CENTRE FOR APPLIED ECONOMIC RESEARCH

WORKING PAPER
(2005/03)

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ISSN 13 29 12 70
ISBN 0 7334 2231 4
www.caer.unsw.edu.au

Faculty of Commerce and Economics

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Malmquist and Törnqvist Productivity Indexes: Returns to scale and technical progress with imperfect competition

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April 2005

Abstract
We propose a method for decomposing the Törnqvist productivity index into technical progress and returns to scale components. We derive our results in the popular Malmquist index framework, without requiring the usual assumption of perfect competition, providing an interesting generalization of previous results. This is very useful when considering methods for assessing the performance of regulated industries. Empirical implementation of the method yields estimates of productivity shocks, which are of interest in many modelling contexts.

Key words: Productivity, index numbers
JEL classification: C43, D24, E23

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* The authors are grateful for helpful comments from Bert Balk and Alice Nakamura, financial support from the Australian Research Council, the SSHRC of Canada, and the Ministry of Education and Science of Spain (Secretaría de Estado de Universidades e Investigatción, SAB2003-0234), along with the hospitality of the University of Valencia. None of the above are responsible for opinions expressed in the paper.

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1 Introduction

This paper extends the results of Caves, Christensen and Diewert (1982) (CCD) in order to demonstrate how a standard Törnqvist productivity index can be decomposed into technical change and returns to scale components. The CCD framework uses the "economic approach" to justifying the choice of index number formulae for calculating aggregate indexes of input, output and productivity. This approach specifies the index of interest in terms of theoretical indexes, assumes a particular functional form to represent the underlying technology, and then derives the index number formula which corresponds to the theoretical index.

The CCD framework does not allow the identification of the contribution of returns to scale when the underlying technology exhibits increasing returns. Our results allow for an empirical assessment of the role of any degree of returns to scale. In addition, the results here are achieved in the context of imperfect competition, meaning that the usual assumption of perfect competition to establish relationships between underlying economic functions and index number formulae is in fact unnecessary. This means that the economic approach to index numbers can be used to justify the use of index number formulae for productivity assessment even in non-competitive environments, such as the case of firms in regulated industries. This greatly strengthens the theoretical underpinnings of empirical analysis in this context.

Finally, empirical implementation of the method for determining the role of returns to scale and technical progress yields statistical errors, which can be interpreted as productivity "shocks" of the type of interest in e.g., many macroeconomic modelling contexts.

2 Malmquist Input, Output and Productivity Indexes

There has been considerable recent interest in, and debate concerning, alternative approaches to decomposing the Malmquist productivity index introduced by CCD; for a review of the issues and the debate, see e.g. Balk (2001) and Grosskopf (2003). Here we give the basic theoretical definitions for Malmquist input, output and productivity indexes, and in the next section we provide a simple method for separating technical progress and returns to scale from a Törnqvist productivity index derived from a Malmquist index using the "economic approach" to index numbers.

The input distance function $D^t(y,x)$ for periods $t=0,1$ can be defined as follows:

$$D^t(y,x) \equiv \max_{\delta} \left\{ \delta : F^t(\tilde{y}, x/\delta) \geq y_1 \right\}, \tag{1}$$

where $F^t(\cdot)$ is a production function, x is a vector of inputs and $\tilde{y}$ is a vector of outputs other than y_1.

A Malmquist input index can then be defined, for $D^0(y^0,x^0)=1$, as follows:

$$\begin{aligned} Q^0(x^1,x^0) &\equiv \frac{D^0(y^0,x^1)}{D^0(y^0,x^0)} = D^0(y^0,x^1) \\ &= \max_{\delta} \left\{ \delta : F^0(\tilde{y}^0, x^1/\delta) \geq y_1^0 \right\} \end{aligned} \tag{2}$$

The interpretation of this index is the maximum value of δ needed to deflate the input vector of period 1 onto the production surface of period 0, given the output vector of period 0. A value of the index greater than one implies that the input vector in period 1 is larger than the input vector in period 0, using the technology of period 0 as the reference technology.

Naturally, the roles of the periods can be reversed, yielding an alternative Malmquist index, for $D^1(y^1,x^1)=1$:

$$\begin{aligned} Q^1(x^1,x^0) &\equiv \frac{D^1(y^1,x^1)}{D^1(y^1,x^0)} = \frac{1}{D^0(y^0,x^1)} \\ &= 1/\max_{\delta} \left\{ \delta : F^1(\tilde{y}^1, x^0/\delta) \geq y_1^1 \right\} \end{aligned} \tag{3}$$

Equations (2) and (3) are theoretical indexes which can be implemented empirically in alternative ways. For example, one way is to use linear programming techniques to estimate the distance functions (e.g. Färe *et al.*, 1994). An alternative is to derive index number formulae from the theoretical indexes; for example, CCD (Theorem 1, page 1398) showed that the geometric mean of the two alternative input indexes (2) and (3) can be expressed as a Törnqvist input index, $Q_T(w^1,w^0,x^1,x^0)$, if the distance functions have the translog functional form (Christensen, Jorgenson and Lau (1973):

$$\begin{aligned}\frac{1}{2}\left[\ln Q^0(x^1,x^0)+\ln Q^1(x^1,x^0)\right] &= \frac{1}{2}\sum_{n=1}^{N}\left[\frac{w_n^0 x_n^0}{w^0\cdot x^0}+\frac{w_n^1 x_n^1}{w^1\cdot x^1}\right]\left[\ln x_n^1-\ln x_n^0\right] \\ &\equiv \ln Q_T(w^1,w^0,x^1,x^0), \qquad (4)\end{aligned}$$

where w^t denotes the vector of input prices corresponding to the inputs x^t, $t = 0, 1$. CCD showed that this result holds without making any assumptions on returns to scale for the translog distance function.

A similar result was shown (Theorem 2, page 1401) using translog output distance functions to derive the Törnqvist output index $Q_T(p^1, p^0, y^1, y^0)$ from the geometric mean of two alternative theoretical Malmquist output indexes, $q^0(y^1, y^0)$ and $q^1(y^1, y^0)$. With the output distance function defined as

$$d^t(y,x) \equiv \min_{\delta}\left\{\delta : g^t(y/\delta,\tilde{x}) \le x_1\right\}, \quad \text{for } t = 0,1, \qquad (5)$$

where $g^t(\cdot)$ is an input requirements function, and $\tilde{x}$ is a vector of outputs other than x_1, then

$$\begin{aligned}\frac{1}{2}\left[\ln q^0(x^1,x^0)+\ln q^1(x^1,x^0)\right] &= \frac{1}{2}\left[\frac{d^0(y^1,x^0)}{d^0(y^0,x^0)}+\frac{d^1(y^1,x^1)}{d^1(y^0,x^1)}\right] \\ &= \frac{1}{2}\sum_{m=1}^{M}\left[\frac{p_i^0 y_i^0}{p^0\cdot y^0}+\frac{p_n^1 y_n^1}{p^1\cdot y^1}\right]\left[\ln y_m^1-\ln y_m^0\right] \\ &\equiv \ln Q_T(p^1,p^0,y^1,y^0), \qquad (6)\end{aligned}$$

where p^t denotes the vector of prices corresponding to the outputs y^t.[1]

One approach to measuring productivity growth is to take ratios of Malmquist output indexes to Malmquist input indexes. Given two possible definitions for both input and output indexes, this leads to four possible productivity indexes. This approach is explained by Hicks (1961, page 22, footnote 2) as follows:[2]

> "This measure of input as a whole is not the same as the measure of output as a whole, as might perhaps be supposed at first sight. In the one case we should be

[1]Theorems 1 and 2 of CCD were generalized by theorems 3.10 and 4.7, respectively, of Balk (1998).

[2]Diewert (1992) refers to this as footnote 4, as it is the fourth footnote in the paper.

asking whether A-outputs could be produced from B-inputs with B-techniques; in the other whether B-inputs would be sufficient to produce A-outputs with A-techniques; and vice versa for the other limb of the comparison. If all went well, the relation between the measure of output and the measure of input ought to give us a measure of the improvement in technique - or, as it might be better to say, a measure of the *efficiency* with which resources are combined on the one occasion compared with the other."

Moorsteen (1961) also suggested taking the ratios of Malmquist indexes, leading Diewert (1992) to label these as "Hicks-Moorsteen" indexes.

Consider taking the geometric means of the alternative Malmquist input and output indexes, respectively, and then taking their ratios. When the distance functions have the translog functional form, as can be seen from equations (4) and (6), this corresponds to taking the ratio of a Törnqvist output index to a Törnqvist input index as a measure of productivity growth; this "standard" Törnqvist productivity index approach is used by e.g., the Bureau of Labor Statistics to construct their productivity estimates for the U.S. manufacturing sector. No assumptions need be made on the returns to scale of the underlying translog functional forms in order to derive this result; the invariance of the Malmquist input and output indexes to returns to scale assumptions was emphasized by CCD and also noted by Bjurek (1996).

However, to examine the contribution of returns to scale to aggregate productivity growth (defined as output growth divided by input growth), this invariance property is a hinderance. Therefore, as in CCD, in the following section we follow a different approach to defining productivity indexes. This allows us to derive a simple method of separating the contributions of technical change and returns to scale to productivity growth, without assuming perfect competition.

3 Returns to Scale, Technical Change and Imperfect Competition

We define the Malmquist productivity index, $\mathcal{M}(x^1, x^0, y^1, y^0)$, in terms of output distance functions, as defined in equation (5), as follows:

$$\mathcal{M}(x^1, x^0, y^1, y^0) \equiv \left[\frac{d^0(y^1, x^1)}{d^0(y^0, x^0)} \frac{d^1(y^1, x^1)}{d^1(y^0, x^0)}\right]^{1/2} \tag{7}$$

Consider the following period 0 translog output distance function, $d^0(y, x)$:

$$\begin{aligned} \ln d^0(y, x) &= \alpha_0^0 + \sum_{m=1}^{M} \alpha_m \ln y_m + \sum_{n=1}^{N} \beta_n \ln x_n \\ &+ \frac{1}{2}\sum_{i=1}^{M}\sum_{j=1}^{M} \alpha_{ij} \ln y_i \ln y_j + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} \delta_{ij} \ln x_i \ln x_j + \sum_{m=1}^{M}\sum_{n=1}^{N} \phi_{mn} \ln y_m \ln x_n. \end{aligned} \tag{8}$$

where $\delta_{ij} = \delta_{ji}$ and $\alpha_{ij} = \alpha_{ji}$ for all i and j. The first line of (8) is a familiar Cobb-Douglas distance function, while the second line adds the second-order terms which make the distance function a flexible translog functional form (Christensen, Jorgenson and Lau, 1973; Diewert, 1976). Following CCD, we do not impose constant returns to scale on this distance function as we are interested in the contribution of returns to scale to a standard index of productivity growth, and this cannot be ascertained if the nature of the returns to scale is imposed *a priori*.

The period 1 translog distance function, $d^1(y, x)$, is the same as $d^0(y, x)$ in equation (8), but with α_0^1 replacing α_0^0. Thus, we can write the following:

$$\ln d^1(y, x) = \ln d^0(y, x) + \alpha_0^1 - \alpha_0^0. \tag{9}$$

For $y_1^t = F^t(\tilde{y}^t, x^t)$, $t = 0, 1$, from (1) we then have

$$\ln d^0(y^0, x^0) = 0 \tag{10}$$

$$\ln d^1(y^1, x^1) = 0. \tag{11}$$

Therefore, from equations (8), (10) and (11), there is positive technical progress if $\alpha_0^0 > \alpha_0^1$.

From CCD (p. 1402), period t local returns to scale, ε^t, can be written as

$$\begin{aligned}
\varepsilon^t &= \left[\left. \frac{\partial d^t(y^t, \lambda x^t)}{\partial \lambda} \right|_{\lambda=1} \right]^{-1} \\
&= \frac{-1}{[d^t(y^t, x^t)]^2} \left. \frac{\partial d^t(y^t, \lambda x^t)}{\partial \lambda} \right|_{\lambda=1} \\
&= -x^t \cdot \bigtriangledown_x d^t(y^t, x^t) \\
&= -\sum_{n=1}^{N} \frac{\partial \ln d^t(y^t, x^t)}{\partial \ln x_n^t} \\
&= -\sum_{n=1}^{N} \beta_n \\
&= \varepsilon, \qquad (12)
\end{aligned}$$

where $\bigtriangledown_x d^t(y^t, x^t)$ denotes a column vector of the partial derivatives of $d^t(y^t, x^t)$ with respect to the vector of variables x, and the last equality in equations (12) comes from the restriction on the distance function in (8) that $\sum_{n=1}^{N} \beta_n$ is a constant.

From page 1404 of CCD, we have

$$\bigtriangledown_x d^t(y^t, x^t) = -\frac{\varepsilon w^t}{w^t \cdot x^t} \qquad (13)$$

where $w^t \cdot x^t$ denotes the inner product, $\sum_{n=1}^{N} w_n^t x_n^t$, for periods $t = 0, 1$.

We write the revenue maximization problem as follows:

$$\max_y \{ \sum_{m=1}^{M} P_m^t(y_m) y_m : g^t(y, \tilde{x}^t) = x_1^t \} \qquad (14)$$

where $P_m^t(y_m)$ is an inverse demand function, $\tilde{x}^t$ is the vector of inputs excluding x_1^t, and $g^t(y, \tilde{x}^t) = x_1^t$ is an input requirements function for period t. The observed price for period t for output m will be $p_m^t \equiv P_m^t(y_m^t)$, for $m = 1, \ldots, M$ and $t = 0, 1$. The nonnegative *ad valorem* monopolistic markup for the mth output in period t can be defined as follows:

$$m_m^t \equiv -\frac{\partial P_m^t(y_m^t)}{\partial y_m^t} \cdot \frac{y_m^t}{p_m^t} \geq 0. \qquad (15)$$

The first-order conditions in equations (26) on page 1400 of CCD for solving the revenue maximization problem, where prices are given exogenously and so there are no markups, are then replaced with

$$p^t[I - m^t] = \lambda^t \bigtriangledown_y g^t(y^t, \tilde{x}^t), \qquad (16)$$

for periods $t = 0, 1$, where $p^t = [p_1^t, \ldots, p_M^t]'$ is a column vector of prices, $m^t = [m_1^t, \ldots, m_M^t]'$ is the corresponding vector of markup factors, and I is an M dimensional column vector of ones. Multiplying both sides of (16) by y^t and solving for λ^t yields

$$\lambda^t = \frac{p^t[I - m^t] \cdot y^t}{y^t \cdot \nabla_y g^t(y^t, \tilde{x}^t)}, \quad t = 0, 1, \tag{17}$$

which, by substitution, allows λ^t to be eliminated from (16). This yields, for $t = 0, 1$,

$$\begin{aligned} \frac{p^t(I - m^t)}{p^t(I - m^t) \cdot y^t} &= \frac{\nabla_y g^t(y^t, \tilde{x}^t)}{y^t \cdot \nabla_y g^t(y^t, \tilde{x}^t)} \\ &= \nabla_y d^t(y^t, x^t), \end{aligned} \tag{18}$$

where the last line follows from equation (19) of CCD, page 1399.

Equations (13) and (18) give the required derivatives of the distance function in terms of "observables." Now, we proceed to re-express the (log of) the Malmquist productivity index in equation (7) as follows:

$$\begin{aligned} \frac{1}{2}[\ln d^0(y^1, x^1) &- \ln d^0(y^0, x^0)] + \frac{1}{2}[\ln d^1(y^1, x^1) - \ln d^1(y^0, x^0)] && (19) \\ &= \frac{1}{2}\ln d^0(y^1, x^1) - \frac{1}{2}\ln d^1(y^0, x^0) \\ &= \frac{1}{2}(\alpha_0^0 - \alpha_0^1) + \frac{1}{2}(\alpha_0^1 - \alpha_0^0) \quad \text{using (9) twice} \\ &= \alpha_0^0 - \alpha_0^1 \\ &\equiv r, \quad \text{a measure of technical change} && (20) \\ &= \frac{1}{2}[\nabla_{\ln y} \ln d^0(y^0, x^0) + \nabla_{\ln y} d^1(y^1, x^1)] \cdot [\ln y^1 - \ln y^0] \\ &\quad + \frac{1}{2}[\nabla_{\ln x} \ln d^0(y^0, x^0) + \nabla_{\ln x} d^1(y^1, x^1)] \cdot [\ln x^1 - \ln x^0] && (21) \\ &= \frac{1}{2}\sum_{m=1}^{M}\left[\frac{p_m^0(1 - m_m^0)y_m^0}{p^0 \cdot (I - m^0)y^0} + \frac{p_m^1(1 - m_m^1)y_m^1}{p^1 \cdot (I - m^1)y^1}\right][\ln y_m^1 - \ln y_m^0] \\ &\quad - \varepsilon \sum_{n=1}^{N}\left[\frac{w_n^0 x_n^0}{w^0 \cdot x^0} + \frac{w_n^1 x_n^1}{w^1 \cdot x^1}\right][\ln x_n^1 - \ln x_n^0] \\ &= \ln Q_T(p^0(I - m^0), p^1(I - m^1), y^0, y^1) - \varepsilon \ln Q_T(w^0, w^1, x^0, x^1), && (22) \end{aligned}$$

where (21) uses the quadratic identity of Diewert (1976, p. 118) applied to (19), $Q_T((p^0 I - m^0), p^1(I - m^1), y^0, y^1)$ is a Törnqvist output index, and $Q_T(w^0, w^1, x^0, x^1)$ is a Törnqvist input index.

Using (20) and (22) we can write the following:

$$\ln Q_T(p^0(I-m^0), p^1(I-m^1), y^0, y^1) = r + \varepsilon \ln Q_T(w^0, w^1, x^0, x^1). \tag{23}$$

Hence, a combination of technical change (r) and returns to scale (ε) will cause output to grow at a different rate to input, and thus drive productivity growth.[3]

For $t = 0, \ldots, T$, where $T \geq 2$, adding a stochastic error term to the right-hand side of (23) allows the estimation of this equation (using ordinary least squares or some other appropriate estimation procedure) to yield estimates of the parameters of interest, r and ε. The error term has the interpretation of unexplained variation in the log of the Törnqvist output index, and so represents the unexplained productivity growth. That is, in each period the error term can be considered as a productivity shock unexplained by returns to scale and technical change. In many macroeconomic models it is productivity shocks such as these which are of interest, rather than secular productivity growth driven by (constant parameter) returns to scale and technical progress. For a firm, sources of these shocks could include changes in efficiency through changing management practices.

The above results were derived using output distance functions. Theoretical indexes defined in terms of input distance functions can be similarly used, yielding

$$\hat{r} = k \ln Q_T(p^0(I-m^0), p^1(I-m^1), y^0, y^1) - \ln Q_T(w^0, w^1, x^0, x^1), \tag{24}$$

or

$$\ln Q_T(w^0, w^1, x^0, x^1) = -\hat{r} + k \ln Q_T(p^0(I-m^0), p^1(I-m^1), y^0, y^1), \tag{25}$$

where $k = 1/\varepsilon$.[4] By multiplying (24) through by ϵ and comparing with (22) we can see that the relationship between the alternative technical change measures can be expressed as $r = \varepsilon\hat{r}$. Which measure of technical change is considered depends on how one wishes to define technical change (as output increasing or input decreasing). Appended with a

[3] Nakamura, Nakamura and Yoshioka (1998) derived a similar expression from a revenue function framework, but assumed perfect competition. See also Diewert, Nakajima, Nakamura and Nakamura (2004).

[4] This result was derived by Diewert and Fox (2004) using a cost function framework, but where the monopolistic markup had been eliminated by assuming constant markups across outputs in each period.

stochastic error term, (25) is an alternative regression equation to (23). We note that in practice the estimated value of k will not equal the inverse of ε in general as the inverse of the estimate of a parameter is not equal to the estimate of the inverse in a linear relationship; see e.g., Bartelsman (1995) and Diewert and Fox (2004). Which regression is run in practice will depend on which variable is believed (more) exogenous and which is believed (more) endogenous in the specific context. Instrumental variables estimation could also be used if appropriate instruments were available (Burnside, 1996; Basu and Fernald, 1997).

Imposing constant returns to scale on the distance functions for each period using the parameter restriction $-\sum \beta_n = 1$ on the translog function form in (8), yields the returns to scale parameters $\varepsilon = k = 1$ from (12). Thus a re-arrangement of (23) or (25) yields the following expression of technical change, or productivity growth, between periods 0 and 1, $PG^{0,1}$:

$$PG^{0,1} = \exp(r) = \exp(\hat{r}) = \frac{Q_T(p^0(I-m^0), p^1(I-m^1), y^0, y^1)}{Q_T(w^0, w^1, x^0, x^1)}. \tag{26}$$

This has the standard form of a productivity index under the assumption of constant returns to scale, of an output index divided by an input index. Thus, we can see that equations (23) and (25) provide a means of separating technical progress and returns to scale contributions to productivity growth. In addition, (23) and (25) have been derived without assuming perfect competition, and returns to scale can be increasing, decreasing or constant.

In order to calculate $PG^{0,1}$ in (26), knowledge of the markup factors is needed along with the usual requirement of knowledge of quantities and their corresponding prices. However, if it can be assumed that for each period markup factors are constant across outputs or we have only one (aggregate) output, then from (22) we can see that the markup factors will cancel from the Törnqvist output quantity index so that we have the following:

$$PG_T^{0,1} = \frac{Q_T(p^0, p^1, y^0, y^1)}{Q_T(w^0, w^1, x^0, x^1)}, \tag{27}$$

which is a standard Törnqvist index of productivity growth.

That is, the usual productivity index can be derived from a theoretical Malmquist index without assuming perfect competition. This is a very useful result, particularly when considering the productivity of regulated industries. It shows that the usual productivity index can be calculated and justified by the economic approach to index numbers, rather than having to rely on e.g., the axiomatic approach to index numbers (see Fisher, 1922).

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