period due to its neglect of (foreseen) obsolescence.

As Hicks (1939; 184) said in his Income chapter: “What a tricky business this all is!”

7. Approximations to the Income Concept

We now relax the perfect foresight assumptions that were made in the previous section. This means that i^0 and ρ^0 defined by (12) and (13) are not known variables; rather ρ^0 is the anticipated (Consumer Price Index) nominal inflation rate for consumption goods and services and i^0 is the anticipated specific asset inflation rate, where the anticipations are formed at the start of period 1. This means that the period 1 real interest rate r^{0*} and real asset inflation rate i^{0*} defined by (18) and (19) are also anticipated variables. Thus we must now address the question as to how exactly these anticipated variables will be estimated in empirical applications of the income concept.²⁵

We will consider three alternative methods for approximating these anticipated variables but the reader will be able to construct many additional approximations, depending on the purpose at hand.

Approximation Method 1:

The two assumptions that are made in order to implement this first method are the following ones:

- Approximate general inflation adjusted beginning of the period price of capital, $(1+\rho^0)P_K^0$ by the period 1 average price for the corresponding investment good P_I^1 and
- Set the anticipated specific asset real inflation rate i^{0*} equal to zero.

Thus we make the following assumptions:

$$(41) (1+\rho^0)P_K^0 = P_I^1 ;$$

$$(42) i^{0*} = 0.$$

²⁴ We regard this invariance property as a fundamental property that any sensible income measure should satisfy.

²⁵ The approximations that we suggest below can be used to implement either Gross Income, Income 1 or Income 3, depending on the user's choice of income concept.

With the above two assumptions, we find that the waiting user cost of capital U^1 defined by (21), the gross rentals user cost of capital u^1 defined by (27) and the price of depreciation P_D^1 defined by (30) simplify as follows:

$$(43) U^1 \equiv r^{0*}(1+\rho^0)P_K^0 = r^{0*}P_I^1 ;$$

$$(44) u^1 \equiv [(1+r^{0*})(1+\rho^0) - (1-\delta)(1+\rho^0)(1+i^{0*})]P_K^0 = [r^{0*} + \delta]P_I^1 ;$$

$$(45) P_D^1 \equiv (1+\rho^0)\pi^1 = (1+\rho^0)[1 - (1+i^{0*})(1-\delta)]P_K^0 = \delta P_I^1 .$$

The only remaining approximation issue is the approximation for the anticipated period 1 real interest rate r^{0*} . There are three obvious possible choices for r^{0*} :

- Calculate the balancing real rate of return that will make the profits of the production unit under consideration equal to zero;²⁶
- Smooth past balancing real rates of return and use the smoothed rate as the predicted rate; or
- Simply pick a plausible constant real rate of return, such as 3 or 4 percent (for annual data).

This method of approximation should be appealing to national income accountants.

Approximation Method 2:

One problem with the previous method is that it neglects *foreseen obsolescence* that is due to anticipated (real) asset price decline.²⁷ For example, for the past 40 years, the real price of computers in constant quality units has steadily declined and these declines are very likely to continue. Hence, the i^{0*} term in our definition of the price of depreciation, P_D^1 defined by (30) and in the revaluation term R^1 defined by (33), should be a negative number if the asset under consideration is computers (or any related asset that has a substantial computer chip component). Thus again make assumption (41) above but now for assets that are expected to decline in price, estimate i^{0*} by smoothed past real declines in the asset price (expressed in constant quality units). With these assumptions, we find that the waiting user cost of capital U^1 defined by (21), the gross rentals user cost of capital u^1 defined by (27) and the price of depreciation P_D^1 defined by (30) simplify as follows:

$$(46) U^1 \equiv r^{0*}(1+\rho^0)P_K^0 = r^{0*}P_I^1 ;$$

$$(47) u^1 \equiv [(1+r^{0*})(1+\rho^0) - (1-\delta)(1+\rho^0)(1+i^{0*})]P_K^0 = [r^{0*} + \delta - (1-\delta)i^{0*}]P_I^1 ;$$

$$(48) P_D^1 \equiv (1+\rho^0)\pi^1 = (1+\rho^0)[1 - (1+i^{0*})(1-\delta)]P_K^0 = [\delta - (1-\delta)i^{0*}]P_I^1 .$$

Examining (47) and (48), it can be seen that if i^{0*} is negative, then the gross rental user cost u^1 and the price of depreciation P_D^1 become *larger* compared to the corresponding

²⁶ Substitute u^1 defined by (44) into equation (28) and solve the resulting equation for the balancing real rate r^{0*} .

²⁷ Recall from chapter VI that not all foreseen obsolescence is due to expected future real price declines; i.e., some models of obsolescence imply contracting asset lives.

values when i^{0*} is set equal to 0. Once an estimate for i^{0*} has been obtained, r^{0*} can be estimated using any of the three methods outlined under Approximation Method 1 above.

However some assets have a long history of real price appreciation (e.g., urban land) and so the question is: for those assets which we expect to appreciate in real terms (i.e., i^{0*} is expected to be positive instead of negative), should we insert these positive expected values into u^1 defined by (47) and P_D^1 defined by (48)?

Symmetry suggests that the answer to the above question is yes.²⁸ However, note that if i^{0*} is large and positive enough, then it could lead to the gross rental price u^1 defined by (47) being *negative*. It is not plausible that expected gross rentals be negative, since under these conditions, we would expect the corresponding asset price to be immediately bid up to eliminate this negative expected rental price. Thus some caution is called for when simply inserting a smoothed value of past real rates of asset price appreciation i^{0*} into formulae (47) and (48): we do not want to insert a value of i^{0*} that is so large that it makes u^1 negative. Hence we want i^{0*} to satisfy the following inequality:

$$(49) \quad i^{0*} < (1-\delta)^{-1}[r^{0*} + \delta].$$

In practical applications of this method, we suggest that for most assets, the assumption that the corresponding anticipated real inflation i^{0*} is zero is appropriate. Only in exceptional cases where we are fairly certain that producers are anticipating real capital gains or losses should we insert a nonzero i^{0*} into formulae (47) and (48).

Approximation Method 3:

Approximation Methods 1 and 2 explained above are suitable for applications of the income concept at the national economy or industry levels where current prices for each class of asset can be obtained.²⁹ However, these methods are not usually suitable for applications at the level of the individual firm or enterprise, because current objective and replicable prices for each asset used by the enterprise will generally not be available. Hence the question arises: how can we approximate the income concept at the firm level?

The simplest and most useful assumptions in this context are the following ones:

$$(50) \quad (1+\rho^0) = P_C^1 / P_C^0 ;$$

$$(51) \quad i^{0*} = 0.$$

Thus we set $1+\rho^0$ equal to the ex post amount of Consumer Price Index inflation that occurred from the beginning to the end of the accounting period and assume that specific real asset price inflation is 0.

²⁸ A positive i^{0*} will have the effect of reducing economic depreciation. This should not bother us unduly; recall our discussion of the basic forms of productive activity in section 4 of chapter II where we concluded that anticipated capital gains were productive.

²⁹ Typically, we will have to rely on national statistical agency index numbers for estimates of current asset prices.

With these assumptions, we find that the waiting user cost of capital U^1 defined by (21), the gross rentals user cost of capital u^1 defined by (27) and the price of depreciation P_D^1 defined by (30) simplify as follows:

$$(52) U^1 = r^{0*}(1+\rho^0)P_K^0$$

$$(53) u^1 = (1+\rho^0)[r^{0*} + \delta]P_K^0 ;$$

$$(54) P_D^1 = (1+\rho^0)\delta P_K^0 .$$

The net effect of our present assumptions is that we can basically use historical cost accounting, except that historical cost depreciation allowances should be escalated each accounting period by the amount of CPI inflation that occurred over the period; i.e., our present approximations lead to *purchasing power adjusted historical cost accounting*, which was studied in section 3 of chapter 3.

The accounting profession is unlikely to embrace the above very simple and straightforward accounting adjustments to historical cost depreciation³⁰ but it is important that the tax authorities recognize the importance of indexing depreciation allowances for general inflation. Using the notation developed in section 6 above, *taxable income* should be defined as follows:

$$(55) \text{Taxable income} \equiv P_C^1 C^1 + P_K^1 I_G^1 - P_D^1 K^0 - W^1 L^1$$

where P_D^1 is defined by (54).³¹ With this definition of taxable income, the real return to capital will be taxed and the ‘unjust’ taxation of inflation inflated “profits” will be prevented.

For additional material on the role of expectations in income measures, the reader is referred to Hill and Hill (2003).

8. Choosing an Income Concept: A Summary

Table 1 in section 6 presented 3 “income” or output concepts:

- Gross output;
- Income 1 or “wear and tear” adjusted net product³² and
- Income 3 or “wear and tear” and “anticipated revaluation” adjusted net product.³³

³⁰ Recall the discussions in chapters II and III.

³¹ If we wanted the business income tax to fall on pure profits or rents, we would also subtract $U^1 K^0$ equal to $r^{0*}(1+\rho^0)P_K^0 K^0$ from the right hand side of (55) where r^{0*} would be a “normal” real rate of return to capital that would be chosen by the tax authorities. The resulting system of business income taxation would lead to minimal deadweight loss.

³² We can associate this income concept with Marshall (1890), Haig (1921), Pigou (1941) and Samuelson (1961).

³³ We can associate this income concept with Hayek (1941), Sterling (1975) and Hill (2000).

Table 1 also indicated that the “traditional” user cost of capital (which approximates a market rental rate for the services of a capital input for the accounting period), u^1 , consists of three additive terms; i.e., we have:

$$(56) u^1 = U^1 + D^1 + R^1$$

where U^1 is the reward for waiting term (interest rate term), D^1 is the cross sectional depreciation term (or wear and tear depreciation term) and R^1 is the anticipated revaluation term, which can be interpreted as an obsolescence charge if the asset is anticipated to fall in price over the accounting period. The Gross output income concept corresponds to the traditional user cost term u^1 . This income measure can be used as an approximate indicator of short run production potential. However, it is not suitable for use as an indicator of sustainable consumption. In order to obtain indicators of sustainable consumption, we turn to Income 1 and 3.

To obtain Income 1, we simply take the wear and tear component of the traditional user cost, D^1 , times the beginning of period corresponding capital stock, K^0 , out of the primary input category and treat it as a negative offset to the period’s gross investment. The resulting Income 1 can be interpreted to be consistent with the position of Pigou (1941), who argued against including any kind of revaluation effect in an income concept. This position can also be interpreted as a *maintenance of physical capital approach* to income measurement. In terms of the Hicks (1961) and Edwards and Bell (1961) Austrian production model, capital at the beginning and end of the period (K^0 and K^1 respectively) are both valued at the end of period stock price for a unit of capital, P_K^1 , and the contribution of capital accumulation to period income is simply the difference between the end of period value of the capital stock and the beginning of the period value (valued at end of period prices), $P_K^1 K^1 - P_K^1 K^0$.³⁴ This difference between end and beginning of period values for the capital stock can be converted into consumption equivalents and then can be added to actual period 1 consumption in order to obtain Income 1. This income concept is certainly defensible.

To obtain Income 3, we subtract both wear and tear depreciation from gross output, $D^1 K^0$, as well as the revaluation term, $R^1 K^0$, and treat both of these terms as negative offsets to the period’s gross investment. The resulting Income 3 can be interpreted to be consistent with the position of Hayek (1941), Sterling (1975) and Hill (2000). This position can also be interpreted as a *maintenance of real financial capital approach* to income measurement. In terms of the Hicks (1961) and Edwards and Bell (1961) Austrian production model, capital stocks at the beginning of the period and end of the period are valued at the prices prevailing at the beginning and the end of the period,³⁵ P_K^0 and P_K^1 respectively, and then these beginning and end of period values of the capital stock are converted into consumption equivalents (at the prices prevailing at the beginning and end of the period). Thus the end of the period value of the capital stock is $P_K^1 K^1$ and this value can be converted into consumption equivalents at the consumption

³⁴ Using Samuelson’s (1961) Figure 1 above, this income can be interpreted as the distance OJ along the consumption axis.

³⁵ Strictly speaking, the end of period price is an expected end of period price.

prices prevailing at the end of the period. The beginning of the period value of the capital stock is $P_K^0 K^0$ but to convert this value into consumption equivalents at end of period prices, we must multiply this value by $(1+\rho^0)$, which is one plus the rate of consumer price inflation over the period. This price level adjusted difference between end and beginning of period values for the capital stock, $P_K^1 K^1 - (1+\rho^0)P_K^0 K^0$, can be converted into consumption equivalents and then can be added to actual period 1 consumption in order to obtain Income 3. Thus the difference between Income 1 and Income 3 can be viewed as follows: Income 1 (asymmetrically) uses the end of period stock price of capital to value both the beginning and end of period capital stocks and then converts the resulting difference in values into consumption equivalents at the prices prevailing at the end of the period whereas Income 3 symmetrically values beginning and end of period capital stocks at the stock prices prevailing at the beginning and end of the period and *directly* converts these values into consumption equivalents and then adds the difference in these consumption equivalents to actual consumption. Thus Income 3 also seems to be a defensible concept.³⁶

In order to highlight the difference between Incomes 1 and 3, use definitions (10), (12) and (14) in order to compute their difference:

$$(57) \text{ Income 1} - \text{Income 3} = P_C^1 C^1 + P_I^1 K^1 - P_I^1 K^0 - [P_C^1 C^1 + P_K^1 K^1 - (1+\rho^0)P_K^0 K^0] \\ = (\rho^0 - i^0)P_K^0 K^0.$$

If ρ^0 (the general consumer price inflation rate) is greater than i^0 (the asset inflation rate) over the course of the period, then there is a negative real revaluation effect (so that obsolescence effects dominate). In this case, Income 3 is less than Income 1, reflecting the fact that capital stocks have become less valuable (in terms of consumption equivalents) over the course of the period. If ρ^0 is less than i^0 over the course of the period, then the real revaluation effect is positive (so that capital stocks have become more valuable over the period). In this case, Income 3 exceeds Income 1, reflecting the fact that capital stocks have become more valuable over the course of the period and this real increase in value contributes to an increase in the period's income which is not reflected in Income 1.

To summarize: both Income 1 and Income 3 both have reasonable justifications. Choosing between them is not a straightforward matter.³⁷

³⁶ Income 1 is much easier to justify to national income accountants because it relies on the standard production function model. On the other hand, Income 3 relies on the Austrian model of production as developed by Hicks (1961) and Edwards and Bell (1961) and this production model is not very familiar. This Austrian model of production has its roots in the work of Böhm-Bawerk (1891), von Neumann (1937) and Malinvaud (1953) but these authors did not develop the user cost implications of the model. On the user cost implications of the Austrian model, see Hicks (1973; 27-35) and Diewert (1977;108-111) (1980; 472-474).

³⁷ However, we lean towards Income 3 over income 1 for three reasons: (i) It seems to us that (expected) obsolescence charges are entirely similar to normal depreciation charges and Income 3 reflects this similarity; (ii) Income 3 does not value the beginning and end of period value of the capital stock in an asymmetric manner as does Income 1 and (iii) it seems to us that waiting services ($U^1 K^0$) along with labour

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services and land rents are natural primary inputs whereas depreciation and revaluation services (D^1K^0 and R^1K^0 respectively) are more naturally regarded as intermediate input charges.

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