Econ 546 Lecture 4

The Basic New Keynesian Model

Michael Devereux

January 2011

Road-map for this lecture

- We are eventually going to get 3 equations, fully describing the NK model
- The first two are just the same as before:
  - The Euler equation for output (consumption) growth as a function of the interest rate
  - The interest rate rule followed by the monetary authority
- The third one describes the path of inflation, as a function of state of economy
- To see, look ahead to “3-Equation New Keynesian..” slide

Road-map for this lecture

- The only essential difference from Lecture 1 is that prices are slow to adjust
  - When we get to the end, make sure that you can show that if you look at the limit of this model where prices are fully flexible, you restore the model of Lecture 1
- Aside from the details of this Lecture, the main thing is to get familiar with the 3 equation model

Basic NK model

- Extend the previous model in two ways so as to make it more realistic and useful for policy evaluation
- 1st : Assume firms cannot change their prices freely
  - Menu costs
  - Information costs
  - Customer resistance
  - Staggered price setting compounds price stickiness
- 2nd : Allow households to consume a variety of different `brands’ of goods
  - a) a non-trivial theory of price setting
  - b) Staggering implies a distribution of prices across firms

Essential Elements of model

- Again, as before, a large number of identical, competitive households
- Households consume a large number of different varieties of goods
- Each variety is supplied by a separate firm, who is a monopolist in that market
  - Model of Monopolistic Competition
- Perfect competition is not a legitimate working assumption in an economy with price stickiness

Price setting assumptions

- Assume that each firm (producing a given brand) can re-set its price with probability 1-θ in a given period
  - No matter how long ago in the past that it fixed its price
- This is unrealistic, but it facilitates a very tractable model at the aggregate level, because it allows for aggregation from the micro to the macro
- `Calvo' model of price setting
- Note, this is a `time-dependent' pricing assumption
  - Price setting based on date only
- Alternative is `state-dependent' pricing
  - Price setting based on state of the world (e.g. High demand, cost episodes)
  - such models are harder to analyze
- From last lecture, we know Calvo is *literally* incorrect, but may be *a reasonable approximation*

Household utility function

- Households maximize expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

- C and N are consumption and labour supply
- Use the same functional forms as before

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\psi}}{1+\psi} \right)$$

Product Differentiation

- There is a continuum of differentiated goods (on line 0-1)
- Elasticity of substitution between each good is ε

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{1}{1-\frac{1}{\varepsilon}}}$$

- Each good has price $P_t(i)$

New household Budget Constraint

- Household purchases all goods

$$\int_0^1 P_t(i)C_t(i)di + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

- Given C, we can compute individual demand for each variety (each firm faces elasticity ε)

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

- Where P is the aggregate price index

$$P_t = \left(\int_{t=0}^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$$

It must be that

- Household purchases all goods

$$\int_0^1 P_t(i)C_t(i)di = P_tC_t$$

- Sum of expenditure on varieties adds up to total consumption expenditure, so at level of aggregate consumption we get exactly previous b.c.

$$P_tC_t + Q_tB_t \leq B_{t-1} + W_tN_t - T_t$$

That means that household's other optimality conditions are same as before

- Optimal Labour Supply

$$-\frac{U_{Nt}}{U_{Ct}} = \frac{W_t}{P_t}$$

- Optimal savings

$$\frac{Q_t U_{Ct}}{P_t} = \beta E_t \frac{U_{Ct+1}}{P_{t+1}}$$

So as before, given the functional form we arrive at

- Log linear labour supply

$$w_t - p_t = \sigma c_t + \psi h_t$$

- Approximate inter-temporal Euler equation

$$c_t = E_t c_{t+1} - \tfrac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho)$$

Each firm i has same production function as before

- Firm producing variety `i':

$$Y_t(i) = AN_t(i)$$

- Note this is constant returns to labour
  - Somewhat unrealistic, but easier than more general case
  - Easier case than in Gali's text
- Calvo pricing structure – firm i can revise its price with probability 1-θ, in each period, *no matter how long ago it sets its price*
- Once it re-sets its price, it can reset it again in each period in the future, with probability 1-θ

Behaviour of the Aggregate Price Level

- By the law of large numbers

$$P_t = (\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{*(1-\varepsilon)})^{\frac{1}{1-\varepsilon}}$$

- Which implies that

$$\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

- Aggregate inflation rate is driven by firms re-setting prices (log approximation is):

$$\pi_t = (1-\theta)(p_t^* - p_{t-1})$$

Firm's price setting

- If the firm had fully flexible prices (Θ=0), then it would set the price as a *mark-up* over marginal cost

$$\hat{P}_t^*(i) = \frac{\varepsilon}{\varepsilon - 1} MC_t(i) = \Omega \frac{W_t}{A_t}, \quad \Omega > 1$$

- This is because monopolist's price is 1/(1-1/elasticity) times MC, where elasticity is ε, and MC=W/A
- Then, in log terms

$$\hat{p}_t^*(i) = \log(\Omega) + w_t - a_t$$

When the firm has to set its price

- (See Gali for details) – Take firm that gets to set its price in any period t. It follows partial adjustment rule
- (log) Price is set as a linear function of optimal desired price under fully flexible prices, and next year's optimal price

$$p_t^*(i) = (1-\beta\theta)\,\hat{p}_t^*(i) + \beta\theta E_t\, p_{t+1}^*(i)$$

  - Ask yourself: why would it not just set $p_t^*(i) = \hat{p}_t^*(i)$?
  - Make sure you understand difference between the hat variable and the others

Then, note following steps:

1. All firms have same optimal price in flexible price environment: $\hat{p}_t^*(i) = \hat{p}_t^*$

2. Then, each firm setting its new price in Calvo model chooses the *same price* as any other firm

$$p_t^* = (1-\beta\theta)(\log(\Omega) + w_t - a_t) + \beta\theta E p_{t+1}^*$$

3. With previous condition, this becomes (will show this),

$$\pi_t = \kappa(\log(\Omega) + w_t - a_t - p_t) + \beta E \pi_{t+1}^*, \quad \kappa \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

More steps:

1. But from labour supply and production function:

$$w_t - a_t = p_t + (\sigma + \psi) y_t - (1 + \psi) a_t$$

2. So we get,

$$\pi_t = \lambda (y_t - \frac{(1+\psi) a_t - \log(\Omega)}{\sigma + \psi}) + \beta E \pi_{t+1}, \quad \lambda \equiv \kappa \left(\sigma + \psi \right)$$

Last step:

Note when prices flexible $\lim_{\theta \to 0}(\lambda) = \infty$, so fully flexible price output is (check with Lecture 1):

$$\hat{y}_t = \frac{(1+\psi)a_t - \log(\Omega)}{\sigma + \psi}$$

So therefore, from previous page, we have

$$\pi_t = \lambda(y_t - \hat{y}_t) + \beta E\pi_{t+1} = \lambda\tilde{y}_t + \beta E\pi_{t+1}$$

Inflation is a function of the output *gap* and expected future inflation

Derive the DIS equation

- Substitute in for c=y

$$y_t = E_t y_{t+1} - \tfrac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho)$$

- Add and subtract away $\hat{y}_t$ and $E_t\ \hat{y}_{t+1}$ from each side to get

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \tfrac{1}{\sigma}(i_t - E_t \pi_{t+1} - \hat{r}_t\ )$$

- Where $\hat{r}_t$ is the *natural* rate of interest

$$\hat{r}_t\ = \rho + \sigma E_t(\hat{y}_{t+1} - \hat{y}_t\ ) = \rho + \sigma \frac{(1+\psi)}{\sigma + \psi} E_t \Delta a_{t+1}$$

Big difference from previous classical model

- Output is no longer independent of the path of inflation
- Given the gap between the actual and natural real interest rate, the path of the output gap is determined
- Given the output gap, the path for inflation is determined
- Need an interest rate policy to complete the model

Assume a simple rule which depends only on inflation

- The same feedback rule as before (recall it affected only inflation and not output or the real interest rate)

$$i_t = \rho + \phi\pi_t$$

- In general, we might expect this would depend on output gap as well
- Combine this with the NKPC and the DIS equation to solve for inflation and the output gap

3-Equation New Keynesian Model

- DIS equation

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - \tfrac{1}{\sigma}(i_t - E_t \pi_{t+1} - \hat{r}_t \ )$$

- Taylor Rule (with added shock to interest rate)

$$i_t = \rho + \phi \pi_t + v_t$$

- NKPC equation

$$\pi_t = \lambda \tilde{y}_t + \beta E_t \pi_{t+1}$$

Look at i.i.d. shock to a, v, and σ=1

- From DIS equation

$$\tilde{y}_t = 0 - (\phi\pi_t + v_t - 0 + a_t)$$

- From NKPC equation

$$\pi_t = \lambda \tilde{y}_t$$

- Solutions for output gap and inflation

$$\tilde{y}_t = -\frac{(a_t + v_t)}{1+\phi\lambda} \quad \pi_t = -\frac{\lambda(a_t + v_t)}{1+\phi\lambda}$$

Conclude

- Shock to a reduces output gap – output falls below the natural rate
  - Because it is deflationary
- Shock to interest rate reduces output gap
  - Because it raises the real interest rate above the natural real interest rate
- Clearly, the monetary policy rule is no longer irrelevant for real outcomes