Estimation of R&D depreciation rates: a suggested methodology and preliminary application

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Abstract. The 2008 version of the SNA has recommended capitalization of R&D expenditures. To implement this recommendation, we need to determine the depreciation rate of R&D capital. In this paper, we develop a simple model, based on a production function method that allows for monopolistic competition, to estimate the annual depreciation rate of R&D capital. We treat R&D capital as a technology shifter instead of as an explicit input factor. Both the R&D stock and the time variable are used to capture technological progress. Estimated R&D depreciation rates and markup factors are presented for the U.S. manufacturing sector and four U.S. knowledge-intensive industries. JEL classification: C32, C43, C81, D24, D43, D92

Estimation des taux de dépréciation pour la R-D : une méthodologie suggérée et une application préliminaire. L'édition 2008 du SCN recommande la capitalisation des dépenses en R-D. Pour mettre en oeuvre cette recommandation, il est nécessaire d'estimer un taux de dépréciation pour la R-D. Dans le présent article, nous développons un modèle simple, basé sur la méthode dite de fonction de production caractérisée par la concurrence monopolistique pour estimer le taux de dépréciation annuel de la R-D. Nous traitons la R-D comme un levier de la technologie plutôt qu'un facteur de production explicite. Le stock de R-D et la variable temps sont utilisés pour traduire le progrès technologique. Le taux de dépréciation estimé de la R-D et un coefficient de 'mark-up' sont présentés pour le secteur manufacturier des États-Unis et pour quatre de leurs industries du savoir.

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1. Introduction

In a recent paper presenting an innovative new indicator of technical change, Michelle Alexopoulos and Jon Cohen (2010) write: 'Although technical change is central in much of modern economics, our ability to identify empirically the factors that shape its pace, nature, and impact are constrained by data limitations.' The relevance of this statement is underlined by both the current imperative for national economies to achieve better productivity growth and new developments on the international scene in accounting for research and development (R&D) in the System of National Accounts (SNA).

Although R&D expenditures account for only a small portion of GDP, their importance in creating new technologies and promoting productivity growth is widely recognized. This necessitates the re-examination of measurement issues related to these expenditures that have broad implications for economists conducting analyses of the contribution of knowledge capital to endogenous growth and to the nation's wealth, as well as for policy makers who may want to subsidize R&D investments in order to maximize economic growth.

A widely used definition of R&D is given in the 1993 Frascati Manual (OECD 1994).[1] In this document, R&D is defined as 'creative work undertaken on a systematic basis in order to increase the stock of knowledge, including knowledge of man, culture and society, and the use of this stock of knowledge to devise new applications.' Three activities are covered: basic research, applied research, and experimental development. The key criteria that distinguish R&D from the other related activities are 'the presence in R&D of an appreciable element of novelty and the resolution of scientific and/or technological uncertainty.'

The SNA 1993 did not treat R&D expenditures as investment. In accord with the international standards that were in effect, in the National Income and Product Accounts (NIPA's) of United States, R&D expenditures are treated as an intermediate input for business, and current consumption for the non-profit institutions and government. However, SNA 2008 that has supplanted SNA 1993 recommends the capitalization of R&D expenditures. Successful R&D investments add to the stock of knowledge, and this stock in turn provides a flow of services over time rather than just in one period. Thus, R&D resembles investment more closely than intermediate input or current consumption.

Although the idea of capitalizing R&D expenditures has been accepted for SNA 2008, it is not easy to implement it. One of the crucial issues is the determination of depreciation rates for R&D capital. Because of the measurement difficulties involved, Nadiri and Prucha (1996) observe that, 'researchers doing applied work typically assume an arbitrary depreciation rate of 10 –15% to construct the stock of R&D capital using the perpetual inventory method.' The

1 This version of the Frascati Manual formed the methodological basis for the influential 2001 OECD STI Scoreboard. See http://library.certh.gr/libfiles/MOBILITY-PORTAL/MON-251-OECD- SCOREBOARD-2001-A92.pdf. The version of the Frascati Manual currently in use is 2002 (OECD 2002).

depreciation rates used by the Bureau of Economic Analysis (BEA) of US in the 2007 R&D Satellite Account were determined by *literature survey* and rates were industry specific: 18% was used for transportation equipment, 16.5% was used for computers and electronics, 11% was used for chemicals, and 15% was used for all the other industries. Mead (2007) provides detail discussion on how these rates were chosen. However, it would be preferable to have a scientific method for determining this depreciation rate.

There have been some attempts to empirically measure R&D depreciation rates. Pakes and Shankerman (1981) estimated the rate of depreciation in the private value of patents using European data on patent renewal fees and rates of renewal. Nadiri and Prucha (1996) measured the depreciation rate of the R&D stock using a factor requirements function and a restricted cost function, treating the R&D capital as a 'normal' reproducible capital input, as is also the case for Bernstein and Mamuneas (2005) who estimated R&D depreciation rates in an intertemporal cost minimization framework. However, the approaches adopted in previous studies generally do not allow for non-R&D effects that improve the technology, such as knowledge diffusion through education and learning by doing. Thus, all of the technological improvement is inappropriately attributed to R&D investment.[2] Also, the estimation is typically conducted in a framework of competitive pricing behaviour. However, private R&D investments are often undertaken with the explicit goal of achieving short-run monopolistic advantages over competitors, in which case, the assumption of competitive behaviour on output markets is unsuitable. Another weakness of most studies is that R&D investments are treated similarly to investments in ordinary physical capital, but R&D investments are quite different in their effects, as is subsequently explained.

In this paper, we do not treat the stock of R&D capital as an explicit input factor. Rather, we define the stock of R&D capital to be a technology index that locates the economy's production frontier. An increase in the stock of R&D shifts the production frontier outwards. In our model, the R&D capital depreciation rate is estimated within a monopolistic competition framework using gross R&D investment data. We estimate R&D depreciation rates for the U.S. total manufacturing sector and four U.S. knowledge intensive industries: chemical products (SIC 28), non-electrical machinery (SIC 35), electrical products (SIC 36) and transportation equipment (SIC 37).

The rest of the paper is organized as follows. Section 2 explains the construction of the R&D stock and the reasons for R&D depreciation. Section 3 develops our basic model for estimating the rate of depreciation for R&D capital. Section 4 presents the estimation methodology and results. Section 5 concludes. Appendix A shows the derivation of the estimating equations. Different sets of break points used in the linear spline model and quadratic spline model are given in appendix B.

2 The work of Bernstein and Mamuneas (2005) is not subject to this criticism.

2. Construction of the R&D stock

According to the 1993 Frascati Manual definition, the objective of conducting R&D is to increase the stock of knowledge. Thus, we define R&D capital as the knowledge asset created by R&D investment. When this perspective is taken, the stock of R&D capital can be regarded as a proxy for society's technological level. R&D investments act as a mechanism for shifting outward society's production possibility frontier. This treatment is the major distinguishing feature of our approach. The studies of others treat R&D capital as an explicit factor input in a manner that is similar to the treatment of ordinary physical capital.

2.1. R&D capital and ordinary physical capital

Physical capital, like machinery, equipment, and buildings, wears out through use and its efficiency tends to decline over time. Successful R&D ventures create new knowledge for the firms conducting the R&D. This new knowledge may be a new cost-saving process, or the technology for an innovative or quality-improved product. R&D capital does not wear out because of its utilization and its absolute efficiency level does not change with the passage of time. However, R&D capital is subject to a type of depreciation because processes and products can be supplanted by still better ones, and can thus become obsolete over time.

Although both physical capital and R&D capital are called 'capital assets,' there are some fundamental differences. First, physical capital, such as machines and equipment, can be reproduced over multiple periods. With physical capital, reproducibility makes it possible for us to observe at the same point in time rental prices of different vintages of a capital asset, and also used asset prices for different vintages. This in turn allows us to estimate depreciation rates for reproducible capital inputs. In contrast, for R&D capital, once, say, a new blueprint has been produced, it can be made available to many economic units without further productive activity. Typically, we cannot collect price information for different vintages of R&D capital at the same point in time[3] . Put another way: we can in principle observe the costs associated with a particular R&D project and if the rights to use the new technology are sold to a third party, we can observe the market value of the output of the project. Since the inputs into an R&D project are the usual inputs into any production function, we can construct a measure of real input into R&D projects using normal index number theory. But there are no natural units which can be used to measure the real output of R&D projects and hence measures of the real output of the R&D producing sector are problematic and the usual methodology used to calculate depreciation rates for reproducible capital inputs cannot be used.

3 However, we can sometimes observe the market price for the rights to some new technology. If the same technology is again sold in a future period, then we could collect price information for different periods and infer a depreciation rate for the R&D investment.

Secondly, physical capital and R&D capital support production in different ways. As Pitzer (2004) pointed out, R&D capital functions as a source of 'recipes' while physical capital is utilized as one of the 'productive' inputs[4] that are 'consumed' during a production process.

Thirdly, the reasons for depreciation are quite different for these two types of capital assets. We will return to this issue subsequently. Because of these differences, we believe that the treatment of R&D assets should necessarily be different from the treatment of reproducible capital assets.

2.2. Constructing the stock of R&D capital

Lacking a good measure of R&D output, we use input information in the form of gross (real) R&D expenditures as a proxy measure. Because all the technologies, new or old, are created by R&D investments, the R&D stock can be written as a function of a series of past R&D investments. Thus, the period t R&D stock is defined as follows:

$$R_t \equiv \theta_1^t I_{t-1} + \theta_2^t I_{t-2} + \theta_3^t I_{t-3} + \theta_4^t I_{t-4} + \theta_5^t I_{t-5} + \cdots, \tag{1}$$

where θ_n^t is the period t efficiency index representing how much the R&D investment that was made n periods before period t contributes to the technology or knowledge stock in period t. The R&D lags and weights are all incorporated in these efficiency indexes.

I_{t-n} is the R&D investment made in period $t - n$. We expect that the further back the R&D investment was made, the less it contributes to the prevailing knowledge. Thus, the following relationships should hold among all the efficiency indexes:[5]

$$\theta_1^t \geq \theta_2^t \geq \theta_3^t \geq \theta_4^t \geq \theta_5^t \geq \cdots. \tag{2}$$

We add the superscript t because these efficiency indexes may vary with time. The speed of the technological upgrading is one factor that helps to determine the size of the efficiency indexes. If the new technology is created quickly, the past investment may become irrelevant at a fast pace, owing to newly updated

4 Pitzer (2004) defines 'productive input' as: 'Fundamental to production is the notion that inputs are proportional in some sense to outputs. Outputs are created by combining a particular collection of inputs in a particular manner ... If more outputs are desired, then more inputs are necessary, and usually, more of all inputs. It may not be necessary to double the inputs to double the outputs, but more inputs are necessary to produce more outputs.'

5 Another way to justify inequality (2) is follows. Suppose that the industry or firm output is not homogeneous but consists of a mix of new and old products. Over time, industry output shifts away from the older more obsolete products and towards the newer, improved products. Hence, the use of the old technology that produces the older obsolete products diminishes over time, and the initial good effects of investments in technological improvements that were made many periods ago gradually wither away; see Diewert and Huang (2010) for a formal model of R&D obsolescence. Thus, we have a justification for why the initial good efficiency effects of R&D investments diminish or depreciate over time.

knowledge. For example, if there were more new knowledge created in year t compared to year $t-1$, we would expect the following inequalities to hold:

$$\theta_1^t > \theta_1^{t-1} \quad \text{and} \quad \theta_2^t < \theta_2^{t-1};\ \theta_3^t < \theta_3^{t-1};\ \theta_4^t < \theta_4^{t-1}; \ldots. \tag{3}$$

These inequalities show us that the previous investments become less important at a faster speed as the pace of technology improvement speeds up. The series of R&D stocks can be written as

$$\begin{aligned} R_t &= \theta_1^t I_{t-1} + \theta_2^t I_{t-2} + \theta_3^t I_{t-3} + \cdots + \theta_{t-1}^t I_1 + \theta_t^t R_0 \\ R_{t-1} &= \theta_1^{t-1} I_{t-2} + \theta_2^{t-1} I_{t-3} + \theta_3^{t-1} I_{t-4} + \cdots + \theta_{t-2}^{t-1} I_1 + \theta_{t-1}^{t-1} R_0 \\ R_{t-2} &= \theta_1^{t-2} I_{t-3} + \theta_2^{t-2} I_{t-4} + \theta_3^{t-2} I_{t-5} + \cdots + \theta_{t-3}^{t-2} I_1 + \theta_{t-2}^{t-1} R_0 \ldots, \end{aligned} \tag{4}$$

where R_0 is the initial knowledge stock. The efficiency index varies with R&D investments and the technology updating frequency. To simplify our analysis, we assume that the efficiency indexes decline at a constant geometric rate; that is, we assume that the following relationships hold among the efficiency indexes:

$$\theta_n = (1-\delta)^{n-1} \quad \text{and} \quad 0 \le \delta \le 1. \tag{5}$$

Based on the above simplifications, the stock of R&D capital can be constructed as follows:

$$R_t = I_{t-1} + (1-\delta) R_{t-1}, \tag{6}$$

where δ can be regarded as the R&D depreciation rate, which is assumed to be constant over time. From (6), we see that the stock of R&D capital in period t is constructed from the previous R&D investment, I_{t-1}, and the depreciated R&D stock of period $t-1$. This is a widely used method to construct capital stocks. However, it can be seen that we need a restrictive assumption about the efficiency index to end up with this simple equation. According to this equation, R&D capital accumulation depends on two opposite forces: the addition of the new knowledge stock, which is created by the current-period R&D investments,[6] and the depreciation of the old knowledge stock.

We use the stock of R&D capital as a technology index, indicating the position of the production frontier. If the newly created knowledge stock is at least as large as the depreciation of the old knowledge stock, we will not have a backward shifting of the production frontier.

It is sometimes argued that the R&D depreciation rate is zero, since old knowledge is preserved. However, new innovations tend to make the innovations from

6 Equation (6) implies that one unit of R&D investment can create one unit of knowledge. This can be regarded as the simplest functional form for a knowledge production function.

pervious periods obsolete. Also, consumer preferences can shift over time, causing a drop in the demand for 'old' products, and leading to obsolescence of past R&D investments.[7] Hence, the older R&D investments do suffer depreciation. Indeed, it is possible that the depreciation of old technologies outweighs the incremental effects of the new technologies and hence results in a net decrease of the R&D capital stock.

R&D investment in the chemical products (SIC 28) and the non-electrical machinery (SIC 35) categories has increased smoothly over the past 40 years, while R&D investment in transportation equipment (SIC 37) has fluctuated more. These different trends in the R&D investment imply different frequencies for creating new technology and result in different depreciation rates for the four industries.

In the rest of this section, we discuss the reasons for R&D capital depreciation in more detail.

2.3. Reasons for R&D capital depreciation

The depreciation of a tangible asset can be estimated by using the information on the price of the used asset (provided that it is not a uniquely constructed tangible asset). If we treat R&D expenditures as investment, there exists a similar problem: the decline in the utilization and efficiency of the knowledge capital, to which we now turn our attention. Although there is no apparent wear or tear of the intangible asset, we cannot assume that the knowledge capital has an infinite service life.

Both price and quantity changes are responsible for changes in the value of R&D. Under our investigation, R&D capital depreciation means the real quantity change due to the change in efficiency and utilization of the knowledge asset. The relative efficiency of the knowledge capital will decline over time, for the following reasons:

- *The obsolescence of the technology*. When newer technologies are created by R&D investment, the old technology may be partly or entirely replaced by the newly created technologies and consequently, the relative efficiency and the utilization of the old knowledge would decline.
- *The changing preferences of consumers*. Consumer's tastes may change for different reasons, such as the implementation of new health restrictions, the emergence of new products, and changes in the consumer's capabilities. Changing tastes may shift away the demand for some products that heavily rely on older technologies, and cause related market shrinkage. Responding to the shifting demand, firms would reduce the utilization level of the older technology that produces the outmoded products.

7 Or, sometimes this effect has been known to go the other way.

Because there are very few observed market prices for old technologies, we cannot observe the depreciation on R&D capital as we can for a tangible asset that is treated on second-hand markets. In the following section, we will set up a framework for estimating the R&D capital depreciation rate.

3. The estimation framework

In our estimation framework, each industry is treated as facing a monopolistic competition environment. In the production function, the R&D stock is treated as a technology index for the position of the production frontier. We use an extension of a model due to Diewert and Lawrence (2005).

3.1. The basic framework

We allow for the possibility of increasing returns to scale in the industry. Successful R&D investments will normally bring monopoly profits to firms that invest in R&D. Therefore, in an environment with R&D investment, competitive pricing behaviour is no longer valid. Thus, the assumption of competitive profit-maximizing behaviour is not suitable for the modelling of the industry's behaviour. Consequently, we treat the industry as engaging in monopolistic profit maximization.

We assume that each industry has an aggregate production function f of the form $y_t = f\,(x_t,\ R_t,\ t)$, so that f is a function that depends on the usual input vector x, the R&D stock R, and the time variable t, which represents non-R&D sources of technical change for the production function.[8] Thus, both R and t shift the production function over time. Defining the production function in this way, we can avoid the overestimation of the effects of R&D capital on technological improvement, compared with production functions that have only an R&D variable R as a shift variable.

The aggregate demand function for the output of an industry in year t is represented by an inverse demand function of the form $p_t = P\,(y_t,\ t)$. Under this situation, each industry solves the following monopolistic profit maximization problem for each period by choosing inputs and the next period's technology level:

$$\max_{\mathrm{x_t}, R_{t+1}} \sum_{t=0}^{\infty} \beta_t \{p_t(y_t,\ t)y_t - w_t \cdot x_t - P_{r,t} I_{r,t}\} \tag{7}$$

subject to: $y_t = f(x_t,\ R_t,\ t)$ and $R_t = I_{r,t-1} + (1-\delta)R_{t-1},\ t = 0, 1, 2 \ldots,$

8 These non-R&D sources of technical progress could include learning-by-doing effects, freely available research, information on new technologies made available at trade fairs, and so on.

where β_t is the period t discount factor; w_t is an input price vector; $I_{r,t}$ is R&D investment in period t; and $P_{r,t}$ is the corresponding price index. In this model, we assume that each industry maximizes the discounted future monopolistic profits with full information about future prices. Ignoring the uncertainty of future prices is not realistic, but it dramatically simplifies the problem.

The first-order necessary conditions for solving the above maximization problem are

$$p_t \nabla_x f(x_t, R_t, t) + [\partial P(y_t, t)/\partial y] y_t \nabla_x f(x_t, R_t, t) = w_t, \quad t = 0, 1, \ldots T, \tag{8}$$

$$\beta_{t+1} \left\{ y_{t+1} \frac{\partial P(\cdot)}{\partial y} \frac{\partial f(\cdot)}{\partial R_{t+1}} + p_{t+1} \frac{\partial f(\cdot)}{\partial R_{t+1}} + P_{r,t+1}(1-\delta) \right\} = \beta_t P_{r,t}, \tag{9}$$

where p_t is the output price and $\nabla_x f\ (x_t, R_t, t)$ is the vector of first-order partial derivatives of the production function with respect to the components of the input vector x. Factoring out p_t and $\nabla_x f\ (x_t, R_t, t)$ on the left-hand side of (8), we obtain the following simplification of equation (8):

$$p_t \times \left(1 + \frac{\partial P(y_t, t)/\partial y_t}{p_t / y_t} \right) \times \nabla_x f(x_t, R_t, t) = w_t. \tag{10}$$

Applying similar algebraic rearrangements to equation (9), we have

$$\begin{aligned} p_{t+1} \times \left[1 + \frac{\partial P_{t+1}(\cdot)/\partial y_{t+1}}{p_{t+1}/y_{t+1}} \right] \times \frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}} \\ = \frac{\beta_t}{\beta_{t+1}} P_{r,t} - (1-\delta) P_{r,t+1}, \end{aligned} \tag{11}$$

where $\beta_t / \beta_{t-1} = 1 + r_t$ and where r_t is the nominal interest rate prevailing at time t.

The term $[\partial P(y, t) / \partial y] / [p / y]$ is the inverse of the price elasticity of demand, and reflects how output (demand) changes with respect to the price change. If we use ε to denote this price elasticity, we can define its inverse as the period t non-negative markup, denoted by m_t, as follows:

$$m_t \equiv -\frac{\partial P(y, t)/\partial y_t}{p_t / y_t} = -\frac{1}{\varepsilon} \geq 0, \tag{12}$$

and the markup factor M_t can be defined as follows:

$$M_t = 1 - m_t = 1 + \frac{1}{\varepsilon} = 1 + \frac{\partial P(y, t)/\partial y_t}{p_t / y_t}. \tag{13}$$

If we assume that the markup factor is constant over time, then we can rewrite (10) and (11) as

$$w_{t,n} = p_t M \left[\partial f(x_t, R_t, t)/\partial x_n\right], \quad n = 1, 2, \&, N; \tag{14}$$

$$(1 + r_t)P_{r,t} - (1 - \delta)P_{r,t+1} = p_{t+1} M \left[\frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}}\right] \tag{15}$$

where n denotes the nth factor in the input vector x. Details of deriving these equations are given in appendix A.

The left-hand side of equation (15) is the *user cost of one unit of R&D investment* purchased in period t.[9] Equations (14) and (15) form our system of estimating equations. Including equation (15) as an extra estimating equation is helpful for distinguishing R and t. However, we may also introduce some estimation problems by using anticipated variables in this equation, where the anticipations are formed in period t when we purchase $I_{r,t}$ at the price $P_{r,t}$. To simplify our analysis, we use the actual data at period $t + 1$ to approximate the predicted variables.

The left-hand side user cost in (15) depends on the depreciation rate δ. In order to compare log likelihoods across alternative depreciation models, we need the left-hand side variable to be constant across models. Thus, we rewrite equation (15) as

$$(1 + r_t)P_{r,t} = (1 - \delta)P_{r,t+1} + p_{t+1} M \left[\frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}}\right]. \tag{15a}$$

Equation (15a) is still not ideal for econometric estimation because it involves a lagged dependent variable. However, if we divide both sides of equation (15a) by $P_{r,t}$, this leads to the following estimating equation:

$$(1 + r_t) = (1 - \delta)\left(\frac{P_{r,t+1}}{P_{r,t}}\right) + \left(\frac{p_{t+1}}{P_{r,t}}\right) M \left[\frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}}\right]. \tag{15b}$$

We treat R&D capital similarly to the time variable in that the R&D stock shifts the technology like other sources of productivity improvement (approximated by the time variable).[10] Hence, in our model, we have labour, intermediate, and

9 It may be surprising initially that the net effect of purchasing an R&D investment in period t can be expressed in such a simple manner as given in equation (15). However, under our perfect foresight assumptions, the producer needs to purchase units of R&D in period t in order to adjust the stock of R&D to precisely the 'right' level in period $t + 1$, the R&D stock for period $t + 2$ can be adjusted to the 'right' level by purchasing additional units of R&D in period $t + 1$, and so on.

10 However, the R&D variable is different from the time variable because we regard the time effect as being entirely exogenous in our model whereas the R&D stock is endogenously determined by producers.

non-R&D capital service inputs. In order to help identify some parameters in the model, we add the production function to the estimating system. Thus, our final estimating system includes the following five equations:

$$\frac{w_{t,1}}{p_t} = M\frac{\partial f(x_t, R_t, t)}{\partial x_1} \tag{16}$$

$$\frac{w_{t,2}}{p_t} = M\frac{\partial f(x_t, R_t, t)}{\partial x_2} \tag{17}$$

$$\frac{w_{t,3}}{p_t} = M\frac{\partial f(x_t, R_t, t)}{\partial x_3} \tag{18}$$

$$(1 + r_t) = (1 - \delta)\left(\frac{P_{r,t+1}}{P_{r,t}}\right) + \left(\frac{p_{t+1}}{P_{r,t}}\right) M \left[\frac{\partial f(x_{t+1}, R_{t+1}, t + 1)}{\partial R_{t+1}}\right] \tag{19}$$

$$y_t = f(x_t, R_t, t). \tag{20}$$

3.2. The choice of the functional form for the production function

To specify estimating equations, we need to choose a functional form for the production function. As a starting point, we use the following variant of a *normalized quadratic functional form*:

$$f(x, R_t, t) \equiv b + c_1x_1 + c_2x_2 + c_3x_3 + g_1x_1t + g_2x_2t + g_3x_3t + h_1x_1R_t + h_2x_2R_t + h_3x_3R_t + e_1t + e_2R_t - (1/2)x^TSx/(\phi_1x_1 + \phi_2x_2 + \phi_3x_3), \tag{21}$$

where x_1 is the labour input, x_2 is the intermediate input, x_3 is the non-R&D capital input, and R is the stock of R&D capital. In addition, $S \equiv [s_{ij}]$ is a 3×3 symmetric positive semi-definite substitution matrix of unknown parameters and the ϕ_i are predetermined positive parameters. In our empirical work, we calculate the sample mean of the x_i, say x_i^*, and then set the ϕ_i equal to $x_i^*/(x_1^* + x_2^* + x_3^*)$. The unknown parameter b determines the degree of returns to scale: if $b = 0$, we have constant returns to scale in production; if b is less than 0, then there are increasing returns to scale; and if b is greater than 0, there are decreasing returns to scale. The two parameters e_1 and e_2 are technical progress parameters.

In order to identify all of the parameters and to reduce multicollinearity, it is necessary to impose some linear restrictions on the matrix S. Our linear restrictions are as follows:[11]

$$\sum_{j=1}^{3} s_{nj} = 0 \quad n = 1, 2, 3. \tag{22}$$

The normalized quadratic production function defined by (21) and (22), with the parameters b, e_1, and e_2 set equal to 0, is flexible in the class of constant return to scale production functions. The additional parameter b allows us to test the degree of local returns to scale. The main advantage of choosing this flexible functional form is that the flexibility properties would not be destroyed by imposing curvature conditions; see Diewert and Wales (1988). In our example, imposing positive semi-definiteness conditions on the symmetric matrix S without destroying the flexibility property can be realized by setting S equal to the following matrix product:

$$S = UU^T, \tag{23}$$

where $U \equiv [u_{ij}]$ is a 3×3 lower triangular matrix and U^T is the transpose of U. The linear restrictions (22) on S can be imposed on U, too; that is, we impose the following restrictions:

$$u_{11} + u_{21} + u_{31} = 0; \quad u_{22} + u_{32} = 0; \quad u_{33} = 0. \tag{24}$$

We find that there are only three independent parameters in the U matrix: u_{21}, u_{31}, and u_{32}. The main diagonal parameters u_{ij} can be represented in terms of the off diagonal parameters u_{ij}.

Partially differentiating the production function defined by equation (21) with respect to the inputs, x_i, and with respect to next period's R&D stock, R_{t+1}, and substituting the resulting derivatives into the estimating equations (16)–(19), we can rewrite the estimating equations of our basic model as follows:

$$w_{t,n}/p_t = M\left\{c_n + g_n t + h_n r - \frac{\sum_{j=1}^{3} s_{nj} x_{t,j}}{\phi^T x_t} + \frac{1}{2}\phi_n \times \frac{x_t^T S x_t}{(\phi^T x_t)^2}\right\} \quad \text{for } n = 1, 2, 3 \tag{25}$$

$$1 + r_t = \frac{p_{t+1}}{P_{r,t}} M\{h_1 x_1 + h_2 x_2 + h_3 x_3 + e_2\} + (1 - \delta)\frac{P_{r,t+1}}{P_{r,t}}, \tag{26}$$

11 Note that a more complex form of technical progress is allowed here than, say, in Diewert et al. (2010) and the literature on which that paper builds. Note also that estimates are needed here not only for the returns to scale parameter but also for the other parameters in the production function defined by (21) and (22).

where

$$\phi^T x_t \equiv \sum_{j=1}^{3} \phi_j x_{t,j}.$$

3.3. The problem of trending elasticities

Diewert and Lawrence (2002) point out that, for the normalized quadratic functional form, the estimated elasticities often have strong trends when there are strong trends in the price and quantity data. They also suggest one way to solve this problem. We adopt their technique here.

In the initial functional form, the substitution matrix S is constant over time. To handle the trending elasticity problem, we let the production function be flexible at two sample points, which means the matrix S is allowed to change over time. We use the following weighted average substitution matrix:

$$S = (1 - (t/T))A + (t/T)B, \quad t = 0, 1, 2, \&, T, \tag{27}$$

where $T + 1$ is the total number of periods covered by the estimation data sample. We have data for the years 1953–2000, so $T = 47$. Using this weighted average substitution matrix, the technological progress captured by the time variable affects not only the constant terms in the estimating system, but also the substitution possibilities.

As in the basic case, we can impose the curvature conditions by setting A and B equal to UU^T and VV^T respectively, where U and V are lower triangular matrices; that is, we set

$$A = UU^T \quad \text{and} \quad B = VV^T. \tag{28}$$

Similarly, we can impose the following normalizations on these two matrices U and V:

$$U^T 1_3 = 0_3 \quad \text{and} \quad V^T 1_3 = 0_3, \tag{29}$$

where 1_3 and 0_3 are three-dimensional vectors of 1s and 0s, respectively. With these constraints, we add only three additional independent parameters to the initial model; these new parameters are v_{21}, v_{31}, and v_{32}. Making these changes, our production function can be written as follows for $t = 0,1,\ldots,47$:

$$f(x, R, t) \equiv b + c^T x + g^T xt + h^T xR + e_1 t + e_2 R - \left\{(1/2)x^T \left[(1 - (t/T))UU^T + (t/T)VV^T\right] x/(\phi_1 x_1 + \phi_2 x_2 + \phi_3 x_3)\right\}. \tag{30}$$

Here we write the production function in matrix form in order to simplify our notation. In equation (30), $c^T \equiv [c_1, c_2, c_3]$, $g^T \equiv [g_1, g_2, g_3]$, and $h^T \equiv [h_1, h_2, h_3]$.

In the estimation, if the trending elasticity problem exists, we will see significant increases in the value of the log-likelihood function as we add the parameters in the V matrix to those in the U matrix. We do see this for our results.

3.4. Problems due to non-smooth technical progress

Another problem related to the initial production function model is that it does not allow for non-smooth change in technical progress. We now add more features to the model in order to capture changes in the direction of technical progress over time.

Technological progress typically does not proceed smoothly. Thus, we add linear splines or quadratic splines in the time variable to allow for the different change patterns of technological progress at different periods. The modified production function in period t can be written as follows:

$$f(x_t, R_t, t) \equiv b + c^T x_t + \sum_{j=1}^{3} g_j(t) x_{j,t} + h^T x_t R_t + e_1(t) + e_2 R_t - (1/2) x_t^T \times [(1 - (t/47)) \times UU^T + (t/47) \times VV^T] x_t / (\phi^T x_t), \quad (31)$$

where $e_1(t)$ and $g_j(t)$ are linear spline functions of time t. The number of spline segments depends on the break points chosen by investigating the plots of preliminary estimations. A break point is a positive integer less than the maximum number of the time variable, so here it is less than 47.

We will illustrate how to define $e_1(t)$ for the case when we choose three break points, $0 < t_1 < t_2 < t_3 < 47$:

$$\begin{aligned} e_1(t) &\equiv e_{11} t && \text{for} \quad t = 0, 1, 2, \ldots t_1 \\ &= e_{11} t_1 + e_{12}(t - t_1) && \text{for} \quad t = t_1 + 1, t_1 + 2, \ldots t_2 \\ &= e_{11} t_1 + e_{12}(t_2 - t_1) + e_{13}(t - t_2) && \text{for} \quad t = t_2 + 1, t_2 + 2, \ldots t_3 \\ &= e_{11} t_1 + e_{12}(t_2 - t_1) + e_{13}(t_3 - t_2) && \text{for} \quad t = t_3 + 1, t_3 + 2, \ldots 47 \\ &\quad + e_{14}(t - t_3). && \quad (32) \end{aligned}$$

In equations (32), the e_{1j} are the unknown parameters to be estimated. From the above example, we know that with n break points, there are $n + 1$ parameters to be estimated. Similarly, we can generate linear splines for the functions $g_j(t)$. The subscript j means we allow different splines for different inputs j.

Adding linear spline induces, perhaps artificially, kinks in the direction of technical change. For smooth change, the linear splines can be replaced by quadratic splines,[12] in which case $e_1(t)$ and the $g_j(t)$ are quadratic spline functions. With

12 On setting up quadratic splines in a normalized quadratic model, see Diewert and Wales (1992). They model normalized quadratic cost functions, whereas we are modelling normalized quadratic production functions.

three break points, the quadratic spline functions can be defined as follows:

$$
\begin{aligned}
e_1(t) &\equiv e_{11}t + (1/2)e_{12}t^2 && 0 \le t \le t_1 \\
&\equiv e_{11}t_1 + (1/2)e_{12}t_1^2 + (t - t_1)(e_{11} + e_{12}t_1) && t_1 < t \le t_2 \\
&\quad + (1/2)e_{13}(t - t_1)^2 \\
&\equiv e_{11}t_1 + (1/2)e_{12}t_1^2 + (t_2 - t_1)(e_{11} + e_{12}t_1) && t_2 < t \le t_3 \\
&\quad + (1/2)e_{13}(t_2 - t_1)^2 + (t - t_2)(e_{11} + e_{12}t_1 + e_{13}t_2) \\
&\quad + (1/2)e_{14}(t - t_2)^2 \\
&\equiv e_{11}t_1 + (1/2)e_{12}t_1^2 + (t_2 - t_1)(e_{11} + e_{12}t_1) && t_3 < t \le 47 \\
&\quad + (1/2)e_{13}(t_2 - t_1)^2 + (t_3 - t_2)(e_{11} + e_{12}t_1 + e_{13}t_2) \\
&\quad + (1/2)e_{14}(t_3 - t_2)^2 + (t - t_3)(e_{11} + e_{12}t_1 + e_{13}t_2 + e_{14}t_3) \\
&\quad + (1/2)e_{15}(t - t_3)^2. && (33)
\end{aligned}
$$

As in the linear spline case, the e_{1j} are the unknown parameters to be estimated. If we choose n break points, then there are $n + 2$ additional parameters that need to be estimated for each equation. Adding splines increases the flexibility of the functional form but at the cost of estimating more technical change parameters. As was the case for linear splines, choosing different break points will generally result in different estimates.

The following features distinguish our model from the previous literature:

- Instead of treating R&D capital as one explicit factor input like ordinary physical capital, we treat R&D capital as a technological index indicating the position of the production frontier. R&D capital, which is the knowledge asset created by the R&D investment, is not 'consumed' like the physical capital in the production; it just indicates the technology level. Holding all the usual flow inputs constant, an increase of the stock of R&D capital would shift the production frontier outwards. Thus, the R&D stock variable is treated in a manner similar to the time variable.
- Both the time variable and the R&D capital stock variable are included in the production function model. The R&D stock frequently grows in a roughly linear fashion, so the inclusion of the variables R and t in the regression equations can lead to a multicollinearity problem. However, if the time variable, t, is dropped from the model, then the R variable becomes the only technical change shift variable, and frequently, the resulting rate of return to R&D investments is unrealistically large. In contrast, by including both the time variable and the R&D variable in the model, we can avoid attributing all of the technological progress to R&D investments and hence can avoid to some extent the overstatement of the effects of R&D investments on both technological progress and on productivity growth.

- The model allows for the possibility of monopolistically competitive behaviour that is consistent with increasing return to scale.[13] For our model, the markup factor and the degree of returns to scale must be estimated along with the R&D depreciation rate.

Because of these different features of our model, our results can be expected to differ from the results obtained from 'traditional' models that treat R&D as just another capital stock.

4. Empirical estimation and results

Here we describe our data and empirical results. We estimate R&D depreciation rates for the period 1953–2000 for U.S. manufacturing and four U.S. technology-intensive industries: chemical and allied products (SIC 28), non-electrical machinery (SIC 35), electrical products (SIC 36), and transportation equipment (SIC 37). In 1998, the R&D expenditures of these four industries accounted for 54.35% of the R&D expenditures of all industries and 76.37% of the manufacturing R&D expenditures.

4.1. Estimation methodology

The basic estimation system with the curvature conditions (23) and linear restrictions (24) imposed is given by equations (21), (25), and (26).[14] To specify these estimating equations, we must define the normalized quantities and the differences for the normalized quantities. The nth normalized quantity, q_n, is

$$q_n \equiv x_n/\phi^T x, \quad n = 1, 2, 3. \tag{34}$$

The differences between the normalized quantities can be defined in the following way:

$$q_{21} \equiv q_2 - q_1; \quad q_{31} \equiv q_3 - q_1; \quad q_{32} \equiv q_3 - q_2. \tag{35}$$

Using the above definitions and substituting restrictions (23) and (24) into equation (25), we can express the first-order necessary conditions for profit maximization with respect to the choice of the different inputs in the following way:

$$w_{t,1}/p_t = M \times \left\{ \begin{array}{c} c_1 + g_1 t + h_1 R + (u_{21} + u_{31})(u_{21}q_{21} + u_{31}q_{31}) \\ +0.5\phi_1[(u_{21}q_{21} + u_{31}q_{31})^2 + (u_{32}q_{32})^2] \end{array} \right\} \tag{36}$$

13 If the estimated markup factor M turns out to equal 1, then we have competitive behaviour.
14 Thus, for this basic model, there are no splines and no trending substitution matrices.

$$w_{t,2}/p_t = M \times \left\{ \begin{array}{c} c_2 + g_2 t + h_2 R - u_{21}(u_{21}q_{21} + u_{31}q_{31}) + (u_{32})^2 q_{32} \\ + 0.5\phi_2[(u_{21}q_{21} + u_{31}q_{31})^2 + (u_{32}q_{32})^2] \end{array} \right\} \tag{37}$$

$$w_{t,3}/p_t = M \times \left\{ \begin{array}{c} c_3 + g_3 t + h_3 R - u_{31}(u_{21}q_{21} + u_{31}q_{31}) - (u_{32})^2 q_{32} \\ + 0.5\phi_3[(u_{21}q_{21} + u_{31}q_{31})^2 + (u_{32}q_{32})^2] \end{array} \right\}. \tag{38}$$

The production function is

$$y = b + c_1x_1 + c_2x_2 + c_3x_3 + g_1x_1t + g_2x_2t + g_3x_3t + h_1x_1R + h_2x_2R + h_3x_3R + e_1t + e_2R - 0.5(\phi_1x_1 + \phi_2x_2 + \phi_3x_3)[(u_{21}q_{21} + u_{31}q_{31})^2 + (u_{32}q_{32})^2]. \tag{39}$$

The above four equations plus (26) form our basic estimating system with 16 parameters to be estimated.

Owing to the non-linearity of the equations, we use the SHAZAM non-linear maximum likelihood estimation option. To find the estimates for the R&D depreciation rates, we construct a grid of depreciation rates: $\delta = 0.01$, $\delta = 0.02$, $\delta = 0.03, \ldots$, and $\delta = 1$. Based on these depreciation rates, we build the initial stock of R&D capital using the following formula:

$$R_0 = I_{r,0}/(\delta + \gamma_r), \quad \delta = 0, 0.01, 0.02, \&, 0.99, 1, \tag{40}$$

where $I_{r,0}$ denotes the R&D investment for the first period, and γ_r denotes the geometric growth rate of R&D investment over the periods spanned by the estimation sample and can be calculated as

$$\gamma r \equiv (I_r, 47/I_r, 0)^{1/47}. \tag{41}$$

For the remaining periods, the R&D stock is calculated using equation (6) in the second section.

Altogether, we have 101 sets of R&D stocks, corresponding to the 101 possible choices for an R&D depreciation rate. Using these alternative R&D stock series, we can estimate the five equations. For each depreciation rate, we obtain the value of the log-likelihood function. Comparing these values of the log-likelihood function, we can locate the depreciation rate corresponding to the maximum likelihood. According to our estimating procedure, we believe that the depreciation rate that maximizes the value of the log-likelihood is the best estimator for the R&D depreciation rate.

4.2. Data construction

To conduct the estimation, we need quantity series and price series for industrial input and output, and price and quantity series for R&D investments. Industrial input and output data other than R&D related data are obtained from the

Multifactor Productivity data sets provided by the Bureau of Labour Statistics (BLS). R&D-related data are derived from the website of the National Science Foundation (NSF).

From the BLS, we obtained value series in current dollar and price index series in 1996 constant dollars.[15] For each industry, we have data on sectoral output, labour input (L), capital service input (K), energy input (E), non-energy materials (M) input, and purchased business services (S) input. The BLS uses the Törnqvist index number formula to construct the aggregate data. Labour is measured as the hours worked by all persons engaged in a sector. Capital input is defined as the flow of services from physical assets, which include equipment, structures, inventories and land. Service flows are assumed to be proportional to stocks. The description of the measures and the methodology for constructing all of these data sets are given in chapters 10 and 11 of *BLS Handbook of Methods*,[16] and in Gullickson and Harper (1987). Our measure of intermediate input is a Törnqvist aggregate of energy, materials and purchased services. With value series and price series, we can construct the implicit quantity series. Industrial input and output data sets are relatively well constructed over the R&D data sets, but we face a double counting problem when we try to explicitly model the role of R&D capital, because R&D expenditures have already been included in the initial (BLS) input data. In our framework, we regard R&D real expenditures as a variable (like the time variable) that shifts the production function for the continuing operations of the industry. Thus, we need to subtract R&D expenditures from the input expenditures of the industry in order to obtain estimates of the inputs that are used to produce the usual outputs of the industry; that is, these usual outputs exclude any R&D outputs of the industry and hence the corresponding R&D inputs should also be excluded.[17]

The industrial R&D expenditure data are obtained from the NSF website. For the years 1953 to 1998, the NSF data are taken from the Industrial R&D Information System (IRIS) that uses the Standard Industrial Classification (SIC) to classify the industry. For other years, the data are obtained from 'Research and Development in Industry,' for the years 2000 and 2001, which uses the North American Industrial Classification System (NAICS). We use the 1998 data as the bridge to link the series based on the different industrial classification and reclassify the data to conform to the SIC. Using $R(S\text{year})$ to denote the data based on SIC and $R(N\text{year})$ to denote the data based on NAICS, the constructed data we need for estimation can be obtained by using the following formula:

$$R(S1999) = R(N1999) \times [R(S1998)/R(N1998)]. \tag{42}$$

15 We thank Mike Harper for providing us with manufacturing data not available on the BLS website.

16 This handbook can be accessed without charge at http://www.bls.gov/opub/hom/homtoc.htm.

17 It should be noted that other accounting frameworks are possible, but these alternative methods are not consistent with our model of the effects of R&D.

Equation (42) describes how we derive the R&D expenditure data based on SIC for the year 1999. Similar adjustments can be made for years 2000 and 2001.

The data on the cost components of R&D expenditures come from various issues of Research and Development in Industry. According to the NSF's classification, the type of R&D expenditure includes wage and salaries, materials, R&D depreciation, and other costs. There are two important problems associated with this data set: one is that the NSF does not use the above classification consistently over years; another is that data are not available for quite a few years. To deal with these problems, we must make assumptions and create approximate data. For example, for the years 1953 to 1961, we do not have data related to the type of cost. Thus, we assume that the cost structure for these years was the same as in 1962. Similarly, for the period 1977 to 1997, we have data every two years. Hence, we use moving-average methods to determine the data for the intervening years when no data are reported. Finally, we group the R&D expenditures into three categories: wage and salaries (labour), materials (intermediates), and capital expenditure. Unfortunately, the cost category – overhead or other costs – accounts for a big portion of the expenditure. We allocate these expenditures to wage and salaries, materials, and capital expenditure, according to the BLS industry cost shares. From the above description, the reader should be aware that our assumptions made to fill in gaps in the R&D data can have a direct effect on the quality of the resulting data sets and on the results of analyses based on these data sets.

After constructing R&D cost component information, we can make adjustments to the initial BLS input data sets. R&D labour cost (wages and salaries) is subtracted from the total labour cost; the material component of R&D expenditure is subtracted from the total intermediate input cost; and the capital expenditure part of R&D is subtracted from the total capital service cost. Dividing these adjusted value series by an implicit price index yields quantity series for labour, intermediate, and capital service inputs. The quantity series of R&D investment is constructed by using the Törnqvist index formula. Price indexes for the three components of R&D expenditure are assumed to be the same as the industrial input price indexes.

Finally, we need nominal interest rates for the years 1953 to 2000 to construct the discount factors. Nominal interest rates are obtained from the online data of the Federal Reserve System. We choose the long-term nominal interest rate: the market yield on U.S. Treasury securities at 10-year constant maturities, quoted on an investment basis, to construct the series of discount factors.

4.3. Estimation results

Maximum likelihood estimation, which is the non-linear option in SHAZAM, is sensitive to the choice of starting point, which also affects the number of iterations. We start from a simple regression with M, b, e_1, and e_2 in equation (26) and equations (36) to (39) initially set equal to zero. The estimated parameters from this regression are used as the starting point for the next regression, which adds additional parameters. As we proceed, we also check for 'big jumps' in the log likelihood of the model. (If the jumps are small, then the inclusion of the extra parameters is not warranted, but, in general, we obtain significant increases in the log likelihood as we add the extra parameters.)

We estimate the following four models:

- Model 1: This is the basic model defined by equation (26) and equations (36) to (39). Without splines and without a weighted substitution matrix, this model may not properly reflect real world complexity. We expected relatively low values of the log-likelihood for this class of model.
- Model 2: This model adds a weighted substitution matrix to equations (36) to (39). It can deal with the possible trending elasticity problem. If our model does have this trending elasticity problem, we would expect to see a big increase in the value of the log-likelihood function.
- Model 3: This model adds linear splines in the time variable based on the last model. Non-smooth change in technical progress can be captured by adding these splines.
- Model 4: This model adds quadratic splines for the time variable to Model 2 (rather than linear splines as in Model 3).

The following table lists the estimates of the depreciation rate, the value of the maximum log-likelihood, and the value of the markup factor for the above four models for U.S. manufacturing and the four selected knowledge intensive industries.[18]

From table 1, we can see that the value of the maximum log-likelihood generally improves considerably from Model 1 to Model 2. This suggests we have trending elasticity problems. With linear splines or quadratic splines added to the model, the value of maximum log-likelihood function increases dramatically. In comparison with the results obtained from the linear spline model (Model 3) and the quadratic spline model (Model 4), the value of the log-likelihood function increases in Model 4 for electrical products, but decreases for chemical products and transportation equipment. For non-electrical machinery and the manufacturing sector, the changes of the log-likelihood value from Model 3 to Model 4 are mixed. We also try different break points for Model 2 and Model 4.[19]

Unfortunately, it turns out that the estimates are sensitive to the choice of the break points. Consequently, in order to choose an appropriate value for the depreciation rate of R&D capital, we have to use our subjective judgment. This is one drawback of our modelling strategy. In our example, we choose the depreciation rate for R&D based on values of both the markup factor and the log-likelihood function. According to the definition of the markup factor given in the second section, a reasonable value should be less than 1. Our final choices of the depreciation rates are given by table 2.

18 The break points for the spline models are reported in appendix B.
19 Different break points corresponding to the estimations listed in table 1 are given in appendix B.

TABLE 1
Depreciation rates maximizing the log-likelihood function

		SIC 28	SIC 35	SIC 36	SIC 37	Manu-facturing
Model 1	Dep rate	0	0.09	0	0.15	0.5
	Markup factor	0.92508	0.92223	0.96331	0.97694	0.95618
	Log-likelihood	266.115	208.271	113.905	306.464	373.561
Model 2	Dep rate	0.14	0.06	0.18	0.12	0.49
	Markup factor	0.90005	0.90465	0.92672	0.98417	0.98372
	Log-likelihood	279.657	221.700	148.229	308.473	394.441
Model 3(a)	Dep rate	0.01	0.03**	0.07	0.06	0.26
	Markup factor	0.97094	0.84863	0.96381	0.90395	0.98041
	Log-likelihood	400.934	343.057	280.239	416.204	530.168
Model 3(b)	Dep rate	0.02	0	0.12	0.27**	0.08
	Markup factor	0.985	0.9439	0.9087	0.96251	0.97252
	Log-likelihood	395.727	400.331	273.373	407.415	532.827
Model 3(c)	Dep rate	0.01**	0	0.09	0.22	0.29**
	Markup factor	0.96872	0.93698	0.86455	1.0087	0.94723
	Log-likelihood	395.907	395.578	236.828	413.723	549.806
Model 4(a)	Dep rate	0	0	0.14**	0.34	0.32
	Markup factor	1.0124	1.1241	0.91897	0.88006	0.93338
	Log-likelihood	378.795	386.034	334.547	406.451	541.723
Model 4(b)	Dep rate	0	0	0.1	0.3	0.38
	Markup factor	1.0179	1.0955	0.93431	0.88437	0.93532
	Log-likelihood	380.397	395.822	285.808	381.842	539.3563
Model 4(c)	Dep rate	0	0	0.05	0.32	0.28
	Markup factor	1.006	1.0898	0.94743	0.88176	0.94824
	Log-likelihood	379.044	397.954	271.822	388.501	544.687

TABLE 2
Depreciation rates and markup factors

	SIC 28	SIC 35	SIC 36	SIC 37	Manufacturing
Depreciation rate[a]	0.01	0.03	0.14*	0.27***	0.29***
	(0.2866)	(1.588)	(3.0986)	(15.3064)	(10.2534)
Markup factor[b]	0.96872	0.84863	0.91897	0.96251	0.94723
	(0.0261)	(0.0494)	(0.0554)	(0.0214)	(0.0191)

a The values in parentheses are log-likelihood statistics. * indicates statistical significance with a 10% critical region; *** indicates statistical significance with a 1% critical region.
b The values in parentheses are standard errors.

From the estimated markup factors, we can derive the markup for the total manufacturing sector and the four industries. The markup is 5.3% for the total manufacturing sector, 3.1% for chemical products, 15.1% for non-electrical machinery, 8.1% for electrical products, and 3.7% for transportation equipment. As shown in table 2, the estimated depreciation rates for SIC 28 and SIC 35 are not significantly different from zero. We may interpret this as the verification of some economist's belief that the knowledge asset should not depreciate over time. Another possible explanation for the small depreciation rates is that our model does not fit the data well for these industries. In order to estimate accurately the

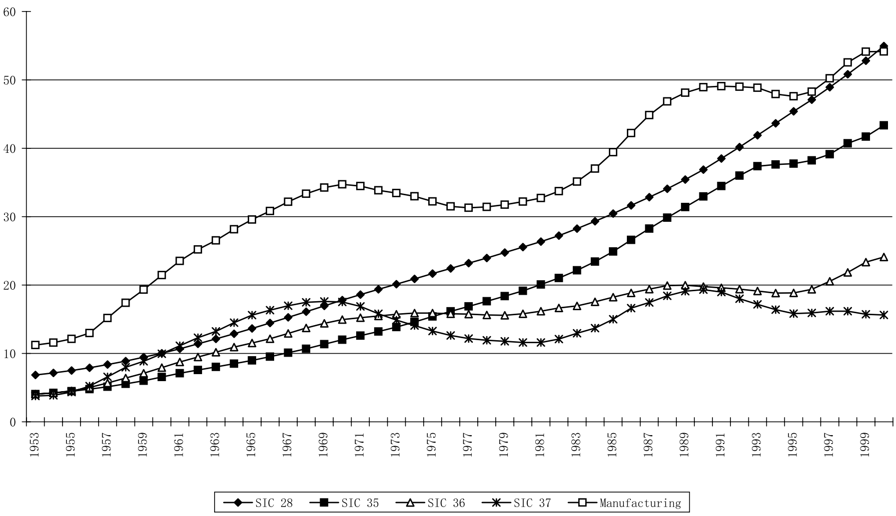

FIGURE 1 R&D stocks, 1953–2000

R&D depreciation rate, we need some large fluctuations in R&D investments. As we pointed out in section 2, R&D investments in SIC 28 and SIC 35 increase relatively smoothly in our sample period. The lack of fluctuations in R&D investments may result in our model failing to find the appropriate depreciation rate. Our other estimates fall in the range of R&D depreciation rates that are found in the literature. Depreciation rates for SIC 37 and Manufacturing are very close to each other, which may be due to the similar patterns in the changes of R&D investments in these two sectors.

The estimated depreciation rates show large variations across industries: for example, the depreciation rate for transportation industry is 27%, while the depreciation rate for chemical products is only 1%. Table 5 in the new OECD (2010) *Handbook on Deriving Capital Measures on Intellectual Property Products*[20] lists the average service lives reported by enterprises in selected industries. The large differences in the service lives – for example, 60 years of major development in chemical products and 5 years of minor improvement in software – imply noticeable variations in the depreciation rates across industries, and this is consistent with our results.

Using our estimates of the depreciation rate for R&D capital, we can construct the series of R&D capital stocks for manufacturing and the four knowledge intensive industries. Figure 1 shows how R&D stocks change over the period 1953–2000.

The geometric growth rate of R&D stocks for manufacturing is 3.4%, and the geometric growth rates of the R&D stocks for the four knowledge intensive

20 It is available under http://www.oecd.org/document/22/0,3343,en_2649_33715_44312278_1_1_1_1,00.html.

industries, namely, SIC 28, SIC 35, SIC 36, and SIC 37, are 4.5%, 5.2%, 3.9%, and 3.1%, respectively.

5. Conclusion

In this paper, we have developed a simple model based on a production function to estimate the depreciation rates of R&D capital for the U.S. total manufacturing sector and the four knowledge-intensive industries, including chemical products (SIC 28), non-electrical machinery (SIC 35), electrical products (SIC 36) and transportation equipment (SIC 37). We treat R&D capital as a technology shifter instead of as an ordinary input in the model. Using both the R&D stock variable and a time variable as technology shifters can avoid overestimation of R&D capital's contribution to productivity growth. Along with the estimation of the depreciation rate, we have estimated the markup factor for the U.S. manufacturing and the four selected industries. The estimated R&D depreciation rate is 29% for U.S. total manufacturing sector, 1% for chemical products, 3% for non-electrical machinery, 14% for electrical products, and 27% for transportation equipment. The corresponding markup is 5.3% for the total manufacturing sector, 3.1% for chemical products, 15.1% for non-electrical machinery, 8.1% for electrical products, and 3.7% for transportation equipment. Based on the estimated depreciation rate, the geometrical growth rate of the R&D stock is 3.4% for the manufacturing sector, 4.5% for chemical products, 5.2% for non-electrical machinery, 3.9% for electrical products, and 3.1% for transportation equipment.

The results reported here are preliminary. We have not incorporated some important features associated with R&D investment, such as the uncertainty of R&D investment and possible externalities of the created knowledge. Also, we have imposed some restrictive assumptions to simplify the problem, such as constant depreciation rates over years, constant markup factors, and full information about the future prices. In addition, the robustness of the results should be checked against alternative functional forms for the production function and against alternative ways of constructing the stock of R&D capital.[21]

Appendix A: The industry's profit maximization problem

Each industry's profit maximization problem can be written as follows:

$$\mathrm{Max}_{x_t}{}'s,\ R_{t+1}{}'s \sum_{t=0}^{\infty} \beta_t \{P_t(y_t, t)y_t - w_t x_\mathrm{t} - P_{r,t} I_t\}, \tag{A1}$$

21 Our results may also be subject to some aggregation bias since we have used industry data instead of firm data.

subject to

$$y_t = f(x_t, R_t, t) \quad \text{and} \quad R_t = I_{r,t-1} + (1-\delta)R_{t-1}.$$

Substituting the constraints into the objective function, we have the following equivalent problem:

$$\begin{aligned} &\mathrm{Max}_{x_t's, R_{t+1}'s} \beta_t \{P_t(f(x_t, R_t, t), t) f(x_t, R_t, t) - w_t x_t - P_{r,t}(R_{t+1} - (1-\delta)R_t)\} \\ &+ \beta_{t+1} \{P_{t+1}(f(x_{t+1}, R_{t+1}, t+1), t+1) f(x_{t+1}, R_{t+1}, t+1) - w_{t+1} x_{t+1} \\ &- P_{r,t+1}(R_{t+2} - (1-\delta)R_{t+1})\} + \beta_{t+2}\{P_{t+2}(f(x_{t+2}, R_{t+3}, t+2), t+2) \\ &\times f(x_{t+2}, R_{t+2}, t+2) - w_{t+2} x_{t+2} - P_{r,t+2}(R_{t+3} - (1-\delta)R_{t+2})\} + \cdots \end{aligned} \quad \text{(A2)}$$

Therefore, the first-order necessary conditions with respect to vector x_t can be written as

$$p_t \nabla_x f(x_t, R_t, t) + [\partial P(y_t, t)/\partial y] y_t \nabla_x f(x_t, R_t, t) = w_t, \quad t = 0, 1, \ldots T, \quad \text{(A3)}$$

where p_t is the output price and $\nabla_x f\ (x_t, R_t, t)$ is the vector of first-order partial derivatives of the period t production function with respect to the components of the input vector x. Factoring out the output price p_t and $\nabla_x f\ (x_t, R_t, t)$ on the left-hand side of equation (A3), we have the following simplified form for equation (A3):

$$p_t \times \left(1 + \frac{\partial P(y_t, t)/\partial y_t}{p_t/y_t}\right) \times \nabla_x f(x_t, R_t, t) = w_t. \quad \text{(A4)}$$

The first-order necessary conditions with respect to the R&D stock variable R_{t+1} are as follows:

$$\beta_{t+1}\left\{y_{t+1}\frac{\partial P(\cdot)}{\partial y}\frac{\partial f(\cdot)}{\partial R_{t+1}} + p_{t+1}\frac{\partial f(\cdot)}{\partial R_{t+1}} + P_{r,t+1}(1-\delta)\right\} = \beta_t P_{r,t}. \quad \text{(A5)}$$

After some rearrangement, we can rewrite the above equation as follows:

$$p_{t+1} \times \left[1 + \frac{\partial P_{t+1}(\cdot)/\partial y_{t+1}}{P_{t+1}/y_{t+1}}\right] \times \frac{\partial f(x_{t+1}, R_{t+1,} t+1)}{\partial R_{t+1}} = \frac{\beta_t}{\beta_{t+1}} P_{r,t} - (1-\delta) P_{r,t+1}. \quad \text{(A6)}$$

Assuming that $\beta_t\ /\beta_{t+1} = 1 + r_t$, where r_t is the nominal interest rate, we can rewrite the above equation as

$$p_{t+1} \times \left[1 + \frac{\partial P_{t+1}(\cdot)/\partial y_{t+1}}{P_{t+1}/y_{t+1}}\right] \times \frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}} = (1+r_t) P_{r,t} - (1-\delta) P_{r,t+1}. \quad \text{(A7)}$$

Define the period t non-negative markup as follows:

$$m_t \equiv -\frac{\partial P(y,t)/\partial y_t}{p_t/y_t} = -\frac{1}{\varepsilon} \geq 0. \tag{A8}$$

The corresponding period t markup factor M_t can be defined as

$$M_t = 1 - m_t = 1 + \frac{1}{\varepsilon} = 1 + \frac{\partial P(y,t)/\partial y_t}{p_t/y_t}. \tag{A9}$$

If we assume that markup factors are constant over time, we can rewrite our system of first-order conditions as

$$w_t = p_t M \nabla_x f(x_t, R_t, t) \tag{A10}$$

$$(1 + r_t)P_{r,t} - (1 - \delta)P_{r,t+1} = p_{t+1} M \frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}}. \tag{A11}$$

Moving the second term to the right-hand side of equation (A11) and dividing through by $P_{r,t}$, we obtain

$$1 + r_t = \frac{p_{t+1}}{P_{r,t}} M \frac{\partial f(x_{t+1}, R_{t+1}, t+1)}{\partial R_{t+1}} + (1 - \delta)\frac{P_{r,t+1}}{P_{r,t}} \tag{A12}$$

Equations (A10) and (A12) form our final system of estimating equations.

Appendix B: Break points

TABLE B1
Break points for Model 3 and Model 4

	Eq.	SIC 28	SIC 35	SIC 36	SIC 37	Manufacturing
Model 3(a) and Model 4(a)	(1)	9, 21, 29, 34, 44	4, 31, 39	20, 38, 46	13, 27, 32, 42	14, 21, 26, 29, 34, 38
	(2)	15, 21, 32, 40	29, 40	9, 19, 27, 39	12, 27, 30, 38	9, 21, 23, 36, 40
	(3)	13, 28, 35, 41	8, 13, 22, 32, 42	12, 22, 38	12, 28, 32, 38	13, 28, 36, 41, 47
	(4)	21, 28, 35, 41	30, 39	26, 40	12, 28, 35, 40	11, 21, 29, 36, 40
Model 3(b) and Model 4(b)	(1)	15, 21, 29, 34, 44	31, 39	20, 32, 38	9, 21, 30, 43	14, 21, 29, 34, 38
	(2)	15, 21, 43	10, 19, 30, 38	9, 22, 36, 41	6, 22, 30,39	9, 21, 23, 36, 40
	(3)	13, 28, 35, 41	8, 13, 22, 38, 42	12, 22, 30, 40, 45	13, 27, 32, 38	13, 22, 36, 41, 47
	(4)	21, 28, 35, 41	19, 30, 39	24, 37	6, 12, 24, 28, 33, 40	11, 21, 29, 36, 40
Model 3(c) and Model 4(c)	(1)	9, 21, 29, 34, 44	31, 39	20, 38	9, 22, 30, 43	15, 21, 29, 34, 39, 44
	(2)	15, 21, 30, 39	10, 19, 30, 38	8, 22, 41	7, 22, 30,37	9, 21, 23, 28, 39, 46
	(3)	13, 28, 35, 45	8, 14, 22, 37, 42	12, 22, 30, 45	12, 28, 32, 38	13, 28, 36, 41, 45
	(4)	21, 28, 35, 41	19, 30, 39	24, 37	13, 28, 35, 39	9, 21, 29, 36, 40

NOTE: Equations (1)–(3) are estimating equations for the three inputs; equation (4) is the production function.

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