Chapter 16

A NOTE ON AGGREGATION AND ELASTICITIES OF SUBSTITUTION*

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One of the most pressing problems facing an applied economist attempting to model the behavior of certain specific markets (such as labor markets) is to determine to what extent other markets, which are of less interest, can be aggregated. Presumably the benefits of more disaggregation of other markets are more accurate predictions of behavior on the markets of primary concern. However, one is led to ask, at what point will the increased benefits of disaggregation be outweighed by the costs of providing the disaggregated information?[1] The purpose of this note is to provide a framework for evaluating the benefits of increased disaggregation of other markets from the viewpoint of modeling behavior on the markets of primary concern.

Suppose that we are given a production function[2] f where $Y = f(L_1, L_2, \ldots, L_M;\ X_1, X_2, \ldots, X_N)$, Y being the output produced during a given time period, L_m the quantity of the mth type of labor services used for $m = 1, 2, \ldots, M$, and X_n the quantity of the nth category of 'other' inputs used for $n = 1, 2, \ldots, N$. Let W_m be the wage rate for one unit of L_m and let P_n be the rental price for one unit of X_n.

Suppose the producer attempts to minimize the cost of producing a given amount of output Y; that is, the producer attempts to solve the following

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[1]In addition to the cost of constructing an extra data series, there will generally be the increased cost of estimating additional unknown parameters. For example, if we are attempting to estimate the parameters of an N factor production function and we introduce an additional input series, then in general, an additional $N + 1$ parameters will have to be estimated if we do not wish *a priori* to constrain elasticities of substitution. Thus as N increases, the number of parameters to be estimated grows as the square of N.

[2]The analysis which follows can also be applied with minor modifications to: (i) multiple output production functions and (ii) systems of consumer demand and labor supply functions.

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minimization problem:

$$\text{(1)} \qquad \text{Minimize} \sum_{m=1}^{M} W_m L_m + \sum_{n=1}^{N} P_n X_n$$

with respect to $L_1, \ldots, L_M$ and $X_1, \ldots, X_N$, subject to $Y = f(L_1, \ldots, L_M; X_1, \ldots, X_N)$.

Define the optimal value of the objective function for (1) to be $C(Y; W_1, \ldots, W_M; P_1, \ldots, P_N)$, the total cost function.

A solution to the cost minimization problem given by (1) is the producer's system of labor demand functions

$$L_m(Y; W_1, \ldots, W_M;\ P_1, \ldots, P_N) \text{ for } m = 1, 2, \ldots, M,$$

and the producer's demand for other input functions

$$X_n(Y; W_1, \ldots, W_M;\ P_1, \ldots, P_N) \text{ for } n = 1, 2, \ldots, N.$$

Suppose that we are primarily interested in the labor demand functions and the response of labor demand to changes in the prices of other inputs; that is, we are interested in the price elasticities:

$$(\partial L_m/\partial P_n)(P_n/L_m) \text{ for } m = 1, 2, \ldots, M \text{ and } n = 1, 2, \ldots, N.$$

It turns out that we can express the above price elasticities in terms of the partial derivatives of the total cost function defined by (1) above. In fact, by Shephard's Lemma:[3]

$$\text{(2)} \qquad L_m(Y^*; W^*; P^*) = \partial C(Y^*; W^*; P^*)/\partial W_m \text{ for } m = 1, 2, \ldots, M,$$

where Y^* is a given output level, $W^* \equiv (W_1^*, W_2^*, \ldots, W_M^*)$ is a given vector of wages, and $P^* \equiv (P_1^*, P_2^*, \ldots, P_N^*)$ is a given vector of prices for other inputs. Equation (2) says that, given a functional form for the total cost function (satisfying the appropriate regularity conditions), we can obtain the demand for labor functions simply by partially differentiating the total cost function with respect to wage rates.

Define $L_m^* \equiv L_m(Y^*; W^*; P^*)$ for $m = 1, 2, \ldots, M$ and $X_n^* \equiv X_n(Y^*; W^*; P^*)$ for $n = 1, 2, \ldots, N$. If we partially differentiate both sides of (2) with respect to P_n, we can express the price elasticity of demand for the mth type of labor with respect to a change in P_n, the price of the nth type of other input, as:

$$\begin{aligned}\frac{\partial L_m(Y^*; W^*; P^*)}{\partial P_n} \frac{P_n^*}{L_m^*} &= \left[\frac{\partial^2 C(Y^*; W^*; P^*)}{\partial P_n \partial W_m}\right]\left[\frac{P_n^*}{L_m^*}\right] \\ &= \left[\frac{\partial^2 C(Y^*; W^*; P^*)}{\partial P_n \partial W_m} \frac{C(Y^*; W^*; P^*)}{L_m^* X_n^*}\right]\left[\frac{P_n^* X_n^*}{C(Y^*; W^*; P^*)}\right] \\ &= \sigma_m^n(Y^*; W^*; P^*)\left[\frac{P_n^* X_n^*}{C(Y^*; W^*; P^*)}\right]\end{aligned}$$

$$\text{(3)} \qquad \text{for } m = 1, 2, \ldots, M; \quad n = 1, 2, \ldots, N,$$

where $\sigma_m^n(Y^*; W^*; P^*)$ is the partial elasticity of substitution[4] between L_m and X_n evaluated at the initial point $(Y^*; W^*; P^*)$.[5]

Define $\sigma_m^{n^*} \equiv \sigma_m^n(Y^*; W^*; P^*)$ and define the share in total cost of the nth other input as $s^{n^*} \equiv P_n^* X_n^*/C(Y^*; W^*; P^*)$. Now equation (3) can be rewritten as:

$$\text{(4)} \qquad [\partial L_m(Y^*; W^*; P^*)/\partial P_n](P_n^*/L_m^*) = \sigma_m^{n^*} s^{n^*} \quad \text{for } m = 1, 2, \ldots, M;\ n = 1, 2, \ldots, N.$$

In words, equation (4) says that the percentage change in the demand for the mth type of labor due to a one percent change in the price of the nth type of other input is equal to the (partial) elasticity of substitution between L_m and X_n times the share in total cost of X_n. Thus partial elasticities of substitution are intimately related to ordinary price elasticities.

We now suppose that the prices of inputs $P \equiv (P_1, P_2, \ldots, P_N)$ vary in strict proportion over time; i.e., that the relative prices of other inputs are constant. If this is the case, it can be shown that, from the viewpoint of modeling labor market behavior, there will be no loss in predictive power due to the effects of aggregation. This result is part of Hicks' [1946; 312–313] Aggregation Theorem.[6] The theorem states that, under the appropriate

[3]See Shephard [1953] or Diewert [1971a; 495–496] for a proof and a statement of the appropriate regularity conditions on the production function f and on the total cost function C which is dual to f.

[4]The partial elasticity of substitution concept was originally defined in terms of the partial derivatives of the production function (see Allen [1938; 504]). However, Uzawa [1962] showed that

$$\sigma_m^n(Y^*; w^*; p^*) \equiv [\partial^2 C(Y^*; w^*; p^*)/\partial p_n \partial w_m][C(Y^*; w^*; p^*)/L_m^* X_n^*].$$

[5]In general, the elasticity of substitution σ_m^n will change as outputs, wages, and prices change. Equation 3 is usually derived using inverses of bordered Hessian matrices; see Allen [1938].

[6]For alternative treatments of Hicks' Aggregation Theorem, see Wold [1953; 109–110], Gorman [1953; 76–77] and Diewert [1978a].

regularity conditions, if the prices of a group of goods change proportionately, then that group of goods behaves just as if it were a single commodity.

Let the proportional variation in the prices of other inputs be expressed as:

$$P_1 = \alpha_1 P_0, \quad P_2 = \alpha_2 P_0, \ \ldots, \ P_N = \alpha_N P_0 \tag{5}$$

where P_0 is the price of the aggregate and the positive constant α_n expresses how the price P_n varies as P_0 varies over time for $n = 1, 2, \ldots, N$ (note that we are regarding the 'micro' prices $P_1, P_2, \ldots, P_N$ as functions of the 'macro' price P_0). We assume that P_0 is initially equal to unity (i.e., $P_0^* = 1$) and thus $P_n^* = \alpha_n \cdot 1$ is the initial price of the nth other input.

Given that the prices of other inputs vary in proportion, we may calculate what percentage change in the demand for the mth type of labor will be induced by a one percent change in the aggregate price index for other inputs; i.e., we may calculate (where $\alpha \equiv (\alpha_1, \alpha_2, \ldots, \alpha_N)$):

$$\begin{aligned}
&\frac{\partial L_m(Y^*; W^*; \alpha P_0^*)}{\partial P_0} \frac{P_0^*}{L_m^*}\Big|_\alpha \\
&= \frac{\partial}{\partial P_0}\left[\frac{\partial C(Y^*; W^*; \alpha P_0^*)}{\partial W_m}\right]\frac{P_0^*}{L_m^*} \\
&= \sum_{n=1}^{N}\left[\frac{\partial^2 C(Y^*; W^*; \alpha P_0^*)}{\partial P_n \partial W_m}\right]\frac{\partial P_n}{\partial P_0}\frac{P_0^*}{L_m^*} \\
&= \sum_{n=1}^{N}\left[\frac{\partial^2 C(Y^*; W^*; \alpha P_0^*)}{\partial P_n \partial W_m}\right]\alpha_n \frac{P_0^*}{L_m^*} \quad \text{(using (5))} \\
&= \sum_{n=1}^{N}\left[\frac{\partial^2 C(Y^*; W^*; P^*)}{\partial P_n \partial W_m}\frac{C(Y^*; W^*; P^*)}{L_m^* X_n^*}\right]\left[\frac{P_n^* X_n^*}{C(Y^*; W^*; P^*)}\right] \\
&= \sum_{n=1}^{N} \sigma_m^{n^*} s^{n^*} \qquad (6)
\end{aligned}$$

where the last equality follows from the fact that equation (3) above is equivalent to equation (4). Thus the response of L_m to a one percent change in P_0, the aggregate price index for other inputs, is a share weighted average of the partial elasticities of substitution of the mth type of labor with the other inputs.

Using equation (3) as a model, we may define the *aggregate elasticity of substitution* of L_m with other inputs as:

$$\sigma_m^{A^*} \equiv \left[\frac{\partial L_m P_0^*}{\partial P_0 L_m^*}\right]\left[\frac{C(Y^*; W^*; P^*)}{\sum_{n=1}^{N} P_n^* X_n^*}\right] \quad \text{for } m = 1, 2, \ldots, M$$

$$= \frac{\sum_{n=1}^{N}\sigma_m^{n^*} s^{n^*}}{\sum_{n=1}^{N} s^{n^*}} \quad \text{using (6) and the definition of } s^{n^*}. \tag{7}$$

Thus the aggregate elasticity of substitution of the mth type of labor with other inputs is equal to a share in total cost weighted average of the micro elasticities of substitution of L_m with X_n divided by the share of other inputs in total cost.[7]

Recall that $\sigma_m^{n^*}$ will be positive if L_m and X_n are substitutes in production and $\sigma_m^{n^*}$ will be negative if the two inputs are complements. Taking a weighted average of both positive and negative[8] micro elasticities of substitution, $\sigma_m^{n^*}$ will tend to give rise to aggregate elasticities of substitution which are considerably *smaller* in magnitude than an average of the absolute values of the micro elasticities of substitution.

Conversely, as we disaggregate, we can expect to encounter increasingly large elasticities of substitution. Two recent empirical papers confirm this statement. Berndt and Christensen [1974] in their 'two types of labor, one type of capital' disaggregation of US manufacturing industries found that the mean partial elasticities of substitution were 7.88 (between blue and white collar workers), 3.72 (between blue collar workers and capital) and −3.77 (between white collar workers and capital). However, when they fitted a model which aggregated the two types of labor into a single labor factor, they found that the aggregate labor capital elasticity of substitution was approximately 1.42, which is considerably smaller than an average of the absolute values of the three 'micro' elasticities of substitution. Similarly, Woodland [1972a] found partial elasticities of substitution in Canadian manufacturing range from −11.16 to 2.18 in his 'four types of capital, one type of labor' disaggregated results, but he found that the aggregate capital-labor elasticity of substitution was only 0.39.

Let us summarize our results thus far. If the prices of other inputs vary in strict proportion, then we may aggregate all the other inputs into a single composite good whose price is represented by a price index P_0 and we can predict what the percentage change in L_m due to a one percent change in the aggregate price of other inputs is by using the following expression (which is

[7] The expression for the aggregate elasticity of substitution given by (7) has been derived by Woodland [1972a; 25, 40] and by Sato and Koizumi [1973]. They did not use Hicks' Aggregation Theorem in their derivations.

[8] It is theoretically possible for all $N(N-1)/2$ partial elasticities of substitution in the N good case to be positive (Hicks [1946; 311]). It is also theoretically possible for all $N(N-1)/2$, save $N-1$, partial elasticities of substitution to be negative (Ibid., 47). From an empirical point of view, it appears that partial elasticities of substitution are more or less randomly distributed with a positive mean and a variance which becomes larger as N increases. See Berndt and Christensen [1974], Gussman [1972] and Woodland [1972a] for empirical tabulations of elasticities of substitution when there are three, eleven, and five goods respectively.

obtained by substituting (7) into (6)):

$$\frac{\partial L_m}{\partial P_0}\,\frac{P_0^*}{L_m^*} = \sigma_m^{A^*}\Big(\sum_{n=1}^{N} s^{n^*}\Big). \tag{8}$$

That is, the price elasticity of the mth type of labor with respect to a change in the price of other inputs is equal to the aggregate elasticity of substitution of L_m with other inputs times the share in total cost of other inputs. Moreover, we can use standard econometric techniques in order to estimate the aggregate elasticities of substitution $\sigma_m^{A^*}$ for $m = 1, 2, \ldots, M$ using only aggregate data on other inputs. We note that our estimated aggregate elasticity of substitution $\sigma_m^{A^*}$ will be a weighted average of micro elasticities of substitution $\sigma_m^{n^*}$ of unknown magnitudes, but for purposes of predicting labor market behavior, a knowledge of the micro parameters $\sigma_m^{n^*}$ is not necessary — provided that the prices of other inputs vary in strict proportion.

Since prices are unlikely to vary *precisely* in proportion over a period of years, one is led to ask whether the results summarized in the previous paragraph are robust; that is, do they carry over *approximately* provided that prices of other inputs vary approximately in proportion. If this is not the case, then Hicks' Aggregation Theorem would be of little empirical significance. Fortunately, the above results do carry over to the approximate case, as we shall now attempt to show.[9]

Suppose that the prices of other inputs have varied in strict proportion according to the relationship given by (5) and that we have initial prices of other inputs given by:

$$P_1^* = \alpha_1, \quad P_2^* = \alpha_2, \ \ldots, \ P_N^* = \alpha_N. \tag{9}$$

Now suppose that, starting from the initial prices given by (9), prices of other inputs vary according to a different proportionality vector; that is, we assume that as P_0 varies from an initial level of unity, the prices of other inputs vary according to the following relationships:

$$P_1 = (\alpha_1 - \beta_1) + \beta_1 P_0,\ P_2 = (\alpha_2 - \beta_2) + \beta_2 P_0,\ \ldots,\quad P_N = (\alpha_N - \beta_N) + \beta_N P_0. \tag{10}$$

[9]The aggregate production function $\hat{f}_\alpha(L_1, \ldots, L_M; X_0)$ can be defined as the dual to the cost function $\widehat{C}_\alpha(Y; W_1, \ldots, W_M; P_0) \equiv C(Y; W; \alpha P_0)$. If α remains constant, $\hat{f}_\alpha$ will be a well defined production function in $(L; X_0)$. Since $\hat{f}_\alpha$ can be shown to be a continuous function of α, small changes in α will lead to small shifts in the aggregate production function $\hat{f}_\alpha$.

Thus when $P_0^* = 1$ initially, the initial prices are given by $P_n^* = \alpha_n$ for $n = 1, \ldots, N$. However, when P_0 increases beyond 1, the old proportionality factors α_n are replaced by the new proportionality factors β_n. We now calculate the response of L_m to a change in the price index of other goods P_0 assuming that the prices of other inputs satisfy the relation (10) rather than (5).[10]

$$\begin{aligned}\frac{\partial L_m(Y^*; W^*; \alpha - \beta + \beta P_0^*)}{\partial P_0}\,\frac{P_0^*}{L_m^*}\Big|_\beta &= \frac{\partial}{\partial P_0}\left[\frac{\partial C(Y^*; W^*; \alpha - \beta + \beta P_0^*)}{\partial W_m}\right]\frac{P_0^*}{L_m^*}\\ &= \sum_{n=1}^{N}\left[\frac{\partial^2 C(Y^*; W^*; \alpha - \beta + \beta P_0^*)}{\partial W_m \partial P_n}\right]\frac{\beta_n}{L_m^*}\\ &= \sum_{n=1}^{N}\sigma_m^{n^*}\left[\frac{\beta_n X_n^*}{C(Y^*; W^*; P^*)}\right]\end{aligned}$$

or

$$\frac{\partial L_m}{\partial P_0}\,\frac{P_0^*}{L_m^*}\Big|_\beta = \sum_{n=1}^{N}\sigma_m^{n^*} s^{n^*}\left[\frac{\beta_n}{\alpha_n}\right] \tag{11}$$

where $\sigma_m^{n^*}$ and s^{n^*} are defined below equation (3), α_n represents the *old* proportionality factor for P_n (the price of one unit of the nth type of other input), and β_n represents the *new* proportionality factor for P_n.

Let us suppose that we have estimated the elasticity $(\partial L_m/\partial P_0)P_0^*/L_m^*$ based on the assumption that the prices of other inputs vary according to (5). Then we will measure (incorrectly) the percentage change in labor demand due to a one percent change in the aggregate price of other inputs as:

$$\left[\frac{\partial L_m}{\partial P_0}\right]\frac{P_0^*}{L_m^*}\Big|_\alpha = \sum_{n=1}^{N}\sigma_m^{n^*} s^{n^*}. \tag{3}$$

The correct percentage change in labor demand is given by (11). There are several points to note in comparing (3) with (11):

(i) Note that (11) coincides with (3) if $\alpha_n = \beta_n$ for $n = 1, 2, \ldots, N$ and, moreover, the closer the numbers β_n are to α_n then the closer will be the incorrectly measured elasticity (3) to the true elasticity given by (11). Thus an approximate version of Hicks' Aggregation Theorem will hold in the sense that prediction errors due to aggregation will be small provided that deviations from the assumption that the prices of other inputs vary in proportion are small.

[10]Recall that $P_0^* \equiv 1$ and $\alpha \equiv (\alpha_1, \alpha_2, \ldots, \alpha_N)$, and define $\beta \equiv (\beta_1, \beta_2, \ldots, \beta_N)$.

(ii) A given percentage change in β_n relative to α_n will generate a bigger prediction error the bigger is the share s^{n^*} of the nth other input in total cost and the bigger the magnitude of the elasticity of substitution $\sigma_m^{n^*}$.

(iii) If we pursue point (ii) a bit further, we see that a major change, say, in the price of the first other input, relative to the general trend in the price level of other inputs, *will generally have a very uneven effect* on the prediction errors for the various labor markets, since micro elasticities of substitution tend to be more or less randomly distributed with a considerable variance.[11] More specifically, let $\beta_n = \alpha_n$ for $n = 2, 3, \ldots, N$ but let the price of the first other input change according to the relation $\beta_1 = \alpha_1(1+\delta)$. Then the prediction error on the mth labor market due to aggregation may be written as the difference between (11) and (3) and is equal to

$$(12) \qquad \sigma_m^{1^*} s^{1^*} \delta$$

when there is a nonproportional variation in the price of the first other input. Thus the error will vary in proportion to the percentage change δ in the price of the first other input relative to the general trend in the price of other inputs, the share in total cost of X_1, and the elasticity of substitution between X_1 and L_m, $\sigma_m^{1^*}$. Since it is unlikely that the elasticities of substitution $\sigma_m^{1^*}$, $m = 1, 2, \ldots, M$, are all equal, we see that the prediction errors generated by aggregating other inputs will not be equal in magnitude for every labor market.

(iv) These points suggest that, in order to minimize prediction errors on disaggregated labor markets, we will have to disaggregate other markets as well. Ideally, we would like each of the other input series X_n to consist of inputs whose prices vary in fixed proportions, at least approximately. In practice, it appears that the more finely we disaggregate labor markets, the more finely we will have to disaggregate other markets in order to keep prediction errors on the disaggregated labor markets down to a satisfactory level.[12] This in turn suggests that it is unlikely that we will be able to predict behavior on very finely disaggregated labor markets.

[11] Actually, this point may be used to explain why Woodland's [1972a] production function results were somewhat unsatisfactory. Woodland used four types of capital and one type of labor. Regard the four types of capital as L_1, L_2, L_3 and L_4 and labor as an aggregate of $X_1, X_2, \ldots, X_N$. Since relative wage rates have not been constant over time, and since the share of labor in value added is large, it can be seen that the prediction errors due to labor aggregation on the capital markets could be fairly large, depending on the magnitudes of the unknown elasticities of substitution.

[12] This follows from our expression for the prediction error given by (12), $\sigma_m^{1^*} s^{1^*} \delta$. As we disaggregate labor markets (i.e., increase M), then the numbers $\sigma_m^{1^*}$ will tend to increase in absolute value, while s^{1^*} and δ remain fixed.

We conclude that the benefits of disaggregating other markets from the viewpoint of predicting behavior on a group of markets that we are primarily interested in will be substantial provided that the prices of the 'other' goods do not move in fixed proportions. For example, if we take Canadian labor markets to be the markets we are primarily interested in, then Woodland's [1972b] data indicate that the rental prices of machinery and equipment, engineering structures, and operating capital do not all vary in strict proportion (even approximately). Furthermore, Woodland's work was based on the first approximation that the prices of outputs and intermediate inputs varied in proportion.[13] One suspects that this first approximation has not been satisfied - the prices of intermediate inputs such as imports and energy have not been varying in proportion with the prices of various outputs (or with the prices of certain other primary inputs such as land and working inventories). Thus it would appear that we will be unable to adequately predict the demand for Canadian labor on a disaggregated basis without disaggregating the 'other inputs' series into 'reasonably' homogeneous groups.[14]

References for Chapter 16

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[13] It turns out that real value added may be used as a proxy for real gross output of an industry (without any prediction errors on labor markets due to the effects of aggregating outputs minus intermediate inputs) provided that the prices of outputs and intermediate inputs vary in strict proportion. This is another version of Hicks' Aggregation Theorem (see Diewert [1978a]).

[14] Just what is a 'reasonable' degree of disaggregation will depend on the application at hand. For example, if we wished to determine the effects on the demand for labor of a *selective* change in tariffs, then it would be necessary to disaggregate an imports inputs series into two imports series: tariff affected imports and nonaffected imports.

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