Estimation of R&D Depreciation Rates: A Suggested Methodology and Preliminary Application

Ning Huang and Erwin Diewert[1]
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Department of Economics
University of British Columbia
Vancouver, Canada V6T 1Z1.

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Email Addresses: Ning.Huang@statcan.gc.ca and diewert@econ.ubc.ca

Abstract

The 2008 version of the SNA will recommend capitalization of R&D expenditures. To implement this recommendation, measures for the stock of R&D capital must be constructed, and this implies a need to determine the depreciation rate of R&D capital. In this paper, we develop a simple model, based on a production function method that allows for monopolistic competition, to estimate the annual depreciation rate of R&D capital. We treat R&D capital as a technology shifter instead of as an explicit input factor. Both the R&D stock and the time variable are used to capture technological progress. Modeling R&D capital in this manner can better represent the role R&D plays in economic growth. Estimated R&D depreciation rates and markup factors are presented for the U.S. manufacturing sector and four U.S. knowledge intensive industries, namely chemical products (SIC 28), non-electrical machinery (SIC 35), electrical products (SIC 36) and transportation equipment (SIC 37).

Keywords

Research and development, R&D depreciation rates, R&D capital stocks, production theory, obsolescence, flexible functional forms, technology shifts, monopolistic competition.

Journal of Economic Literature Classification Codes

C32, C43, C81, D24, D43 D92.

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1. Introduction

In a recent paper presenting an innovative new indicator of technical change, Michelle Alexopoulos and Jon Cohen (2008) write that: "Although technical change is central in much of modern economics, our ability to identify empirically the factors that shape its pace, nature, and impact are constrained by data limitations." The relevance of this statement is underlined by both the current imperative for national economies to achieve better productivity growth and new developments on the international scene in accounting for R&D in the System of National Accounts.

Although research and development (R&D) expenditures account for only a small portion of GDP, their importance in creating new technologies and promoting productivity growth is widely recognized. This necessitates the re-examination of measurement issues related to these expenditures that have broad implications for economists conducting analyses of the contribution of knowledge capital to endogenous growth and to the nation's wealth, as well as for policy makers who may want to subsidize R&D investments in order to maximize economic growth.

A widely used definition of R&D is given in the 1993 Frascati Manual (OECD, 1994).[2] In this document, R&D is defined as "creative work undertaken on a systematic basis in order to increase the stock of knowledge, including knowledge of man, culture and society, and the use of this stock of knowledge to devise new applications". Three activities are covered: basic research, applied research, and experimental development. The key criteria that distinguish R&D from the other related activities are "the presence in R&D of an appreciable element of novelty and the resolution of scientific and/or technological uncertainty."

The SNA 1993, which is still in effect as the international standard, does not treat R&D expenditures as investment. Thus, in the National Income and Product Accounts (NIPA's) of United States, R&D expenditures are treated as an intermediate input for business and current consumption for the non-profit institutions and government. However, the SNA 2008 that will soon supplant the SNA 1993 does recommend the capitalization of R&D expenditures. Successful R&D investments add to the stock of knowledge, and this stock in turn provides a flow of services over time, rather than in one period. Thus, R&D resembles investment more closely than intermediate input or current consumption.

However, though the idea of capitalizing R&D expenditures has been accepted in principle for the SNA 2008, implementation requires the determination of depreciation rates for R&D capital. Due to the measurement difficulties involved, Nadiri and Prucha (1996) observe that, "researchers doing applied work typically assume an arbitrary depreciation rate of 10 to 15 percent to construct the stock of R&D capital using the perpetual inventory method." However, it would be preferable to have a scientific method for determining this depreciation rate.

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[2] This version of the Frascati Manual formed the methodological basis for the influential 2001 OECD STI Scoreboard. See http://library.certh.gr/libfiles/MOBILITY-PORTAL/MON-251-OECD-SCOREBOARD-2001-A92.pdf. The version of the Frascati Manual currently in use is 2002 (OECD 2002).

There have been some attempts to empirically measure R&D depreciation rates. Pakes and Shankerman (1981) estimated the rate of depreciation in the private value of patents using European data on patent renewal fees and rates of renewal; Nadiri and Prucha (1996) measured the depreciation rate of R&D stock using a factor requirements function and restricted cost function, and treating the R&D capital as a "normal" reproducible capital input, as is also the case for Bernstein and Mamuneas (2005) who estimated R&D depreciation rates in an intertemporal cost minimization framework. However, the approaches adopted in previous studies generally do not allow for non-R&D effects that improve the technology, such as knowledge diffusion through education and learning by doing. Thus all of the technological improvement is inappropriately attributed to R&D investment.[3] Also, the estimation is typically conducted in a framework of competitive pricing behaviour. However, private R&D investments are often undertaken with the explicit goal of achieving short-run monopolistic advantages over competitors. Thus the assumption of competitive behaviour on output markets is unsuitable. Another weakness with most studies is that R&D investments are treated similarly to investments in ordinary physical capital, but R&D investments are quite different in their effects, as is explained subsequently.

In this paper, we propose to treat the stock of R&D capital not as an explicit input factor; instead we define the stock of R&D capital to be the technology index that locates the economy's production frontier. An increase in the stock of R&D shifts the production frontier outwards. In our model, the R&D capital depreciation rate is estimated within a monopolistic competition framework using gross R&D investment data. We estimate R&D depreciation rates for the U.S. total manufacturing sector and four U.S. knowledge intensive industries: chemical products (SIC 28), non-electrical machinery (SIC 35), electrical products (SIC 36) and transportation equipment (SIC 37).

The rest of the paper is organized as follows. Section 2 explains the construction of the R&D stock and the reasons for R&D depreciation. Section 3 develops our basic model for estimating the rate of depreciation for R&D capital. Section 4 presents the estimation methodology and results. Section 5 concludes. Appendix A shows the derivation of the estimating equations. Different sets of break points used in the linear spline model and quadratic spline model are given in appendix B.

2. Construction of the R&D Stock

According to the 1993 Frascati Manual definition, the objective of conducting R&D is to increase the stock of knowledge. Thus we define R&D capital as the knowledge asset created by R&D investment. Hence the stock of R&D capital can be regarded as a proxy for society's technological level. R&D investments act as a mechanism for shifting outward society's production possibility frontier. This treatment is the major distinguishing feature of our approach. The studies of others treat R&D capital as an explicit factor input in a manner that is similar to the treatment of ordinary physical capital.

2.1 R&D Capital and Ordinary Physical Capital

[3] The work of Bernstein and Mamuneas (2005) is not subject to this criticism.

Physical capital like machinery, equipment and buildings wears out through use and the efficiency tends to decline over time. Successful R&D ventures create new knowledge for the firms conducting the R&D. This new knowledge can be either a new cost-saving process, or a new technology for an innovative or quality-improved product. R&D capital does not wear out because of its utilization and its absolute efficiency level does not change with the passage of time. However, R&D capital is subject to a type of depreciation because processes or products can become obsolete over time.

Although physical capital and R&D capital are both called "capital assets," there are some fundamental differences. First, physical capital, such as machines and equipment, can be reproduced over multiple periods. With physical capital, reproducibility makes it possible for us to observe at the same point in time rental prices of different vintages, and also used asset prices of different vintages, of a capital asset, and this in turn allows us to estimate depreciation rates. In contrast, for R&D capital, once, say, a new blueprint has been produced, it can be made available to many economic units without further productive activity. Typically, we cannot collect price information for different vintages of R&D capital at the same point in time[4]. Secondly, physical capital and R&D capital support production in different ways. As Pitzer (2004) pointed out, R&D capital acts as if it were producing "recipes" while physical capital is regarded as one of the "productive" inputs[5] that are "consumed" during a production process. Thirdly, the reasons for depreciation are different for these two types of capital assets. We return to this issue subsequently. Because of these differences, we believe that the treatment of R&D assets should necessarily be different from the treatment of reproducible capital assets.

2.2 Constructing the Stock of R&D Capital

Lacking a good measure of R&D output, we use input information in the form of gross (real) R&D expenditures as a proxy measure. Because all the technologies, new or old, are created by R&D investments, the R&D stock can be written as a function of a series of past R&D investments. Thus, the period t R&D stock is defined as follows:

$$(1)\ R_t \equiv \theta_1^t I_{t-1} + \theta_2^t I_{t-2} + \theta_3^t I_{t-3} + \theta_4^t I_{t-4} + \theta_5^t I_{t-5} + \ldots$$

where θ_n^t is the period t efficiency index representing how much the R&D investment that was made n periods before period t contributes to the technology or knowledge stock in period t. The R&D lags and weights are all incorporated in these efficiency indexes.

[4] However, we can sometimes observe the market price for the rights to some new technology. If the same technology is again sold in a future period, then we could collect price information for different periods and infer a depreciation rate for the R&D investment

[5] Pitzer (2004) defines "productive input" as: "Fundamental to production is the notion that inputs are proportional in some sense to outputs. Outputs are created by combining a particular collection of inputs in a particular manner. … If more outputs are desired, then more inputs are necessary, and usually, more of all inputs. It may not be necessary to double the inputs to double the outputs, but more inputs are necessary to produce more outputs."

I_{t-n} is the R&D investment made in year t–n. We expect that the further back the R&D investment was made, the less it contributes to the prevailing knowledge. Thus the following relationships should hold among all the efficiency indexes[6]:

(2) $\theta_1^t \geq \theta_2^t \geq \theta_3^t \geq \theta_4^t \geq \theta_5^t \geq \ldots$.

We add the superscript t because these efficiency indexes may vary with time. The speed of the technological upgrading is one factor which helps to determine the size of the efficiency indexes. If the new technology is created quickly, the past investment may become irrelevant at a fast pace with the newly updated-knowledge. For example, if there were more new knowledge created in year t compared to year t−1, we would expect the following inequalities to hold:

(3) $\theta_1^t > \theta_1^{t-1}$ and $\theta_2^t < \theta_2^{t-1}$; $\theta_3^t < \theta_3^{t-1}$; $\theta_4^t < \theta_4^{t-1}$; ... .

These inequalities show us that the previous investments become less important at a faster speed as the pace of technology improvement speeds up. The series of R&D stocks can be written as:

(4) $R_t = \theta_1^t I_{t-1} + \theta_2^t I_{t-2} + \theta_3^t I_{t-3} + \ldots + \theta_{t-1}^t I_1 + \theta_t^t I_{t-5} R_0$;
$R_{t-1} = \theta_1^{t-1} I_{t-2} + \theta_2^{t-1} I_{t-3} + \theta_3^{t-1} I_{t-4} + \ldots + \theta_{t-2}^{t-1} I_1 + \theta_{t-1}^{t-1} R_0$;
$R_{t-2} = \theta_1^{t-2} I_{t-3} + \theta_2^{t-2} I_{t-4} + \theta_3^{t-2} I_{t-5} + \ldots + \theta_{t-2}^{t-2} I_1 + \theta_{t-2}^{t-2} R_0$;
...

where R_0 is the initial knowledge stock. The efficiency index varies with R&D investments and the technology updating frequency. To simplify our analysis, we assume that the efficiency indexes decline at a constant geometric rate; that is, we assume that the following relationships hold among the efficiency indexes:

(5) $\theta_n = (1-\delta)^{n-1}$ and $0 \leq \delta \leq 1$.

Based on the above simplifications, the stock of R&D capital can be constructed as follows:

(6) $R_t = I_{t-1} + (1-\delta)R_{t-1}$

where δ can be regarded as the R&D depreciation rate, which is assumed to be constant over time. From (6), we see that the stock of R&D capital in period t is constructed from the previous R&D investment, I_{t-1}, and the depreciated R&D stock of period t−1. This is a widely used method to construct capital stocks. However, it can be seen that we need a restrictive assumption about the efficiency index to end up with this simple equation. According to this equation, R&D capital accumulation depends on two opposite forces: the addition of the new knowledge stock,

[6] Another way to justify inequality (2) is follows. Suppose that the industry or firm output is not homogeneous but consists of a mix of new and old products. Over time, industry output shifts away from the older more obsolete products and towards the newer improved products. Hence the use of the old technology that produces the older obsolete products diminishes over time and the initial good effects of investments in technological improvements that were made many periods ago gradually wither away. Thus we have a justification for why the initial good efficiency effects of R&D investments diminish or depreciate over time.

which is created by the current period R&D investments,[7] and the depreciation of the old knowledge stock.

We use the stock of R&D capital as a technology index, indicating the position of the production frontier. If the newly created knowledge stock is at least as large as the depreciation of the old knowledge stock, we will not have a backward shifting of the production frontier.

It is sometimes argued that the R&D depreciation rate is zero, since old knowledge is preserved. However, new innovations tend to make the innovations from pervious periods obsolete. Also, consumer preferences can shift over time, causing a drop in the demand for "old" products, and leading to obsolescence of past R&D investments. Hence the older R&D investments do suffer depreciation. Indeed, it is possible that the depreciation of old technologies outweighs the incremental effects of the new technologies and hence results in a net decrease of the R&D capital stock.

R&D investment in the Chemical Products (SIC 28) and the Non-electrical Machinery (SIC 35) categories has increased smoothly over the past 40 years, while R&D investment in Transportation Equipment (SIC 37) has fluctuated more. These different trends in the R&D investment imply different frequencies for creating new technology and result in different depreciation rates for the four industries.

In the rest of this section, we discuss the reasons for R&D capital depreciation in more detail.

2.3 Reasons for R&D Capital Depreciation

The depreciation of a tangible asset can be estimated by using the information on the price of the used asset (provided that it is not a uniquely constructed tangible asset). If we treat R&D expenditures as investment, there exists a similar problem: the decline in the utilization and efficiency of the knowledge capital. Although there is no apparent wear or tear of the intangible asset, we cannot assume that the knowledge capital has an infinite service life.

Both price and quantity changes are responsible for changes in the value of R&D. R&D capital depreciation, under our investigation, means the real quantity change due to the change in efficiency and utilization of the knowledge asset. The relative efficiency of the knowledge capital will decline over time due to the following reasons:

- The obsolescence of the technology. When newer technologies are created by R&D investment, the old technology may be partly or entirely replaced by the newly created technologies and consequently, the relative efficiency and the utilization of the old knowledge would decline.
- The changing preferences of consumers. Consumer's tastes may change for different reasons, such as the implementation of new health restrictions, the emergence of new products, and changes in the consumer's capabilities. Changing tastes may shift away the demand for some products that heavily rely on older technologies, and cause related

[7] Equation (6) implies that one unit of R&D investment can create one unit knowledge. This can be regarded as the simplest functional form for a knowledge production function.

market shrinkage. Responding to the shifting demand, firms would reduce the utilization level of the older technology that produces the outmoded products.

Because there are very few observed market prices for old technologies, we cannot observe the depreciation on R&D capital as we can for a tangible asset which trades on second hand markets. In the following section, we will set up a framework for estimating the R&D capital depreciation rate.

3. The Estimation Framework

In our estimation framework, each industry is treated as facing a monopolistic competition environment. In the production function, the R&D stock is treated as a technology index for the position of the production frontier. We use an extension of a model due to Diewert and Lawrence (2005).

3.1 The Basic Framework

We allow for the possibility of increasing returns to scale in the industry. Because the assumption of competitive profit maximizing behaviour is not suitable for the modelling of the industry's behaviour, we treat the industry as engaging in monopolistic profit maximization.

We assume that each industry has an aggregate production function f, with the form $y_t = f(x_t,R_t,t)$ so that f is a function that depends on the usual input vector x, the R&D stock R, and the time variable t which represents non R&D sources of technical change for the production function.[8] Thus both R and t shift the production function over time. Defining the production function in this way, we can avoid the overestimation of the effects of R&D capital on technological improvement, compared to production functions that have only an R&D variable R as a shift variable.

The aggregate demand function for the output of an industry in year t is represented by an inverse demand function of the form $p_t = P(y_t,t)$. Under this situation, each industry solves the following monopolistic profit maximization problem at each period by choosing inputs and the next period's technology level:

$$\begin{aligned} &\underset{x_t,R_{t+1}}{\mathrm{Max}} \sum_{t=0}^{\infty} \beta_t \{p_t(y_t,t)y_t - w_t \cdot x_t - P_{r,t}I_{r,t}\} \\ &\text{subject to: } \; y_t = f(x_t,R_t,t) \;\text{ and }\; R_t = I_{r,t-1} + (1-\delta)R_{t-1}, \qquad t = 0,1,2.... \end{aligned} \tag{8}$$

where β_t is the period t discount factor, w_t is an input price vector, $I_{r,t}$ is R&D investment in period t, and $P_{r,t}$ is the corresponding price index. In this model, we assume that each industry maximizes the discounted future monopolistic profits with full information about future prices. Ignoring the uncertainty of future prices is not realistic, but it dramatically simplifies the problem.

[8] These non R&D sources of technical progress could include learning by doing effects, freely available research, information on new technologies made available at trade fairs and so on.

The first order necessary conditions for solving the above maximization problem are:

(9) $$p_t \nabla_x f(x_t, R_t, t) + [\partial P(y_t, t)/\partial y] y_t \nabla_x f(x_t, R_t, t) = w_t, \qquad t = 0,1,...T,$$

(10) $$\beta_{t+1}\left\{ y_{t+1} \frac{\partial P(\cdot)}{\partial y} \frac{\partial f(\cdot)}{\partial R_{t+1}} + p_{t+1} \frac{\partial f(\cdot)}{\partial R_{t+1}} + P_{r,t+1}(1-\delta) \right\} = \beta_t P_{r,t},$$

where p_t is the output price and $\nabla_x f(x_t,R_t,t)$ is the vector of first order partial derivatives of the production function with respect to the components of the input vector x. Factoring p_t and $\nabla_x f(x_t,R_t,t)$ out on the left hand side of (9), we obtain the following simplification of equation (9):

(11) $$p_t \times (1 + \frac{\partial P(y_t, t)/\partial y_t}{p_t / y_t}) \times \nabla_x f(x_t, R_t, t) = w_t.$$

Applying similar algebraic rearrangements to equation (10), we have:

(12) $$p_{t+1} \times \left[1 + \frac{\partial P_{t+1}(\cdot)/\partial y_{t+1}}{p_{t+1}/y_{t+1}} \right] \times \frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}} = \frac{\beta_t}{\beta_{t+1}} P_{r,t} - (1-\delta) P_{r,t+1},$$

where $\beta_t/\beta_{t-1} = 1+r_t$ and where r_t is the nominal interest rate prevailing at time t.

The term $[\partial P(y,t)/\partial y]/[p/y]$ is the inverse of the price elasticity of demand, and reflects how output (demand) changes with respect to the price change. If we use ε to denote this price elasticity, we can define its inverse as the period t nonnegative markup, denoted by m_t, as follows:

(13) $$m_t \equiv -\frac{\partial P(y,t)/\partial y_t}{p_t / y_t} = -\frac{1}{\varepsilon} \geq 0$$

and the markup factor M_t can be defined as follows:

(14) $$M_t = 1 - m_t = 1 + \frac{1}{\varepsilon} = 1 + \frac{\partial P(y,t)/\partial y_t}{p_t / y_t}.$$

If we assume that the markup factor is constant over time, then we can rewrite (11) and (12) as:

(15) $$w_{t,n} = p_t M [\partial f(x_t,R_t,t)/\partial x_n], \qquad n = 1,2,...,N;$$

(16) $(1+r_t)P_{r,t} - (1-\delta)P_{r,t+1} = p_{t+1} M [\partial f(x_{t+1},R_{+1},t+1)/\partial R_{t+1}]$

where n denotes the n-th factor in the input vector x. Details of these equations are given in appendix A.

The left hand side of equation (16) is the *user cost of one unit of R&D investment* purchased in period t.[9] Equations (15) and (16) form our system of estimating equations. Including equation (16) as an extra estimating equation is helpful for distinguishing R and t. However we may also introduce some estimation problems by using anticipated variables in this equation, where the anticipations are formed in period t when we purchase $I_{r,t}$ at the price $P_{r,t}$. To simplify our analysis, we use the actual data at period t+1 to approximate the predicted variables.

The left hand side user cost in (16) depends on the depreciation rate δ. In order to compare log likelihoods across alternative depreciation models, we need the left hand side variable to be constant across models. Thus we rewrite equation (16) as:

(16a) $(1+r_t)P_{r,t} = (1-\delta)P_{r,t+1} + p_{t+1}\, M\, [\partial f(x_{t+1},R_{+1},t+1)/\partial R_{t+1}]$.

Equation (16a) is still not ideal for econometric estimation because it involves a lagged dependent variable. However, if we divide both sides of equation (16a) by $P_{r,t}$, this leads to the following estimating equation:

(16b) $(1+r_t) = (1-\delta)(P_{r,t+1}/P_{r,t}) + (p_{t+1}/P_{r,t})\, M\, [\partial f(x_{t+1},R_{+1},t+1)/\partial R_{t+1}]$.

We treat R&D capital similarly to the time variable in that the R&D stock shifts the technology like other sources of productivity improvement (approximated by the time variable).[10] Hence, in our model, we have labour, intermediate, and non-R&D capital service inputs. In order to help identify some parameters in the model, we add the production function to the estimating system. Thus our final estimating system includes the following five equations:

(17) $w_{t,1}/p_t = M\, \partial f(x_t,R_t,t)/\partial x_1$;
(18) $w_{t,2}/p_t = M\, \partial f(x_t,R_t,t)/\partial x_2$;
(19) $w_{t,3}/p_t = M\, \partial f(x_t,R_t,t)/\partial x_3$;
(20) $(1+r_t) = (1-\delta)(P_{r,t+1}/P_{r,t}) + (p_{t+1}/P_{r,t})\, M\, [\partial f(x_{t+1},R_{+1},t+1)/\partial R_{t+1}]$;
(21) $y_t = f(x_t,Rt,t)$.

3.2 The Choice of the Functional Form for the Production Function

To specify estimating equations, we need to choose a functional form for the production function. As a starting point, we use the following variant of a *normalized quadratic functional form*:

[9] It may be a be surprising initially that the net effect of purchasing an R&D investment in period t can be expressed in such a simple manner as is given in equation (16). However, under our perfect foresight assumptions, the producer needs to purchase units of R&D in period t in order to adjust the stock of R&D to precisely the "right" level in period t+1; the R&D stock for period t+2 can be adjusted to the "right" level by purchasing additional units of R&D in period t+1 and so on.

[10] However, the R&D variable is different from the time variable because we regard the time effect as being entirely exogenous in our model whereas the R&D stock is endogenously determined by producers.

(22) $f(x,R,t) \equiv b + c_1x_1 + c_2x_2 + c_3x_3 + g_1x_1t + g_2x_2t + g_3x_3t + h_1x_1R + h_2x_2R + h_3x_3R + e_1t + e_2R - \{(1/2)x^TSx/(\phi_1x_1 + \phi_2x_2 + \phi_3x_3)\}$

where x_1 is the labour input, x_2 is the intermediate input, x_3 is the non-R&D capital input, and R is the stock of R&D capital. In addition, $S \equiv [s_{ij}]$ is a 3 by 3 symmetric positive semi-definite substitution matrix of unknown parameters and the ϕ_i are predetermined positive parameters. In our empirical work, we calculate the sample mean of the x_i, say x_i^*, and then set the ϕ_i equal to $x_i^*/(x_1^* + x_2^* + x_3^*)$. The unknown parameter b determines the degree of returns to scale: if b = 0, we have constant returns to scale in production; if b is less than 0, then there are increasing returns to scale; and if b is greater than 0, there are decreasing returns to scale. The two parameters e_1 and e_2 are technical progress parameters.

In order to identify all of the parameters and to reduce multi-collinearity, it is necessary to impose some linear restrictions on the matrix S. Our linear restrictions are as follows:

(23) $\sum_{j=1}^{3} s_{nj} = 0$; $\qquad$ n = 1,2,3.

The normalized quadratic production function defined by (22) and (23), with the parameters b, e_1 and e_2 set equal to 0, is flexible in the class of constant return to scale production functions. The additional parameter b allows us to test the degree of local returns to scale. The main advantage of choosing this flexible functional form is that the flexibility properties would not be destroyed by imposing curvature conditions; see Diewert and Wales (1988). In our example, imposing positive semi-definiteness conditions on the symmetric matrix S means that we can write S in term of the following matrix product:

(24) $S = UU^T$

where $U \equiv [u_{ij}]$ is a 3 by 3 lower triangular matrix and U^T is the transpose of U. The linear restrictions (23) on S can be imposed on U too; that is, we impose the following restrictions:

(25) $u_{11} + u_{21} + u_{31} = 0$; $u_{22} + u_{32} = 0$; $u_{33} = 0$.

We find that there are only three independent parameters in the U matrix: u_{21}, u_{31} and u_{32}. The main diagonal parameters u_{ij} can be represented in terms of the off diagonal parameters u_{ij}.

Partially differentiating the production function defined by equation (22) with respect to the inputs, x_i, and with respect to next period's R&D stock, R_{t+1}, and substituting the resulting derivatives into the estimating equations (17) to (20), we can rewrite the estimating equations of our basic model as follows:

(26) $$w_{t,n}/p_t = M\left\{c_n + g_nt + h_nr - \frac{\sum_{j=1}^{3} s_{nj}x_{t,j}}{\phi^T x_t} + \frac{1}{2}\phi_n \times \frac{x_t^T S x_t}{(\phi^T x_t)^2}\right\}$$ for n = 1, 2, 3, and

$$(27)\quad 1+r_t = \frac{p_{t+1}}{P_{r,t}} M\{h_1x_1 + h_2x_2 + h_3x_3 + e_2\} + (1-\delta)\frac{P_{r,t+1}}{P_{r,t}} \quad \text{where } \phi^T x_t \equiv \sum_{j=1}^{3} \phi_j x_{t,j} .$$

3.3 The Problem of Trending Elasticities

Diewert and Lawrence (2002) point out that, for the normalized quadratic functional form, the estimated elasticities often have strong trends when there are strong trends in the price and quantity data. They also suggest one way to solve this problem. We adopt their technique here.

In the initial functional form, the substitution matrix S is constant over time. To handle the trending elasticity problem, we let the production function be flexible at two sample points, which means the matrix S is allowed to change over time. We use the following weighted average substitution matrix:

(28) $S = (1-(t/T))A + (t/T)B$; $\quad t = 0,1,2,...,T$

where T+1 is the total number of periods covered by the estimation data sample. We have data for the years 1953-2000, so T = 47. Using this weighted average substitution matrix, the technological progress captured by the time variable not only affects the constant terms in the estimating system, but also the substitution possibilities.

As in the basic case, we can impose the curvature conditions by setting A and B equal to UU^T and VV^T respectively, where U and V are lower triangular matrices; that is, we set:

(29) $A = UU^T$ and $B = VV^T$; U and V are lower triangular.

Similarly, we can impose the following normalizations on these two matrices U and V:

(30) $U^T 1_3 = 0_3$ and $V^T 1_3 = 0_3$

where 1_3 and 0_3 are 3-dimensional vectors of 1's and 0's, respectively. With these constraints, we only add three additional independent parameters to the initial model; these new parameters are v_{21}, v_{31} and v_{32}. Making these changes, our production function can be written as follows for t =0,1,...,47:

$$(31)\quad f(x,R,t) \equiv b + c^Tx + g^Tx\, t + h^Tx\, R + e_1t + e_2R - \{(1/2)x^T[(1-(t/T))UU^T + (t/T)VV^T]x/(\phi_1x_1 + \phi_2x_2 + \phi_3x_3)\}.$$

(31) Here we write the production function in matrix form in order to simplify our notation. In equation (31), $c^T \equiv [c_1,c_2,c_3]$, $g^T \equiv [g_1,g_2,g_3]$ and $h^T \equiv [h_1,h_2,h_3]$. In the estimation, if the trending elasticity problem exists, we will see significant increases in the value of the log-likelihood function as we add the parameters in the V matrix to those in the U matrix. We do see this for our results.

3.4 Problems due to Non-Smooth Technical Progress

Another problem related to the initial production function model is that it does not allow for non-smooth change in technical progress. We now add more features to the model in order to capture changes in the direction of technical progress over time.

Technological progress typically does not proceed smoothly. Thus we add linear splines or quadratic splines in the time variable to allow for the different change patterns of technological progress at different periods. The modified production function in period t can be written as follows:

$$(32)\ f(x_t,R,t) \equiv b + c^T x_t + \sum_{j=1}^{3} g_j(t)\, x_{j,t} + h^T x_t R + e_1(t) + e_2 R_t - \{(1/2)x_t^T[(1-(t/T))UU^T + (t/T)VV^T]x_t/\phi^T x_t\}.$$

where $e_1(t)$ and $g_j(t)$ are linear spline functions of time t. The number of spline segments depends on the break points chosen by investigating the plots of preliminary estimations. A break point is a positive integer less than the maximum number of the time variable; so here, it is less than 47.

We will illustrate how to define $e_1(t)$ if we choose three break points, $0 < t_1 < t_2 < t_3 < 47$:

$$(33)\ \begin{aligned} e_1(t) &\equiv e_{11}t && \text{for } t = 0,1,2,\ldots,t_1\ ; \\ &\equiv e_{11}t_1 + e_{12}(t - t_1) && \text{for } t = t_1+1, t_1+2,\ldots,t_2\ ; \\ &\equiv e_{11}t_1 + e_{12}(t_2 - t_1) + e_{13}(t - t_2) && \text{for } t = t_2+1, t_2+2,\ldots,t_3\ ; \\ &\equiv e_{11}t_1 + e_{12}(t_2 - t_1) + e_{13}(t_3 - t_2) + e_{13}(t - t_3) && \text{for } t = t_3+1, t_3+2,\ldots,47. \end{aligned}$$

In equations (33), the e_{1j} are the unknown parameters to be estimated. From the above example, we know that with n break points, there are n+1 parameters to be estimated. Similarly, we can generate linear splines for the functions $g_j(t)$. The subscript j means we allow different splines for different inputs j.

Adding linear spline induces, perhaps artificially, kinks in the direction of technical change. For smooth change, the linear splines can be replaced by quadratic splines,[11] in which case $e_1(t)$ and the $g_j(t)$ are quadratic spline functions. With three break points, the quadratic spline functions can be defined as follows:

[11] On setting up quadratic splines in a normalized quadratic model, see Diewert and Wales (1992). Normalized quadratic cost functions are modeled there whereas we are modeling normalized quadratic production functions

$$\begin{aligned}
(34)\ e_1(t) &\equiv e_{11}t + (1/2)e_{12}t^2 ; && 0 \le t \le t_1 ; \\
&\equiv e_{11}t_1 + (1/2)e_{12}t_1^2 + (t-t_1)(e_{11} + e_{12}t_1) + (1/2)e_{13}(t-t_1)^2 ; && t_1 < t \le t_2 ; \\
&\equiv e_{11}t_1 + (1/2)e_{12}t_1^2 + (t_2-t_1)(e_{11} + e_{12}t_1) + (1/2)e_{13}(t_2-t_1)^2 \\
&\quad + (t-t_2)(e_{11} + e_{12}t_1 + e_{13}t_2) + (1/2)\, e_{14}(t-t_2)^2 ; && t_2 < t \le t_3 ; \\
&\equiv e_{11}t_1 + (1/2)e_{12}t_1^2 + (t_2-t_1)(e_{11} + e_{12}t_1) + (1/2)e_{13}(t_2-t_1)^2 \\
&\quad + (t_3-t_2)(e_{11} + e_{12}t_1 + e_{13}t_2) + (1/2)e_{14}(t_3-t_2)^2 \\
&\quad + (t-t_3)(e_{11} + e_{12}t_1 + e_{13}t_2 + e_{14}t_3) + (1/2)e_{15}(t-t_3)^2 ; && t_3 < t \le 47.
\end{aligned}$$

As in the linear spline case, the e_{1j} are the unknown parameters to be estimated. If we choose n break points, then there are n+2 additional parameters that need to be estimated for each equation. Adding splines increases the flexibility of the functional form but at the cost of estimating more technical change parameters. As was the case for linear splines, choosing different break points will generally result in different estimates.

The following features distinguish our model from the previous literature:

- Instead of treating R&D capital as one explicit factor input like ordinary physical capital, we treat R&D capital as a technological index indicating the position of the production frontier. R&D capital, which is the knowledge asset created by the R&D investment, is not "consumed" like the physical capital in the production; it just indicates the technology level. Holding all the usual flow inputs constant, an increase of the stock of R&D capital would shift the production frontier outwards. Thus the R&D stock variable is treated in a manner similar to the time variable.
- Both the time variable and R&D capital stock variable are included in the production function model. The R&D stock frequently grows in a roughly linear fashion, so the inclusion of the variables R and t in the regression equations can lead to a multi-collinearity problem. However, if the time variable, t, is dropped from the model, then the R variable becomes the only technical change shift variable, and frequently, the resulting rate of return to R&D investments is unrealistically large. Therefore, including both the time variable and the R&D variable in the model, we will generally not attribute all of the technological progress to R&D investments, and avoid overstatement of the effects of R&D investments on both technological progress and on productivity growth to some extent.
- The model allows for the possibility of monopolistically competitive behaviour that is consistent with increasing return to scale.12 Thus our model estimates the markup factor and the degree of returns to scale along with the R&D depreciation rate.

Because of these different features of our model, our results may be different from the results obtained from "traditional" models that treat R&D as just another capital stock.

4. Empirical Estimation and Results

[12] If the estimated markup factor M turns out to equal 1, then we have competitive behavior.

Here we describe our data and empirical results. We estimate R&D depreciation rates for the period of 1953-2000 for U.S. manufacturing and four U.S. technology intensive industries: chemical and allied products (SIC 28), non-electrical machinery (SIC 35), electrical products (SIC 36) and transportation equipment (SIC 37). In 1998, the R&D expenditures of these four industries accounted for 54.35% of the R&D expenditures of all industries and 76.37% of the manufacturing R&D expenditures.

4.1 Estimation Methodology

The basic estimation system with the curvature conditions (24) and linear restrictions (25) imposed is given by equations (22), (26) and (27).[13] To specify these estimating equations, we must define the normalized quantities and the differences for the normalized quantities. The nth normalized quantity, q_n, is:

(35) $q_n \equiv x_n/\phi^T x$; $\qquad$ n = 1,2,3.

The differences between the normalized quantities can be defined in the following way:

(36) $q_{21} \equiv q_2 - q_1$; $q_{31} \equiv q_3 - q_1$; $q_{32} \equiv q_3 - q_2$.

Using the above definitions and substituting restrictions (24) and (25) into equation (26), we can express the first order necessary conditions for profit maximization with respect to the choice of the different inputs in the following way:

(37) $$w_{t,1}/p_t = M \times \begin{Bmatrix} c_1 + g_1 t + h_1 R + (u_{21} + u_{31})(u_{21}q_{21} + u_{31}q_{31}) \\ + 0.5\phi_1[(u_{21}q_{21} + u_{31}q_{31})^2 + (u_{32}q_{32})^2] \end{Bmatrix},$$

(38) $$w_{t,2}/p_t = M \times \begin{Bmatrix} c_2 + g_2 t + h_2 R - u_{21}(u_{21}q_{21} + u_{31}q_{31}) + (u_{32})^2 q_{32} \\ + 0.5\phi_2[(u_{21}q_{21} + u_{31}q_{31})^2 + (u_{32}q_{32})^2] \end{Bmatrix}, \text{ and}$$

(39) $$w_{t,3}/p_t = M \times \begin{Bmatrix} c_3 + g_3 t + h_3 R - u_{31}(u_{21}q_{21} + u_{31}q_{31}) - (u_{32})^2 q_{32} \\ + 0.5\phi_3[(u_{21}q_{21} + u_{31}q_{31})^2 + (u_{32}q_{32})^2] \end{Bmatrix}.$$

The production function is:

(40) $$\begin{aligned} y = {} & b + c_1x_1 + c_2x_2 + c_3x_3 + g_1x_1t + g_2x_2t + g_3x_3t + h_1x_1R + h_2x_2R + h_3x_3R \\ & + e_1t + e_2R - 0.5(\phi_1x_1 + \phi_2x_2 + \phi_3x_3)[(u_{21}q_{21} + u_{31}q_{31})^2 + (u_{32}q_{32})^2]. \end{aligned}$$

The above four equations plus (27) form our basic estimating system with 16 parameters to be estimated.

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[13] Thus for this basic model, there are no splines and no trending substitution matrices.

Due to the non-linearity of the equations, we use non-linear maximum likelihood estimation option in the SHAZAM. To find the estimates for the R&D depreciation rates, we construct a grid of depreciation rates: $\delta = 0.01$, $\delta = 0.02$, $\delta = 0.03$, … and $\delta = 1$. Based on these depreciation rates, we build the initial stock of R&D capital using the following formula:

(41) $R(0) = I_{r,0}/(\delta+\gamma_r)$; $\delta = 0, 0.01, 0.02, ..., 0.99, 1$

where $I_{r,0}$ denotes the R&D investment at the first period, and γ_r denotes the geometric growth rate of R&D investment over the sample period and can be calculated as:

(42) $\gamma_r \equiv (I_{r,47}/I_{r,0})^{1/47}$.

For the remaining periods, the R&D stock is calculated using equation (6) in the second section.

All together, we have 101 sets of R&D stocks, corresponding to the 101 possible choices for an R&D depreciation rate. Using these alternative R&D stock series, we can estimate the five equations. For each depreciation rate, we obtain the value of the log-likelihood function. Comparing these values of the log-likelihood function, we can locate the depreciation rate corresponding to the maximum value of the likelihood function. According to our estimating procedure, we believe that the depreciation rate that maximizes the value of log-likelihood is the best estimator for the R&D depreciation rate.

4.2 Data Construction

To conduct the estimation, we need quantity series and price series for industrial input and output, and price and quantity series for R&D investments. Industrial input and output data other than R&D related data are obtained from the Multifactor Productivity data sets provided by the Bureau of Labour Statistics (BLS). R&D related data are derived from the website of the National Science Foundation (NSF).

From the BLS, we obtained value series in current dollar and price index series in 1996 constant dollar.[14] For each industry, we have data on sectoral output, labour input (L), capital service input (K), energy input (E), non-energy materials (M) input, and purchased business services (S) input. The BLS uses the Törnqvist index number formula to construct the aggregate data. Labour is measured as the hours worked by all persons engaged in a sector. Capital input is defined as the flow of services from physical assets, which include equipment, structures, inventories and land. Service flows are assumed to be proportional to stocks. The description of the measures and the methodology for constructing all of these data sets are given in Chapter 10 and 11 of "BLS Hand Book of Methods", and in Gullickson and Harper (1987). Our measure of intermediate input is a Törnqvist aggregate of energy, material and purchased services. With value series and price series, we can construct the implicit quantity series. Industrial input and output data sets are relatively well constructed over the R&D data sets, but we face a double counting problem when we try to explicitly model the role of R&D capital because R&D expenditures have already been included in the initial (BLS) input data.

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[14] We thank Mike Harper for providing us with manufacturing data not available on the BLS website.

The industrial R&D expenditure data are obtained from the website of NSF. For the years 1953-1998, the data are taken from the Industrial R&D Information System (IRIS) that uses the Standard Industrial Classification (SIC) to classify the industry. For other years, the data are obtained from "Research and Development in Industry", for the year 2000 and 2001. These two publications use the North American Industrial Classification System (NAICS). We use the 1998 data as the bridge to link the series based on the different industrial classification and reclassify the data to conform to the SIC. Using R(Syear) to denote the data based on SIC and R(Nyear) to denote the data based on NAICS, the constructed data we need for estimation can be obtained by using the following formula:

(43) $R(S1999) = R(N1999)\times[R(S1998)/R(N1998)]$.

Equation (43) describes how the two data sets are linked for 1999. Similar adjustments can be made for 2000 and 2001.

The data on the cost components of R&D expenditures come from various issues of Research and Development in Industry. According to the NSF's classification, the type of R&D expenditure includes: wage and salaries, materials, R&D depreciation, and other costs. There are two important problems associated with this data set: one is that the NSF does not use the above classification consistently over years; another problem is that data are not available for quite a few years. To deal with these problems, we must make assumptions and create approximate data. For example, for the years 1953 to 1961, we do not have data related to the type of cost. Thus, we assume that the cost structure for these years was the same as that in 1962. Similarly, for the period 1977 to 1997, we have data every two years. To obtain data for the missing years, we use moving average methods to determine the missing data. Finally, we group the R&D expenditures into three categories: Wage and Salaries (labour), Materials (intermediates) and Capital expenditure. Unfortunately, the cost category, overhead or other costs, accounts for a big portion of the expenditure. We allocate these expenditures to wage and salaries, materials and capital expenditure according to the BLS industry cost shares. From the above descriptions, we should be aware that assumptions made to fill in gaps in the R&D data can have a direct effect on the quality of the resulting data sets and on the results of analyses based on these data sets.

After constructing R&D cost component information, we can make adjustments to the initial BLS input data sets. R&D labour cost, wage and salaries, is subtracted from the total labour cost; the material component of R&D expenditure is subtracted from the total intermediate input cost; and the capital expenditure part of R&D is subtracted from the total capital service cost. Dividing these adjusted value series by an implicit price index, yields quantity series for labour, intermediate and capital service input. The quantity series of R&D investment is constructed by using the Törnqvist index formula. Price indexes for the three components of R&D expenditure are assumed to be same as industrial input price indexes.

Finally, we need nominal interest rates for the year 1953 to 2000 to construct the discount factors. Nominal interest rates are obtained from the on-line data of Federal Reserve System. We choose the long-term nominal interest rate: market yield on U.S. Treasury securities at 10-year constant maturities, quoted on an investment basis, to construct the series of discount factors.

4.3 Estimation Results

Maximum likelihood estimation, which is the non-linear option in SHAZAM, is sensitive to the choice of starting point, which also affects the number of iterations. We start from a simple regression with M, b, e_1 and e_2 in equation (27) and equations (36) to (40) set equal to zero initially. The estimated parameters from this regression are used as the starting point for the next regression, which adds additional parameters. As we proceed, we also check for "big jumps" in the log likelihood of the model (if the jumps are small, then the inclusion of the extra parameters is not warranted but in general, we obtain significant increases in the log likelihood as we add the extra parameters).

We estimate the following four models:

- Model I: This is the basic model defined by equation (27) and equations (36) to (40). Without splines and without a weighted substitution matrix, this model may not properly reflect real world complexity. We expected relatively low values of the log-likelihood for this class of models.
- Model II: This model adds a weighted substitution matrix to equations (36) to (40). It can deal with the possible trending elasticity problem. If our model does have this trending elasticity problem, we would expect to see a big increase in the value of the log-likelihood function.
- Model III: This model adds linear splines in the time variable based on the last model. Non-smooth change in technical progress can be captured by adding these splines.
- Model IV: This model adds quadratic splines for the time variable to Model II (rather than linear splines as in Model III).

The following table lists the estimates of the depreciation rate, the value of maximum log-likelihood, and the value of markup factor for the above 4 models for U.S. manufacturing and the four selected knowledge intensive industries.[15]

Table1. Depreciation Rates Maximizing the Log-likelihood Function

		SIC 28	SIC 35	SIC 36	SIC 37	Manuf
Model 1	Dep Rate	0	0.09	0	0.15	0.5
	Markup Factor	0.92508	0. 92223	0.96331	0.97694	0.95618
	Log-likelihood	266.115	208.271	113.905	306.464	373.561
Model 2	Dep Rate	0.14	0.06	0.18	0.12	0.49
	Markup Factor	0.90005	0.90465	0.92672	0.98417	0.98372
	Log-likelihood	279.657	221.700	148.229	308.473	394.441
Model 3(a)	Dep Rate	0.01	0.03**	0.07	0.06	0.26
	Markup Factor	0.97094	0.84863	0.96381	0.90395	0.98041
	Log-likelihood	400.934	343.057	280.239	416.204	530.168
Model 3(b)	Dep Rate	0.02	0	0.12	0.27**	0.08
	Markup Factor	0.985	0.9439	0.9087	0.96251	0.97252

[15] The break points for the spline models are reported in Appendix B.

	Log-likelihood	395.727	400.331	273.373	407.415	532.827
Model 3(c)	Dep Rate Markup Factor Log-likelihood	0.01** 0.96872 395.907	0 0.93698 395.578	0.09 0.86455 236.828	0.22 1.0087 413.723	0.29** 0.94723 549.806
Model 4(a)	Dep Rate Markup Factor Log-likelihood	0 1.0124 378.795	0 1.1241 386.034	0.14** 0.91897 334.547	0.34 0.88006 406.451	0.32 0.93338 541.723
Model 4(b)	Dep Rate Markup Factor Log-likelihood	0 1. 0179 380.397	0 1.0955 395.822	0.1 0.93431 285.808	0.3 0.88437 381.842	0.38 0.93532 539.3563
Model 4(c)	Dep Rate Markup Factor Log-likelihood	0 1.006 379.044	0 1.0898 397.954	0.05 0.94743 271.822	0.32 0.88176 388.501	0.28 0.94824 544.687

From the above table, we can see that the value of the maximum log-likelihood generally improves considerably from Model I to Model II. This suggests we have trending elasticity problems. With linear splines or quadratic splines added to the model, the value of maximum log-likelihood function increases dramatically. In comparison with the results obtained from the linear spline model (Model III) and the quadratic spline model (Model IV), the value of the log-likelihood function increases in Model IV for Electrical Products, and decreases for Chemical Products and Transportation Equipment. For Non-electrical Machinery and the manufacturing sector, the changes of the log-likelihood value from Model III to Model IV are mixed. We also try different break points for Model III and Model IV[16].

Unfortunately, it turns out that the estimates are sensitive to the choice of the break points. Consequently, in order to choose an appropriate value for the depreciation rate of R&D capital, we have to use our subjective judgement. This is one drawback of our modeling strategy. In our example, we choose the depreciation rate for R&D based on values of both the markup factor and the log-likelihood function. According to the definition of markup factor given in the second section, a reasonable value should be less than 1. Our final choices of the depreciation rates are given by table 2.

Table 2. Depreciation Rates and Markup Factors

	SIC 28	SIC 35	SIC 36	SIC 37	Manufacturing
Depreciation Rate	0.01 (0.2866)1	0.03 (1.588)	0.14* (3.0986)	0.27*** (15.3064)	0.29*** (10.2534)
Markup Factor	0.96872 (0.0261)2	0.84863 (0.0494)	0.91897 (0.0554)	0.96251 (0.0214)	0.94723 (0.0191)

Notes: 1: The values in the brackets are log-likelihood statistics, which is defined as: $\chi^2 = -2 \times [\lg(L_0) - \lg(L_1)]$.

*: Statistically significant with 10% confidence level;

***: Statistically significant with at least 1% confidence level.

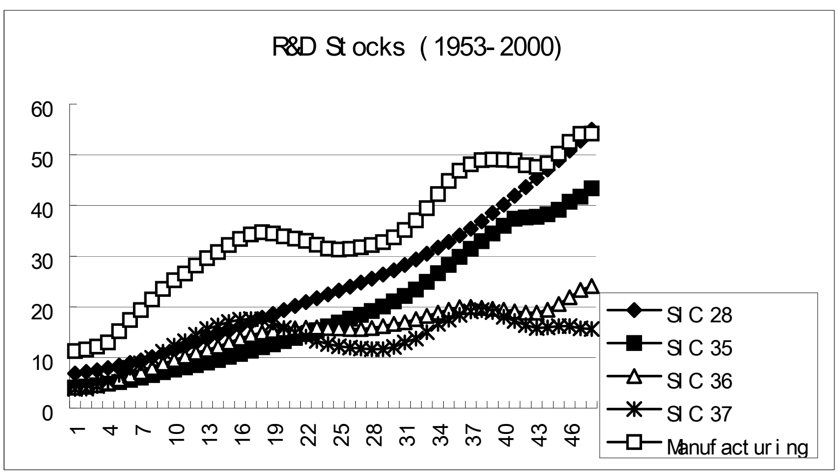
[16] Different break points corresponding to the estimations listed in Table 1 are given in the appendix B.

2: The values in the brackets are the standard errors.

From the estimated markup factors, we can derive the markup for the total manufacturing sector and the four industries. The markup is 5.3 percent for the total manufacturing sector, 3.1 percent for chemical products, 15.1 percent for non-electrical machinery, 8.1 percent for electrical products, and 3.7 percent for transportation equipment. As shown in table 2, the estimated depreciation rates for SIC 28 and SIC 35 are not significantly different from zero. We may interpret this as the verification of some economist's belief that the knowledge asset should not depreciate over time. Another possible explanation for the small depreciation rates is that our model does not fit the data well in these industries. In order to estimate accurately the R&D depreciation rate, we need some large fluctuations in R&D investments. As we have pointed our in section 2, R&D investments in SIC 28 and SIC 35 increase relatively smoothly in our sample period. The lack of fluctuations in R&D investments may cause our model fail to find the appropriate depreciation rate. Our other estimates fall in the range of R&D depreciation rates that are found in the literature. Depreciation rates for SIC 37 and Manufacturing are very close to each other, which may due to the similar pattern in the changes of R&D investments in these two sectors.

Using our estimates of the depreciation rate for R&D capital, we can construct the series of R&D capital stocks for manufacturing and the four knowledge intensive industries. The following figure shows how R&D stocks change over the period of 1953-2000.

Figure 1. R&D Stocks (1953-2000)



The geometric growth rate of R&D stocks for manufacturing is 3.4%, and the geometric growth rates of the R&D stocks for the four knowledge intensive industries, namely SIC 28, SIC 35, SIC 36 and SIC 37, are 4.5%, 5.2%, 3.9% and 3.1%, respectively.

5. Conclusion

In this paper, we have developed a simple model based on a production function to estimate the depreciation rates of R&D capital for the U.S. total manufacturing sector and the four knowledge intensive industries, including chemical products (SIC 28), non-electrical machinery (SIC 35), electrical products (SIC 36) and transportation equipment (SIC 37). We treat R&D capital as a technology shifter instead of as an ordinary input in the model. Using both the R&D stock variable and time variable as technology shifters can avoid the overestimation of R&D capital's contribution to productivity growth. Along with the estimation of the depreciation rate, we have estimated the markup factor for the U.S. manufacturing and the four selected industries. The estimated R&D depreciation rate is 29 percent for U.S. total manufacturing sector, 1 percent for chemical products, 3 percent for non-electrical machinery, 14 percent for electrical products and 27 percent for transportation equipment. The corresponding markup is 5.3 percent for the total manufacturing sector, 3.1 percent for chemical products, 15.1 percent for non-electrical machinery, 8.1 percent for electrical products, and 3.7 percent for transportation equipment. Based on the estimated depreciation rate, the geometrical growth rate of the R&D stock is 3.4 percent for the manufacturing sector, 4.5 percent for chemical products, 5.2 percent for non-electrical machinery, 3.9 percent for electrical products, and 3.1 percent for transportation equipment.

The results reported here are preliminary. We have not incorporated some important features associated with R&D investment, such as the uncertainty of R&D investment and the externality of the created knowledge. Also, we have imposed some restrictive assumptions to simplify the problem, such as constant depreciation rates over years, constant markup factors, and full information about the future prices. In addition, the robustness of the results should be checked against alternative functional forms for the production function and against alternative ways of constructing the stock of R&D capital.[17]

Appendix A: The Industry's Profit Maximization Problem

Each industry's profit maximization problem can be written as follows:

(A1) $$\text{Max}_{x_t\text{'s},R_{t+1}\text{'s}} \sum_{t=0}^{\infty} \beta_t \left\{P_t(y_t,t)y_t - w_t x_t - P_{r,t} I_t\right\} \text{ subject to}$$
$$y_t = f(x_t, R_t, t) \text{ and } R_t = I_{r,t-1} + (1-\delta)R_{t-1}.$$

Substituting the constraints into the objective function, we have the following equivalent problem:

$$\text{Max}_{x_t\text{'s},R_{t+1}\text{'s}} \beta_t \{P_t(f(x_t,R_t,t),t)f(x_t,R_t,t) - w_t x_t - P_{r,t}(R_{t+1} - (1-\delta)R_t)\}$$
$$+\beta_{t+1}\{P_{t+1}(f(x_{t+1},R_{t+1},t+1),t+1)f(x_{t+1},R_{t+1},t+1) - w_{t+1}x_{t+1} - P_{r,t+1}(R_{t+2} - (1-\delta)R_{t+1})\}$$
$$+\beta_{t+2}\{P_{t+2}(f(x_{t+2},R_{t+3},t+2),t+2)f(x_{t+2},R_{t+2},t+2) - w_{t+2}x_{t+2} - P_{r,t+2}(R_{t+3} - (1-\delta)R_{t+2})\}$$
(A2) $$+ \ldots$$

Therefore the first order necessary conditions with respect to vector x_t can be written as:

(A3) $$p_t \nabla_x f(x_t,R_t,t) + [\partial P(y_t,t)/\partial y] y_t \nabla_x f(x_t,R_t,t) = w_t, \qquad t=0,1,\ldots T,$$

[17] Our results may also be subject to some aggregation bias since we have used industry data instead of firm data.

where p_t is the output price and $\nabla_x f(x_t,R_t,t)$ is the vector of first order partial derivatives of the period t production function with respect to the components of the input vector x. Factoring out the output price p_t and $\nabla_x f(x_t,R_t,t)$ on the left hand side of equation (A3), we have the following simplified form for equation (A3):

(A4) $$p_t \times \left(1+\frac{\partial P(y_t,t)/\partial y_t}{p_t/y_t}\right)\times\nabla_x f(x_t,R_t,t)=w_t.$$

The first order necessary conditions with respect to R&D stock variable R_{t+1} are as follows:

(A5) $$\beta_{t+1}\left\{y_{t+1}\frac{\partial P(\cdot)}{\partial y}\frac{\partial f(\cdot)}{\partial R_{t+1}}+p_{t+1}\frac{\partial f(\cdot)}{\partial R_{t+1}}+P_{r,t+1}(1-\delta)\right\}=\beta_t P_{r,t}.$$

After some rearrangement, we can rewrite the above equation as follows:

(A6) $$p_{t+1}\times\left[1+\frac{\partial P_{t+1}(\cdot)/\partial y_{t+1}}{P_{t+1}/y_{t+1}}\right]\times\frac{\partial f(x_{t+1},R_{t+1},t+1)}{\partial R_{t+1}}=\frac{\beta_t}{\beta_{t+1}}P_{r,t}-(1-\delta)P_{r,t+1}.$$

Assuming that $\beta_t/\beta_{t+1} = 1+r_t$ where r_t is the nominal interest rate, the above equation can be rewritten as:

(A7) $$p_{t+1}\times\left[1+\frac{\partial P_{t+1}(\cdot)/\partial y_{t+1}}{P_{t+1}/y_{t+1}}\right]\times\frac{\partial f(x_{t+1},R_{t+1},t+1)}{\partial R_{t+1}}=(1+r_t)P_{r,t}-(1-\delta)P_{r,t+1}.$$

Define the period t non-negative markup as follows:

(A8) $$m_t\equiv-\frac{\partial P(y,t)/\partial y_t}{p_t/y_t}=-\frac{1}{\varepsilon}\geq 0.$$

The corresponding period t markup factor M_t can be defined as:

(A9) $$M_t=1-m_t=1+\frac{1}{\varepsilon}=1+\frac{\partial P(y,t)/\partial y_t}{p_t/y_t}.$$

If we assume that markup factors are constant over time, we can rewrite our system of first order conditions as:

(A10) $w_t=p_t M\nabla_x f(x_t,R_t,t)$, and

(A11) $$(1+r_t)P_{r,t}-(1-\delta)P_{r,t+1}=p_{t+1}M\frac{\partial f(x_{t+1},R_{t+1},t+1)}{\partial R_{t+1}}.$$

Moving the 2nd term to the right hand side of equation (A11) and dividing through by $P_{r,t}$, we obtain:

(A12) $$1+r_t = \frac{p_{t+1}}{P_{r,t}} M \frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}} + (1-\delta)\frac{P_{r,t+1}}{P_{r,t}}.$$

Equations (A10) and (A12) form our final system of estimating equations.

Appendix B: Break Points

Table B1. Break Points for Model III and Model IV

	Eq.	SIC 28	SIC 35	SIC 36	SIC 37	MANF
Model 3(a) and Model 4(a)	1	9, 21, 29, 34, 44	4, 31, 39	20, 38, 46	13, 27, 32, 42	14, 21, 26, 29, 34, 38
	2	15, 21, 32, 40	29, 40	9, 19, 27, 39	12, 27, 30, 38	9, 21, 23, 36, 40
	3	13, 28, 35, 41	8, 13, 22, 32, 42	12, 22, 38	12, 28, 32, 38	13, 28, 36, 41, 47
	4	21, 28, 35, 41	30, 39	26, 40	12, 28, 35, 40	11, 21, 29, 36, 40
Model 3(b) and Model 4(b)	1	15, 21, 29, 34, 44	31, 39	20, 32, 38	9, 21, 30, 43	14, 21, 29, 34, 38
	2	15, 21, 43	10, 19, 30, 38	9, 22, 36, 41	6, 22, 30,39	9, 21, 23, 36, 40
	3	13, 28, 35, 41	8, 13, 22, 38, 42	12, 22, 30, 40, 45	13, 27, 32, 38	13, 22, 36, 41, 47
	4	21, 28, 35, 41	19, 30, 39	24, 37	6, 12, 24, 28, 33, 40	11, 21, 29, 36, 40
Model 3(c) and Model 4(c)	1	9, 21, 29, 34, 44	31, 39	20, 38	9, 22, 30, 43	15, 21, 29, 34, 39, 44
	2	15, 21, 30, 39	10, 19, 30, 38	8, 22, 41	7, 22, 30,37	9, 21, 23, 28, 39, 46
	3	13, 28, 35, 45	8, 14, 22, 37, 42	12, 22, 30, 45	12, 28, 32, 38	13, 28, 36, 41, 45
	4	21, 28, 35, 41	19, 30, 39	24, 37	13, 28, 35, 39	9, 21, 29, 36, 40

Note: Equations (1)-(3) are estimating equations for the three inputs; equation (4) is the production function.

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