CHAPTER 8. UNCERTAINTY AND INFORMATION

- Games with incomplete information about players.
  - Incomplete information about players’ preferences can be symmetric or asymmetric.
  - To analyze games with asymmetric information, we will introduce Bayesian Nash equilibrium and perfect Bayesian equilibrium.
8.1 Uncertainty and risk

- An Okanagan Valley winery’s annual income in dollars can vary substantially depending on weather and other factors, so owner faces income uncertainty or risk ahead of season.
  - Expected income, or mean income, is probability-weighted average of prospective incomes.
  - Example: expected income is 50 if income can be either 0 or 100 with equal probabilities, or equally likely to be any number between 0 and 100.
• Owner is risk-neutral if owner makes decisions based only on expected income.

  – Owner is a risk-neutral player in a game if prospective incomes enter payoff only through expected income.

  – Example: an investment changing prospective income from 0 and 100 with equal probabilities to 0 and 400 with equal probabilities is worth 150.

  – A risk-neutral player treats risky income as same as the mean, and does not care about the variance.
• Owner is risk-averse if expected payoff from risky income is lower than payoff from expected income.

  – A risk-averse player uses non-linear scale called payoff function to convert income to payoff.

  – Expected payoff from risky income is lower than payoff from expected income because the payoff function is concave — it’s flatter at higher incomes.

  – Example: if payoff function is square root function, then expected payoff from prospective incomes of 0 and 100 with equal probabilities is 5, which is lower than \( \sqrt{50} \).
Figure 1. Concave payoff function.
• Certainty equivalent.
  
  – Certainty equivalent of uncertain income is certain or riskless income that gives the same expected payoff.

  – Risk aversion means certainty equivalence of uncertain income is smaller than the expected income.

  – Certainty equivalent does not change if we change the payoff function by multiplying it by a positive number and then adding another number to it.
• Degree of risk-aversion.
  
  – A risk averse person is willingness to pay a premium to eliminate risk in uncertain income.
  
  – Difference between certainty equivalent of an uncertain income and the expected income is the risk premium.
  
  – A more concave payoff function produces a greater risk premium, and represents a higher degree of risk-aversion.
Figure 2. Risk premium and degrees of risk aversion.
• Insurance contract.

  – A mutually beneficial deal exists between a risk-averse agent and an insurance company, which is risk neutral with respect to profits because it holds large portfolios.

  – Any deal should eliminate risk for risk-averse agent.

  – In the example, winery will receive some payment $x$ from insurance company when realized income is 0 and make a payment equal to $100 - x$ to insurance company when realized income is 100, with $x$ between 25 and 50.
• Pooling of risks.

  – Instead of purchasing insurance, a risk-averse agent can reduce risks by entering into risk-sharing contracts with other risk-averse agents.

  – If Okanagan winery can find an Ontario winery with perfectly negatively correlated risky income, then they can each get the certain income of 50.

  – Risk-sharing is possible unless incomes are perfectly positively correlated: for example, with independence, each winery can get the income of 50 half of the time.
8.2 Asymmetric information: basic ideas

- In many games, some players have more information than others about underlying strategic situation.
  - Examples: adverse selection in insurance, arms race.

- Bayesian Nash equilibrium is just Nash equilibrium under asymmetric information.
  - In simultaneous-move games, informed players will choose their strategies based on their superior information, but this is anticipated by uninformed players, and so on.
• Informed players may communicate, manipulate, or signal their information, while uninformed players may screen in order to reduce their information disadvantage.

  – Perfect Bayesian equilibrium extends rollback method and Bayesian Nash equilibrium to sequential-move games under asymmetric information.

  – Bayes’ rule is used by uninformed players in making inference about the information of informed players by observing latter’s actions.
8.4 Adverse selection

- Okanagan Valley winery with square root payoff function and risk-neutral insurance company.

  - Facing risky income of 100 with probability 0.5 and 0 with probability 0.5, winery owner is willing to enter insurance contract of receiving $x$ when realized income is 0 and paying $100 - x$ when realized income is 100, so long as $x$ is greater than certainty equivalent of 25.
- If risky income is 100 with probability 0.9 and 0 with probability 0.1, owner will enter insurance contract so long as $x$ is greater than 81.

- Asymmetric information: suppose that insurance company does not know which type of risky income the winery faces.
  - If insurance company believes that each type is equally likely, then it will offer an insurance contract only if $x$ is smaller than 70.
• Adverse selection: risky income of 100 with probability 0.5 and 0 with probability 0.5 is bad risk type, because expected income is 50 compared to expected income of 90 for the good risk type, but any $x$ that attracts the good risk type will also attract the bad risk type.

  – Adverse selection closes market for the good risk type: insurance company could offer $x$ between 25 and 50 and deal with bad risk type only.
• Fix any $x$ and consider a simultaneous-move game in which Firm chooses whether or not to offer an insurance contract with $x$ and Winery chooses whether to accept or reject it.

  – If Firm chooses Don’t or if Winery chooses Reject, Firm’s payoff is 0, while Winery’s payoff is 5 for bad risk type and 9 for good risk type.

  – If Firm chooses Offer and Winery chooses Accept, Firm’s payoff is $0.5 \cdot (-x) + 0.5 \cdot (100 - x)$ if Winery is bad risk type and $0.1 \cdot (-x) + 0.9 \cdot (100 - x)$ if Winery is good risk type, while Winery’s payoff is always $\sqrt{x}$. 
• Firms’ strategy is *Offer* or *Don’t*; Winery’s strategy specifies a choice between *Accept* and *Reject* for each risk type.

  – More generally, in any simultaneous-move game with asymmetric information, private information of informed player is represented by having multiple types.

  – Strategies of uninformed players are defined in the same way as before, while a strategy of an informed players must specify what each type will choose.
• There is no $x$ such that it is a Bayesian Nash equilibrium in the game with $x$ for Firm to choose *Offer* and for both risk types of Winery to choose *Accept*; for $x$ between 25 and 50, it is a Bayesian Nash equilibrium in the game with $x$ for Firm to choose *Offer*, for bad risk type to choose *Accept* and for good risk type to choose *Reject*.

  – Bayesian Nash equilibrium: each player’s equilibrium strategy is a best response to equilibrium strategies of opponents; for an informed player, this means each type is best responding.
- Adverse selection is a form of market failure.
  - It occurs in other markets (used cars, laid off workers).
  - Market responses include signaling by informed player (warranty, education) and screening by uninformed player (incomplete insurance, probationary employment).
  - Signaling and screening differ in timing of moves, but share same goal of separating different types of informed player and same mechanism of different costs.
8.3 Cheap talk communication

- Informed player often may want to transmit information to uninformed player, and sometimes may succeed in doing so with cheap talk.

- Pure coordination with asymmetric information.
  - Branco and Steffi must simultaneously choose where to meet between Granville Island and Metrotown, but only Branco knows which of the two is better for both, while Steffi knows only they are equally likely.
- Payoffs are, respectively, when *Island* or *Metrotown* is better:

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<tr>
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<th>Island</th>
<th>Metrotown</th>
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<tr>
<td><strong>Branco</strong></td>
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<tr>
<td><em>Island</em></td>
<td>2,2</td>
<td>0,0</td>
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<tr>
<td><em>Metrotown</em></td>
<td>0,0</td>
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Figure 3. Pure coordination with asymmetric information.
Without cheap talk, in the simultaneous-move game, Branco is informed and has two types.

- Branco’s strategy specifies a choice between Island and Metro when Island is better and when Metro is better, while Steffi’s is just a single choice.

- One Bayesian Nash equilibrium is Branco always chooses Island, and Steffi chooses Island; another equilibrium is Branco always chooses Metro, and Steffi chooses Metro.
Cheap talk adds one stage after Nature’s move but before the above simultaneous-move game, in which Branco can text Steffi a message about which of two locations is better.

- Branco’s strategy now specifies message choice between Island and Metro and location choice for each possible message, both when Island is better and when Metro is better; while Steffi’s strategy specifies a location choice for each of the two messages she receives from Branco.
Figure 4. Pure coordination with cheap talk.
• There is a perfect Bayesian equilibrium in which Branco’s message choice coincides with Nature’s move and location choice coincides with message choice; while Steffi’s location choice coincides with Branco’s message choice.

  – Since talk is cheap, there is another equilibrium with the same outcome but opposite message choice by Branco.

  – There is also an equilibrium in which Branco’s message conveys no information to Steffi.
• Perfect Bayesian equilibrium.

  - As in any Nash equilibrium, each player’s equilibrium strategy is a best response to equilibrium strategies of other players, and as in Bayesian Nash equilibrium, this requires each type of informed player to best respond.

  - There is no subgame in sequential-move games with asymmetric information, but the idea of rollback still applies: uninformed player is required to form belief about type of informed player at each information set and must best respond given the belief.
- Bayes’ rule.
  
  - Uninformed player’s belief is a probability for each type such that probabilities sum up to 1 over all types.
  
  - Uninformed player uses given prior belief and strategy of informed player to update at each information set.
  
  - Updated probability of any type at an information set is ratio of probability of this type reaching it and total probability over all types reaching it.
  
  - Bayes’ rule does not apply if information set is not reached.
• Example of Bayesian Nash equilibrium that is not perfect.

  – Branco’s message choice is *Island* and location choice given message *Island* is *Island* regardless of type, but his location choice is *Metro* given message *Metro*; while Steffi’s location choice is *Island* regardless of Branco’s message choice.

  – As in Nash equilibrium that is not subgame perfect, Bayesian Nash equilibrium does not check if players’ equilibrium strategies are best responses to each other for information sets that are never reached.
8.5 Labor market signaling

- In entry-level job market, workers often have asymmetric information about their innate ability that firms care about.
  - Given the same productivity, firms are willing to pay more for high-ability workers.
  - Cheap talk communication does not work here because workers all want higher pay regardless of their ability.
  - Education emerges as market response to asymmetric information.
• Adverse selection in labor market.

  – Asymmetric information: Worker knows whether he has high ability of low ability, but Firm only knows prior probability of high ability is $p$.

  – Firm is willing to pay up to $w_H$ for high ability, but only $w_L < w_H$ for low ability.

  – Worker’s outside option is $u_H$ between $w_L$ and $w_H$ for high-ability type and $u_L$ between $w_L$ and $u_H$ for low ability, and so if information is symmetric, Worker will be hired by Firm only if he has high ability.
- For given wage offer \( w \) between \( w_L \) and \( w_H \), consider a simultaneous-move game in which Worker chooses between Accept and Reject and Firm chooses between Offer and Don’t.

- If \( u_H > pw_H + (1 - p)w_L \), there is no Bayesian Nash equilibrium where only Worker with high ability chooses Accept and Firm chooses Offer: Firm has to offer at least \( u_H \) to attract high-ability type, but this will also attract low ability, and so if \( u_H \) is higher than Firm’s average willingness to pay, labor market fails.
• Education as a costly signal.

- Worker can get education at cost $c_H$ for high ability type and $c_L$ for low ability, before Firm decides to whether or not make wage offer $w$ between $w_L$ and $w_H$.

- This is a game with asymmetric information where Worker is informed with two types and Firm is uninformed, and Worker moves first and Firm moves second after observing Worker’s move but not Worker’s type.
Figure 5. Job market signaling.
• Separating equilibrium: perfect Bayesian equilibrium in which different types of the informed player make different choices.

  – Equilibrium strategies: high-ability chooses *Education*, and low-ability chooses *No*; Firm chooses *Offer* after seeing *Education* and *Don’t* after seeing *No*.

  – Verification: for high-ability type, \( w - c_H \geq u_H \); for low-ability type, \( u_L \geq w - c_L \); for Firm after *Education* through Bayes’ rule, \( w_H - w \geq 0 \); and for Firm after *No* through Bayes’ rule, \( w_L - w \leq 0 \).

  – Such \( w \) exists if \( u_H + c_H \leq w_H \) and \( u_H + c_H \leq u_L + c_L \).
• Pooling equilibrium: perfect Bayesian equilibrium in which different types of the informed player make the same choices.
  
  – Equilibrium strategies \( w \geq u_H \): both types choose \( \text{No} \); Firm chooses \( \text{Offer} \) after seeing either \( \text{Education} \) or \( \text{No} \).

  – Verification: for high ability type, \( w \geq w - c_H \); for low ability, \( w \geq w - c_L \); for Firm after \( \text{No} \) through Bayes’ rule, \( pw_H + (1 - p)w_L - w \geq 0 \); for Firm after \( \text{Education} \), \(qw_H + (1 - q)w_L - w \geq 0\), where \( q \) is updated belief that Worker has high ability (Bayes’ rule does not apply).

  – Such \( w \) exists if \( pw_H + (1 - p)w_L \geq u_H \).
• Pooling equilibrium when $pw_H + (1 - p)w_L < u_H$.

  – Equilibrium strategies: both types choose No; Firm chooses Don’t after either Education or No.

  – Verification: for high ability type, $u_H \geq u_H - c_H$; for low-ability, $u_L \geq u_L - c_L$; for Firm after No through Bayes’ rule, $0 \geq pw_H + (1 - p)w_L - w$; and for Firm after Education, $0 \geq qw_H + (1 - q)w_L - w$ for some $q$.

  – Question: shouldn’t $q = 1$ if $u_H + c_H < w_H < u_L + c_L$, and if so, is the pooling equilibrium reasonable?