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The Gains from Trade and Policy Reform Revisited

*W. Erwin Diewert and Alan D. Woodland**

Abstract

The primary purpose of the paper is to provide characterizations of the conditions for welfare improvements in several situations that have received very little attention in the existing literature. The first aim is to exhibit the gains that can accrue to a country from the elimination of excess supplies as a result of a policy move from autarky to free trade. The second aim is to characterize the conditions under which the introduction of new goods into the economy will generate welfare gains. The third main area discussed is the extension of the authors' methodology to a large open economy that can influence its terms of trade. The techniques used to illustrate the gains from eliminating excess supplies and from the introduction of new goods have a much wider applicability; they may be used to obtain and synthesize several welfare results from the literature.

1. Introduction

In this paper, we consider questions that are related to a very old question: what are the gains to a country of opening up its borders to international trade?[1] We also consider the more modern and general question: what are the welfare effects of some tax policy change or of some other exogenous shock to the economy?

The primary purpose of the paper is to provide characterizations of the conditions for welfare improvements in several situations that have received very little attention in the existing literature. The first of these is to exhibit the gains that can accrue to a country from the elimination of excess supplies as a result of a policy move from autarky to free trade. This is a source of gain that has been overlooked in the recent literature, with the exception of Ohyama (1972, p. 49) and Neary and Schweinberger (1986, p. 428), but has its origins in the "vent for surplus" idea developed by Myint (1958). This source of gains accrues to the country where the autarky equilibrium has some free goods (i.e., goods that are in excess supply), and the opening up of trade eliminates these excess supplies. These gains, which are not the usual consumption and production gains that are discussed by Dixit and Norman (1980, p. 78) and Woodland (1982, p. 267), are illustrated in section 4 using the general methods developed in section 3.

The second contribution of the paper is to characterize the conditions under which the introduction of new goods into the economy will generate welfare gains. While the appearance of new goods is a fundamental feature of the modern economy and has been the subject of analysis in various "endogenous growth" models, there has been little attention paid to their role in the gains from trade and policy reform literature.

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* Diewert: Department of Economics, University of British Columbia, #997-1873 East Mall, Vancouver, BC V6T 1Z1, Canada. Tel: (604) 822-2544; Fax: (604) 822-5915; E-mail: diewert@econ.ubc.ca. Woodland: Econometrics and Business Statistics, School of Economics & Political Science, University of Sydney, Sydney, NSW 2006, Australia. Tel: 61 (2) 9351-6825; Fax: 61 (2) 9036-9121; E-mail: A.Woodland@econ.usyd.edu.au. We thank, without implicating, Kala Krishna, Michihiro Ohyama, and the referees of this journal for valuable comments on a previous draft. Financial support from the Australian Research Council, the Social Sciences and Humanities Research Council of Canada, and the University of Sydney Modeling of Economic and Financial Behaviour Research Cluster Fund is gratefully acknowledged.

Accordingly, our analysis of the gains from the introduction of new goods in section 5 is offered as a step in the direction of redressing this deficiency in the literature, as discussed by Romer (1994). Our analysis of this source of welfare gain requires a formulation of the model to allow consumer-specific and producer-specific "taxes," and so the traditional formulation of trade gains is not sufficient for our purposes.

The third main area discussed is the extension of our methodology to a large open economy that can influence its terms of trade. Our measure of welfare improvement now includes terms reflecting a shift in the foreign offer set and the optimality (or otherwise) of tariff and tax policies. We develop, in section 6, a new sufficiency condition for a welfare improvement that applies if the country imposes optimal tariffs in the new situation.

The techniques that we use to illustrate the gains from eliminating excess supplies and from the introduction of new goods have a much wider applicability. Thus, in sections 2 and 3 we set out a general-equilibrium model of an open economy that can be adopted to study a wide range of problems involving the welfare consequences of discrete changes in the economy. Our main task in these sections is the derivation of alternative expressions for the decomposition of a measure of welfare change, which we call the "aggregate quasi-variation." This variation gives cardinal measures of changes in consumers' welfare between two periods and is evaluated at an arbitrarily given vector of reference prices. Our main special cases of this general expression are the identities for the aggregate quasi-equivalent variation and for the aggregate compensating variation, evaluated at the initial- and second-period world market prices, respectively. These quasi-variations reduce to the usual compensating and equivalent variations originally defined by Hicks (1942), provided that there are no consumer commodity tax distortions in the economy. Our formulas for these quasi-variations bear some similarities to identities derived by Grinols and Wong (1991), who, as in the early welfare analysis of Ohyama (1972), decomposed aggregate changes in real income into various components of endowment growth, production growth, tax revenue growth, and changes in the terms-of-trade. Our analysis is similar to these authors in that we also provide decomposition. However, the focus of our paper is the application of our welfare identities to the treatment of new goods and goods in excess supply.

2. Model of a Small Open Economy

We assume that there are N internationally traded goods and domestic goods and/or resources in the economy. There are H households or consumers, K firms, and one (consolidated) government sector in the economy. It is further assumed that the economy is small in the usual sense that all agents (households, firms, and the government) take world prices as exogenously given.

The primary aim of the paper is to compare welfare in two situations or periods, which are distinguished by superscript $t \in T \equiv \{0, 1\}$. For period $t \in T$ and household $h \in H$, let the consumption vector for all goods (both internationally traded and do↑mestic) be c^{ht}, where factor supplies are indexed with negative signs.[2] Let y^{kt} denote the observed period t net supply vector for firm k (input demands are indexed with negative signs). The net export vector for the economy in period $t \in T$ is denoted as x^t (if $x_i^t < 0$, then the ith good is imported into the economy in period t). The vector v^t represents the sum of the economy's endowments of goods less the government's (fixed) net demand vector for goods.

In period t, the market balance conditions for the economy for goods are given by

$$\sum_{h\in H} c^{ht} = \sum_{k\in K} y^{kt} - x^t + v^t - e^t, \quad t \in T. \tag{1}$$

The vector e^t is the period t excess supply vector. We assume, consistent with market equilibrium, that these vectors are nonnegative; i.e.,

$$e^t \geq 0 \quad \text{for } t \in T, \tag{2}$$

and that e_i^t can be positive only if all demanders of the ith good face zero prices for this good.

Turning now to the price side of the economy, we assume that the period t price vector for goods is $p^t \geq 0$. Since the economy has been assumed to be a small open economy, the prices for internationally traded goods are given exogenously, while the market prices for domestic goods are endogenously determined to clear domestic markets.[3] In period $t \in T$, household h faces the price vector $p^t + t^{ht} \geq 0$, where t^{ht} should usually be interpreted as a vector of commodity tax distortions faced by household h in period t, but other interpretations are possible as we shall see later. In period $t \in T$, firm k faces the price vector $p^t + \tau^{kt} \geq 0$, where τ^{kt} should usually be interpreted as a vector of commodity tax distortions faced by firm k in period t. In the usual case considered in international trade theory, there are no domestic tax distortions and a common tariff vector is faced by all consumers and producers in period t. However, our general formulation allows us to deal with household- and industry-specific taxes and subsidies.

Competitive optimizing behavior on the part of consumers and producers is assumed in both periods. We assume that household h's preferences over various combinations of goods can be represented by means of the utility function f^h. The utility level attained in period t by household h is $u^{ht} \equiv f^h(c^{ht})$, where c^{ht} is the observed period t consumption vector for household h. Expenditure-minimizing behavior for household h in each period is assumed, whence

$$\begin{aligned} m^h(u^{ht}, p^t + t^{ht}) &\equiv \min_c \{(p^t + t^{ht}) \cdot c : f^h(c) \geq u^{hr}\} \\ &\equiv (p^t + t^{ht}) \cdot c^{ht} \qquad\qquad t \in T, \quad h \in H, \end{aligned} \tag{3}$$

where $p^t \cdot c$ denotes the inner product of the vectors p^t and c, and m^h is the expenditure function for household h.[4]

Turning now to producers, we assume that firm $k \in K$ has a feasible set of net output vectors, S^{kt}, in period $t \in T$. The assumption of profit-maximizing behavior for firm k in period t may be represented as

$$\pi^k(p^t + \tau^{kt}; S^{kt}) \equiv \max_y \{(p^t + \tau^{kt}) \cdot y : y \in S^{kt}\} \equiv (p^t + \tau^{kt}) \cdot y^{kt}, \quad t \in T, \quad k \in K, \tag{4}$$

where (4) defines the profit function π^k for firm k, and y^{kt} is the firm k observed period t net output vector.[5]

3. Welfare Identities

Producer and Consumer Substitution Functions

Before we define our main welfare identities, it is useful to detour briefly and define various consumer and producer substitution functions. These functions characterize the gains from substitution that consumers and producers experience as a result of price changes in the economy. These functions are then used to help decompose aggregate welfare changes into easily interpreted components. They are evaluated at an arbitrarily given reference price vector, specific choices of which will be considered further below.

Consider the problem of minimizing the expenditure required for household h to attain its period r utility level, u^{hr}, but using the price vector p instead of the actual period r prices faced by household h, $p^r + t^{hr}$. Define the *household h substitution function* $s_{hr}(p)$ (hr refers to the utility level attained by household h in period r) by

$$s_{hr}(p) \equiv p \cdot c^{hr} - m^h(u^{hr}, p) \geq 0, \quad r \in T, \quad h \in H, \tag{5}$$

where the inequality follows since c^{hr} is feasible for the minimization problem (3) but is not necessarily optimal. This expression gives the cost, at the reference price vector p, of the consumption vector actually consumed in period r minus the minimum expenditure needed to attain period r utility at this reference price vector. It therefore represents a measure of the gains from substitution around the period r indifference curve.[6]

Turning to the producer side of the model, consider a hypothetical sector k profit-maximization problem where firm k faces the price vectors p and has available the period r technology set S^{kr}. Define the *firm k substitution function* $\sigma_{kr}(p)$ (kr refers to the firm k technology set in period r, S^{kr}) by

$$\sigma_{kr}(p) \equiv \pi^k(p; S^{kr}) - p \cdot y^{kr} \geq 0, \quad r \in T, \quad k \in K, \tag{6}$$

where the inequality follows since y^{kr} is feasible for the maximization problem but is not necessarily optimal. This expression is the profit attained in period r at reference price p minus the profit (at this same reference price vector) attained using the actual period r production point. It therefore represents a measure of the gains from substitution around the period r transformation frontier for firm k.

There is one additional set of definitions that we require in subsequent sections. We first relate the firm k production possibilities set in period 0, S^{k0}, to the firm k production possibilities set in period 1, S^{k1}, by defining the *firm k technological change function* α_k, using the reference prices p, by

$$\alpha_k(p) \equiv \pi^k(p; S^{k1}) - \pi^k(p; S^{k0}), \quad k \in K. \tag{7}$$

It can be seen that $\alpha_k(p)$ is a measure of the expansion (if positive) or contraction (if negative) in firm k's production possibilities set going from period 0 to 1. In general, the firms' production possibilities sets may be quite different in the two periods under consideration, and so the sign of the function $\alpha_k(p)$ is ambiguous. However, as a special case of some interest, it may be assumed that there is no technological regress; i.e., that

$$S^{k0} \text{ is a subset of } S^{k1}, \quad k \in K. \tag{8}$$

Using this assumption, it is easy to see that

$$\pi^k(p; S^{k1}) \equiv \max_y\{p \cdot y : y \in S^{k1}\} \geq \max_y\{p \cdot y : y \in S^{k0}\} \equiv \pi^k(p; S^{k0}), \tag{9}$$

and hence that the firm k technological progress function is nonnegative, $\alpha_k \geq 0$. In this special case, it can be seen that $\alpha_k(p)$ is a measure of the *expansion* in firm k's production possibilities set going from period 0 to 1.

It is useful to record the situations where the various functions defined above take zero values. For future reference, we note that

$$
\begin{aligned}
&s_{hr}(p) \equiv p \cdot c^{hr} - m^h(u^{hr}, p) = 0 && \text{if } p = p_C^{hr} \equiv p^r + t^{hr}; \quad h \in H, \quad r \in T \\
&\sigma_{kr}(p) \equiv \pi^k(p; S^{kr}) - p \cdot y^{kr} = 0 && \text{if } p = p_S^{kr} \equiv p^r + \tau^{kr}; \quad k \in K, \quad r \in T \\
&\alpha_k(p) \equiv \pi^k(p; S^{k1}) - \pi^k(p; S^{k0}) = 0 && \text{if } S^{k1} = S^{k0}; \quad k \in K.
\end{aligned} \tag{10}
$$

Thus, the consumer substitution function is zero if it is evaluated at the period r price vector facing that consumer, while the producer substitution function is zero if it is evaluated at the period r price vector facing that producer. If the technology does not change then the technology progress function is zero for any reference price vector.

Now we are ready to derive our main results.

The Basic Identities

Define the *money metric change in utility*[7] for household h going from period 0 to 1 using the reference prices, p, by

$$
V^h(p) \equiv m^h(u^{h1}, p) - m^h(u^{h0}, p), \quad h \in H. \tag{11}
$$

If we choose $p = p_C^{h1} \equiv p^1 + t^{h1}$, then (11) becomes Hicks' (1942, p. 128) *compensating variation* for household h; and if we choose $p = p_C^{h0} \equiv p^0 + t^{h0}$, then (11) reduces to Hicks' (1942, p. 128) *equivalent variation* for household h. Each of these money metric measures is a valid measure of individual welfare change. More generally, it is straightforward to show that, under suitable regularity conditions, a necessary and sufficient condition for $u^{h1} > u^{h0}$ is that $V^h(p) > 0$ for some $p \gg 0$.

To handle an economy consisting of many households, we consider an aggregation of household variation measures. Thus, the *aggregate variation* $V(p)$ is defined by

$$
V(p) \equiv \sum_{h \in H} V^h(p) \equiv \sum_{h \in H} \{m^h(u^{h1}, p) - m^h(u^{h0}, p)\}, \tag{12}
$$

which is the sum of households' money metric changes in utility. If all consumers face the same price vector, $p = p_C^t$, the functions $V^h(p_C^0)$ and $V^h(p_C^1)$ are the aggregate equivalent variation and the aggregate compensating variation, respectively.

This aggregate measure of welfare change is valid in a multihousehold economy if lump-sum transfers between the government and consumers are permitted and if prices of goods that enter households' utility functions do not change as a result of the transfers that take place.[8] In our small open economy context, in which the prices of internationally traded goods are exogenously given, this latter condition will be satisfied if the prices of nontraded goods are determined solely by international prices and the technology and not by household preferences or the income distribution. This property occurs in a final-goods production model when the number of produced goods is at least as great as the number of fully used fixed factors (Woodland, 1982, pp. 227–32). This assumption, commonly employed in the literature, has been used, for example, by Wong (1991, p. 51) in his analysis of welfare comparisons.

The expression for the aggregate variation may be rewritten in several different ways that allow for interesting economic interpretations. Using (5), the aggregate variation may be expressed as the identity

$$
V(p) \equiv \sum_{h \in H} V^h(p) = \sum_{h \in H} s_{h0}(p) - \sum_{h \in H} s_{h1}(p) + \sum_{h \in H} p \cdot (c^{h1} - c^{h0}). \tag{13}
$$

If we now replace the aggregate consumption vectors by equivalent vectors using the material balance equations in (1) for periods $t = 0$ and $t = 1$, we obtain the following identity:

$$V(p) \equiv \sum_{h\in H} s_{h0}(p) - \sum_{h\in H} s_{h1}(p) + \sum_{k\in K} p\cdot(y^{k1} - y^{k0}) - p\cdot(x^1 - x^0) + p\cdot(v^1 - v^0) - p\cdot(e^1 - e^0). \quad (14)$$

Now eliminate the production vectors y^{kt} from (14) using definitions (6) and (7). We thus obtain the third identity:

$$V(p) = \sum_{h\in H} s_{h0}(p) - \sum_{h\in H} s_{h1}(p) + \sum_{k\in K} \sigma_{k0}(p) - \sum_{k\in K} \sigma_{k1}(p) + \sum_{k\in K} \alpha_k(p) - p\cdot(x^1 - x^0) + p\cdot(v^1 - v^0) - p\cdot(e^1 - e^0). \quad (15)$$

Expressions (13)–(15) provide three interesting identities for the aggregate variation. The first identity (13) shows that the aggregate variation equals a difference of nonnegative consumer substitution functions plus an index of consumption growth, using as weights the reference prices. The second identity (14) replaces the index of consumption growth by indices of growth in its various components: output, net imports (the negative of net exports), endowments, and excess demands (negative of excess supplies). In the third identity (15), the aggregate variation of output growth on the right-hand side of (14) has been replaced by the following three sets of terms: (i) a nonnegative sum of producer substitution functions, $\Sigma_{k\in K}\sigma_{k0}(p)$; (ii) a nonpositive sum of producer substitution functions, $-\Sigma_{k\in K}\sigma_{k1}(p)$; and (iii) a sum of technical change effects, $\Sigma_{k\in K}\alpha_k(p)$, that are nonnegative if there is no technical regress.

The identity (15) is one of the main results in the paper. In a one-consumer economy, it decomposes an exact index of welfare change into various consumer and producer substitution functions and additional components that are sources of growth, including technical progress, growth in real imports, growth in endowments, and reductions in excess supplies. This identity is similar in some respects to one derived by Ohyama (1972, p. 47).[9] However, Ohyama deals exclusively with consumer prices and does not explicitly define the consumer and producer substitution functions. Our identity is also similar to an expression derived by Grinols and Wong (1991, pp. 431–4) as an exact measure of welfare change. While Grinols and Wong also define consumer and producer substitution functions, they use domestic market prices for evaluation. As will be seen below, it is convenient to deal with the general identity above, and then consider particular choices of reference prices to establish particular results.

4. Gains from Eliminating Excess Supplies

In this section, we use the welfare decomposition developed above to characterize the gains from trade that can arise from the elimination of excess supplies. Some goods that may be in excess supply, and hence have zero prices, in the autarky equilibrium may command positive prices in a free-trade situation due to strong foreign demand. The result is that there is a welfare gain arising from this elimination of the excess supply.

Assume that autarky equilibrium occurs in period $t = 0$ and that free trade occurs in period $t = 1$. Furthermore, assume for simplicity that there are no taxes before or after trade, no technical change, and no endowment changes. We now consider the case where some goods are free in the autarky equilibrium, so that there are excess supplies of some commodities in the period $t = 0$ equilibrium, but that there are no excess supplies in the free-trade equilibrium, so that $e^1 = 0$. Under these conditions, we have $p^0 \cdot e^0 = 0$ and the aggregate quasi-compensating variation may be expressed as

$$V(p^1) = \sum_{h \in H} s_{h0}(p^1) + \sum_{k \in K} \sigma_{k0}(p^1) + p^1 \cdot e^0. \tag{16}$$

We see that the traditional consumer and producer gains, given by the first two non-negative terms, are now augmented by an additional term that reflects the disappearance of free goods. If the vector of excess supplies in autarky, e^0, contains positive elements and the corresponding elements of the free trade price vector, p^1, are positive then the last term in (16) will be positive, hence providing a sufficient condition for a welfare improvement. Thus we have the following proposition.

PROPOSITION 1. *Assume that a small open economy moves from autarky to free trade and that there are no taxes before or after trade, no technical change, and no endowment changes. A sufficient condition for a welfare gain is that there is some good that is in excess supply in autarky, but trades at a positive price under free trade.*

This can be an important source of gains that seems to have been overlooked in the traditional trade literature, with the exception of the papers by Ohyama (1972, p. 49) and Neary and Schweinberger (1986, p. 428). Neary and Schweinberger point out the potential gains from trade due to previously free factors being positively priced in free trade. However, as expected, our formulation applies equally to the elimination of surpluses in either factors or goods.[10,11] It also applies to any change in circumstance or policy, not just to the move from autarky to free trade. Accordingly, it provides a formalization and generalization of (at least one aspect of) the idea of a vent for surplus' gain from trade as initially conceived by Adam Smith and further developed by Myint (1958) and Findlay (1970, pp. 70–6).

The gains-from-trade identity is illustrated in Figure 1 for the special case where there are two traded goods, one household and one firm ($N = 2$, $H = 1$, and $K = 1$) and where nontraded goods do not enter the utility function and hence are in fixed supply or demand. The economy's production possibilities set for traded goods is represented by the curve D′D. The autarky consumption vector is c^{h0} and the autarky production vector is y^{k0}. It can be seen that good 1 is in excess supply in the autarky equilibrium and its price is zero. When the region is opened up to trade at positive prices, the production vector becomes y^{k1} and the consumption vector becomes c^{h1}.

The measure of trading gains, $V(p^1)$, uses the international prices under free trade, p^1, as reference prices. If we measure the gains in terms of good 1, we have that $V(p^1)$ = AE. This consists of $s_{h0}(p^1)$, the consumption substitution gain AB, plus $\sigma_{k0}(p^1)$, the production substitution gain CE, plus $p^1 \cdot e^0$, the elimination of excess supply gain BC.

Turning to the equivalent variation measure of trading gains represented by (15), evaluated at the autarky prices, we find that we can no longer measure gains and losses in terms of good 1 because good 1 is a free good in the autarky equilibrium. However, if we measure the gains in terms of good 2, we find that $V(p^0)$ = D′E′ = $p^0 \cdot (-x^1)$. This consists of the net import vector valued at autarky prices given by the distance C′F′, less $s_{h1}(p^0)$, the consumer substitution function E′F′, less $\sigma_{k1}(p^0)$, the producer substitution function C′D′. Note that the excess supply elimination gain does not explicitly show up in this decomposition of the gains from trade.

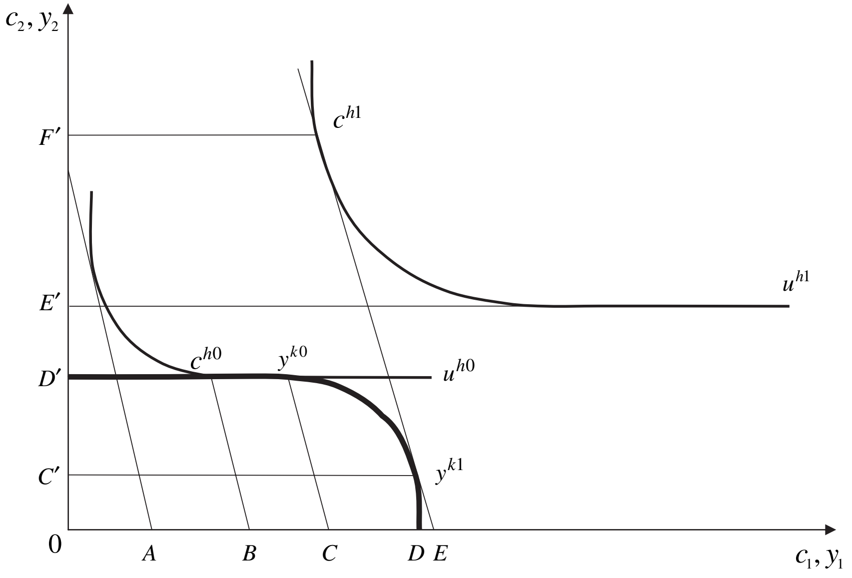

Figure 1. The Gains from Elimination of Excess Supplies

5. Gains from the Introduction of New Goods

Nowadays goods are being introduced into most economies at an increasingly rapid pace. Despite this, the literature on the role of new goods in international trade is rather sparse. A discussion of why this may be so, and of the importance of developing the theoretical analysis of new goods in the analysis of trade and trade policy, has been provided by Romer (1994). In addition, Feenstra (1994) provides an analysis of the role that new goods play in empirical research in international trade, specifically in the measurement of price indices. In this section, we indicate how our gains-from-trade framework may be adapted to deal with the introduction of a new good into an open economy.

While we do not explain the creation of new goods, we model their introduction as a result of profit- and utility-maximizing choices: new goods are introduced when current prices yield positive production, while previous prices did not. Our characterization of the gains from the introduction of new goods in terms of the aggregate quasi-variation developed above thus requires us to determine shadow prices for new goods before they were "introduced." These shadow prices are constructed using the concept of restricted profit and expenditure functions and, since they are firm- and consumer-specific, our general formulation, involving firm- and consumer-specific taxes, is required for the analysis to proceed.

In period 1, a new internationally traded good, say good 1, is introduced into the world economy. We assume for simplicity that there are no commodity tax distortions in either period so that, in particular, we have

$$t^{h1} = 0, \quad \tau^{k1} = 0, \quad h \in H; \quad k \in K. \tag{17}$$

The vector of period 1 equilibrium prices is $p^1 \equiv (p_1^1, \tilde{p}^1)$, where $\tilde{p}^1 \equiv (p_2^1, \ldots, p_N^1)$ is defined as the vector of period 1 prices excluding the new good. The vector of period 0 equilibrium prices is $\tilde{p}^0 \equiv (p_2^0, \ldots, p_N^0)$.

We assume that consumers have preferences over the new good even before it is introduced but they are restricted to consume zero units of it in period 0.[12] We now look for a shadow or virtual price for good 1 that will just induce household h to consume zero units of the new good in period 0.[13] How can we find these shadow prices?

For $h \in H$, define the *restricted expenditure function* for household h as[14]

$$\tilde{m}^h(u^h, c_1, \tilde{p}) \equiv \min_{\tilde{c}}\{\tilde{p} \cdot \tilde{c} : f^h(c_1, \tilde{c}) \geq u^h\}, \tag{18}$$

where f^h is the household h utility function, $\tilde{p}$ and $\tilde{c}$ are vectors of prices and quantities, c_1 is consumption of the new good, and u^h is a reference utility level. We assume that the observed period 0 consumption vector $\tilde{c}^{h0}$ for household h solves (18) when $u^h = u^{h0}$, $c_1 = 0$, $\tilde{p} = \tilde{p}^0$; i.e., we have

$$\tilde{m}^h(u^{h0}, 0, \tilde{p}^0) = \tilde{p}^0 \cdot \tilde{c}^0, \quad h \in H. \tag{19}$$

Assuming that the derivative (from the right) exists, the appropriate household h shadow price for good 1 in period 0 may be defined as[15]

$$p_1^{h0} \equiv \partial\tilde{m}^h(u^{h0}, 0, \tilde{p}^0)/\partial c_1, \quad h \in H. \tag{20}$$

Using expression (20) for the appropriate household h shadow price for good 1 in period 0, the household-specific distortion vectors are defined by choosing the reference price $p_1^0 \equiv p_1^1$ for the new good, and "taxes" according to

$$t_1^{h0} \equiv p_1^{h0} - p_1^0, \quad t_i^{h0} \equiv 0, \quad i \neq 1, \quad i \in N, \quad h \in H. \tag{21}$$

On the producers' side of the economy, we assume that the technology includes the new good even before it is introduced but producers are restricted to produce zero units of it in period 0. Accordingly, we look for a period 0 shadow price for good 1 that would induce firm k to supply a zero quantity of the new good. To do this we define the *restricted profit function* for firm $k \in K$ as[16]

$$\tilde{\pi}^k(y_1, \tilde{p}, S^{k0}) \equiv \max_{\tilde{y}}\{\tilde{p} \cdot \tilde{y} : (y_1, \tilde{y}) \in S^{k0}\}, \tag{22}$$

where $\tilde{y}$ is a vector of quantities excluding the first good. We assume that the observed period 0 production vector $\tilde{y}^{h0}$ for firm k solves (22) when $y_1 = 0$ and $\tilde{p} = \tilde{p}^0$; i.e., we have

$$\tilde{\pi}^k(0, \tilde{p}, S^{k0}) \equiv \tilde{p}^0 \cdot \tilde{y}^0, \quad k \in K. \tag{23}$$

Assuming that the derivative (from the right) exists, the appropriate firm k shadow or virtual price for good 1 in period 0 may be defined as[17]

$$p_1^{k0} \equiv -\partial\tilde{\pi}^k(0, \tilde{p}, S^{k0})/\partial y_1, \quad k \in K. \tag{24}$$

Using the shadow prices defined by (24), the firm-specific distortion vectors are defined by using the reference price $p_1^0 \equiv p_1^1$ for the new good, and "taxes" according to

$$\tau_1^{k0} \equiv p_1^{k0} - p_1^0, \quad \tau_i^{k0} \equiv 0, \quad i \neq 1, \quad i \in N, \quad k \in K. \tag{25}$$

With these household- and firm-specific distortions defined, the reference prices determine the welfare effects in the usual way, as is now illustrated. To focus exclusively on the introduction of a new good we assume no technical progress, so that $S^{k0} = S^{k1}$ for each k, no endowment change, and no excess supply of goods in each period.

Then expression (15) for the aggregate variation, evaluated at period 1 world prices, reduces to

$$V(p^1)=\sum_{h\in H} s_{h0}(p^1)+\sum_{k\in K} \sigma_{k0}(p^1)-p^1\cdot(x^1-x^0), \tag{26}$$

since $s_{h1}(p^1) = 0$, $\sigma_{k1}(p^1) = 0$, and $\alpha_k(p^1) = 0$ for all households and firms, $v^1 = v^0$ and $e^1 = e^0$. By a similar argument, expression (15) for the aggregate variation, evaluated at period 0 world prices, reduces to[18]

$$V(p^0)=\sum_{h\in H} s_{h0}(p^0)-\sum_{h\in H} s_{h1}(p^0)+\sum_{k\in K}\sigma_{k0}(p^0)-\sum_{k\in K}\sigma_{k1}(p^0)-p^0\cdot(x^1-x^0). \tag{27}$$

In the following, it is assumed, for simplicity of exposition, that international prices are constant ($p^0 = p^1$). In this case, the substitution terms $s_{h1}(p^0)$ and $\sigma_{k1}(p^0)$ vanish from (27) and the two aggregate variation measures above are seen to be identical. If, further, there is a zero trade balance in the two periods ($p^t\cdot x^t = 0$), then the terms $-p^1\cdot(x^1 - x^0)$ and $-p^0\cdot(x^1 - x^0)$ equal 0. Under these conditions, (26) shows that the country will unambiguously gain from the introduction of the new good from abroad if any of the producer or consumer substitution functions are positive. Accordingly, we have established the following proposition.

PROPOSITION 2. *Assume that a new good is introduced into a small open economy from abroad and that there are no taxes, no technical change, no excess supplies, zero trade balances in each period, and no changes in the terms of trade. The positivity of any producer or consumer substitution function in the aggregate quasi-compensating variation (26) is sufficient for a welfare improvement as a result of the introduction of the new good from abroad.*

The special case where there are two traded goods, one household, one firm, no price changes, and zero balances of trade is illustrated in Figure 2. The initial consumption vector is $c^{h0} = y^{k0}$ at C′, where no units of good 1 are consumed or produced. When the new good is introduced, production moves to y^{k1} and consumption moves to c^{h1}. The constant world prices are $p_1^1 = p_1^0$ and $p_2^1 = p_2^0$ and the lines C′C, D′D and B′B have slopes equal to $-p_1^1/p_2^1$. The slope of the line C′A equals $-p_1^{h0}/p_2^0$ and the slope of the line C′E equals $-p_1^{k0}/p_2^0$. Under our simplifying assumptions, the aggregate variation (26) becomes

$$\begin{aligned} V(p^1) &= s_{h0}(p^1)+\sigma_{k0}^0(p^1)\\ &= \text{BC}+\text{CD}\\ &= \text{BD}, \end{aligned} \tag{28}$$

where the substitution terms are $s_{h0}(p^1) \equiv p^1\cdot c^{h0} - m^h(u^{h0}, p^1) = \text{BC}$ and $\sigma_{k0}(p^1) \equiv \pi^k(p^1; S^{k0}) - p^1\cdot y^{k0} = \text{BD}$. Due to the simplicity of the model and our assumption that the terms of trade are constant, our two measures of gain coincide in this case and aggregate variation (27) is readily shown to yield the same outcome.

The above model assumed that the technology was capable of producing the new good in period 0 but producers were constrained to supply zero units of it; the introduction of the new good involved no technical change. An alternative model assumption is that the introduction of the new good involves a change in the technology—possibly affecting the producer's ability to produce all goods.[19] Under this assumption, producers are simply unable to produce the new good in period 0 but tech-

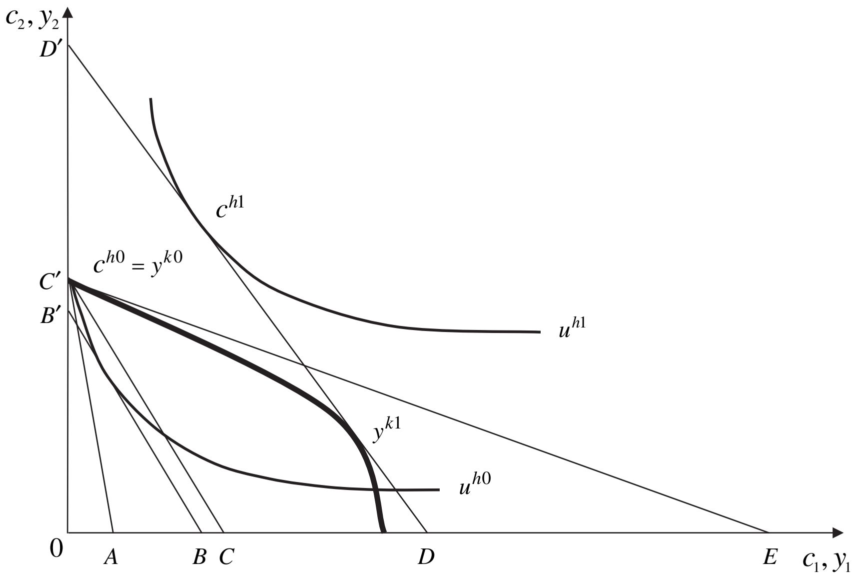

Figure 2. The Gains from New Goods

nological change occurs just before period 1, allowing producers to produce and utilize the new commodity in period 1.

This new set of assumptions concerning the nature of the technological change can be accommodated using our existing notation by adding the following restriction to the period 0 technology sets S^{k0}:

$$(y_1, \tilde{y}) \in S^{k0} \text{ implies } y_1 = 0, \quad k \in K. \tag{29}$$

It can also be seen that the previously defined firm k substitution terms continue to satisfy the inequalities in (6), specifically that $\sigma_{k0}(p) \equiv \pi^k(p; S^{k0}) - p \cdot y^{k0} \geq 0$ if $y^{k0} \in S^{k0}$. We also make the assumption that there is no technological regress as a result of the introduction of the new commodity and the associated new technology in period 1. That is, we assume that $S^{k0} \subset S^{k1}$, $k \in K$, as given by (8). Assumptions (29) and (8) on technological change imply that the introduction of the new good is costless.

In addition to the above assumptions, assume constant endowments, no excess supplies in each period, constant world prices ($p^0 = p^1$), and balanced trade in each period. Under these assumptions, expression (15) for the aggregate variation, evaluated at the (constant) world prices, reduces to

$$V(p^1) = \sum_{h \in H} s_{h0}(p^1) + \sum_{k \in K} \sigma_{k0}(p^1) + \sum_{k \in K} \alpha_k(p^1). \tag{30}$$

Under these conditions, (30) shows that the country will unambiguously gain from the combined effects of technological change and the introduction of the new commodity if any of the consumer substitution terms, producer substitution terms, or the technical change terms are positive. Accordingly, we have established the following proposition.

PROPOSITION 3. *Assume that there is costless technological change between two periods that allows for the production of new commodities introduced from abroad. In*

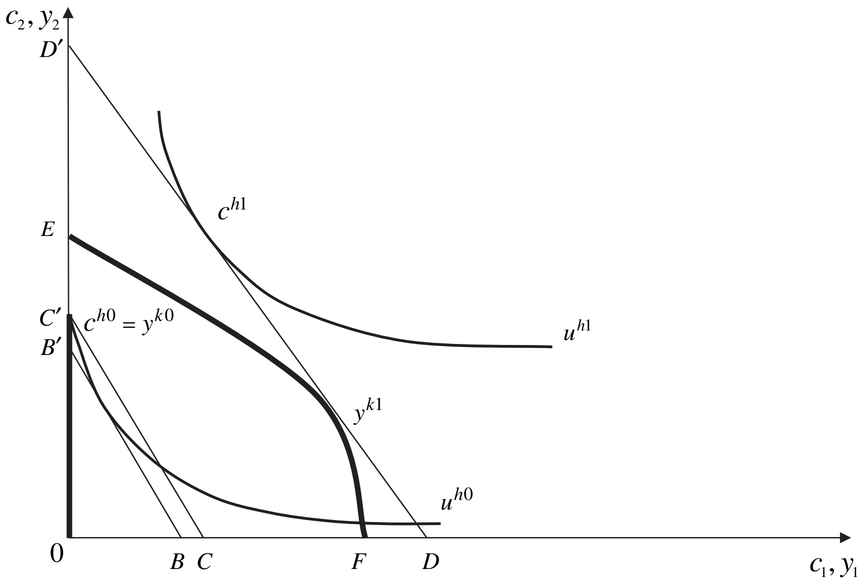

Figure 3. The Gains from a New Goods Technology

addition, assume no changes in endowments, constant world prices, no excess supplies, no taxes, and a zero balance of trade in each period. Then the positivity of any producer, consumer, or technical progress terms in the aggregate quasi-compensating variation (30) is sufficient for a welfare improvement.

Figure 3 illustrates this proposition by extending the example illustrated in Figure 2 where there are two traded goods, one household, one firm, no price changes, and zero balances of trade. In period 0, the production possibilities set is given by the line segment OC′. The period 0 consumption and production points satisfy $c^{h0} = y^{k0}$, where no units of good 1 are consumed or produced. When the new good and associated technology are introduced, the production possibilities set becomes OEF. Production moves to y^{k1} and consumption moves to c^{h1}. The constant world prices are $p_1^1 = p_1^0$ and $p_2^1 = p_2^0$ and the lines B′B, C′C, D′D and E′E have slopes equal to $-p_1^1/p_2^1$. Under our simplifying assumptions, the aggregate variation (30) becomes

$$\begin{aligned} V(p^1) &= s_{h0}(p^1) + \sigma_{k0}(p^1) + \alpha_k(p^1) \\ &= \mathrm{BC} + 0 + \mathrm{CD} \\ &= \mathrm{BD}, \end{aligned} \tag{31}$$

where the consumer substitution, producer substitution, and technical change terms are $s_{h0}(p^1) \equiv p^1 \cdot c^{h0} - m^h(u^{h0}, p^1) = \mathrm{BC}$, $\sigma_{k0}(p^1) \equiv \pi^k(p^1; S^{k0}) - p^1 \cdot y^{k0} = 0$, and $\alpha_k(p^1) \equiv \pi^k(p^1; S^{k1}) - \pi^k(p^1; S^{k0}) = \mathrm{CD}$. Thus, the gain in welfare is measured as the sum of the consumer substitution term BC and the technical change term CD. The latter term embodies the inextricably linked introduction of the new good and the associated change in technology.

The above analysis essentially assumed that the technology for producing the new commodity can be costlessly imported from abroad; that is, resident producers did not have to incur any research and development costs in order to produce the new commodity. In the case where the technology to produce the new good requires the expenditure of domestic resources, the introduction of the new goods will no longer necessarily increase welfare.

For example, consider the case where there are only two outputs in the economy in period 1 and the new commodity is not producible in period 0. In this case, the output production possibilities sets in period 0 would simply be line segments along the y_2 axis emanating from the origin; i.e., no units of y_1 would be producible in period 0. In this case, we simply set $y_1 = 0$ in definition (22). Since the derivatives defined by (24) would not exist in this case, we simply replace the p_1^{k0} defined in (24) by the reference price p_1^0. With these alternative definitions, the firm-specific "taxes" τ_1^{k0} defined by (25) all become zero. This is as it should be, since, under the assumption that the technology for producing the new good in period 0 did not exist, there are no producer substitution effects in period 0. However, it is no longer the case that the technological change functions, $\alpha^k(p^1)$, are necessarily zero, since the existence of research and development costs means that we cannot assume that S^{k0} is a subset of S^{k1} for each k. Thus, the expression (15) for the aggregate variation, evaluated at period 1 prices, now reduces to

$$V(p^1) = \sum_{h \in H} s_{h0}(p^1) + \sum_{k \in K} \sigma_{k0}(p^1) + \sum_{k \in K} \alpha_k(p^1) - p^1 \cdot (x^1 - x^0). \tag{32}$$

Again consider the simplified model that was described in Figure 2 but now assume that the period 0 production possibilities sets is the line segment OA in Figure 4. If the ratio of international prices in period 1 is such that the slope of the line segment AD is equal to $-p_1^1/p_2^1$, then the technological change function $\alpha_k(p^1)$ will equal zero and our previous analysis carries through. If $-p_1^1/p_2^1$ increases, then it can be seen that the domestic production point in period 1 will be somewhere along DC and $\alpha_k(p^1)$ will be strictly positive. Again, our previous analysis goes through.

However, if $-p_1^1/p_2^1$ decreases, so that the tangent line AD rotates towards the y_2 axis, then $\alpha_k(p^1)$ becomes negative and the welfare effects of the new good are

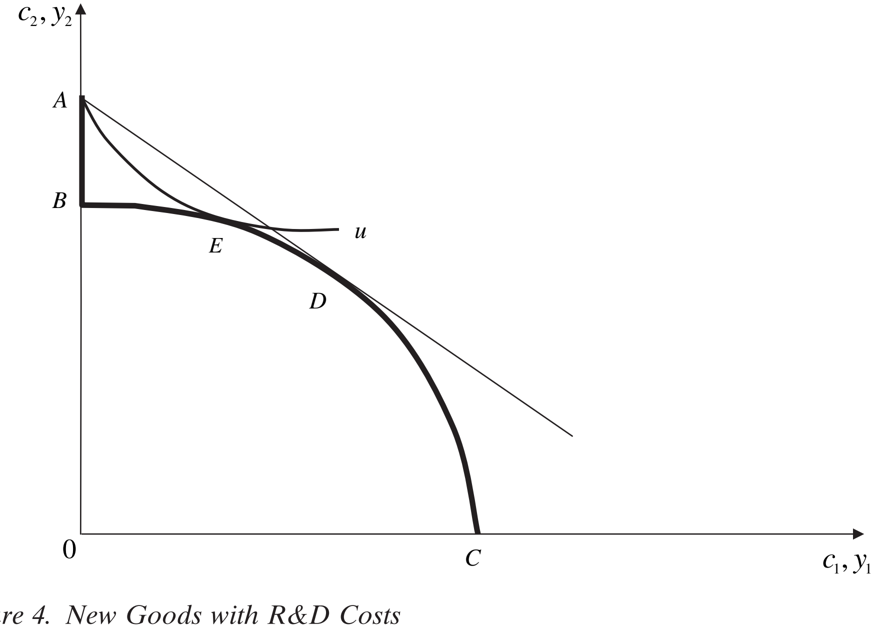

Figure 4. New Goods with R&D Costs

indeterminate. If the consumer substitution effects are strong enough, then this negative technological change effect can be overcome, for a net welfare gain, but this happy outcome is not guaranteed. Thus, it is possible to over-invest in the development of new goods. Thus, we have the following proposition.

PROPOSITION 4. *Assume that new goods are introduced into a small open economy and that they incur research and development costs. Assume further that there are no taxes, no technical change, and no excess supplies. The introduction of new goods will be welfare-improving if* $V(p^1) > 0$*; welfare may fall if the producer and consumer substitution effects are not sufficiently strong to outweigh any negative welfare effects due to research and development costs.*

6. Extension to a Large Open Economy

We now extend the ideas developed above to a large open economy that can influence its terms of trade by its tariff and tax policies. First, we add terms to our expressions for welfare gains to deal with changes in the world trade environment and with the optimality of the tariff policy. Second, we use these expressions to develop a new sufficiency condition for a welfare improvement.

To this end, we define the net trade value function

$$\beta(p, X) \equiv \max_z \{p \cdot z : z \in X\}, \tag{33}$$

which is the maximum value of net imports (foreign country net exports), z, that can be attained at reference price vector, p, when the country faces the foreign country's offer set X. This function is analogous to a profit function, recognizing that X may be interpreted as the "production possibilities set from trade." Using this function, we define

$$\delta(p) \equiv \beta(p, X^1) - \beta(p, X^0) \tag{34}$$

as the difference in the maximum values arising from a shift in the foreign offer set. This function will be zero if there is no change in the foreign offer set, and will be non-negative if the offer set is enlarged. Finally, we define

$$\gamma_r(p) \equiv \beta(p, X^r) - p \cdot z^r \geq 0. \tag{35}$$

Function $\gamma_r(p)$ is the maximum value of attainable net imports minus the cost, at the reference prices p, of the actual net import vector z^r in period r. It therefore measures the gains from substitution around the foreign offer set and is called the *trade substitution function*. If there are no consumer-specific taxes, optimality of the home country's tariff vector implies that the (common) consumer price vector, p_C^r, satisfies the equation $\gamma_r(p_C^r) = 0$, which is the usual first-order characterization of optimal tariffs.

Using these definitions, the aggregate variation may be expressed alternatively as

$$V(p) = \sum_{h \in H} s_{h0}(p) - \sum_{h \in H} s_{h1}(p) + \sum_{k \in K} p \cdot (y^{k1} - y^{k0}) + p \cdot (v^1 - v^0) - p \cdot (e^1 - e^0) + \gamma_0(p) - \gamma_1(p) + \delta(p) \tag{36}$$

$$V(p) = \sum_{h \in H} s_{h0}(p) - \sum_{h \in H} s_{h1}(p) + \sum_{k \in K} \sigma_{k0}(p) - \sum_{k \in K} \sigma_{k1}(p) + \sum_{k \in K} \alpha_k(p) + p \cdot (v^1 - v^0) - p \cdot (e^1 - e^0) + \gamma_0(p) - \gamma_1(p) + \delta(p). \tag{37}$$

These two expressions for the aggregate variation differ from their counterparts (14) and (15) in that the term $-p \cdot (x^1 - x^0)$ has been replaced by $\gamma_0(p) - \gamma_1(p) + \delta(p)$. This new term describes welfare gains from movements around, and shifts of, the foreign offer curve.

Assuming for simplicity that there is no change in technologies, endowments, excess supplies, or the foreign offer set, expression (36) reduces to

$$V(p) = \sum_{h \in H} s_{h0}(p) - \sum_{h \in H} s_{h1}(p) + \sum_{k \in K} p \cdot (y^{k1} - y^{k0}) + \gamma_0(p) - \gamma_1(p). \tag{38}$$

If, in addition, it is assumed that tariffs are optimal in period 1 (and that there are no consumer-specific domestic taxes), a further interesting simplification may be established. To obtain this simplification, we choose consumer prices as reference prices for the evaluation of the aggregate variation. Then, since the optimality of tariffs in period 1 implies that $\gamma_1(p_C^1) = 0$, where p_C^1 is the consumer price vector in period 1, expression (38) reduces to

$$V(p_C^1) = \sum_{h \in H} s_{h0}(p_C^1) + \sum_{k \in K} p_C^1 \cdot (y^{k1} - y^{k0}) + \gamma_0(p_C^1). \tag{39}$$

Since the first and third terms on the right-hand side of this expression are nonnegative, a sufficient condition for a welfare improvement is that

$$\sum_{k \in K} p_C^1 \cdot (y^{k1} - y^{k0}) > 0. \tag{40}$$

This result appears to be new. It establishes the following proposition.

PROPOSITION 5. *Assume that an open economy imposes optimal tariffs following some exogenous change and that there is no technical change, no changes in endowments, and no excess supplies. A sufficient condition for a welfare improvement following some exogenous change (such as a change in consumer taxes) is that the value of production, at the period 1 consumer price vector, increases.*

The assumptions behind the scenes are important. The crucial one is that the country imposes optimal tariffs against the rest of the world in period 1. If tariffs are not optimal, then $\gamma_1(p_C^1) > 0$ and hence the result does not follow. Notice, also, that if consumer and producer prices are equal in period 1, sufficiency condition (40) for a welfare improvement automatically holds.

7. Conclusions

The basic welfare identities developed above have been shown to be useful in measuring not only the gains from trade and from policy reform, but also the gains (or losses) from other sources of economic change. Our paper considered some of these, including, for example, the gains from reducing excess supplies and from introducing a new internationally traded product. However, the welfare measures and identities we have presented have much wider applicability and can, in principle, be used to measure the welfare effects of a discrete change in any policy instrument or other exogenous variable in our model. Moreover, they are useful in synthesizing what may appear to be disparate results on welfare and trade that have appeared in the literature.[20]

However, our model and analysis are subject to a number of limitations, which must be kept in mind in evaluating our approach to discrete welfare change. In particular, (i) we are limited to an analysis of *ex post* data; (ii) the same households are assumed to exist in both periods;[21] (iii) constancy of tastes is assumed; (iv) competitive price-taking behavior is assumed; and (v) the model is static.[22] Finally, in the context of many consumers, it should be noted that our aggregate variation measures might not be good measures of aggregate welfare change if welfare of the economy depends upon the distribution of income.

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Notes

1. The modern literature on this subject begins with Samuelson (1939) and includes Kemp (1962), Johnson (1965), Ohyama (1972), Kemp and Wan (1972), and Smith (1982). For textbook accounts, see Kemp (1969), Dixit and Norman (1980, pp. 74–80), and Woodland (1982, pp. 256–72).
2. We let the number of households and the set of households be identified by the same notation. Firms and goods are similarly treated.
3. While nontraded goods' prices adjust to clear the domestic markets, it is assumed further below that the domestic prices are solely determined by the traded goods' prices and the technology and not by household preferences or the income distribution.
4. This function was introduced to the economics literature by Hicks (1946, p. 331). The mathematical properties of expenditure functions are discussed in Diewert (1982, pp. 553–6). We assume that the minima in (3) exist.
5. Hicks (1946, pp. 319–25), Gorman (1968), McFadden (1978), and Diewert (1973) discuss properties of profit functions. We assume that the maxima in (4) exist.
6. The concept of a household substitution function, and that of a firm substitution function introduced below, is due to Grinols (1987). See, also, the unpublished paper by Diewert (1987). Similar ideas lie behind Krishna's (1992) concept of indices of structural adjustment.
7. The term is due to Samuelson (1974).
8. Specifically, if $V(p) > 0$ for some price vector p, then there is sufficient income to be reallocated in a lump-sum fashion to allow every household to consume its initial consumption vector with some extra income left over. Provided that this price vector, p, is faced by households and does not change as a result of the transfers, this extra income can be redistributed to achieve a strict Pareto improvement in welfare. This argument is no longer valid if prices alter; additional argument is needed to establish whether a Pareto improvement has occurred or can be generated. Alternatively, a representative consumer model, which is isomorphic to a one-household model, could be assumed.
9. Our identity has no hypothetical tax revenue terms. However, tax distortions do play a role in (15) via the definitions of the producer and consumer substitution terms.
10. Think of transportation improvement that allows the natural resources of a previously isolated region to be exploited.
11. Ohyama's (1972) Proposition 2 on the gains from self-financing trade covers the "vent for surplus" argument for commodities.
12. Thus, it is assumed that the introduction of new goods is not associated with a change in household preferences. The analysis of changes in preferences due to the new good or to advertising introduces new issues that go beyond the scope of this paper.

13. The basic idea is due to Hicks (1940). Also see Neary and Roberts (1980), who introduced the term "virtual price" for the shadow price that induces zero consumption.
14. For the properties of restricted expenditure functions, see Diewert (1986, pp. 170–6) and Neary and Roberts (1980).
15. To see why the shadow prices defined by (20) do the job, consider the following period 0 expenditure minimization problem for household h:

$$\begin{aligned} m^h\left(u^{h0}, p_1^{h0}, \tilde{p}^0\right) &\equiv \min_{c_1,\tilde{c}}\left\{p_1^{h0}c_1 + \tilde{p}^0\cdot\tilde{c}^0 : f^h(c_1,\tilde{c}) \geq u^{h0}\right\} \\ &= \min_{c_1}\left\{p_1^{h0}c_1 + \tilde{m}^h\left(u^{h0}, c_1, \tilde{p}^0\right)\right\} \\ &= p_1^{h0}c_1^{h0} + \tilde{p}^0\cdot\tilde{c}^{h0} = \tilde{p}^0\cdot\tilde{c}^{h0}, \end{aligned}$$

which uses the definition of the restricted expenditure function, (18). It should be evident that the first-order necessary condition for a solution to this problem is satisfied by $c_1 = 0$.
16. For the properties of restricted profit functions, see Gorman (1968), Diewert (1973), and McFadden (1978).
17. To see why the shadow prices defined by (24) do the job, consider the following period 0 profit-maximization problem for firm k:

$$\begin{aligned} \pi^k\left(w_1^{k0}, \tilde{p}^0, S^{k0}\right) &\equiv \max_{y_1,\tilde{y}}\left\{w_1^{k0}y_1 + \tilde{p}^0\cdot\tilde{y} : (y_1,\tilde{y}) \in S^{k0}\right\} \\ &= \max_{y_1}\left\{w_1^{k0}y_1 + \tilde{\pi}^k\left(y_1, \tilde{p}, S^{k0}\right)\right\} \\ &= w_1^{k0}y_1^{k0} + \tilde{p}\cdot\tilde{y}^{k0} = \tilde{p}\cdot\tilde{y}^{h0}, \end{aligned}$$

using the definition of the restricted profit function, (22). It should be evident that the first-order necessary condition for a solution to this problem is satisfied by $y_1 = 0$.
18. In equation (27), the terms $s_{h0}(p^0)$ and $\sigma_{k0}(p^0)$ do not vanish as they are evaluated at world prices, which differ from the period 0 household- and firm-specific prices that include the shadow price for the new good.
19. We are indebted to an anonymous referee for pointing out this extension of the model.
20. A synthesis of the results established by Ohyama (1972) and Wong (1991) is provided in an earlier version of this paper.
21. If a household existed in only one of the two periods under consideration, we would have to absorb its nonzero consumption vectors into the appropriate net endowment vectors.
22. Accordingly, there is no modeling of saving and investment behavior, no expectations about future prices are formed, and so on. Our results do apply, of course, if there exists a full set of futures markets in a finite-horizon model.