The Model:
N individuals
N/2 of each sex, N/2 households
Infinitely lived households, discrete time, discount rate $\rho$
People get utility from: $c^i_t, e^i_t, H^i_t$ — consumption, effort at work, effort in
the household (respectively).
They are constrained: $e^i + H^i \leq e^{max}$
Utility function

$$U(c^i) + V(e^i + H^i)$$

$$U' > 0, V' < 0, U'' < 0, V'' < 0$$

$$\lim_{c^i \to 0} U'(c^i) = \infty, \lim_{e^i + H^i \to e^{max}} = -\infty$$

Price of consumption normalized to 1
Savings not allowed (simplifying)
The Household
Each individual has to undertake $\bar{H}$ units of household tasks
– cleaning, shopping, cooking, childcare etc. etc.
– don’t model utility benefit from these (simplifying)
– private goods, not public
Exists an external labor market where these services can be bought
– by paying $w^e$ per unit of services, can reduce tasks by $\gamma < 1$ per unit of labor hired.
Note assumption of household specific human capital implicit in setting $\gamma < 1$.
– more effort required from an outsider to do your household tasks, than from any member of the household.
So, if I pay $w^e H^e$ to outsiders, I only get $\gamma w^e H^e$ household work done.
But I’d get $w^e H^e$ from my spouse.
But I can also get some of my $H^e$ work done in the household by my spouse
– pay $p^n H^{\text{max}}$ to my spouse, I get $H^{\text{max}}$ work done.
– How much you buy internally, externally, do yourself, depends on your income, the prices of these things etc. etc.
Firms
Piece-rate firms produce goods sold at price $p'$
- one unit of labor input produces one unit of output
- like a backstop technology, always available
Sophisticated product firms produce good sold at $p^g$
Assume there are $F$ of these, and each wants to hire $n$ workers
- $e < \bar{e}$ effort produces 0 output
- $e \geq \bar{e}$ effort produces 1 unit of output
- $e$ is not observable, and not contractible
Problem of providing incentives is solved by an efficiency wage
- efficiency wage: payment greater than the market price, so that threat of getting fired has impact – threat of losing this nice high wage provides incentive to not shirk.
Call efficiency wage jobs “good” jobs.

Assume more than enough workers of each sex to fill all the good jobs –

\[ nF < N/2 \]
Labor Market Equilibrium
At $t = 0$, each “good” firm calls wage $w^g$ for good workers
– workers then apply
– firms hire applicants
– applicants are all identical, but differ only in gender, which is observable
The implicit contract between worker and firm:
– firm pays $w^g$ up front
– if worker’s output is positive (implies $e \geq \bar{e}$) then re-hires worker next period
– if not, worker is dismissed, and never again re-hired
Now show that there is a type of dependence of hiring decisions across firms
First calculate individual optimal labor supply decisions. For individuals NOT in good jobs:
– given wage \( w^e = p' \) (which must be true) individuals indifferent to providing work on the household labor market or to working in piece-rate sector
– in some households a person’s spouse may wish to buy household services from them (at some internally negotiated price)
– possible “gains from trade” in household because piece rate worker gets \( p' \) for “outside” work, but a household member buying in household work must pay \( p'/\gamma \) (denoted \( \bar{p} \)) for each unit of household work they buy in.
– any internally negotiated price \( p^n \in (p', \bar{p}) \) will make both better off
– assume they can always come to some terms on this price
Maximization problem for individual under this price, $p^n$:

$$\max_{e_i, H_i^S} U (e_i p' + H_i^S p^n) + V (e_i + \bar{H} + H_i^S) + \Psi_S (e_i, 0)$$

subject to:

$$e_i \geq 0$$

$$0 \leq H_i^S \leq H_{\text{max}}(p^n)$$

where $H_{\text{max}}(p^n)$ denotes the upper bound on household services that the person’s spouse is willing to buy at price $p^n$.

– denote solutions to this problem $H_s^*$ and $\hat{e}$.
– clearly for any $p^n > p'$, $H_{s^*} > 0$, whereas $\hat{e}$ may equal 0.

Let $\Psi^S(p^n, H_{\text{max}}(p^n))$ be the indirect utility function obtained from this maximization.

Let $\Psi^S(\cdot, 0)$ denote the indirect utility function for a person who does not have the opportunity to sell household services to their spouse (i.e., both have identical external labor markets).
Since $H^{s*} > 0$ you’re always better off with the opportunity to provide household services with your spouse:

$$
\Psi^S (p^n, H^{\max}(p^n)) > \Psi^S (\cdot, 0)
$$

– internal trades are mutually beneficial.

Who demands these services?
– people with “good” jobs

Good jobs lead to either working $\bar{e}$ or shirking $e = 0$.
– Let $\Psi^{B,\bar{e}}$ denote indirect utility of a non-shirker, and $\Psi^{B,0}$ that of a shirker.
To work these out. First, for a non-shirker, maximize utility by choosing $H^B$; household services to buy internally, $H^e$ household services to buy externally

$$\max_{H^B, H^e} U \left( w^g \bar{e} - p^n H^B - p' H^e \right) + V \left( \bar{e} + \bar{H} - H^B - \gamma H^e \right)$$

s.t.

$$H^B \leq \lim_{p^n} (p^n)$$

$$H^B + \gamma H^e \leq \bar{H}$$

$$p^n H^B + p' H^e \leq w^g \bar{e}$$

where $\lim_{p^n} (p^n)$ is the most labor that $i$’s spouse will supply at $p^n$. Solutions are denoted $H^B*$ and $H^e*$. If you want to buy household services you ALWAYS want to do that from your spouse first — lower effective price.
Assume Good jobs are arduous:
For a person working in a good job, the exeternal price of household services is less than the MRS of effort for income, i.e.:

\[ p' < -\frac{V' (\bar{e} + \bar{H})}{U' (w^g \bar{e})}. \]

People working these jobs would always want to buy in household help. An immediate implication is that being able to trade household services internally (at \( p^n < p' \)) makes a non-shirker strictly better off:

\[ \Psi^{B,\bar{e}} (p^n, H^{lim} (p^n), w^g) > \Psi^{B,\bar{e}} (\cdot, 0, w^g). \]

But it’s not necessarily true that a non-shirker will trade household services internally. A non-shirker will always trade more than shirker (weakly):

\[ H_{\bar{e}}^{B*} \geq H_0^{B*}. \]
Intuitively, you spend less effort at work, you want to buy off no more of your effort in the household. But then it follows that not being able to buy household services makes a non-shirker at least as much worse off as a shirker:

\[
\psi^{B,\bar{e}}(p^n, w^g) - \psi^{B,\bar{e}}(\cdot, w^g) \geq \psi^{B,0}(p^n, w^g) - \psi^{B,0}(\cdot, w^g).
\]
Wages Sufficient to Stop Shirking
Consider two different wages:
– 1. For a person in a good job, whose spouse is a piece-rate worker
– 2. For a person in a good job, whose spouse has a good job too.
In situation 1:

\[
\Psi^{B,0} (p^n, w^g) + \frac{\rho \Psi^{S} (\cdot)}{1 - \rho} \leq \frac{\Psi^{B,\bar{e}} (p^n, w^g)}{1 - \rho}
\]

recall that \( \Psi^{S} (\cdot) \) is the indirect utility function for a (potential) seller of household services who has no one to sell to. Let the wage solving this with equality be denoted \( w^g_d \)
In Situation 2:

\[
\Psi^{B,0} (\cdot, w^g) + \frac{\rho \Psi^{S} (p^n)}{1 - \rho} \leq \frac{\Psi^{B,\bar{e}} (\cdot, w^g)}{1 - \rho}.
\]

Let the wage solving this with equality be denoted \( w^g_n \).
Both of these wages are well defined and unique.
PROPOSITION 1: $w_d^g < w_n^g$.
Intuition – good jobs require higher effort, pay higher income
Income gives more consumption and allows you to buy off some
housework.
If your spouse does not have a good job, this creates gains from trade.
If you lose your good job, you lose the higher income, and the gains from
trade.
Conversely, if you spouse has a good job already. Your higher income
generates no gains from trade. Losing a good job, doesn’t cost you any
gains from trade, it actually means that you can get gains from trade.
DISCRIMINATION EQUILIBRIUM

PROPOSITION 2. If the employment status of a person's spouse is not freely observable, and if all other firms discriminate, then each firm finds it profitable to also discriminate.
NON-DISCRIMINATION EQUILIBRIUM
The nature of such an equilibrium depends on the optimal hiring strategies of firms.
If the firm sets $w = w^g_d$, only workers with spouses not in good jobs do not shirk.
If $\alpha$ denotes the probability of hiring someone whose spouse also has a good job (assuming they randomly allocate the shirking in the household) then expected profits are:
$$\Pi^d = n[p^g(1 - \alpha/2) - w^g_d],$$
If the firm sets $w = w^g_n$ all workers would not shirk. This of course costs more in terms of wage, but leads to more certainty in production.
$$\Pi^n = n[p^g - w^g_n],$$
The latter strategy is preferred if and only if $\Pi^d < \Pi^n$, i.e.:
$$w^g_n - w^g_d < \alpha p^g / 2 \quad (1)$$
Let $D$ denote the number of firms discriminating. Then we have:

- # of women in good jobs: $(F - D)n/2$.
- # of men in good jobs: $(F - D)n/2 + Dn = (F + D)n/2$.

Since $N/2$ of each sex, this implies prob. of man with spouse in good job: $(n/N)(F - D)$
for women this is: $(n/N)(F + D)$.

PROPOSITION 3. A Nash equilibrium in which firms do not discriminate always exists. If and only if:

$$w_n^g - w_d^g < (n/2N)Fp^g$$

(2)

do firms follow the high wage policy in this equilibrium.

PROPOSITION 4: A discrimination equilibrium always exists.
So it seems we should expect to see either. But not so fast.

Let’s now consider “stability” of the system.
Stability:
This depends on how the policies of other firms affect those of a particular firm.
Consider a figure depicting costs and benefits of the high v. low wage strategy.
Cost of low wage strategy – chance of hiring a shirker.
Benefit of low wage strategy – pay low wages.
See Figure.
PROPOSITION 5: If firms follow the low wage policy in a non-discrimination equilibrium, it is unstable. Hence unlikely to see it in reality.
Policies:
Affirmative Action:
PROPOSITION 6: If and only if the non-discrimination equilibrium is stable, a policy of affirmative action, if applied widely enough, can end discrimination in the labor market.
Child Care subsidies:
Affect $w_s^g < w_n^g$.

PROPOSITION 7: If affirmative action does not work on its own, a policy of childcare subsidies coupled with affirmative action, can end discrimination in the labor market.
Single People:

\[ w_d^g < w_s^g < w_n^g \]

In a discrimination equilibrium, these inequalities can explain why:

- married men get the good jobs first,
- why if you need to extend beyond that, you will then include single people, and there are no reasons to discriminate,
- but why once a woman is married she then has a lower probability of promotion to the good jobs.