

Economics 546

Question Sheet 2

1. Take the model of inflation determination given by:

$$\pi_t = \lambda x_t + \beta E_{t-1} \pi_{t+1} + u_t + \kappa \quad u_t = \rho u_{t-1} + \varepsilon_t, \quad E_{t-1} \varepsilon_t = 0.$$

Here, the variable u denotes a temporary deviation of the natural rate of output from the desired level. The variable κ indicates a permanent gap between the natural rate of output and the desired level.

Assume that the policy-maker's loss function is:

$$W = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \alpha_x x_t^2)$$

Until part e), assume that $\kappa=0$.

- a) Under policy discretion, compute the equilibrium rate of inflation and output gap. Will inflation and the output gap have any persistence if $\rho=0$?
 - b) Now assume that the policy-maker can pre-commit. Compute the optimal pre-commitment policy. Show that both inflation and the output gap will be persistent even if $\rho=0$.
 - c) Denote $q_t = p_t - p_{-1}$ as the normalized price level (i.e. the price in any period t normalized by the period -1 price). Write out the optimal pre-commitment policy as a 2nd order difference equation in q_t . Assume that the solution to this difference equation has the form $q_t = aq_{t-1} + bu_t$. Show how to solve for a and b . (Hint: a is the smallest root of a quadratic equation).
 - d) Assume that $\alpha_x = 2$, $\lambda = .1$, $\beta = .9$. Compute the value of the root a from the previous part. Now for these numerical values, compare the impact of a one time shock in ε , at time $t=0$ in the two cases (discretion and commitment). Illustrate in a chart (e.g. excel chart) the response of inflation and the output gap to the shock under (i) the assumption that $\rho=0$, and (ii), the assumption that $\rho=0.75$. Explain the difference between the discretion and commitment outcomes.
 - e) Now, assume no shocks to the economy at all, so $u=0$, but assume that $\kappa>0$. Compute the rational expectations equilibrium rate of inflation under the discretionary policy. Compute also the *long-run* rational expectations equilibrium rate of inflation under commitment. Explain the difference.
2. Assume that the economy fluctuates between two states; one where the nominal interest rate is positive, and another where the nominal interest rate is at its zero bound. In state 1, the nominal interest rate is positive, and given by $r_t^n + \phi\pi$, so that the real interest rate 'gap' in the DIS equation is $i_t - E_t \pi_{t+1} - r_t^n = \phi\pi_t - E_t \pi_{t+1}$

In state 2, the nominal interest rate is zero, and then the equivalent real interest rate gap is given as follows, where

$$i_t - E_t \pi_{t+1} - r_t^n = 0 - E_t \pi_{t+1} - r_L, \quad r_L < 0$$

Assume state 1 occurs with probability p , and state 2 with probability $1-p$.

- a) Compute the values of the output gap and the inflation rate in each state.
- b) Using the numerical values from the previous question, and the additional assumptions that $\sigma=2$, $p=0.75$, and $r_L=-1$, show that inflation and the output gap are negative in *both* states, but more negative in zero interest rate states.