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Index Number Issues in the Consumer Price Index

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The Boskin Commission has fulfilled a useful purpose. There is now unprecedented public interest in price measurement issues and an increased awareness on the part of academics about the difficult problems involved in measuring price change in a dynamic economy. There is also increased cooperation between economists and statisticians in attempting to solve these problems. This comment offers some perspectives on the magnitude of the differences between the fixed base Laspeyres price index that statistical agencies produce and a theoretical cost of living index, along with some thoughts about what can be done to reduce these differences or "biases."

What is an Appropriate Concept for the Price Index?

Defining a true cost of living index must begin on the household level, and then move to the social level. The Konüs (1939) true cost of living index for a single household is defined as the ratio of the minimum costs of achieving a certain reference utility level in a base period, given the prices prevailing at that time, and at a later "current" period, given whatever changes in prices had occurred in the interval. An appropriate generalization of the Konüs cost of living concept to the case of many households is Pollak's (1981, p. 328) social cost of living index, which is the ratio of the total minimum cost or expenditure required to enable each of the households present in the two periods to attain their reference utility levels in both time periods.

As economists have long known, a Laspeyres index, which finds the cost of

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purchasing a fixed basket of goods representing the base period and then the cost of buying the same basket in the present, tends to overstate the rise in the cost of living by not allowing any substitution between goods to occur. Conversely, a Paasche index, which finds the cost of purchasing a fixed basket of goods representing the present and then the cost of buying that same basket in the past, tends to understate the rise in the cost of living. Diewert (1983, p. 191) showed that the (unobservable) Pollak-Konüs true cost of living index was between the (observable) Paasche and Laspeyres price indexes. An implication of this result is that some average of the Paasche and Laspeyres aggregate price indexes should provide a reasonably close approximation to the underlying true cost of living. Note that this argument does not rely on any particular assumption about the form of the household preferences; in particular, it does not assume that indifference curves are homothetic (that is, shaped so that the slope of the indifference curves will be the same along the path of a ray extending out from the origin).

One strong candidate for an average of the Laspeyres and Paasche indexes is the Fisher (1922) ideal price index, which is the geometric average of the Laspeyres and Paasche indexes (that is, the square root of their product). This choice can be defended from at least four different perspectives. First, it is evident that the base period basket used in the Laspeyres index is just as valid as the current period basket used in the Paasche index. Hence it makes sense to take an even-handed average of the two. The geometric mean is more desirable than other simple averages, like the arithmetic mean, because it has a time reversibility property: using the Fisher formula, price change going from the current period to the base period is the reciprocal of the original price change (Diewert, 1997). Note that the Paasche and Laspeyres indexes also do not satisfy this time reversal test. This leads to a second justification for the Fisher formula: it satisfies more reasonable "tests" or "axioms" than any of its competitors (Diewert, 1992). The test approach to index number theory, initiated by Walsh (1901) and Fisher (1922), looks at an index number formula from the viewpoint of its mathematical properties. For example, if current period prices increase, does the price index increase? Does the price index lie between the Paasche and Laspeyres indexes? If current period prices increase by a common factor of proportionality, does the price index increase by that same factor of proportionality? These reasonable tests are all satisfied by the Fisher formula.[1] A third justification for the use of the Fisher formula is the fact that it is exact for (that is, consistent with) a homothetic preference function that can approximate arbitrary homothetic preferences. Diewert (1976) calls index number formulae that have this property "superlative". The Törnqvist index which is discussed by the Boskin Commission is an example of another superlative for-

[1] The Fisher Formula satisfies all 20 of Diewert's (1992) desirable tests while the Törnqvist formula satisfies only 13 of these tests. Thus the Fisher formula seems preferable from the axiomatic point of view. However, Diewert (1978) shows that numerically, there will be only small differences between the two formulae.

mula.[2] A final justification for the use of the Fisher formula rests on its consistency with revealed preference theory (Diewert, 1976, p. 137).

Thus in what follows, we will take the Fisher formula as our best "practical" approximation to a theoretical cost of living index or "true" price index.

Substitution Biases

Most statistical agencies have used a fixed-base Laspeyres index to approximate the true price index. The fixed-base Laspeyres index does not allow for substitution between goods, which can ameliorate the effect of price increases on household utility, and it has a hard time dealing with changes in the quality of goods. This section takes up three kinds of substitution: at the elementary index level, at the commodity level, and between outlets. The next section takes up the issue of quality improvement and new goods.

The discussion in the previous section assumed that it was straightforward to obtain a single price for a commodity. In reality, households may purchase a commodity at a variety of prices at different outlets. This price heterogeneity at the lowest level of aggregation has to be summarized as a single price so that it can be inserted into an index number formula. The appropriate price that should be inserted into an index number formula at the lowest level of aggregation appears to be an "outlet unit value," defined as the total value of the commodity sold during the time period divided by the corresponding quantity sold at an outlet. As Diewert (1995, pp. 20–24) discusses, this approach was first proposed by Walsh (1901, p. 96).

To aggregate the price and quantity information for a particular commodity over many outlets, we can either: i) construct a unit value for the commodity over all outlets for each period and use the resulting average price and total quantity sold of the good as an input into an index number formula at the next level of aggregation; or ii) use the Fisher index formula to aggregate the outlet-specific prices and quantities into an aggregate price and quantity for that commodity. If the outlet characteristics do not seem important in the final consumption of the commodity—for example, a can of beer yields the same utility to me at home no matter where I purchased it—then the first approach of averaging over all outlets can be justified. But if the choice of outlet does matter, then using the Fisher ideal index has a better economic justification, since it allows for substitution between outlets (Diewert, 1995, pp. 17–20).

Until very recently, many statistical agencies substituted "representative" outlet price quotations for a broader calculation of outlet unit values and they used a modification of the Laspeyres formula to aggregate the outlet prices. The modification typically assumes that outlet expenditure shares are approximately equal in

[2] Theil (1967, pp. 136–7) provided a strong justification for the use of the Törnqvist formula from the viewpoint of the stochastic approach to index number theory; see also Diewert (1997).

the base period. (This assumption is not as restrictive as it appears, since the sampling of outlets can be carried out so that the assumption of equal expenditure shares holds roughly true.) The statistical agencies then aggregate the elementary price indexes for each commodity class into an overall index, again using a Laspeyres-style fixed basket approach, but this time weighting according to the consumer expenditures on each commodity class. The double use of the fixed-base Laspeyres index, at the elementary and the commodity aggregation level, does not allow substitution between elementary goods at the lower level, nor substitution between commodities at the higher level. Thus, it creates elementary index bias and commodity substitution bias.

Elementary substitution bias B_E can be defined as the difference between the fixed base Laspeyres index P_L and the corresponding Fisher index P_F where the prices in these indexes refer to some homogeneous component of the CPI. The Appendix shows that this bias will be approximately equal to one-half the Laspeyres price index P_L times the variance of the inflation-adjusted percentage changes in prices among the goods examined:

$$B_E \equiv P_L - P_F \cong (1/2)(1 + i)\mathrm{Var}(\varepsilon),$$

where i is the inflation rate in the CPI component as measured by its Laspeyres price index; that is, $1 + i = P_L$. For the purposes of illustration, assume that the variance of the percentage change in prices is .005, which is a plausible amount of variation. Then, according to the formula, inflation rates at the present level of about 2 percent will imply that the Laspeyres index is upwardly biased by .00255, or .255 percentage points.

In the case of commodity substitution bias, this calculation can be repeated, except that, in this case, the aggregation is happening across different commodity prices instead of prices for the same commodity across different outlets. It is difficult to say a priori whether the variability of outlet prices for the same commodity is greater or less than the variability of prices across commodities. But if we stick with a variance estimate of .005 and an inflation estimate of 2 percent, then commodity substitution bias would be another .255 percentage points. These two effects taken together would represent an upward bias of .5 percentage points—which is quite similar to the Boskin Commission estimate.

Now let us return to the subject of outlet substitution bias, which was assumed away by the earlier formulation. Suppose that discount outlets move into a market area and capture market share from traditional high cost retailers. *If* differences in the services provided by discount and traditional retailers can be neglected (a controversial assumption), then a reasonable concept of the "true" price index is the average price (or unit value) paid by consumers over all outlets (Reinsdorf, 1993; Hill, 1993, p. 399; Diewert, 1995; p. 28). In this case, the relationship between the Laspeyres index and the true price index can be defined as:

$$P_T \equiv (1 - s)(1 + i) + s(1 + i)(1 - d),$$

where $(1 + i) = P_L$ is the Laspeyres price index for the traditional retailers in the current period, s is the market share captured by low cost retailers in the current period and d is the percentage discount of the low cost retailer over traditional retailers.[3] Essentially, this formula says that to take the discount stores into account, one must weight the existing Laspeyres index measure of inflation by the discount store share of the market and their lower prices. The outlet substitution bias—that is, the gap between the true index and the original Laspeyres index—then works out to be:

$$B_O \equiv P_L - P_T = (1 + i)sd.$$

For a back-of-the-envelope estimate of the outlet substitution bias, assume that the increased market share (s) captured by low cost retailers in a given year is 2 percent, a rather conservative estimate, and that the percentage discount (d) of the low cost retailer over traditional retailers is 20 percent, which is consistent with some limited Canadian evidence reported by White (1997). Then, if the Laspeyres index $1 + i$ was 1.02, the upward bias of this measure over a "true" index would be .0041, or .41 percentage points.

This figure is higher than the Boskin Commission's estimate of .1 percentage points for outlet substitution bias, in part because the commission was conservative in its estimates,[4] and in part because the method here assumes no quality difference between the different types of stores, which presumably overestimates the change in utility that consumers receive from buying at outlets that provide a lower quality of service.

In contrast to substitution and elementary index bias, which can continue forever as long as there is dispersion in relative prices, outlet substitution bias must end when low-cost retailers capture the entire market. For example, Reinsdorf (1993) found no evidence of outlet substitution bias in the United States in the 1960s, since at that time the transition from general stores and corner stores to department stores and supermarkets had largely been completed. However, in the 1980s when the impact of large discount stores was felt, Reinsdorf found substantial evidence of outlet substitution bias. In recent years in the United States and Canada, discount chains and specialty megastores have continued to gain market share. In the future, new outlet competition will come from discount selling of goods over the Internet.

Quality Change and New Goods Bias

Every year, statistical agencies find that some of the commodities that they are pricing in various outlets disappear. Although disappearance from one outlet does

[3] This formula implicitly assumes that the discount is constant in the two periods and the period-to-period trend in discount retail prices is the same as the traditional retailer's trend.

[4] Using scanner data, Saglio (1994) found that the outlet substitution bias for milk chocolate bars in France averaged .8 percentage points per year. Neglecting service quality differences, Reinsdorf (1993) found outlet substitution bias in the United States to be about .25 percentage points per year.

not mean that the good has vanished altogether from the market, statistical agencies typically require that the good be found and priced in the same outlet, as a way of minimizing any variation in price related to location or quality of service at various outlets. The typical disappearance rate of goods from the outlet where they were previously surveyed is about 20 percent per year. Some of these disappearances are due to seasonal factors and temporary inventory outages, but a substantial fraction result from improved models. Examples of this phenomenon include faster computing, light bulbs that last longer and use less energy; television sets that no longer have tubes that burn out frequently; new automobiles that require less maintenance and have a higher fuel efficiency; hand-held video cameras that now offer color viewfinders and double the zoom capability at 60 percent of the price of a comparable model of three years ago; and so on (Gordon, 1981, pp. 130–133; 1990; 1993, p. 242). The question then becomes how the index should capture these quality changes.

In many cases, the statistical agency will simply "link in" the new model, a process which involves looking at price changes in the old model up to a point in time, and then after that point, looking at price changes in the new model. After two periods of pricing the new model, the price ratio for the new model can be aggregated or "linked" in with the price ratios of old models that have not disappeared. This approach works if any quality differences between the two models are reflected by the price difference between them. But more typically, the new model has improved efficiency which is not fully offset by its price. For a rough measure of the bias created here, the true price index is assumed to be

$$P_T \equiv (1 - s)(1 + i) + s(1 + i)/(1 + e),$$

where $P_L = (1 + i)$ is the Laspeyres index calculated by the statistical agency, s is the share of commodities that have been replaced by new models and e is the percentage increase in the efficiency of new models that is missed when the new models are linked into the index. Notice that this formulation is parallel to the earlier discussion of outlet substitution bias: there, the weights were the growing market share of discount stores and their price difference; here, the weights are the market share of the new models and their efficiency (or quality) difference.

The quality change bias B_Q is then the difference between P_L and P_T, which is:

$$B_Q \equiv P_L - P_T = (1 + i)se/(1 + e).$$

Assume that inflation as measured by the Laspeyres index is 2 percent, so that $(1 + i) = 1.02$. Assume further that the share of commodities that have been replaced by new models (s) is .1, which may be too high for many commodity categories, and that the percentage increase in the efficiency of new models (e) which was missed by the linking procedure is .05, which will be too low for many classes of electronic goods. Then the quality change bias will be .0049, or .49 percentage points, which is quite similar to the .4 percentage point estimate of quality change from the Boskin Commission.

The appearance of new goods offers an additional problem for a fixed-weight index. Again, such goods can be "linked" into the index over time, but it often takes a period of years before the new good is actually included in the basket. As Alfred Marshall (1887, p. 373) observed many years ago, when a new product is introduced into the market, it generally has a high price which is reduced in subsequent periods. Since statistical agencies do not introduce new goods into their commodity baskets until the new product has become important in the market, they often miss this early decline in price. In addition, for the period before the new good appears, we can follow Hicks (1940, p. 114) and imagine an imputed price for the new good that would cause consumers to demand zero units of it. Statistical agencies also miss the (imputed) price decline of a new product in the period when the new good makes its first appearance. Hausman (1996, 1997) econometrically estimated these imputed prices for cellular telephones and for certain brands of breakfast cereals; for new cereals, he found that the average imputed price was approximately double the introduction price.

In the spirit of the earlier estimates, a rough estimate of the neglect of new goods begins with a true price index P_T, defined by

$$P_T \equiv (1 - (1/2)s)(1 + i) + (1/2)s(1 + i)(1 - d),$$

where $1 + i = P_L$ is the Laspeyres estimate of overall price change, s is the market share of new goods which have not yet been introduced into the basket of commodities and d is the percentage decline in the prices of the new goods from their initial imputed prices.[5] The new goods bias B_N will be the difference between P_L and P_T:

$$B_N \equiv P_L - P_T = (1/2)(1 + i)sd.$$

Again assume that the inflation rate is 2 percent, so that $P_L = (1 + i)$ is 1.02. Assume that the share of new commodities that are not in the statistical agency basket is .05 and that the average decline in price that was missed was 20 percent. Then, the new goods bias is .0051, or .51 percentage points.

New goods bias is an even more pervasive phenomenon than it may appear at first sight. From the viewpoint of the local market place, the introduction of an increased selection of commodities creates new goods bias even though the newly available commodities are not "new" in a global sense.[6] Increased product variety

[5] The Paasche index under our assumptions is $P_P \equiv \{(1 - s)(1 + i)^{-1} + s[(1 + i)(1 - d)]^{-1}\}^{-1}$. We approximate this weighted harmonic mean by the corresponding weighted arithmetic mean so that $P_P \cong (1 - s)(1 + i) + s(1 + i)(1 - d)$. Finally, define the true index P_T to be the Fisher index $P_F = \sqrt{(P_L P_P)}$ and approximate the geometric mean by the arithmetic mean so that $P_T \cong (\frac{1}{2})P_L + \frac{1}{2}P_P \cong (\frac{1}{2})(1 + i) + (\frac{1}{2})[1 - s)(1 + i)(1 - d)]$ which simplifies to the formula in the text. For alternative models of the true price index under these conditions, see Diewert (1987, p. 378).

[6] Thus, a household with a credit card and a new Internet connection can achieve a vast increase in its choice set.

thus leads to goods bias. Transportation and communication improvements also lead to larger choice sets and greater variety, a point emphasized by Marshall (1887, pp. 373–374).

What Can Be Done to Reduce Biases?

Recent developments in index number theory should help statistical agencies address some of the bias problems discussed in previous sections.

The most significant new development is the application by Shapiro and Wilcox (1997) of an index number formula that was independently proposed by Lloyd (1975) and Moulton (1996). This formula offers the promise of overcoming the major practical difficulty of using a superlative index like the Fisher ideal index: that information on quantities currently being consumed is typically not available for a lag of a year or more—which clearly makes it unsuitable for producing a monthly estimate like the Consumer Price Index. However, Lloyd and Moulton developed a formulation for a true cost-of-living index in the case where consumer preferences can be represented by a constant elasticity of substitution utility function.[7] Then, Shapiro and Wilcox showed that this cost-of-living index, based on a constant elasticity of substitution parameter of 0.7, could exactly capture the trend rate of growth of a U.S. superlative index (like the Fisher ideal) *while using the lagged consumer expenditure weights of two years ago.* Hence, this method can predict a superlative index on a monthly basis, using data that are presently available to statistical agencies. This methodology could be used to reduce both elementary index bias and commodity substitution bias. It should be mentioned that the superlative indexes constructed by Shapiro and Wilcox averaged .3 percentage points per year below the corresponding fixed basket Laspeyres index during the period 1984–94; therefore, an estimate of the commodity substitution bias in the U.S. CPI during this period is .3 percentage points per year.

A second major recent development is the willingness of statistical agencies to experiment with scanner data, which are the electronic data generated at the point of sale by the retail outlet and generally include transaction prices, quantities, location, date and time of purchase and the product described by brand, make or model. Such detailed data may prove especially useful for constructing better indexes at the elementary level. Recent studies that use scanner data in this way include Silver (1995), Reinsdorf (1996), Bradley, Cook, Leaver and Moulton (1997), Dalén (1997), de Haan and Opperdoes (1997) and Hawkes (1997). Some estimates of elementary index bias (on an annual basis) that emerged from these

[7] If consumer preferences over N commodities can be represented by a Constant Elasticity of Substitution utility function, then Lloyd and Moulton showed that the corresponding true cost of living index is equal to $[\sum_{n=1}^{N} s_n^0 r_n^{1-\sigma}]^{1/(1-\sigma)}$ where $\sigma \geq 0$ is the elasticity of substitution between the commodities. The base period expenditure share s_n^0, for commodity n and the price relative for commodity n, r_n, are defined as in the Appendix. Note that when the elasticity substitution σ is equal to zero, this index number formula reduces to the Laspeyres index defined by (1) in the Appendix.

studies were: 1.1 percentage points for television sets in the United Kingdom; 4.5 percentage points for coffee in the United States; 1.5 percentage points for ketchup, toilet tissue, milk and tuna in the United States; 1 percentage point for fats, detergents, breakfast cereals and frozen fish in Sweden; 1 percentage point for coffee in the Netherlands and 3 percentage points for coffee in the United States, respectively. These bias estimates incorporate both elementary and outlet substitution biases and are significantly higher than our earlier ballpark estimates of .255 and .41 percentage points. On the other hand, it is unclear to what extent these large bias estimates can be generalized to other commodities.

A third significant new development is the willingness of statistical agencies to consider using hedonic regression techniques on a more widespread basis to adjust for quality change. These methods use the characteristics of goods (either over time or in different models) as the independent variables to explain its price in a regression, and thus estimate the value of particular qualities along with a measure of general price change. For example, see Moulton, La Fleur and Moses (1997) who found an upward bias of approximately 3 percentage points per year in the U.S. CPI component for television sets using hedonic techniques.

The final type of bias, new goods bias, can be controlled by introducing new products into the CPI more quickly—scanner data will be helpful in doing this—and perhaps by utilizing the econometric techniques for estimating the imputed price decline of a new good when it first appears that were pioneered by Hausman (1996, 1997).

Conclusion

The previous section may have left the impression that it is relatively easy for statistical agencies to control for biases in the CPI. However, all of the methods discussed in the previous section that could be used to estimate biases suffer from a lack of reproducibility. As the Bureau of Labor Statistics (1997, p. 19) has noted, it must use methods that are objective, reproducible and verifiable. The basic problem is that traditional index number theory assumes that the set of commodities is fixed and unchanging from period to period, so that like can be compared to like. Unfortunately, the real world is not so accommodating: new products, outlets and consumers appear; other products, outlets and consumers disappear. It is simply impossible to decompose period-to-period value changes into completely objective price and quantity components (Dalén, 1997; Hawkes, 1997). These problems intensify when using econometric techniques for estimating new goods bias, because different econometricians will make alternative statistical specifications and thus, obtain different quality adjustments using the same data.

The numerical estimates of the biases made by the Commission were based on the information available to them at the time of writing and seem reasonable to me. The additional evidence that has come to light generally supports their contention that their estimates were relatively conservative. The present paper can be viewed as a complement to the work of the Boskin Commission in that some very rough and ready models of the determining factors of the size of the biases have been presented.[8]

I conclude with a few words of praise for both the BLS and the Boskin Commission. I believe that the origins of the current interest in price measurement can be traced back to the work of BLS researcher Marshall Reinsdorf (1993). Other prominant BLS researchers past and present who have contributed to the current debate include Aizcorbe, Armknecht, Dean, Fixler, Greenlees, Harper, Jackman, McDonald, Manser, Moulton, Pollak, Stewart, Triplett and Zieschang. On the other hand, with a total budget of $25,000, Boskin, Dulberger, Griliches, Gordon and Jorgenson have probably written the most important measurement paper of the century in terms of its impact: Every statistical agency in the world is reevaluating its price measurement techniques as a direct result of their report and the widespread publicity it has received.

Appendix: An Approximate Substitution Bias Formula

Assume that the statistical agency has collected N price quotations p_n^t, $n = 1, \ldots, N$, for two periods, $t = 0, 1$, for some homogeneous component of the CPI. Associated with each price quotation p_n^t is the amount of q_n^t of the commodity sold in period t in the nth outlet. Define the nth price ratio $r_n \equiv p_n^1/p_n^0$ for $n = 1, \ldots, N$. These price ratios can be used to define the Laspeyres, Paasche and Fisher ideal price indexes as follows:

$$P_L \equiv \sum_{n=1}^{N} s_n^0 r_n; \tag{1}$$

$$P_P \equiv \left[\sum_{n=1}^{N} s_n^1 r_n^{-1} \right]^{-1}; \tag{2}$$

$$P_F \equiv (P_L P_P)^{1/2} \tag{3}$$

where the period t outlet expenditure shares s_n^t are defined as $s_n^t \equiv p_n^t q_n^t / \Sigma_{i=1}^N p_i^t q_i^t$ for $n = 1, \ldots, N$ and $t = 0, 1$.

Statistical agencies typically use sampling techniques to ensure that outlet expenditure shares are approximately equal in the base period. We make the further simplifying assumption that outlet expenditure shares are equal in both periods; i.e., we assume that

$$s_n^t = 1/N \quad \text{for} \quad n = 1, \ldots, N \quad \text{and} \quad t = 0, 1. \tag{4}$$

Strictly speaking, outlet expenditure shares would remain constant only if con-

[8] Our models can be used to answer the question: do the biases increase as the inflation rate increases? The answer is: not very much, since each bias formula has the term $(1 + i)$ in it so a change in i from 2% to 10% will have little effect on the magnitude of the biases.

sumer preference functions over outlets were Cobb-Douglas; see Diewert (1995; p. 18). Lloyd (1975) developed more complex bias formulae when preferences are CES.

Under assumptions (4), the Laspeyres index P_L defined by (1) is equal to

$$(5) \qquad P_L = \sum_{n=1}^{N} (1/N) r_n \equiv (1 + i)$$

where we have defined $(1 + i)$ to be the arithmetic mean of the price relatives and we interpret i to be the measured inflation rate. We can now define the residual variation of each price ratio r_n deflated by its mean $1 + i$ to be ε_n:

$$(6) \qquad 1 + \varepsilon_n \equiv r_n/(1 + i); \qquad n = 1, \ldots, N.$$

Using (5), we have $\Sigma_{n=1}^{N} \varepsilon_n = 0$. Now substitute (4) and (6) into (2) in order to obtain the following formula for the Paasche price index:

$$(7) \qquad P_P = (1 + i)\left[\sum_{n=1}^{N} (1/N)(1 + \varepsilon_n)^{-1}\right]^{-1} \cong (1 + i)[1 - \mathrm{Var}(\varepsilon)]$$

where $\mathrm{Var}(\varepsilon) \equiv \Sigma_{n=1}^{N} (1/N)\varepsilon_n^2$ is the variance of the residuals $\varepsilon_1, \ldots, \varepsilon_N$ defined by (6). The approximation (7) follows from the line above by taking a second order Taylor series approximation of $[\Sigma_{n=1}^{N} (1/N)(1 + \varepsilon_n)^{-1}]^{-1}$ around $\varepsilon_n = 0$ for $n = 1, \ldots, N$; see Dalén (1992; pp. 146–147) and Diewert (1995; pp. 28–29) for examples of similar second order approximations. Note that (5) and (7) imply that $P_P \lesssim P_L$ with a strict inequality unless all $\varepsilon_n = 0$, in which case all price ratios r_n are equal and $P_P = P_L = 1 + i$. Intuitively, P_P is less than P_L because the harmonic mean of N price ratios is less than the corresponding arithmetic mean.

In a similar manner, substituting (4) and (6) into (3) and taking a second order Taylor series approximation, we obtain the following approximation to the Fisher ideal price index:

$$(8) \qquad P_F \cong (1 + i)[1 - (1/2)]\mathrm{Var}(\varepsilon).$$

Substituting (5) and (8) into $B_E \equiv P_L - P_F$ yields the approximate bias formula $B_E \cong (1/2)(1 + i)\mathrm{Var}(\varepsilon)$ which appears in the text.

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References

Boskin, Michael J., Ellen Dulberger, Robert Gordon, Zvi Griliches, and Dale Jorgenson, "Toward a More Accurate Measure of the Cost of Living." *Final Report to the U.S. Senate Finance Committee,* Washington, D.C., December 1996.

Bradley, Ralph, Bill Cook, Sylvia E. Leaver, and Brent R. Moulton, "An Overview of Research on Potential Uses of Scanner Data in the U.S. CPI," Third Conference of the International Working Group on Price Indexes, Statistics Netherlands, Voorburg, April 16–18, 1997.

Bureau of Labor Statistics, "Measurement Issues in the Consumer Price Index," BLS, Washington, D.C., June 1997.

Dalén, Jörgen, "Computing Elementary Aggregates in the Swedish Consumer Price Index," *Journal of Official Statistics,* 1992, *8,* 129–47.

Dalén, Jörgen, "Experiments with Swedish Scanner Data," Third Conference of the International Working Group on Price Indexes, Statistics Netherlands, Voorburg, April 16–18, 1997.

de Haan, Jan, and Eddy Opperdoes, "Estimation of the Coffee Price Index Using Scanner Data: Simulation of Official Practices," Third Conference of the International Working Group on Price Indexes, Statistics Netherlands, Voorburg, April 16–18, 1997.

Diewert, W. Erwin, "Exact and Superlative Indexes," *Journal of Econometrics,* 1976, *4,* 115–45.

Diewert, W. Erwin, "Superlative Index Numbers and Consistency in Aggregation," *Econometrica,* July 1978, *46*:4, 883–900.

Diewert, W. Erwin, "The Theory of the Cost of Living Index and the Measurement of Welfare Change." In Diewert, W. Erwin and Claude Montmarquette, eds. *Price Level Measurement.* Ottawa: Statistics Canada, 1983, 163–233.

Diewert, W. Erwin, "Index Numbers." In Eatwell, J., M. Milgate, and P. Newman, eds. *The New Palgrave: A Dictionary of Economics.* London: The Macmillan Press, 1987, 767–80.

Diewert, W. Erwin, "Axiomatic and Economic Approaches to Elementary Price Indexes," Discussion Paper No. 95-01, Department of Economics, University of British Columbia, Vancouver, Canada, 1995, 1–60.

Diewert, W. Erwin, "Commentary." *Federal Reserve Bank of St. Louis Review,* May/June 1997, forthcoming.

Fisher, Irving, *The Making of Index Numbers.* Boston: Houghton-Mifflin, 1922.

Gordon, Robert J., "The Consumer Price Index: Measuring Inflation and Causing It," *The Public Interest,* Spring 1981, *63,* 112–34.

Gordon, Robert J., *The Measurement of Durable Goods Prices,* National Bureau of Economic Research, Chicago: University of Chicago Press, 1990.

Gordon, Robert J., "Measuring the Aggregate Price Level: Implications for Economic Performance and Policy." In Shigehara, K., ed. *Price Stabilization in the 1990s.* London: Macmillan, 1993, 233–76.

Hausman, Jerry, "Cellular Telephone, New Products and the CPI," Department of Economics, MIT, Cambridge MA, February 1997.

Hausman, Jerry, "Valuation of New Goods Under Perfect and Imperfect Competition." In Bresnahan, Timothy and Robert Gordon, eds. *The Economics of New Products.* Chicago: The University of Chicago Press, 1996.

Hawkes, William J., "Reconciliation of Consumer Price Index Trends in Average Prices For Quasi-Homogeneous Goods Using Scanning Data," Third Conference of the International Working Group on Price Indexes, Voorburg, Netherlands, April 1997, 16–18.

Hicks, John R., "The Valuation of the Social Income," *Economica,* 1940, *7,* 105–40.

Hill, Peter, "Price and Volume Measures," *System of National Accounts 1993,* Eurostat, IMF, OECD, UN and World Bank: Brussels, Luxembourg, New York and Washington, D.C., 1993, 379–406.

Konüs, Alexander A., "The Problem of the True Cost of Living," *Econometrica,* 1939, *7,* 10–29.

Lloyd, Peter J., "Substitution Effects and Biases in Nontrue Price Indices," *American Economic Review,* 1975, *65,* 301–13.

Marshall, Alfred, "Remedies for Fluctuations of General Prices," *Contemporary Review,* 1887, *51,* 355–75.

Mills, Frank C., *The Behaviour of Prices.* New York: National Bureau of Economic Research, 1927.

Moulton, Brent R., "Constant Elasticity Cost-of-Living Index in Share Relative Form," Bureau of Labor Statistics, Washington, D.C., December 1996.

Moulton, Brent R., Timothy J. LaFleur, and Karin E. Moses, "Research on Improved Quality Adjustment in the CPI: The Case of Televisions," Bureau of Labor Statistics, Washington, D.C., July 1997.

Pollak, Robert A., "The Social Cost-of-Living Index," *Journal of Public Economics,* 1981, *15,* 311–36.

Reinsdorf, Marshall, "The Effect of Outlet Price Differentials in the U.S. Consumer Price Index." In Foss, M. F., M. E. Manser, and A. H. Young, eds. *Price Measurement and Their Uses,* NBER Studies in Income and Wealth, 1993, *57,* 227–54.

Reinsdorf, Marshall, "Constructing Basic Component Indexes for the U.S. CPI from Scanner Data: A Test Using Data on Coffee," BLS Working Paper 277, Bureau of Labor Statistics, Washington, D.C., April 1996.

Shapiro, Mathew D., and David W. Wilcox, "Alternative Strategies for Aggregating Prices in the CPI," *Federal Reserve Bank of St. Louis Review,* May/June 1997, forthcoming.

Theil, Henri, *Economics and Information Theory.* Amsterdam: North-Holland Publishing Co., 1967.

Silver, Mick, "Elementary Aggregates, Micro-Indices and Scanner Data: Some Issues in the Compilation of Consumer Price Indices," *Review of Income and Wealth,* 1995, *41,* 427–38.

Walsh, Correa M., *The Measurement of General Exchange Value.* New York: Macmillan, 1901.

White, Alan, "Outlet Types and the Canadian CPI," Department of Economics, University of British Columbia, Vancouver, Canada, 1997.