

Methods of Aggregation above the Basic Heading Level: Linking the Regions

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Abstract

The paper explains part of the methodology that was used in the 2005 International Comparison Program (ICP) that compared the relative price levels and GDP levels across 146 countries and 5 regions. Each of the regions constructed its own set of Purchasing Power Parities (PPPs) and relative country expenditure volumes. The paper studies various methods for constructing a global set of relative country volumes and PPPs while respecting the regional parities—the fixity constraint. Two methods emerge as having good properties: a method due to Heston and Dikhanov and an alternative method due to Eurostat, the OECD and Robert Hill. Some small numerical examples illustrate the differences between the various methods.

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Index numbers, multilateral comparison methods, Purchasing Power Parities, fixity constraint, GEKS, Geary-Khamis, Heston-Dikhanov, Eurostat-OECD-Hill, similarity linking, dissimilarity measures.

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1. Introduction

In this chapter, various possible methods for linking the five regions in the ICP in such a way that the relative country volumes within each region are preserved will be discussed.²

The ICP 2005 round of international comparisons was quite different from earlier ICP rounds for a number of reasons:

- Each of the five regions prepared their own lists of representative (for their region) products which were priced by each country in the region over the reference year, whereas in previous ICP comparisons, there was a single product list for all countries over all regions.
- Each region was allowed to use their preferred multilateral method to make comparisons within their region. Thus four of the five regions chose to use the GEKS method while the African region decided to use the additive IDB method.³
- Each region was also allowed to use slightly different methods for constructing Basic Heading PPPs across the countries in their respective regions.⁴
- In order to link the five regions, another separate product list was developed and these products were priced by *18 ring countries* where there were two or more of these ring countries in each region.⁵ This additional set of prices across regions allowed the 155 BH PPPs to be linked across regions.
- At the final stage of aggregation (the subject of this chapter), the within region aggregate volumes for each country in a region were linked across the five regions, giving rise to a complete set of 146 aggregate PPPs and relative volumes for each country in ICP 2005.

The final stage in the above process which involved linking the regions proved to be the most challenging: the various regions involved in ICP 2005 wanted the final linking procedure to leave unchanged the relative volumes for the countries in their respective regions. This is called the *fixity constraint* on the final step which links the regional parities into a set of global parities.

Diewert (2004; 46-47) suggested a class of methods for linking the regions in the 2005 ICP round that would preserve relative volumes for country aggregates within a region

² An early discussion of the issues surrounding the linking of regions while preserving within region relative volumes can be found in Kravis, Heston and Summers (1982).

³ These multilateral methods were described in Chapter 7 above.

⁴ The Country Product Dummy (CPD) method was used by the African, Asian Pacific and West Asian regions, the Extended Country Product Dummy (CPRD) method was used by South America and the EKS* method was used by the OECD-Eurostat-CIS region. See Chapter 6 for a review of these methods. For the details on how the commodity lists were chosen and how the Basic Heading PPPs were linked across regions, see Chapters 4 and 6 above. For a summary of new methodology used in ICP 2005, see Diewert (2010).

⁵ The 18 ring countries were: Brazil, Cameroon, Chile, Egypt, Estonia, Hong Kong, Japan, Jordan, Kenya, Malaysia, Oman, Philippines, Senegal, Slovenia, South Africa, Sri Lanka, United Kingdom and Zambia.

but at the same time would link the various regions so that volumes could be compared across all countries in the ICP comparison in a consistent manner; i.e., his suggested interregional linking method would satisfy the fixity constraint. The basic idea behind the method is as follows: once information on country expenditures for each Basic Heading (BH) category in each country in a region is determined along with the 155 corresponding Basic Heading Purchasing Power Parities (PPPs), then BH expenditures in the local currencies can be divided by the corresponding PPPs in order to obtain country volumes in consistent comparable units by Basic Heading category. These volumes (or loosely speaking, quantities) can then be added across countries within the region in order to obtain *total regional volumes* or “quantities” by Basic Heading category. Then with an appropriate choice for BH prices for each region, these BH regional “prices” and “quantities” can then be linked using any multilateral method such as the GEKS method⁶ studied in chapter 7. Once regional real expenditure shares have been determined by the chosen multilateral method, they can be combined with each region’s country shares of regional real expenditures to give each country’s share of world real expenditures. Note that this method will not affect the country shares of regional expenditures that are determined independently by each region. Note also that this method is very similar to the methods used by national income accountants to determine the annual real output of a nation by aggregating over quarterly data.

The problem with the above class of methods for linking the regions is this: how exactly should the *regional* Basic Heading PPPs be chosen? These regional aggregate “price” or PPP vectors should be chosen so that the overall method leads to country (and regional) relative volumes that are independent of the choice of both the numeraire region and the numeraire countries within each region. In section 2 below, several alternative choices of the regional BH PPPs will be explored. Option 1 explained in section 2 does lead to relative volumes which are independent of the choice of numeraire regions and numeraire countries within each region but the resulting method involves the use of country exchange rates which is not a desirable feature. Option 2 explained in section 2 does not involve the use of exchange rates but the resulting method is dependent on the choice of the regional numeraire countries, which again, is not a desirable feature.⁷ Finally, Option 3 explained in section 2, which was suggested by Sergeev (2009b), does lead to relative volumes which are independent of the choice of numeraire regions and numeraire countries within each region and this method for linking the regions does not involve the use of exchange rates.

It should be noted that all of the variants of the above method for linking the regions are based on two important properties within each region:

- Volumes are added up across countries in the region to obtain total regional volumes by BH category of expenditure and
- A single set of BH reference PPPs or “prices” is applied to the above regional volumes for each region.

⁶ This method is due to Gini (1924) (1931), Eltetö and Köves (1964) and Szulc (1964).

⁷ Sergey Sergeev (2009b) first noticed this problem with this method, which was the method used to link the regions in ICP 2005.

As mentioned above, the first property is analogous to practices in the System of National Accounts where annual quantities by expenditure category are obtained by adding up quarterly or monthly quantities and so this aspect of the linking method is not necessarily problematic. However, in the national accounting context, the structure of prices tends to be very similar going from quarter to quarter within a year. Thus in the national accounts, the annual price for a commodity is usually taken to be an annual unit value; i.e., total expenditure or revenue for the commodity is summed up over say quarters and divided by the total quantity used or produced during the year and this unit value is used as the annual price of the commodity.⁸ Using an annual unit value price in the context of aggregating over time is generally not a problem, since the price of a commodity within a year generally does not vary too much. However, when one is aggregating quantities across countries within a region, variations in commodity prices can be very large and generally speaking, there will *not* be a single vector of commodity prices that are *representative* for the structure of commodity prices for *every* country in the region. Thus the property of these methods which calculates a regional quantity vector by addition over country quantity vectors means that all of these methods are essentially *additive methods* (within each region) and hence are subject to *substitution bias*.⁹ Thus for ICP 2011, other methods for linking the regions, while respecting within region parities, should be considered. In sections 3 and 4 below, alternative linking methods will be considered.

In section 3, another method for constructing interregional parities while respecting within region parities will be discussed. This method is due to Alan Heston (1986) and Yuri Dikhanov (2007), who suggested that the GEKS method be used for all countries in the comparison, which will generate shares of world real expenditures (or a subaggregate) for each country. Now simply add up the shares of world real expenditures of the countries in each region in order to obtain regional shares. These between region shares plus the within region shares (determined independently by the regions) will generate an overall set of shares that respect the within region parities.

In section 4, a final method for constructing interregional parities while respecting within region parities will be discussed. This method is based on a statistical model that attempts to reconcile the global volume parities with the regional parities. This method (or a variant of it) was originally used by Eurostat and the OECD in order to impose fixity of the European Union country parities within a broader OECD comparison; see Sergeev (2005). Its properties were systematically studied by Robert Hill (2011a) (2011b), who called it the EOH method.

In section 5, a variant of the method used in section 3 will be discussed: instead of using GEKS to generate individual country shares for each country in the comparison, one could use Robert Hill's Minimum Spanning Tree (MST) or similarity linking method¹⁰ to

⁸ If there is high inflation within the year for the country, then finding an appropriate annual price is more complicated; see Hill (1996).

⁹ See the discussion in Chapter 7 on additive methods and substitution bias.

¹⁰ See Robert Hill (1999a) (1999b) (2001) (2004). Fisher (1922; 271-274) is a precursor to the work of Hill.

form the shares of each country's output in the world aggregate. As was noted in Chapter 7, the basic idea behind this method is to link countries through a series of bilateral comparisons where the overall comparison rests on a chain of bilateral comparisons between countries which are most similar in their (relative) price structures.

In section 6, regional parities are computed using the methods suggested in sections 2-4 for a small artificial data set.

In section 7, another numerical example based on a subset of the 1985 ICP data is computed to again illustrate the fact that different methods can give quite different results.

It should be noted that the methods that are suggested in this chapter that could be used to form aggregate PPPs for the 146 countries in the comparisons (over the entire set of 155 Basic Heading product groups) while respecting the fixity within regions constraint can also be used to form PPPs for *subsets* of the 155 BH product groups; e.g., the same techniques can be used to form a set of consumption PPPs for the 146 countries.

Section 8 concludes.

2. Variants of Diewert's Suggested Method for Linking the Regions

This section begins by defining what basic data are needed in order to link the regions in a fashion that will respect the parities and relative volumes that have been determined within each region. It is assumed that there are R regions in the comparison and region r has $C(r)$ countries in it for $r = 1, 2, \dots, R$. In ICP 2005, R was equal to 5 with varying numbers of countries in each region. Assume that there are N Basic Heading commodity groups and that each country in the comparison has collected expenditure data on these N commodity groups in their own currencies. In ICP 2005, N was equal to 155. Let E_{rcn} be the expenditure (in the currency of country c in region r) on commodity class n for the reference year for $r = 1, \dots, R$; $c = 1, \dots, C(r)$; $n = 1, \dots, N$. Assume that these country expenditure data have been collected.

Assume also that each region r has constructed a *Purchasing Power Parity* (PPP) for each BH commodity group n and each country c in the region r , α_{rcn} say, for $r = 1, \dots, R$; $c = 1, \dots, C(r)$ and $n = 1, \dots, N$. Assume that country $c = 1$ is the numeraire country in each region and so the PPP for this numeraire country is set equal to unity so that:

$$(1) \alpha_{r1n} = 1 ; \quad r = 1, \dots, R; n = 1, \dots, N.$$

Thus for $c \neq 1$, α_{rcn} is the price in the currency of country c in region r of a bundle of the products in BH commodity group n which costs one currency unit in the currency of the numeraire country (country 1) for region r .

The next assumption is that the central office has constructed a set of *interregional linking PPPs*, β_m , that link the PPPs of the numeraire country in each region with the

numeraire country in the numeraire region, which is assumed to be region 1. Thus assume that

$$(2) \beta_{1n} = 1 ; \quad n = 1, \dots, N.$$

For $r \neq 1$, β_{rn} is the price in the currency of country 1 in region r of a bundle of commodity n which costs one currency unit in the currency of the numeraire country 1 in region 1, the numeraire region.

Now the two sets of PPPs can be multiplied together to form a consistent set of *global Basic Heading PPPs*, γ_{rcn} , defined as follows:

$$(3) \gamma_{rcn} = \beta_{rn} \alpha_{rcn} ; \quad r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N.$$

Thus γ_{rcn} is the price in the currency of country c in region r of a bundle of BH commodity group n which costs one currency unit in the currency of the numeraire country (country 1) in the numeraire region (region 1).

If the numeraire region is changed, what happens to the global BH PPPs γ_{rcn} defined by (3)? Thus suppose region 2 becomes the numeraire region in place of region 1. In this case, the original linking BH PPPs (the β_{rn}) should be replaced by the new linking PPPs defined as

$$(4) \beta_{rn}^* = \beta_{rn} / \beta_{2n} ; \quad r = 1, \dots, R; n = 1, \dots, N$$

and the original set of global BH PPPs, the γ_{rcn} defined by (3) should be replaced by the following new set of global BH PPPs:

$$(5) \gamma_{rcn}^* = \beta_{rn}^* \alpha_{rcn} = \gamma_{rcn} / \beta_{2n} ; \quad r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N.$$

Thus the new set of global BH PPPs is equal to the old set of global BH PPPs except for a divisor that depends on the BH commodity n under consideration. Hence the net effect of switching the numeraire region is to leave the PPPs unchanged except that the new set of PPPs are measured in a *new system of units*; instead of measuring commodity units in terms of a dollar's worth of purchases of BH commodity n in the numeraire country of region 1, now commodity units are measured in terms of a peso's worth of purchases of BH commodity n in the numeraire country of region 2.

Now suppose that the numeraire country in a region is changed. If the change of numeraire country is outside region 1, (say in region 2 where there is a switch from country 1 to country 2 as the numeraire country), then it can be seen that nothing will happen to the global BH PPPs γ_{rcn} : the old $\gamma_{rcn} = \beta_{rn} \alpha_{rcn}$ will remain the same for all regions r except for $r = 2$ and for the second region, α_{2cn} will be replaced by $\alpha_{2cn} / \alpha_{22n}$ for $n = 1, \dots, N$ and $c = 1, \dots, C(2)$ and β_{2n} will be replaced by $\beta_{2n} \alpha_{22n}$ for $n = 1, \dots, N$. Thus the new γ_{rcn}^* will still equal the old γ_{rcn} for all r including $r = 2$.

If the change of numeraire country is within region 1 so that the new numeraire country in region 1 is country 2 in place of country 1, then the within region parities for region 1 become $\alpha_{1c_n}/\alpha_{12n}$ for $n = 1, \dots, N$ and the new set of interregional linking BH PPPs become β_{2n}/α_{12n} for $n = 1, \dots, N$. Thus in this case, the new set of world BH PPPs becomes:

$$(6) \gamma_{rcn}^* = \gamma_{rcn}/\alpha_{12n}; \quad r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N.$$

Thus in this case of a change of the numeraire country within the numeraire region, the new set of global BH PPPs are equal to the old set of global BH PPPs except for a divisor that depends on the BH commodity n under consideration. Hence as was the case with a change in the numeraire region, the net effect of switching the numeraire country within the numeraire region is to leave the BH PPPs unchanged except that the new set of BH PPPs are measured in a *new system of units*.

Recall a knowledge of the expenditures (in the local currencies) of country c in region r by BH commodity class n , E_{rcn} , is assumed for $r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N$. This expenditure information is used along with the consistent set of world BH PPPs, the γ_{rcn} defined by (3) above, in order to define consistent (across countries) *volumes* or *imputed quantities*, Q_{rcn} , for each BH commodity group n and each country c in each region r as follows:

$$(7) Q_{rcn} \equiv E_{rcn}/\gamma_{rcn}; \quad r = 1, \dots, R; c = 1, \dots, C(r); n = 1, \dots, N.$$

These imputed quantities or volumes are measured in units which are comparable across countries and regions. Thus these “quantities” can be added across countries within a region and the resulting regional totals, Q_{rn} , are also comparable across regions; i.e., define *regional total volumes by commodity class* as follows:

$$(8) Q_{rn} \equiv \sum_{c=1}^{C(r)} Q_{rcn}; \quad r = 1, \dots, R; n = 1, \dots, N.$$

Now form *regional volume* or *imputed quantity* vectors from the Q_{rn} :

$$(9) Q^r \equiv [Q_{r1}, \dots, Q_{rN}]; \quad r = 1, \dots, R.$$

Note that if the numeraire region or the numeraire countries within a region are changed, then provided that the global BH PPPs are changed in a consistent manner, the regional volume vectors will be identical to the initial regional volume vectors defined by (7)-(9) except that the units of measurement for these vectors may have been changed by the change in numeraires; i.e., a change in the numeraire region or numeraire country within a region will lead to identical regional quantity vectors Q^r or to a new set of Q^{r*} which are equal to $\hat{\delta} Q^r$ where $\hat{\delta}$ is a diagonal matrix where the diagonal elements reflect changes in the units of measurement of the N commodity groups.

Regional BH PPP or “price” vectors P^r are now needed to match up with the above regional volume vectors Q^r defined by (9). There are at least three possible strategies at this point and each will be discussed in turn.

Option 1: The Conversion to Regional Currency Unit Values Option:

The strategy using this option will be to convert country expenditures in each BH commodity classification category into common regional expenditures (in a numeraire country's currency) using market exchange rates for the reference year and then deflate these regional commodity expenditures by the corresponding regional quantities defined by (8) in order to form regional unit value prices, which will be used as the regional prices. As mentioned in the introduction, this is the same type of strategy that is used by national income accountants in forming annual price and quantity vectors from subannual information.

Suppose the *reference year exchange rate* for country c in region r is ε_{rc} for $r = 1, \dots, R$ and $c = 1, \dots, C(r)$. Assume that country 1 in each region is the numeraire region as usual so that

$$(10) \quad \varepsilon_{r1} = 1 ; \quad r = 1, \dots, R.$$

Thus ε_{rc} for $c \neq 1$ tells us how many units of the numeraire (country 1) currency in region r is equal to one unit of the currency unit of country c in region r . These market exchange rates can be used to convert country expenditures (in the country's currency) on commodity class n within region r into region r numeraire currency units, $V_{rcn} \equiv \varepsilon_{rc} E_{rcn}$, and then these *country expenditures on BH commodity class n in a common regional currency* can be summed to regional totals V_m defined as follows:

$$(11) \quad V_m \equiv \sum_{c=1}^{C(r)} \varepsilon_{rc} E_{rcn} ; \quad r = 1, \dots, R ; n = 1, \dots, N.$$

Now *regional unit value PPPs*, P_m , can be formed by taking the regional values defined by (11) and dividing them by the corresponding regional quantity totals Q_m defined by (8):

$$(12) \quad P_m \equiv V_m / Q_m ; \quad r = 1, \dots, R ; n = 1, \dots, N.$$

Finally, form *regional unit value PPP vectors* from the components, P_m :

$$(13) \quad P^r \equiv [P_{r1}, \dots, P_{rN}] ; \quad r = 1, \dots, R.$$

Now consider what happens to the regional total expenditures on commodity class n , V_m , if a numeraire country within a region is changed or there is a change the numeraire region. If the numeraire region is changed, nothing happens to regional expenditures defined by (11) since it is not necessary to relate the regional exchange rates across regions. However, if the numeraire country within a region r is changed, then it can be seen that that all of the regional values for this region will change by a scalar factor; i.e., if in region r , the numeraire country is changed from country 1 to country 2, then the new market exchange rates will be $\varepsilon_{rc}/\varepsilon_{r2}$ for $c = 1, \dots, C(r)$ and hence the new regional totals for region r will be

$$(14) V_m^* \equiv \sum_{c=1}^{C(r)} [\varepsilon_{rc}/\varepsilon_{r2}] E_{rcn} = V_m/\varepsilon_{r2}; \quad n = 1, \dots, N.$$

Thus the new regional expenditure totals on the N commodity groups in region r, V_m^* , will be equal to a scalar multiple ($1/\varepsilon_{r2}$) times the old regional expenditure totals in region r, V_m .

The above is a careful exposition of the regional unit value method for forming regional price and quantity vectors that could be used to link the regions by using a multilateral index number method with the above regional price and quantity vectors as the input vectors to the method.

Now consider linking the R regions using a *multilateral index number method*.¹¹ Diewert (1999) defined such methods in terms of share functions for the R regions; i.e., he looked at a system of *shares of global output or real expenditures*, $s_1(P^1, \dots, P^R; Q^1, \dots, Q^R), \dots, s_R(P^1, \dots, P^R; Q^1, \dots, Q^R)$, where $s_r(P^1, \dots, P^R; Q^1, \dots, Q^R)$ is region r's share of global output and considered the axiomatic properties of such multilateral systems. The next question that needs to be addressed is: what properties does the multilateral method have to satisfy in order to ensure that the regional shares are independent of the choice of the numeraire region and the numeraire countries within the regions?

From the above discussion, it is clear that the multilateral method must satisfy the following two properties:

Property 1: Invariance to changes in the units of measurement.

Thus let $\delta_n > 0$ for $n = 1, \dots, N$ and define $\hat{\delta}$ as the N by N matrix with the elements δ_n running down the main diagonal. Then this property requires that the multilateral share system satisfy the following equations:

$$(15) s_r(\hat{\delta} P^1, \dots, \hat{\delta} P^R; \hat{\delta}^{-1} Q^1, \dots, \hat{\delta}^{-1} Q^R) = s_r(P^1, \dots, P^R; Q^1, \dots, Q^R); \quad r = 1, \dots, R.$$

Property 2: Homogeneity of degree zero in the regional price vectors.

Let $\lambda_1 > 0, \dots, \lambda_R > 0$. Then this property requires that the multilateral share system satisfy the following equations:

$$(16) s_r(\lambda_1 P^1, \dots, \lambda_R P^R; Q^1, \dots, Q^R) = s_r(P^1, \dots, P^R; Q^1, \dots, Q^R); \quad r = 1, \dots, R.$$

This homogeneity property means that it is relative regional prices that affect the interregional volume comparisons and not the absolute level of regional prices.

It can be seen that the above two properties are also sufficient to imply that a multilateral method using the unit value regional price and quantity vectors defined above will

¹¹ See the material in Chapter 7.

generate regional shares that are independent of the choice of the numeraire region and the numeraire countries within the regions. It should be noted that the GEKS multilateral method satisfies these two properties; see Diewert (1999; 33).

A drawback of this method for linking the regions is that it brings market exchange rates into the picture. Since market exchange rates are often far removed from their corresponding PPPs, it is desirable to avoid their use in constructing the interregional PPPs and relative volumes. The following two methods make use of the regional “quantity” vectors Q^r defined by (8) but the corresponding regional “price” vectors P^r do not make use of exchange rates.

Option 2: The Use of Regional Numeraires as Regional Price Weights

It is not necessary to bring in market exchange rates to convert regional expenditures into a common currency. Rather than doing this, one can simply use the PPPs for each numeraire country 1 in each region r , relative to the numeraire country in the numeraire region, the β_m described above, as the price for commodity n in region r ; i.e., define the regional price for commodity n in region r as follows:

$$(17) P_m \equiv \beta_m ; \quad r = 1, \dots, R ; n = 1, \dots, N.$$

Then use (13) to form the *regional PPP vectors* P^r in the usual way. The regional total volume or quantity vectors, Q^r , are defined as before by (7)-(9) and these equations do not involve exchange rates. Finally, use these regional PPP and volume vectors, P^r and Q^r , in order to calculate the share functions for the R regions using a favoured multilateral method, $s_1(P^1, \dots, P^R; Q^1, \dots, Q^R), \dots, s_R(P^1, \dots, P^R; Q^1, \dots, Q^R)$, where $s_r(P^1, \dots, P^R; Q^1, \dots, Q^R)$ is region r 's share of world output.

Unfortunately, as Sergeev (2009b) pointed out, this method is not invariant to the choice of the numeraire countries within the regions. Thus this method should not be used in ICP 2011.

Option 3: The Use of the Geometric Average of the Regional Numeraires as Regional Price Weights

Sergeev (2009b) suggested a way to avoid the lack of numeraire invariance in Option 2: within each region, take the *geometric mean of the country parities* over all countries in the region. Thus equations (17) are replaced by the following equations:¹²

¹² This method of aggregation within a region is related to the geometric average price multilateral method originally suggested by Walsh (1901; 381 and 398) which was noted by Gini (1924; 106) and implemented by Gerardi (1982; 387). These authors used reference world prices that were the geometric mean over all countries in the world that were applied to all countries so that the resulting volume estimates are additive over all countries and all regions. In section 6 when the Sergeev method parities are computed using a subset of the ICP 2005 data, the regional parities price parities P_m are equal to a regional constant times the parities defined by (18). This will not affect the final regional parities.

$$(18) P_m \equiv \prod_{c=1}^{C(r)} \gamma_{rcn}^{1/C(r)} ; \quad r = 1, \dots, R ; n = 1, \dots, N.$$

Recall that γ_{rcn} is the country c in region r PPP for commodity group n and these parities are relative to the global numeraire country, country 1 in region 1. Thus (18) takes the geometric mean of all of these parities over all countries c in the region r . Now use (13) to form the *regional price vectors* P^r in the usual way. Finally, use these regional “price” and “quantity” vectors defined by (9), P^r and Q^r , in order to calculate the share functions for the R regions using a favoured multilateral method, $s_1(P^1, \dots, P^R; Q^1, \dots, Q^R)$, ... , $s_R(P^1, \dots, P^R; Q^1, \dots, Q^R)$, where $s_r(P^1, \dots, P^R; Q^1, \dots, Q^R)$ is region r 's share of global output. If the multilateral method satisfies Property 1 above (Invariance to changes in the units of measurement), then the resulting regional shares will be invariant to the choices of both numeraire countries and the numeraire region.

As mentioned above, Option 2 can be ruled out as a method for linking the regions for ICP 2011 due to its lack of country numeraire invariance within the regions. However, there is a good case for ruling out Options 1 and 3 as well. This is due to the fact that all of the methods in this section *impose a set of common prices to add up the volumes or quantities within a region*; i.e., the methods impose a form of *additivity*. Additive multilateral methods are subject to *substitution bias* if there are three or more countries in the comparison.¹³

Thus in the following three sections, *nonadditive methods* for linking the regions will be discussed which avoid the substitution bias that is inherent in the methods discussed in the present section.

3. The Heston Dikhanov Method for Harmonizing Global Volumes with Regionally Determined Volumes

Recall definitions (7) which defined the volume or “quantity” of commodity n , Q_{rcn} , that was finally demanded by purchasers in country c in region r . Define the country c in region r volume vector, Q^{rc} , in the usual way, using definitions (7) to define the components, Q_{rcn} :

$$(19) Q^{rc} \equiv [Q_{rc1}, \dots, Q_{rcN}] ; \quad r = 1, \dots, R ; c = 1, \dots, C(r).$$

The country PPP vectors P^{rc} that correspond to the country volume vectors Q^{rc} defined by (19) are defined using the global set of PPPs, γ_{rcn} , defined by equations (3) above. Recall that γ_{rcn} is the price in the currency of country c in region r of a bundle of commodity n which costs one currency unit in the currency of the numeraire country (country 1) in the numeraire region (region 1). The Basic Heading PPP vector for country c in region r , P^{rc} , is defined as follows:

$$(20) P^{rc} \equiv [\gamma_{rc1}, \dots, \gamma_{rcN}] ; \quad r = 1, \dots, R ; c = 1, \dots, C(r).$$

¹³ For an explanation of the problem, see Marris (1984; 52), Diewert (1999; 48-50) or Chapter 7.

Thus there are country PPP or “price” and volume or “quantity” vectors (P^{rc}, Q^{rc}) for all $C(1) + C(2) + \dots + C(R)$ countries in the ICP.

At this stage, any multilateral method could be used in order to form price comparisons between each individual country participating in the International Comparisons Project.

One such multilateral method which has good axiomatic and economic properties is the GEKS method.¹⁴ The algebra for this method works as follows. First define the *Fisher* (1922) *quantity index* for country c in region r relative to country d in region s as follows:¹⁵

$$(21) Q_F(rc/sd) \equiv [P^{rc} \cdot Q^{rc} P^{sd} \cdot Q^{rc} / P^{rc} \cdot Q^{sd} P^{sd} \cdot Q^{sd}]^{1/2} ; \\ r = 1, \dots, R; c = 1, \dots, C(r); s = 1, \dots, R; d = 1, \dots, C(s)$$

where $P^{rc} \cdot Q^{rc} \equiv \sum_{n=1}^N P_{rcn} Q_{rcn}$ denotes the inner product between the vectors P^{rc} and Q^{rc} . If the base country is fixed (i.e., fix the region s and the country d in region s) and let $r = 1, \dots, R$ and $c = 1, \dots, C(r)$, then the Fisher indexes defined by (21) can be interpreted as the volume of each country rc in the comparison relative to the base country sd and then these relative volumes can be normalized into a set of shares of global product using country sd as the base country; i.e., a set of country “star” shares are obtained for each rc with country sd as the “star” country.¹⁶ The GEKS method then takes the geometric mean of all of these country parities over all possible “star” bases. Thus define these geometric mean relative parities as follows:

$$(22) Q(rc) \equiv [\prod_{s=1}^R \prod_{d=1}^{C(s)} Q_F(rc/sd)]^{1/[C(1) + \dots + C(R)]} ; \quad r = 1, \dots, R; c = 1, \dots, C(r).$$

Now normalize the relative parities given by (22) into country shares of global product. Thus define the sum of the parities defined by (22) as σ :

$$(23) \sigma \equiv \sum_{r=1}^R \sum_{c=1}^{C(r)} Q(rc).$$

The *GEKS share of global real final expenditures for country c in region r* can now be defined as $Q(rc)$ divided by σ :

$$(24) s_{rc} \equiv Q(rc) / \sigma ; \quad r = 1, \dots, R; c = 1, \dots, C(r).$$

¹⁴ See Gini (1924) (1931), Eltetö and Köves (1964) and Szulc (1964) and Diewert (1999; 31-37) for the properties of this method.

¹⁵ Instead of using the Fisher ideal quantity index as the basic building block for this method, any other superlative quantity index could be used in this multilateral method. However, the Fisher index is generally preferred due to its strong axiomatic and economic properties; see Diewert (1976) (1992).

¹⁶ Kravis (1984; 10) introduced this “star” terminology.

Following Heston (1986; 3) and Dikhanov (2007)¹⁷, one can aggregate over the individual country shares of world real final expenditures defined by (24) within each region in order to obtain the following *GEKS regional shares of global real expenditures*:

$$(25) S_r \equiv \sum_{c=1}^{C(r)} s_{rc} ; \quad r = 1, \dots, R.$$

The between region shares S_1, S_2, \dots, S_R defined by (25) can be used in conjunction with the within region shares in each region in order to obtain a system of global expenditure shares for each country and the resulting parities will respect the regional parities that are independently determined by the regions. An explicit formula for these *Heston Dikhanov global relative volume shares of final expenditures* (that respect regional parities) can be obtained as follows. Let s_{rc}^* be the *regional volume shares* for region $r = 1, \dots, R$ and country c in region r for $c = 1, \dots, C(r)$ with $\sum_{c=1}^{C(r)} s_{rc}^* = 1$ for each r . Define the s_{rc} and S_r by (24) and (25) above. Then the *Heston Dikhanov (HD) share of country c in region r of global real expenditures*, $s_{rc}(\text{HD})$, is defined as follows:

$$(26) s_{rc}(\text{HD}) \equiv S_r s_{rc}^* ; \quad r = 1, \dots, R; c = 1, \dots, C(r).$$

Why are the GEKS regional shares defined by (25) to be preferred over the various regional shares defined in the previous section? The reason for this preference is that the GEKS shares are consistent with broader patterns of substitutability between commodities; i.e., if the preferences of each country can be represented by certain homothetic preferences (that can approximate arbitrary homothetic preferences to the second order), then the GEKS country shares will give exactly the “right” relative volumes across countries.¹⁸

Strictly speaking, the Heston Dikhanov method for harmonizing a set of global volume shares with a set of regional volume shares does not require the use of the GEKS multilateral method for the global and regional multilateral comparisons: any set of global volume shares s_{rc} and regional volume shares s_{rc}^* can be inserted into equations (25) and (26). However, the GEKS multilateral method has good axiomatic and economic properties and so its use can be recommended. In section 5 below, another alternative to using GEKS at the global level will be suggested. This alternative method also has some good economic properties.

In the following section, an alternative to the Heston Dikhanov method for reconciling regional volume parities with global parities will be discussed. This alternative method was developed by the OECD, Eurostat and Robert Hill who called it the OEH method.

¹⁷ The basic idea of the method was already suggested by Kravis, Heston and Summers (1982; 7-8) except that they favoured the use of the GK method for making multilateral comparisons.

¹⁸ On the other hand, the additive methods discussed in the previous section are consistent with homothetic preferences that can provide only a first order approximation to arbitrary homothetic preferences. For further explanation of this point, see Diewert (1999; 31) who introduced the concept of a superlative multilateral system. The GEKS system is a superlative method, whereas the additive methods are not. Balk (2009;82) provided a recent overview of various multilateral methods and endorsed the GEKS-Fisher method as a centre stage method, particularly from the economic approach to international comparisons.

4. The EOH Method for Harmonizing Global Volumes with Regionally Determined Volumes

The Eurostat, the OECD¹⁹ and Robert Hill (2011a) (2011b) recently developed a method for linking the regions while respecting within region parities. Hill provided several derivations of the EOH method but perhaps the simplest derivation is the result of considering a simple statistical model that is related to the derivation of the GEKS parities provided by Eltetö and Köves (1964) and Szulc (1964).²⁰ Thus let s_{rc} be the *global final expenditure volume share* of country c in region r and let s_{rc}^* be the *regional final expenditure volume share* of country c in region r for $r = 1, \dots, R$ and $c = 1, \dots, C(r)$.²¹ If there were R positive numbers β_r for $r = 1, \dots, R$ such that the following equations (27) held, then there would be no conflict between preserving the regional relative volume parities s_{rc}^* while being consistent with the global volume parities s_{rc} :

$$(27) \quad s_{rc} = \beta_r s_{rc}^* ; \quad r = 1, \dots, R; c = 1, \dots, C(r).$$

In practice, equations (27) will not hold for any choice of the β_r . A reasonable way for approximately reconciling the global parities with the regional parities is to choose the β_r so as to minimize some distance measure between the s_{rc} and the $\beta_r s_{rc}^*$. Thus we could choose to minimize with respect to the β_r the sum of squared residuals $\sum_{r=1}^R \sum_{c=1}^{C(r)} [s_{rc} - \beta_r s_{rc}^*]^2$. However, it proves to be more convenient²² to choose the β_r to minimize the following sum of squares:

$$(28) \quad \min_{\beta_1, \dots, \beta_R} \sum_{r=1}^R \sum_{c=1}^{C(r)} [\ln s_{rc} - \ln \beta_r - \ln s_{rc}^*]^2.$$

The first order necessary conditions for solving (28) lead to the following solutions for the β_r :

$$(29) \quad \beta_r^* \equiv \left[\prod_{c=1}^{C(r)} s_{rc} \right]^{1/C(r)} / \left[\prod_{c=1}^{C(r)} s_{rc}^* \right]^{1/C(r)} ; \quad r = 1, \dots, R.$$

Thus for each region r , the β_r which solves the least squares problem (28) is equal to the geometric mean of the *global* country volume parities for region r , $\left[\prod_{c=1}^{C(r)} s_{rc} \right]^{1/C(r)}$, divided by the geometric mean of the *regional* country volume parities, $\left[\prod_{c=1}^{C(r)} s_{rc}^* \right]^{1/C(r)}$.

¹⁹ The method is described in Sergeev (2005). The method was originally developed for use with the Geary-Khamis multilateral method in the early 1980s by Peter Hill and Michael Ward of the OECD at the urging of Hugo Krijnse-Locker of Eurostat. Other OECD economists who worked on the method included Derek Blades, John Dryden, Micheline Harary, Francette Koechlin and David Roberts.

²⁰ The EOH method is also related to Summer's (1973) country product dummy method; see also Diewert (2005). The derivations presented in this section basically follow those of Hill (2011a) but differ in a few details.

²¹ Thus $\sum_{r=1}^R \sum_{c=1}^{C(r)} s_{rc} = 1$ and $\sum_{c=1}^{C(r)} s_{rc}^* = 1$ for $r = 1, \dots, R$. The world shares s_{rc} and the regional shares s_{rc}^* do not have to be derived using the GEKS method; they can be derived by any preferred method. Hill (2011a) derived his method assuming that GEKS is the preferred method.

²² If the world shares s_{rc} and the regional shares s_{rc}^* are both independent of the choice of numeraire countries, then it can be shown that the EOH global shares $s_{rc}(\text{EOH})$ defined by (30) (which respect regional volume parities) are also independent of the choice of numeraire countries.

first region acts as the numeraire country in the set of world price parities (so that $p_{11} = 1$) and the first country in each region is used as the numeraire country in each regional comparison (so that $p_{r1}^* = 1$ for $r = 1, \dots, R$). However, for present purposes, it will be more convenient to match up these global and regional PPPs with the corresponding shares of global and regional volumes (the s_{rc} and s_{rc}^* respectively) so that the product of each PPP with the corresponding volume equals the *country expenditure* E_{rc} in national currency units. Thus it is assumed that the global PPPs (the p_{rc}) and the global volume shares (the s_{rc}) satisfy the following global comparison equations:²⁵

$$(32) \quad p_{rc} s_{rc} = E_{rc} ; \quad r = 1, \dots, R; c = 1, \dots, C(r).$$

It is also assumed that the regional PPPs (the p_{rc}^*) and the corresponding regional volume shares (the s_{rc}^*) satisfy the following regional comparison equations:

$$(33) \quad p_{rc}^* s_{rc}^* = E_{rc} ; \quad r = 1, \dots, R; c = 1, \dots, C(r).$$

The Hill method for constructing a global set of PPPs that respect regional PPPs proceeds as follows. If there were R positive numbers α_r for $r = 1, \dots, R$ such that the equations $p_{rc} = \alpha_r p_{rc}^*$ for $r = 1, \dots, R; c = 1, \dots, C(r)$, then there would be no conflict between preserving the regional relative PPP parities p_{rc}^* while being consistent with the world PPP parities p_{rc} . In practice, these equations will not hold for any choice of the α_r . A reasonable way for approximately reconciling the global parities with the regional parities is to choose the α_r so as to minimize the following sum of squares:

$$(34) \quad \min_{\alpha_1, \dots, \alpha_R} \sum_{r=1}^R \sum_{c=1}^{C(r)} [\ln p_{rc} - \ln \alpha_r - \ln p_{rc}^*]^2.$$

The first order necessary conditions for solving (34) lead to the following solutions for the α_r :

$$(35) \quad \alpha_r^* \equiv \left[\prod_{c=1}^{C(r)} p_{rc} \right]^{1/C(r)} / \left[\prod_{c=1}^{C(r)} p_{rc}^* \right]^{1/C(r)} ; \quad r = 1, \dots, R.$$

Thus for each region r , the α_r^* which solves the least squares problem (34) is equal to the geometric mean of the *global* country PPPs for region r , $\left[\prod_{c=1}^{C(r)} p_{rc} \right]^{1/C(r)}$, divided by the geometric mean of the *regional* country PPPs for region r , $\left[\prod_{c=1}^{C(r)} p_{rc}^* \right]^{1/C(r)}$.

Using (35), the *EOH harmonized PPP for country c in region r that respects regional parities*, $p_{rc}(\text{EOH})$, is defined as follows:

$$(36) \quad p_{rc}(\text{EOH}) \equiv \alpha_r^* p_{rc}^* \quad r = 1, \dots, R; c = 1, \dots, C(r).$$

Assume that the global PPPs, p_{rc} , and the regional PPPs, p_{rc}^* , are given and the EOH harmonized global PPPs, $p_{rc}(\text{EOH})$, are defined by (35) and (36). Define the *EOH imputed harmonized country imputed volumes* $v_{rc}(\text{EOH})$ that match up with country expenditures E_{rc} and the OEH harmonized PPPs $p_{rc}(\text{EOH})$ defined by (36) as follows:

²⁵ Thus p_{rc} and s_{rc} are essentially price and quantity indexes that satisfy Frisch's (1930; 399) *product test*.

$$(37) v_{rc}(\text{EOH}) \equiv E_{rc}/p_{rc}(\text{EOH}) \quad r = 1, \dots, R; c = 1, \dots, C(r) \\ = E_{rc}/\alpha_r^* p_{rc}^* \quad \text{using definitions (36).}$$

But the global and regional parities, p_{rc} and p_{rc}^* , along with equations (32) and (33) can be used to define (nonharmonized) global and regional volume parities, s_{rc} and s_{rc}^* , as follows:

$$(38) s_{rc} \equiv E_{rc}/p_{rc}; s_{rc}^* \equiv E_{rc}/p_{rc}^*; \quad r = 1, \dots, R; c = 1, \dots, C(r).$$

Once the s_{rc} and s_{rc}^* have been determined, the direct harmonizing factors of proportionality β_r^* can be defined by equations (29) above and the EOH direct harmonized global volume shares $s_{rc}(\text{OEH})$ can be defined by equations (30). How do these direct harmonized volume shares compare with the imputed harmonized shares defined by (37)? Hill (2011a) showed that they are identical. In order to prove this result, we require a preliminary result: namely that the α_r^* defined by (35) are equal to the reciprocals of the β_r^* defined by (29). From definitions (35), we have:

$$(39) \alpha_r^* = \left[\prod_{c=1}^{C(r)} p_{rc} \right]^{1/C(r)} / \left[\prod_{c=1}^{C(r)} p_{rc}^* \right]^{1/C(r)}; \quad r = 1, \dots, R \\ = \left[\prod_{c=1}^{C(r)} E_{rc}/s_{rc} \right]^{1/C(r)} / \left[\prod_{c=1}^{C(r)} E_{rc}/s_{rc}^* \right]^{1/C(r)} \quad \text{using (32) and (33)} \\ = 1 / \left\{ \left[\prod_{c=1}^{C(r)} s_{rc} \right]^{1/C(r)} / \left[\prod_{c=1}^{C(r)} s_{rc}^* \right]^{1/C(r)} \right\} \\ = 1/\beta_r^* \quad \text{using definitions (29).}$$

Using (37), we have:

$$(40) v_{rc}(\text{EOH}) = E_{rc}/\alpha_r^* p_{rc}^* \quad r = 1, \dots, R; c = 1, \dots, C(r) \\ = \beta_r^* E_{rc}/p_{rc} \quad \text{using (39)} \\ = \beta_r^* s_{rc} \quad \text{using (33)} \\ = s_{rc}(\text{EOH}) \left(\sum_{s=1}^R \beta_s^* \right) \quad \text{using definitions (30).}$$

Thus the EOH harmonized imputed volume index for country c in region r $v_{rc}(\text{EOH})$ that was defined indirectly using the region r price proportionality factor α_r^* defined by (35) is equal to the EOH harmonized volume index for country c in region r $s_{rc}(\text{EOH})$ that was defined directly using the region r volume proportionality factor β_r^* defined by (29) times an inconsequential constant, $(\sum_{s=1}^R \beta_s^*)$.

Combining the above result (40) with the numerical approximation result (31) shows that the EOH harmonized global shares $v_{rc}(\text{EOH})$ (which are generated via a price harmonization approach) times an inconsequential constant will be approximately equal to the Heston Dikhanov world shares $s_{rc}(\text{HD})$ defined earlier by (26).

The above EOH approach to the harmonization of regional volume shares with globally generated volume shares can be regarded as a *democratic procedure* in the sense that each country in a region is given equal weight in the computation of the harmonizing factors of proportionality defined by (29) (which define the β_r^*) or by (35) (which define

the α_r^*). It would be possible to modify the EOH harmonization method into a procedure where larger countries play a larger role in the determination of the harmonizing factors of proportionality by using a weighted least squares approach instead of the unweighted approaches that are described by (28) and (34).²⁶

An advantage of the EOH method for reconciling a set of global volume shares with regionally determined volume shares is that standard errors for the regional factors of proportionality (the β_r^*) can be determined using the EOH method whereas this cannot be done using the Heston Dikhanov method. However, the EOH method does have a possible disadvantage as compared to the HD method. Presumably, the global volume shares determined by the central office (the s_{rc}) are “best” in some sense. If this is the case, then the regional shares of global real expenditures, the $S_r \equiv \sum_{c=1}^{C(r)} s_{rc}$ determined by equations (25) above, are also the “best” regional shares of global product that can be determined. The statistically determined EOH regional shares are in general not equal to these “best” regional shares.

In the following section, an alternative method for constructing global volume shares will be discussed.

5. Spatial Comparisons Based on Similar Price Structures

The GEKS multilateral method treats each country “star” parity as being equally valid and hence an averaging of the parities is appropriate under this hypothesis. However, is it really the case that all bilateral comparisons of volume between two countries are equally accurate? One could argue that the answer to this question is that the answer is no: if the relative prices in countries A and B are very similar, then the Laspeyres and Paasche quantity indexes will be very close to each other and hence it is likely that the “true” volume comparison between these two countries (using the economic approach to index number theory) will be very close to the Fisher volume comparison. On the other hand, if the structure of relative prices in the two countries is very different, then it is likely that the structure of relative quantities in the two countries will also be different and hence the Laspeyres and Paasche quantity indexes will likely differ considerably and we can no longer be certain that the Fisher quantity index will be close to the “true” volume comparison. The above considerations suggest that a more accurate set of world product shares could be constructed if an initial bilateral comparison was made between the two countries which had the most similar relative price structures. Then look for a third country which had the most similar relative price structure to the first two countries and link in this third country to the comparisons of volume between the first two countries and so on. At the end of this procedure, a *minimum spanning tree* (MST) would be constructed: a path between all countries that minimized the sum of the relative price similarity measures. This MST linking methodology has been developed by Robert Hill (1999a) (1999b) (2001) (2004) (2009).

²⁶ This weighted approach is developed in Hill (2011b). Other weighted least squares approaches are described in Rao (1990), Balk (1996) and Diewert (2004).

A key aspect of this methodology is the choice of the measure of similarity (or dissimilarity) of the relative price structures of two countries. Various measures of the similarity or dissimilarity of relative price structures have been proposed by Aten and Heston (2009), Diewert (2009), Hill (2009) and Sergeev (2001) (2009a). The following *weighted log quadratic measure of relative price dissimilarity*, $\Delta_{PLQ}(p^1, p^2, q^1, q^2)$, was suggested by Diewert (2009; 207) (the smaller the measure, the more similar is the structure of relative prices between the two countries):

$$(41) \Delta_{WLQ}(p^1, p^2, q^1, q^2) \equiv \sum_{n=1}^N (1/2)(s_n^1 + s_n^2) [\ln(p_n^2/p_n^1 P_F(p^1, p^2, q^1, q^2))]^2$$

where $P_F(p^1, p^2, q^1, q^2) \equiv [(p^2 \cdot q^1 / p^2 \cdot q^2) / (p^1 \cdot q^1 / p^1 \cdot q^2)]^{1/2}$ is the Fisher ideal price index between countries 2 and 1 and $s_n^c \equiv p_n^c q_n^c / p^c \cdot q^c$ is the country c expenditure share on Basic Heading category n for $c = 1, 2$ and $n = 1, \dots, N$.

Diewert (2009; 208) also suggested the following measure of relative price similarity, the *weighted asymptotically quadratic measure of relative price dissimilarity*, $\Delta_{WAQ}(p^1, p^2, q^1, q^2)$.²⁷

$$(42) \Delta_{WAQ}(p^1, p^2, q^1, q^2) \equiv \sum_{n=1}^N (1/2)(s_n^1 + s_n^2) \{ [(p_n^2/p_n^1 P_F(p^1, p^2, q^1, q^2)) - 1]^2 + [(P_F(p^1, p^2, q^1, q^2) p_n^1 / p_n^2) - 1]^2 \}.$$

It can be seen that if prices are proportional for the two countries so that $p^2 = \lambda p^1$ for some positive scalar λ , then $P_F(p^1, p^2, q^1, q^2) = \lambda$ and the measures of relative price dissimilarity defined by (41) and (42) will equal its minimum of 0. Thus the smaller are $\Delta_{WLQ}(p^1, p^2, q^1, q^2)$ or $\Delta_{WAQ}(p^1, p^2, q^1, q^2)$, the more similar is the structure of relative prices in the two countries. These two measures of price dissimilarity were Diewert's (2009) preferred measures of relative price dissimilarity and they will be used in the following two sections in numerical examples to illustrate the MST method for making multilateral comparisons. We will also use the following measure of relative price dissimilarity in our numerical examples which was suggested by Kravis, Heston and Summers (1982; 105) and Robert Hill (1999a) (1999b) (2001) (2004). This measure is essentially a normalization of the relative spread between the Paasche and Laspeyres price indexes, so it is known as the *Paasche and Laspeyres Spread relative price dissimilarity measure*, $\Delta_{PLS}(p^1, p^2, q^1, q^2)$:

$$(43) \Delta_{PLS}(p^1, p^2, q^1, q^2) \equiv \max \{ P_L / P_P, P_P / P_L \} - 1$$

where $P_L \equiv p^2 \cdot q^1 / p^1 \cdot q^1$, and $P_P \equiv p^2 \cdot q^2 / p^1 \cdot q^2$. As was noted in Chapter 7, a major problem with this measure of relative price dissimilarity is that it is possible for P_L to equal P_P but yet p^2 could be very far from being proportional to p^1 .

Hill's MST method of similarity or spatial linking using the above three measures of relative price dissimilarity will be illustrated in the following two sections using small

²⁷ For a discussion on how to deal with the problems with (41) and (42) that arise if any of the prices p_n^1 are 0, see Chapter 7.

numerical examples. Basically, instead of using the GEKS country shares s_{rc} defined by (24) in section 3, the shares generated by the Minimum Spanning Tree (using the three dissimilarity measures) are used to link all of the countries in the ICP. Once these country shares s_{rc} have been defined, again use equations (25) in order to form the regional shares S_r .

In order to get an impression as to how the different methods suggested in this section and the previous sections perform in practice, two small numerical examples will be computed in the following two sections. The first example uses an artificial data set and the second example uses a small subset of the ICP 1985 data.

6. An Artificial Data Set Numerical Example

In this example, there are only 4 countries and only 2 commodities. There are two regions where region 1 consists of the first two countries and region 2 consists of the second two countries. The basic data for the countries is the within region r Basic Heading PPP for commodity n for country c in region r , α_{rcn} , the expenditure on commodity class n for country c in region r in domestic currency, E_{rcn} , and the market exchange rate for country c in region r , ε_{rc} (which does not depend on the commodity n), for $r = 1,2$, $c = 1,2$ and $n = 1,2$. The Basic Heading interregional PPPs for the numeraire countries in each region are also needed, the β_{rn} , which are the parities for commodity n for region r relative to region 1. See Table 1 below for a listing of the data.

Table 1: Data for Four Countries in Two Regions

	Region 1						Region 2						Regional	
	Country 1		Country 2		Country 1		Country 2		Country 1		Country 2		BH Parities	
n	α_{11n}	E_{11n}	ε_{11}	α_{12n}	E_{12n}	ε_{12}	α_{21n}	E_{21n}	ε_{21}	α_{22n}	E_{22n}	ε_{22}	β_{1n}	β_{2n}
1	1	10	1	20	2	5	1	20	3	10	20	18	1	2
2	1	10	1	4	8	5	1	20	3	4	160	18	1	4

There are 8 world Basic Heading PPPs that are obtained using equations (3), $\gamma_{rcn} = \beta_{rn}\alpha_{rcn}$. Thus for commodity class 1, the following world BH PPPs are obtained: $\gamma_{111} = 1$; $\gamma_{121} = 20$; $\gamma_{211} = 2$; $\gamma_{221} = 20$. For commodity class 2, the following world BH PPPs are obtained: $\gamma_{112} = 1$; $\gamma_{122} = 4$; $\gamma_{212} = 4$; $\gamma_{222} = 16$.

Working through the algebra in section 2, the Option 1 shares of world output for Regions 1 and 2 turn out to equal 0.45134 for Region 1 and 0.54866 for Region 2. As expected, the same regional shares are obtained no matter which region is chosen as the numeraire region and no matter which country is chosen as the numeraire country within a region.

The Option 2 regional shares turn out to depend on the choice of the regional numeraire countries as expected. Letting Country 1 in each region be the numeraire country, 0.45676 and 0.54324 are obtained as the two regional shares of world product. Letting

Country 2 in Region 1 be the numeraire country and maintaining Country 1 in Region 2 as the numeraire country leads to 0.46287 and 0.53713 as the new regional shares of world product. Letting Country 1 in Region 1 be the numeraire country and letting Country 2 in Region 2 be the numeraire country leads to 0.46041 and 0.53959 as the new regional shares of world product. The bottom line is that the Option 2 regional shares are not invariant to the choice of the numeraire countries in the regions.

The Option 3 regional shares²⁸ (Sergeev Option) turn out to be S_1 equal to 0.46186 and S_2 equal to 0.53814. Recall that the Option 1 regional shares were 0.45134 for Region 1 and 0.54866 for Region 2. Thus the Option 1 and 3 regional shares differ by about 2.3%.

The regional shares for the methods explained in section 3 above will now be calculated. The individual country shares of world output using each of the four countries as the “star” in bilateral Fisher index number comparisons are listed in the first four rows of Table 2. It can be seen that there are some relatively large differences between these world shares, particularly for the small country, Country 2 in Region 1. The GEKS country shares of world product are listed in the last line of Table 2.

Table 2: Star and GEKS Country Shares of Global Real Expenditures

	Region 1		Region 2	
	Country 1 s_{11}	Country 2 s_{12}	Country 1 s_{21}	Country 2 s_{22}
Region 1, Country 1 Star	0.43523	0.02879	0.30775	0.22823
Region 1, Country 2 Star	0.47078	0.03114	0.32258	0.17550
Region 2, Country 1 Star	0.42439	0.02897	0.30009	0.24655
Region 2, Country 2 Star	0.44331	0.04125	0.28296	0.23248
HDGEKS	0.44436	0.03226	0.30387	0.21951

The GEKS volume shares in the last row of the above table are defined by equations (24); i.e., the entries for this last row are the GEKS shares s_{11} , s_{12} , s_{21} and s_{22} respectively. Using these country shares, the *Heston Dikhanov regional shares* S_r using GEKS are defined by (25) so that using the entries in the last line of Table 2, $S_1(\text{HDGEKS})$ and $S_2(\text{HDGEKS})$ are defined as follows:

$$(44) S_1(\text{HDGEKS}) = s_{11} + s_{12} = 0.47662 ; S_2(\text{HDGEKS}) = s_{21} + s_{22} = 0.52338.$$

Thus the HDGEKS share of global output for Region 1 is 0.47662 which is higher than the corresponding Region 1 shares for the Option 1 method (0.45134) and for the Option 3 method (0.46479).

The GEKS global country shares s_{rc} can be found in Table 2 above. In order to implement Hill’s method for harmonizing global country volume shares with regional shares which was explained in section 4 above (using GEKS globally and regionally), it is necessary to have the global GEKS shares s_{rc} (which are listed in Table 2 above) and the country

²⁸ The Sergeev regional price and quantity vectors P^r and Q^r for $r = 1, 2$ turn out to be $P^1 = [4.472, 2.000]$, $P^2 = [6.325, 8.000]$, $Q^1 = [10.1, 12.0]$ and $Q^2 = [11.0, 15.0]$.

GEKS shares of regional product, the s_{rc}^* for $r = 1,2$ and $c = 1,2$. Since there are only two countries in each region, it can be verified that the GEKS regional country shares for regions 1 and 2 are defined by (45) and (46) below:

$$(45) s_{11}^* = 1/(1+Q_F(12/11)) = 0.91598 ; s_{12}^* = Q_F(12/11)/(1+Q_F(12/11)) = 0.08402 ;$$

$$(46) s_{21}^* = 1/(1+Q_F(22/21)) = 0.62830 ; s_{22}^* = Q_F(22/21)/(1+Q_F(22/21)) = .37170$$

where the Fisher bilateral quantity indexes $Q_F(rc/sd)$ are defined by (21) above. Recall that the regional proportionality factors for the EOH harmonization method (the β_r^* for $r = 1,2$) were defined by equations (29). Using the data for the present numerical example, it is found that β_1^* equals 0.43160 and β_2^* equals 0.53443. Using these country shares, the *EOH regional shares* S_r (using GEKS at the global and regional levels) are defined by equations (30) for $r = 1,2$:

$$(47) S_1(\text{EOHGEKS}) = \beta_1^*/(\beta_1^* + \beta_2^*) = 0.44677 ; S_2(\text{EOHEKS}) = \beta_1^*/(\beta_1^* + \beta_2^*) = 0.55323.$$

Comparing (47) with (44), it can be seen that the harmonized EOH share of global product of region 1 (0.44677) is 6.7% lower than the harmonized Heston Dikhanov share of global product for region 1 (0.47662) where both harmonization methods use GEKS to compute global and regional shares. This is a substantial difference!²⁹

Finally, the MST or similarity or spatial linking method regional shares will be calculated using the three dissimilarity measures defined by (41) (WLQ), (42) (WAQ) and (43) (PLS).

The first measure of relative price dissimilarity is the Weighted Log Quadratic measure of relative price dissimilarity between countries 1 and 2, $\Delta_{WLQ}(p^1, p^2, q^1, q^2)$ defined by (41) above. The 4 by 4 matrix of relative price dissimilarity measures is listed in Table 3 below. For convenience in labeling the countries, set country 1 equal to Country 1 in Region 1, country 2 to Country 2 in region 1, country 3 to Country 1 in Region 2 and country 4 to Country 2 in Region 2.

Table 3: Weighted Log Quadratic Relative Price Dissimilarities between Countries i and j

	Country 1	Country 2	Country 3	Country 4
Country 1	0.00000	0.59465	0.12011	0.01057
Country 2	0.59465	0.00000	1.22741	0.25253
Country 3	0.12011	1.22741	0.00000	0.17933
Country 4	0.01057	0.25253	0.17933	0.00000

²⁹ The geometric mean of s_{11} and s_{12} is 0.11973 whereas the corresponding arithmetic mean is 0.23831 and the ratio of the arithmetic mean to the geometric mean is 1.99. The geometric mean of s_{11}^* and s_{12}^* is 0.27742 whereas the corresponding arithmetic mean is 0.50000 and the ratio of the arithmetic mean to the geometric mean is 1.80. If these ratios of the two types of mean were equal to each other, then the EOH and Heston Dikhanov harmonized parities would coincide, but they differ substantially which explains the differences in the two sets of harmonized global volumes.

From the above Table, it can be seen that countries 1 and 4 have the most similar structures of relative prices with a dissimilarity measure equal to 0.01057. The next pair of countries that have most similar structure of relative price is countries 1 and 3 with a dissimilarity measure equal to 0.12011. Thus countries 3 and 4 can be linked to country 1 using the bilateral Fisher quantity index between 4 and 1 and between 3 and 1. The next most similar in structure pair of countries is 3 and 4 with a dissimilarity measure equal to 0.17933. But these two countries are already linked so the next lowest measure of relative price dissimilarity is considered. The next most similar pair of countries is 2 and 4 with a dissimilarity measure equal to 0.25253. The bilateral Fisher quantity index is used to link country 2 to country 4 and now all of the countries in the global comparison have been linked using bilateral links. The volumes of all 4 countries relative to country 1 turn out to be 1.00000, 0.09305, 0.70711 and 0.52440. This leads to the following vector of country shares of global output: 0.43019, 0.04003, 0.30419, 0.22559. Adding up the shares of the countries in each region leads to the following *Heston Dikhanov similarity linked regional shares of world output*:

$$(48) S_1(\text{HDS}) = s_{11} + s_{12} = 0.47022 ; S_2(\text{HDS}) = s_{21} + s_{22} = 0.52978.$$

Thus the similarity linking method at the global level using the dissimilarity measure defined by (41), combined with the Heston Dikhanov method for harmonizing regional volume shares with global volume shares, leads to 0.47022 as the estimate of Region 1's share of world output. This estimate should be compared to the Heston-Dikhanov-GEKS estimate of 0.47662, the Sergeev Option 3 estimate of 0.46479 and the Hill-GEKS estimate of 0.44677.

The second measure of relative price dissimilarity is the Weighted Asymptotically Quadratic measure of relative price dissimilarity, $\Delta_{\text{WAQ}}(p^1, p^2, q^1, q^2)$ defined by (42) above. The corresponding 4 by 4 matrix of relative price dissimilarity measures is listed in Table 4 below. The labeling of countries is the same as in Table 3.

Table 4: Weighted Asymptotically Quadratic Relative Price Dissimilarities between Countries i and j

	Country 1	Country 2	Country 3	Country 4
Country 1	0.00000	1.74216	0.25736	0.02135
Country 2	1.74216	0.00000	4.94541	0.91013
Country 3	0.25736	4.94541	0.00000	0.41607
Country 4	0.02135	0.91013	0.41607	0.00000

From the above Table, it can be seen that countries 1 and 4 have the most similar structures of relative prices with a dissimilarity measure equal to 0.02135. The next pair of countries that have most similar structure of relative price is countries 1 and 3 with a dissimilarity measure equal to 0.25736. The next most similar in structure pair of countries is 3 and 4 with a dissimilarity measure equal to 0.41607. But these two countries are already linked so the next lowest measure of relative price dissimilarity is 2

and 4 with a dissimilarity measure equal to 0.91013. The bilateral Fisher quantity index is used to link country 2 to country 4 and now all of the countries in the “world” have been linked using bilateral links. Note that we end up with exactly the same Minimum Spanning Tree that we ended up with using the Weighted Log Quadratic measure of dissimilarity so all of the further analysis of this case is the same as in the previous case. It is encouraging that Diewert’s two most preferred measures of relative price dissimilarity give rise to exactly the same set of links.

The third measure of relative price dissimilarity is the Paasche and Laspeyres Spread measure of relative price dissimilarity, $\Delta_{PLS}(p^1, p^2, q^1, q^2)$ defined by (43) above. The 4 by 4 matrix of relative price dissimilarity measures generated by this measure is listed in Table 5 below. The labeling of countries is the same as in Table 3.

Table 5: Paasche and Laspeyres Spread Relative Price Dissimilarities between Countries i and j

	Country 1	Country 2	Country 3	Country 4
Country 1	0.00000	1.52000	0.12500	0.10000
Country 2	1.52000	0.00000	3.51000	0.13333
Country 3	0.12500	3.51000	0.00000	0.63333
Country 4	0.10000	0.13333	0.63333	0.00000

Working through the information in the above table, it can be seen that the Minimum Spanning Tree using the Paasche and Laspeyres Spread measure of dissimilarity is exactly the same as the MST using the previous two dissimilarity measures. Thus for this example, all three dissimilarity measures generate the same set of bilateral linkages and hence the same estimates of country and region relative volumes.

Recall that all three of the dissimilarity measures led to the following global country shares of global real expenditures: $s_{11} = 0.43019$, $s_{12} = 0.04003$, $s_{21} = 0.30419$, $s_{22} = 0.22559$. Recall also that the GEKS regional country shares for regions 1 and 2, were defined by (45) and (46). Since there are only 2 countries in each region, these regional shares are both the GEKS shares and the similarity method shares. Now apply the EOH harmonization method to these shares; i.e., recall equations (29) which define the regional proportionality factors, the β_r^* for $r = 1, 2$. Using the share data defined in this paragraph, it is found that β_1^* equals 0.47302 and β_2^* equals 0.54207. Using these country shares, the *EOH regional shares* S_r (using similarity linking at the global and regional levels) are defined by equations (30) for $r = 1, 2$:

$$(49) S_1(\text{EOHS}) = \beta_1^*/(\beta_1^* + \beta_2^*) = 0.46599 ; S_2(\text{EOHS}) = \beta_2^*/(\beta_1^* + \beta_2^*) = 0.53401.$$

Recall that the EOH harmonization method share of region 1 in world product when GEKS was used at the global and regional levels was 0.44677, which was a much lower share than those region 1 shares derived by all other methods. However, when the EOH harmonization procedure is applied to global and regional shares that are generated by similarity linking, the share of region 1 in world product is substantially increased to

0.46599, which is much closer to the corresponding Heston Dikhanov GEKS estimate of 0.47662.

A summary of the Region 1 share of world output using the various methods can be found in Table 6 below.

Table 6: The Share of World Output for Region 1 using Various Methods

	Region 1 Share of World Output
Option 1 (Regional Unit Values Method)	0.45134
Option 2: Base Countries: 1 in Region 1; 1 in Region 2	0.45676
Option 2: Base Countries: 2 in Region 1; 1 in Region 2	0.46287
Option 2: Base Countries: 1 in Region 1; 2 in Region 2	0.46041
Option 3: Geometric Mean Average Prices in each Region	0.46186
Heston-Dikhanov-GEKS	0.47662
EOH-GEKS	0.44677
Heston-Dikhanov-Spatial Linking	0.47022
EOH-Spatial Linking	0.46599

The differences between the various methods are fairly substantial: a 6.7% difference in the share of Region 1 for the highest share method (the Heston Dikhanov harmonization method using GEKS global and regional shares) versus the lowest share method (the EOH harmonization method using GEKS global and regional shares).

In the following section, another numerical example based on actual ICP 1985 data is considered that leads to even bigger numerical differences between the various methods.

7. A Numerical Example Based on ICP 1985 Data

Yuri Dikhanov generated some highly aggregated data (across Basic Heading groups) from ICP 1985 on 5 consumption components for 8 countries. The 8 countries are:

- 1 = Hong Kong;
- 2 = Bangladesh;
- 3 = India;
- 4 = Indonesia;
- 5 = Brazil;
- 6 = Japan;
- 7 = Canada and
- 8 = US.

Note that the countries Hong Kong, Japan, Canada, and the US can be considered to be “rich” countries while Bangladesh, India, Indonesia, and Brazil can be considered to be “less rich”.

The 5 commodity groups were:

- 1 = durables;
- 2 = food, alcohol and tobacco;
- 3 = other nondurables excluding food, alcohol, tobacco and energy;
- 4 = energy and
- 5 = services.

The expenditure data (converted to US dollars at market exchange rates) and the volume or “quantity” data for the 8 countries are listed in the following Tables 7 and 8.

Table 7: Expenditures in US Dollars for Eight Countries and Five Consumption Categories

	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
1	14320	1963	23207	8234	52722	307547	94121	967374
2	10562	24835	176782	83882	105527	448995	82056	778665
3	14951	5100	60748	15158	60798	272875	69461	992761
4	2619	3094	42126	17573	39933	125835	43342	524288
5	62124	11627	166826	61248	273669	1736977	379629	5559458

Table 8: Quantities (Volumes) in Comparable Units for Eight Countries and Five Consumption Categories

	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
1	15523	2312	30189	9781	46146	280001	81021	967374
2	9164	47509	356756	138273	163868	251846	63689	778665
3	317564	10588	180964	29879	65274	200614	58261	992761
4	1095	3033	38377	22084	23963	59439	35714	524288
5	81148	47611	786182	223588	541236	1695136	417210	5559458

If the entries in Table 7 (expenditures converted to US dollars at market exchange rates) are divided by the entries in Table 8 (quantities in comparable units), the Basic Heading PPPs (converted into US dollars at market exchange rates) for each commodity class for each country are obtained. These PPPs are listed in Table 9 below.

Table 9: PPPs of Consumption Components in US Dollars for Eight Countries and Five Consumption Categories

	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
1	0.92250	0.84905	0.76872	0.84184	1.14250	1.09838	1.16169	1.0
2	1.15255	0.52274	0.49553	0.60664	0.64398	1.78282	1.28839	1.0
3	0.85123	0.48168	0.33569	0.50731	0.93143	1.36020	1.19224	1.0
4	2.39178	1.02011	1.09769	0.79573	1.66644	2.11704	1.21359	1.0
5	0.76556	0.24421	0.21220	0.27393	0.50564	1.02468	0.90992	1.0

Thus the US price level for each commodity group is set equal to 1 and the other prices are the average domestic prices for the commodity group converted into US dollars at the 2005 market exchange rates. Note that for durables, India has the lowest price level at 0.77 and Canada has the highest level at 1.16; for food, again India has the lowest prices at 0.49 while Japan has the highest at 1.78; for other nondurables, India has the lowest price level at 0.33 while Japan has the highest level at 1.36; for energy, Indonesia has the lowest price level at 0.79 while Japan has the highest at 2.12 and for services, India has the lowest price level at 0.21 while Japan has the highest level at 1.02. Thus the amount of price level variation across countries ranges from 51% for durables to 471% for services.

The above data are used to compute relative consumption volumes for the 8 countries using various multilateral methods. Instead of normalizing the relative volumes into shares of “world” consumption, the consumption of each country relative to the consumption of the US will be calculated. This is simply an alternative normalization of the country relative volumes.

The star method for constructing relative volumes for the 8 countries was explained briefly in section 3 above. Basically, one country is chosen as the “star” country and the Fisher quantity index of all other countries is calculated relative to the star country, which gives the volume of all 8 countries relative to the star country. In Table 10 below, these star relative volumes are listed except that they have been normalized so that country 8’s volume (for the U.S.) has been set equal to unity. This will give the reader some indication of the variability in the data.

Table 10: Fisher Star Volumes Relative to the U.S.

	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
Star 1	0.01346	0.01367	0.16021	0.05158	0.09192	0.27530	0.07444	1.00000
Star 2	0.01257	0.01277	0.14351	0.04660	0.08984	0.24629	0.07129	1.00000
Star 3	0.01275	0.01350	0.15178	0.04984	0.09040	0.25596	0.07328	1.00000
Star 4	0.01277	0.01341	0.14902	0.04894	0.09141	0.25496	0.07262	1.00000
Star 5	0.01323	0.01284	0.15169	0.04837	0.09035	0.26357	0.07372	1.00000
Star 6	0.01355	0.01437	0.16439	0.05322	0.09504	0.27724	0.07464	1.00000
Star 7	0.01343	0.01331	0.15387	0.05007	0.09105	0.27596	0.07429	1.00000
Star 8	0.01346	0.01277	0.15178	0.04894	0.09035	0.27724	0.07429	1.00000

The Fisher star parities for the 7 countries relative to the US had the following relative volume ranges: Hong Kong: 0.01257 to 0.01355 (7.7% variation); Bangladesh: 0.01277 to 0.01437 (12.5%); India: 0.14351 to 0.16439 (14.5%); Indonesia: 0.04660 to 0.05322 (14.2%); Brazil: 0.08984 to 0.09504 (5.8%); Japan: 0.24629 to 0.27724 (12.6%) and Canada: 0.07129 to 0.07464 (4.7%). Thus the variation in relative volumes is quite large, depending on which country is used as the base country in a comparison based on the use of Fisher star parities.

The GEKS, GK and IDB methods for comparing relative volumes were explained in previous sections and in Chapter 7. Consumption volumes (relative to the U.S.) for the 8 countries were computed using these methods and they are listed in Table 14 below.

The Hill similarity linking or Minimum Spanning Tree (MST) method was also used to compute relative volumes using the 3 dissimilarity measures (41)-(43) above. The first measure of relative price dissimilarity is the Weighted Log Quadratic (WLQ) measure of relative price dissimilarity defined by (41) above. The 8 by 8 matrix of relative price dissimilarity measures is listed in Table 11 below.

Table 11: Weighted Log Quadratic Relative Price Dissimilarities between Eight Countries

	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
HK	0.00000	0.10056	0.11017	0.09067	0.07011	0.01381	0.03660	0.06143
BGD	0.10056	0.00000	0.01188	0.01165	0.05632	0.10506	0.13237	0.23223
INDI	0.11017	0.01188	0.00000	0.03133	0.08980	0.13429	0.18955	0.29841
INDO	0.09067	0.01165	0.03133	0.00000	0.07084	0.07726	0.09610	0.19600
BRA	0.07011	0.05632	0.08980	0.07084	0.00000	0.09146	0.08770	0.14328
JPN	0.01381	0.10506	0.13429	0.07726	0.09146	0.00000	0.01904	0.05322
CAN	0.03660	0.13237	0.18955	0.09610	0.08770	0.01904	0.00000	0.02020
US	0.06143	0.23223	0.29841	0.19600	0.14328	0.05322	0.02020	0.00000

Looking at the above Table, it can be seen that the 8 countries group themselves into two groups that have similar price structures: the rich countries HK, JPN, CAN and US form one group and the less rich countries BGD, INDIA, INDO and BRA form the other group. The linking between the two groups took place via Hong Kong and Brazil.³⁰ The details of the spatial linking process are as follows. Country 7 (CAN) is linked to 8 (US) (the WLQ dissimilarity measure Δ_{WLQ} equals 0.0202) and 7 (CAN) is linked to 6 (JPN) as well ($\Delta = 0.019$). Then country 6 (JPN) is linked to 1 (HK) ($\Delta = 0.0138$) and this completes the linking of the rich countries. Country 2 acts as a star country for the poorer countries: 2 (BGD) is linked to 4 (INDO) ($\Delta = 0.0116$); 2 (BGD) is linked to 3 (INDIA) ($\Delta = 0.0118$) and 2 (BGD) is linked to 5 (BRAZIL) ($\Delta = 0.056$). Finally, the two groups of countries are linked via countries 1 (HK) and 5 (BRAZIL) ($\Delta = 0.070$). The resulting MST volumes relative to the US are listed in Table 14 below.

The second measure of relative price dissimilarity is the Weighted Log Quadratic measure of relative price dissimilarity defined by (42) above. The 8 by 8 matrix of relative price dissimilarity measures is listed in Table 12 below.

Table 12: Weighted Asymptotic Quadratic Relative Price Dissimilarities between Eight Countries

³⁰ Note that another possible bilateral link between the two regions would be via Indonesia and Japan which have a dissimilarity measure equal to 0.07726 which is a bit higher than the Hong Kong and Brazil dissimilarity measure which was equal to 0.07011.

	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
HK	0.00000	0.24097	0.27260	0.20885	0.15436	0.02914	0.10005	0.18062
BGD	0.24097	0.00000	0.02444	0.02455	0.11694	0.28243	0.32119	0.58729
INDI	0.27260	0.02444	0.00000	0.07050	0.19169	0.36590	0.50994	0.85586
INDO	0.20885	0.02455	0.07050	0.00000	0.15038	0.20144	0.21328	0.45346
BRA	0.15436	0.11694	0.19169	0.15038	0.00000	0.21447	0.20876	0.36462
JPN	0.02914	0.28243	0.36590	0.20144	0.21447	0.00000	0.04023	0.11820
CAN	0.10005	0.32119	0.50994	0.21328	0.20876	0.04023	0.00000	0.04106
US	0.18062	0.58729	0.85586	0.45346	0.36462	0.11820	0.04106	0.00000

Note that the WAQ dissimilarity measures listed in Table 12 are roughly two to three times the size of the WLQ measures listed in Table 11. The lowest measure of dissimilarity is between Bangladesh and India ($\Delta_{\text{WAQ}} = 0.02444$) and then between Bangladesh and Indonesia. ($\Delta = 0.02444$). Then there is a shift to the rich countries where the next lowest measure of dissimilarity is between Hong Kong and Japan ($\Delta = 0.02914$). The next lowest measure is between Japan and Canada ($\Delta = 0.04023$) and then between Canada and the US. ($\Delta = 0.04106$). Thus the rich countries are linked: Hong Kong, then Japan to Canada and then Canada to the US. The next lowest measure of dissimilarity is between 3 and 4 but we have already linked Bangladesh to both India and Indonesia so we move to the next lowest measure of dissimilarity which is between Bangladesh and Brazil ($\Delta = 0.11694$). Thus now all of the poor countries are linked: Bangladesh is a poor country star, directly linked to India, Indonesia, and Brazil. Now we need to link the rich and poor countries and the lowest dissimilarity measure between these two groups is again between Hong Kong and Brazil. Thus the MST generated by the Weighted Asymptotic Quadratic measure of relative price dissimilarity is exactly the same as the tree generated by the Weighted Log Quadratic measure. Thus the MST (WLQ) relative volume parities will be exactly the same as the MST (WAQ) parities; see Table 14 below.

The third measure of relative price dissimilarity is the Paasche and Laspeyres Spread measure of relative price dissimilarity defined by (28) above. The 8 by 8 matrix of relative price dissimilarity measures is listed in Table 13 below.

Table 13: Paasche and Laspeyres Spread Relative Price Dissimilarities between Eight Countries

	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
HK	0.00000	0.06486	0.06860	0.08716	0.12845	0.01907	0.08539	0.09904
BGD	0.06486	0.00000	0.02484	0.02354	0.09876	0.09420	0.06389	0.02905
INDI	0.06860	0.02484	0.00000	0.04596	0.00007	0.10421	0.13058	0.08134
INDO	0.08716	0.02354	0.04596	0.00000	0.05308	0.06806	0.02150	0.03897
BRA	0.12845	0.09876	0.00007	0.05308	0.00000	0.09387	0.11206	0.11177
JPN	0.01907	0.09420	0.10421	0.06806	0.09387	0.00000	0.01615	0.03390
CAN	0.08539	0.06389	0.13058	0.02150	0.11206	0.01615	0.00000	0.00133
US	0.09904	0.02905	0.08134	0.03897	0.11177	0.03390	0.00133	0.00000

The lowest measure of dissimilarity is between countries India and Brazil ($\Delta_{PLS} = 0.00007$) and then between Canada and the U.S. ($\Delta = 0.00133$). The next lowest measure of dissimilarity is between Japan and Canada ($\Delta = 0.01615$) and then the next lowest measure is between Japan and Hong Kong ($\Delta = 0.01907$). Thus the rich countries are linked: Hong Kong to Japan, then Japan to Canada and then Canada to the U.S., which is exactly the same set of linkages generated by the WLQ and WAQ measures of dissimilarity. The next lowest measure of dissimilarity is between Indonesia and Canada ($\Delta = 0.02150$) so the rich and poor countries are now linked by Indonesia and Canada! Recall that in the previous two spanning trees, the rich and poor countries were linked by Hong Kong and Brazil. The next lowest measure of dissimilarity is between Bangladesh and Indonesia ($\Delta = 0.02354$) and then between Bangladesh and India ($\Delta = 0.02484$) Thus now all of the poor countries are linked: Indonesia to Bangladesh; Bangladesh to India and India to Brazil. As mentioned above, rich and poor countries are linked via countries Indonesia and Canada. Thus the MST generated by the Paasche and Laspeyres Spread measure of relative price dissimilarity is quite different from the trees generated by the WLQ and WAQ measures. The MST (PLS) relative volume parities are reported in Table 14 below.

The country consumption volumes relative to the US for the 6 multilateral methods are listed in Table 14 below.

Table 14: Country Consumption Volumes Relative to the US Using Six Multilateral Methods

Method	HK	BGD	INDIA	INDO	BRA	JPN	CAN	US
GEKS	0.01315	0.01332	0.15317	0.04966	0.09128	0.26556	0.07357	1.0
MTS (WLQ)	0.01349	0.01310	0.14720	0.04779	0.09214	0.27596	0.07429	1.0
MTS (WAQ)	0.01349	0.01310	0.14720	0.04779	0.09214	0.27596	0.07429	1.0
MTS (PLS)	0.01349	0.01372	0.15420	0.05007	0.09184	0.27596	0.07429	1.0
GK	0.01386	0.01357	0.16258	0.05057	0.09613	0.27814	0.07431	1.0
IDB	0.01346	0.01392	0.16187	0.05143	0.09441	0.27076	0.07417	1.0

The above volume parities for the 7 countries relative to the US had the following ranges: Hong Kong: 0.01315 to 0.01386 (5.4% variation); Bangladesh: 0.01310 to 0.01392 (6.3%); India: 0.14720 to 0.16258 (10.4%); Indonesia: 0.04779 to 0.05143 (7.6%); Brazil: 0.09128 to 0.09613 (5.3%); Japan: 0.26556 to 0.27814 (4.7%) and Canada: 0.07357 to 0.07439 (1.1%). Thus the variation in relative volumes is quite large, depending on which multilateral method is used.

It can be seen that the relative consumption volumes generated by the four methods based on the use of a bilateral superlative index (the GEKS and the MTS or similarity linking methods) are fairly close to each other and the relative consumption volumes generated by the two additive methods (GK and IDB) are also fairly close to each other but the

additive methods tend to overstate the consumption levels of the poorer countries (Bangladesh, India, Indonesia and Brazil) relative to the US.³¹

From the above Table, it is difficult to choose between GK and IDB if an additive method is required: both methods tend to overstate the relative volume of poor countries relative to rich countries but the degree of overstatement seems to vary between poor countries.

Turning to methods based on the economic approach to multilateral comparisons, the MST method based on the Paasche and Laspeyres spread is not recommended since this measure of dissimilarity does not adequately distinguish dissimilar price vectors. In the above empirical example, the WLQ and WAQ measures of dissimilarity gave rise to the same set of comparisons and so for this example, these two variants of the MST method cannot be distinguished from one another. The differences between the GEKS volume estimates and the MST(WLQ) estimates are smaller than the differences between the GEKS estimates and the two additive methods but there are some significant differences.³²

What are the advantages and disadvantages of using GEKS versus MST(WLQ) or MST(WAQ)? The GEKS method has the advantage that it makes use of all possible bilateral comparisons between each pair of countries in the comparison and thus it is more robust to data problems in any one country. On the other hand, the MST method is very dependent on each set of bilateral comparisons in the final tree of comparisons and so poor quality data for any single country could adversely affect the overall quality of the comparison. But if the quality of data is roughly the same across countries, the MST method is the spatial counterpart to the use of the chain principle in annual intertemporal comparisons; i.e., using the MST method, the countries which have the most similar structure of relative prices are compared and bilateral comparisons are generally regarded as being more accurate if the structure of relative prices is similar. Thus in the example above, the U.S. and Canada (which have very similar structures of relative prices) are linked directly via the Fisher index between these two countries using the MST method, whereas using GEKS, links involving all other countries enter the comparison. Thus if data quality were uniformly high across countries, the MTS method would be preferred over GEKS.³³

³¹ The GK volumes relative to the GEKS volumes (with U.S. volumes normalized to equal 1) were all higher for the seven non U.S. countries by the following percentages: 5.4% for Hong Hong, 1.9% for Bangladesh, 6.1% for India, 1.8% for Indonesia, 5.3% for Brazil, 4.7% for Japan and 1.0% for Canada. The IDB volumes relative to the GEKS volumes (with U.S. volumes normalized to equal 1) were also all higher for the seven non U.S. countries by the following percentages: 2.3% for Hong Hong, 4.5% for Bangladesh, 5.7% for India, 3.6% for Indonesia, 3.4% for Brazil, 2.0% for Japan and 0.8% for Canada.

³² The MTS(WLQ) volumes relative to the GEKS volumes (with U.S. volumes normalized to equal 1) differed by the by the following percentages: 2.6% for Hong Hong, -1.7% for Bangladesh, -3.9% for India, -3.8% for Indonesia, 0.9% for Brazil, 3.9% for Japan and 1.0% for Canada.

³³ However, data quality is not uniformly high across countries so this argument for the use of the MTS method is not decisive.

Now consider the problems associated with forming regional shares of “world” consumption where the “world” is simply the 8 countries listed above. Suppose that the first four countries form an “Asian” region, Region 1 and the remaining four countries form a “Rest of the World” region, Region 2. Obviously, the Heston Dikhanov method for forming regional shares can be applied to the data listed in Table 14 above. The consumption volumes (relative to the US) listed there for the GEKS, the three MST methods, GK and the IDB methods can be converted into shares of “world” consumption and then the first four country shares can be summed to form the Region 1 shares, S_1 and S_2 . The resulting Region 1 shares for the six methods are listed below in Table 19.

It is also useful to calculate the Region 1 shares of “world” consumption for the Option 1 (Diewert) and Option 3 (Sergeev) methods. Equations (7) and (8) are first used to calculate the country and regional quantity vectors where the country expenditures E_{rcn} are listed in Table 7 above and the country Basic Heading PPPs (which can be set equal to the γ_{rcn}) are listed in Table 9 above. The resulting matrix of regional consumption volumes or “quantities” defined by (8), Q_{rn} , are listed in Table 15 below.

Table 15: Regional Consumption Quantities or Volumes Q_{rn} by Commodity

Commodity	1	2	3	4	5
Region 1	57805	551702	238995	64589	1138529
Region 2	1374542	1258068.	1316910	643404	8213040

The regional unit value commodity prices p_n for each region r , P_{rn} , defined by (12) are listed in Table 16 below.

Table 16: Regional Unit Value Consumption Basic Heading PPPs P_{rn} by Commodity

Commodity	1	2	3	4	5
Region 1	0.82560	0.53663	0.40150	1.01274	0.26510
Region 2	1.03435	1.12493	1.05998	1.13987	0.96794

As could be expected, the Region 2 unit value commodity prices are all relatively close to the US prices (which are all equal to unity) since 3 out of 4 of the Region 2 countries are “rich” and hence have price structures similar to the US structure. The Region 1 unit value prices are all lower than the corresponding Region 2 prices and for Commodity 5, Services, the Region 1 unit value price is considerably lower. Now the Fisher quantity index for Region 2 relative to Region 1 can be calculated using the regional price and quantity data listed in Tables 15 and 16 and the resulting index is equal to 6.26177. Thus the relative regional consumption volumes are 1 and 6.17739 and when these volumes are converted into shares, it is found that the Region 1 share of world consumption is 0.13771 and the Region 2 share is 0.86229.

The Option 3 additive method which was suggested by Sergeev (2009) is also listed in Table 19 below. In order to implement this method, it is necessary to compute the geometric mean of the regional Basic Heading prices that are listed in Table 8; recall

equations (18) above. These regional geometric mean prices P_m^* are listed in Table 17 below.³⁴

Table 17: Regional Geometric Mean Prices P_m^* by Commodity

Commodity	1	2	3	4	5
Region 1	0.84377	0.65236	0.51405	1.20824	0.32287
Region 2	1.09882	1.10282	1.10861	1.43846	0.82863

The pattern of regional commodity prices is fairly similar in Tables 16 and 17. To complete the analysis for this case, the Fisher quantity index for Region 2 relative to Region 1 is calculated using the regional price and quantity data listed in Tables 17 and 15 and the resulting index is equal to 6.17739, which is very similar to the corresponding Option 1 Fisher index which was equal to 6.26177. Thus the relative regional consumption volumes for the Sergeev method are 1 and 6.17739 and when these volumes are converted into shares, then the Region 1 share of world consumption is 0.13933 and the Region 2 share is 0.86067.

Recall that the EOH harmonized shares of global output of country c in region r , $s_{rc}(\text{EOH}) = \beta_r^* s_{rc}^* / \sum_{s=1}^R \beta_s^*$, were defined by equations (30) in section 4 above where the regional proportionality factors (the β_r^*) were defined by equations (29). It can be seen that region r 's share of global real expenditures is equal to $S_r(\text{EOH}) = \beta_r^* / \sum_{s=1}^R \beta_s^*$ for $r = 1, \dots, R$. In the present situation, there are only two regions so $R = 2$. The regional GEKS shares for the two regions, s_{rc}^* , turn out to have the following values:

Table 18: Regional GEKS Volume Shares s_{rc}^*

	Country 1	Country 2	Country 3	Country 4
Region 1	0.05690	0.05889	0.66691	0.21731
Region 2	0.06369	0.18997	0.05158	0.69476

Inserting the above regional GEKS shares along with the global GEKS shares listed in Table 14 into equations (29) leads to the following regional proportionality factors: $\beta_1^* = 0.13789$ and $\beta_2^* = 0.85804$. Thus share of region 1 in global product using the EOH harmonization method is $S_1 = \beta_1^* / \sum_{s=1}^2 \beta_s^* = 0.13846$. This EOH harmonized region 1 share differs from the corresponding harmonized share for region 1 using the Heston

³⁴ For comparison purposes, the vector of international prices generated by the GK method (with the price of commodity 1 normalized to equal 1) is [1.00000, 1.16099, 0.99626, 1.22554, 0.86834] and the vector of international prices generated by the IDB method is [1.00000, 1.10438, 0.92265, 1.53623, 0.66525]. Note that the U.S. prices are all equal to 1. Thus when the price of durables is set equal to unity for all three price vectors, the GK and IDB price levels for food and for energy are above the corresponding U.S. price levels while the GK and IDB price levels for other nondurables and services are below the U.S. levels. Note also that as expected, the GK prices are closer to the structure of U.S. prices whereas the IDB prices are a more "democratic" average of U.S. and poorer country prices.

Dikhanov method which turns out to equal 0.13815. These two shares are listed in Table 19 below as the EOH-GEKS and HD-GEKS region 1 shares respectively.

Instead of applying the EOH harmonization method to the global and regional GEKS shares, the method can be applied to the global MST shares using either the WLQ or WAQ measures of dissimilarity and using the regional GEKS shares. These global MST shares are listed in Table 14 and the regional GEKS shares are listed in Table 18. Inserting these shares into equations (29) leads to the following regional proportionality factors: $\beta_1^* = 0.13518$ and $\beta_2^* = 0.86825$. Thus share of region 1 in global product using the EOH harmonization method is $S_1 \equiv \beta_1^* / \sum_{s=1}^2 \beta_s^* = 0.13472$. This EOH harmonized region 1 share differs from the corresponding harmonized share for region 1 using the Heston Dikhanov method on the same data which turns out to equal 0.13316. These two shares are listed in Table 19 below as the EOH-MTS(WLQ) and HD-MTS(WLQ) region 1 shares respectively.

The resulting Region 1 shares of “world” consumption that have been generated by the various methods discussed above are summarized in Table 19 below.

Table 19: The Share of World Output for Region 1 using Various Methods

	Region 1 Share of Consumption
Option 1 (Regional Unit Values Method)	0.13771 (3.4%)
Option 3: Geometric Mean Average Prices in each Region	0.13933 (4.6%)
HD-GEKS	0.13815 (3.7%)
HD-GK	0.14243 (7.0%)
HD-IDB	0.14326 (7.6%)
HD-MST(PLS)	0.13831 (3.9%)
HD-MTS(WLQ) and MTS(WAQ)	0.13316
EOH-GEKS	0.13846 (4.0%)
EOH-MTS(WLQ) and EOH-MTS(WAQ)	0.13472 (1.1%)

While the variations in the Region 1 shares that the various methods generate are not huge, they are certainly not negligible. The percentage differences between the various estimated shares and the preferred HD-MST(WLQ) and HD-MST(WAQ) shares are listed in brackets in the above Table. It can be seen that the preferred HD-MST measure of Region 1 “global” share of real consumption is 1.1% to 7.6% below the Region 1 shares generated by the other methods for linking the regions.

It is of interest to compare the Heston Dikhanov and EOH methods for reconciling a global set of volume parities with a set of regional parities. When the input data into the harmonization process are the global and regional GEKS parities, the EOH region 1 share of global product is 0.13846 which is only 0.2% higher than the corresponding HD share of 0.13815. When the regional volume parities are GEKS parities but the global parities are the similarity linked parities using the WLQ or WAQ dissimilarity measures, then the EOH region 1 share of global product is 0.13472 which is 1.2% higher than the corresponding HD share of 0.13316. Thus it appears that the EOH and HD reconciliation

methods can give virtually identical results but it also appears that there can be significant differences between the two methods. It also appears that the choice of method for making the multilateral comparisons (GEKS, MST, IDB and GK) can lead to much larger differences between the final volume parities between countries.

8. Conclusion

Some tentative conclusions that can be drawn from the analysis and examples presented above are as follows:

- The Option 2 method should be ruled out for ICP 2011 since as Sergeev has pointed out, it is not invariant to the choice of the numeraire countries in the regions.
- The Option 1 method should also be ruled out for ICP 2011 since it depends on market exchange rates which are not reliable and hence their use should be avoided if possible.
- The Option 3 method is not fully consistent with the economic approach to index number theory because the method is inherently an additive method (when constructing regional volume aggregates) and additive methods are subject to substitution bias.
- The use of GK or IDB should perhaps be ruled out for “headline” estimates for country and interregional parities due to their inherent substitution biases. These methods however could be used to provide users with analytical tables where the users demand an additive method.³⁵
- GEKS remains a viable method for constructing regional shares in a consistent manner.
- Various forms of spatial linking could also be considered for ICP 2011 but the use of this method should await more experimental results using the 2005 data base.
- The Heston Dikhanov method for reconciling global volume shares with regionally determined volume shares is probably a preferred method over the Eurostat OECD Hill reconciliation method.

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³⁵ Based on some empirical evidence developed by Deaton and Heston (2010) using the entire set of ICP 2005 data, if it desired to have the additive method parities approximate the GEKS parities, then IDB appears to be better than GK. The axiomatic properties of IDB may be more attractive to users than the axiomatic properties of GK; see Chapter 7 for a discussion of the axiomatic properties of the GK and IDB methods.

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