Exchange Rate Policy and Endogenous Price Flexibility

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Abstract

Most theoretical analysis of flexible versus fixed exchange rates takes the degree of nominal rigidity to be independent of the exchange rate regime choice itself. However, informal policy discussion often suggests that a credible exchange rate peg may increase internal price flexibility. This paper explores the relationship between exchange

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rate policy and price flexibility, in a model where price flexibility itself is an endogenous choice of profit-maximizing firms. A fixed exchange rate can affect the optimal degree of price flexibility by altering the volatility of nominal demand facing price-setting firms. We find that a unilateral peg, such as a Currency Board, adopted by a single country, will increase internal price flexibility, perhaps by a large amount. On the other hand, when an exchange rate peg is supported by bilateral participation of all monetary authorities such as in a Monetary Union, price flexibility may actually be less than under freely floating exchange rates. Quantitatively, we find that the endogenous increase in price flexibility following a unilateral peg might be large enough that output volatility is no greater than it would be under a floating exchange rate regime. (JEL: F0, F4)
1 Introduction

The classic argument for flexible exchange rates is that they enhance the ability of the economy to respond to shocks, in the presence of nominal rigidities (e.g. Friedman 1953). However, almost all theoretical discussion of the trade-off between fixed and flexible exchange rates takes the degree of nominal rigidities to be independent of the regime choice itself. In policy circles, however, it is often suggested that by eliminating the use of the exchange rate as a mechanism for adjustment, an exchange rate peg may increase internal price flexibility. This has been especially important in the analysis of the conditions for small countries to operate Currency Board or similar ‘hard-peg’ rules (e.g. Latter 2002). Since these countries will generally not have access to compensating policy responses from the country to which they are pegging, the need to increase internal price flexibility after a peg becomes more critical. Another area where this discussion is important is that of the impact of a Monetary Union on flexibility. If a single currency encourages price flexibility within the different regions of the currency area, this will reduce the loss from the absence of exchange rate adjustment. To this extent, the economic case for a Monetary Union may be enhanced by the formation of the Union itself, as suggested by
Frankel and Rose (1998)\textsuperscript{1}.

Is price flexibility likely to take place automatically in response to changes in monetary policy, through the decisions of individual price setters? We could think of price stickiness as being determined by the trade-off between ‘costs of flexibility’ (information or planning costs, for instance) and benefits of ex-post price adjustment. The more volatile the environment within which a price setter operates, the higher the benefits. If an exchange rate peg substantially increases the volatility of demand for their product, the elimination of the exchange rate as a policy lever may lead price setters to adjust more frequently.

This paper provides a theoretical investigation of the implications of exchange rate rules for the flexibility of nominal prices. The analysis is based on a two-country model, with shocks to relative national demands, and country-specific velocity shocks. Given this uncertainty, profit-maximizing firms may choose ex-ante to incur a cost so as to have the flexibility to adjust their prices ex-post. Within this setting, we ask a) what features determine the equilibrium degree of price flexibility, and b) in what way does an exchange rate peg affect the degree of price flexibility?

\textsuperscript{1}In the discussion of Economic and Monetary Union, the likelihood of wage and price flexibility being enhanced by the single currency was considered, for example OECD (1999).
goods, the higher the incentive for ex-post price flexibility. An increase in monetary variability will increase the variance of nominal demand. However, the variance of nominal demand will also depend on the degree of price flexibility itself. This introduces a strategic interaction between the pricing decisions of firms. We find that, for standard parameter values, the incentive for flexibility is increasing in the total number of firms who choose to adjust their ex-post prices; there is a strategic complementarity in the choice of flexibility. If only a small number of price setters adjust their price, then there may be little incentive for the marginal price setter to have price flexibility. However, if all price setters choose to adjust, the volatility of prices will increase the overall volatility of demand facing a price setter, increasing the incentive for a given firm to adjust.

How does exchange rate policy affect the degree of price flexibility? The key feature of a fixed exchange rate is that it requires that monetary policy adjust to internal and external shocks in lieu of exchange rate adjustment. This enhances price flexibility whenever the policy increases the volatility of a firm’s demand. In turn, this depends on the type of shocks that occur, and the way in which the fixed exchange rate system operates.

We first focus on a one-sided peg, which describes a situation where one country fixes its exchange rate against a trading partner, and accepts sole responsibility for maintaining the peg. A Currency Board is an ex-
ample of such an arrangement. Our model predicts unambiguously that a one-sided peg will increase internal price flexibility in the pegging country, while leaving foreign price flexibility unchanged. In a one-sided peg, the domestic monetary authority must respond to all shocks, domestic and foreign, in order to protect the peg. This must lead to an increase in the overall volatility of nominal demand facing firms. Therefore, more firms will choose to incur the costs of price flexibility.

However, a cooperative peg, involving active participation of all monetary authorities (in our model, this is equivalent to a Monetary Union), has an ambiguous effect on price flexibility, depending on the source of shocks. If most shocks are ‘real’, coming from fluctuations in demand for one country’s goods relative to another, then a cooperative peg will also increase price flexibility in all countries. In contrast, if most shocks are ‘monetary’ coming from exogenous shocks to the velocity of money, then a cooperative peg will reduce price flexibility in all the pegging countries.

How large is the impact of an exchange rate change on price flexibility? The presence of strategic complementarity allows for changes in the external environment to have a potentially very large effect on the equilibrium degree of price flexibility. We illustrate this point by exploring the effect of the exchange system on output and relative price volatility. Holding the degree of price flexibility constant, a unilateral peg will lead to
a substantial increase in output volatility, and a large fall in terms of trade volatility. However, the peg itself increases the incentive for firms to have flexible prices. For a standard parameterization of the model, we find that this indirect effect of the exchange rate peg on price flexibility can be of the same order of magnitude as the direct effect of the peg itself on output volatility. As a result, the volatility of GDP remains essentially unchanged after a move from floating exchange rates to a unilateral peg. The volatility of the terms of trade, however, is substantially reduced. The endogenous adjustment in price flexibility, therefore, can explain why a comparison of fixed and floating exchange rates for a small economy might show little differences in the behavior of GDP, but substantial differences in relative price variability.\footnote{Baxter and Stockman (1989), Flood and Rose (1995), and others have highlighted the empirical ‘puzzle’ that, across exchange rate regimes, there are large differences in real exchange rate volatility, but negligible differences in output volatility, and volatility of other fundamentals.}

How does the presence of endogenous price flexibility affect optimal monetary policy rules? In general, if monetary authorities wish to target the flexible price equilibrium allocation, they will set policy so that firms never have to adjust prices. As a result, the possibility of endogenous price flexibility will not affect the optimal monetary rule (see Dotsey, King and Wolman 1999). In our model, the flexible price allocation is inefficient due
to the absence of complete international financial markets. As a result, an optimal monetary rule does not replicate the flexible price allocation. In principle, this could mean that the presence of endogenous price flexibility significantly alters optimal monetary rules, relative to an economy with exogenously sticky prices. However, in our calibrated model, we find that optimal monetary rules are close to those in an economy with exogenously sticky prices. Moreover, in our benchmark calibration, an optimal monetary policy in the model allows for only a very small degree of price flexibility.

This paper is related to a large recent literature evaluating the effects of monetary rules in sticky price equilibrium models. It differs in allowing for the degree of price stickiness itself to be an endogenous variable. In this respect, the paper is closer to the literature on state-dependent pricing and menu costs of price change (see Ball and Romer 1991, Dotsey, King and Wolman, 1999). The model is most closely related to Ball and Romer (1991). They show the possibility of multiple equilibrium, in an environment where price setters can choose ex-post whether to adjust prices, given a common menu cost of price change, within a one-country environment. Our analysis differs because we allow a distribution of firm-specific menu costs, and we assume that price setters choose in advance whether or not to have the ex-post flexibility to adjust price. This is more in line with
the view that a large change in monetary policy regime (e.g. fixing the exchange rate) may lead to structural changes in the flexibility of contracts within a monetary economy. Moreover, our focus is not primarily on multiple equilibrium, but more on the role of strategic complementarity in the choice of flexibility. Finally of course, we use a two-country model.

The next section sets out the basic technology of endogenous price flexibility for a given firm. Section 3 incorporates this model into a two-country general equilibrium environment. Section 4 examines the link between price flexibility and the exchange rate regime. Section 5 investigates the predictions of the model for output and relative price volatility, while Section 6 discusses the optimal monetary policy under endogenous price flexibility. Some conclusions then follow in Section 7. An appendix contains detailed proofs.

2 The Firm and the Choice of Price Flexibility

We first describe the decision faced by a single firm with respect to the choice of price flexibility. In the typical model of state-dependent pricing\(^3\), a firm chooses whether or not to adjust its price ex-post, given information on its demand and costs, by trading off the benefits of price adjustment relative to the direct (e.g. menu) costs of price change. By contrast, our

\(^3\)See, for example, Dotsey, King and Wolman (1999).
assumption is that the firm must invest ex-ante in flexibility. That is, a firm must choose ex-ante whether to have the flexibility to adjust its price ex-post, after observing the realized state of the world. The firm incurs a fixed (labor) cost in order to have this flexibility. Roughly speaking, this decision may correspond to the way in which changes in monetary policy or other structural features of the economy would impact on the institutional characteristics of nominal price or wage setting.

Let a firm $i$ have the production function

$$Y_i = (H_i - D_i \Phi_i)\alpha, \quad 0 < \alpha \leq 1,$$

where $Y_i$ is the firm’s output, $H_i$ is total employment, and $\Phi_i$ is a firm-specific fixed cost of flexibility. Assume that the firm knows $\Phi_i$. We let $D_i$ be an indicator variable, whereby $D_i = 1$ ($D_i = 0$) if the firm chooses to (not to) incur the cost of ex-post price flexibility. The firm’s production function indicates that it has some firm-specific factor of production, together with which it combines labor to produce output for sale.

Assume that the firm faces market demand

$$X_i = \left(\frac{P_i}{P}\right)^{-\lambda} X,$$

where $P_i$ is the firm’s price, $P$ is the (possibly stochastic) industry price, $\lambda > 1$ is the firm’s own elasticity of demand, and $X$ is the stochastic total.

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4 We use this form of demand because it is obtained from the general equilibrium model analyzed below.
market demand shock. The wage $W$ is stochastic. From the production technology (1), the firm’s total operating cost is

$$W \left( Y_i \right)^{1/\alpha} + WD_i \Phi_i. \quad (3)$$

The firm evaluates its expected profits using a stochastic discount factor\(^5\) $\Gamma$. Then discounted expected profits may be written as

$$E_{\Gamma} \left[ P_i \left( \frac{P_i}{P} \right)^{-\lambda} X - W \left( \left( \frac{P_i}{P} \right)^{-\lambda} X \right)^{1/\alpha} - WD_i \Phi_i \right]. \quad (4)$$

We further assume that the firm knows the distribution of the discount factor, the market demand, and the wage.

The firm chooses $P_i$ to maximize (4). If $D_i = 1$, then the firm can choose its price after observing $P$, $X$, and $W$, and it sets the following price:

$$\tilde{P}_i = \delta \left[ W^{\alpha} \left( \hat{X} \right)^{1-\alpha} \right] \omega, \quad (5)$$

where $\delta = \left\{ \lambda / \left( \alpha (\lambda - 1) \right) \right\}^{\omega \omega}, \omega = 1 / \left( \alpha + \lambda (1 - \alpha) \right)$, and $\hat{X} = P^\lambda X$. When $\alpha = 1$, the firm’s price is a constant markup over the wage. However, when $\alpha < 1$, the optimal price will depend on a geometric average of the wage and market demand.

When $D_i = 0$, the firm must choose the price ex-ante. In that case,\(^5\) in the next section, we determine $\Gamma$ from the preferences of the firm’s household shareholders.
its optimal price is given by

$$\bar{P}_{i} = \delta \frac{E[\Gamma W(\hat{X})^{1/\alpha}]^{\omega}}{E[\Gamma X]^{\omega}}.$$  \hfill (6)

When the wage and market demand are known ex-ante, (5) and (6) give the same answer. However, in general the two prices will differ, even in expectation, as the distribution of market demand and wages will influence the mean pre-set price that the firm sets.

Now substituting (5) and (6) respectively into the expected profit function (4), we may evaluate the firm’s expected profits (excluding fixed costs) under $D_{i} = 1$ and $D_{i} = 0$. Let $\Theta = \{\Gamma, W, \hat{X}\}$. Then

$$\tilde{V}(\Theta) = \Psi E\Gamma(W^{\alpha(1-\lambda)}\hat{X})^{\omega},$$  \hfill (7)

$$\bar{V}(\Theta) = \Psi (E\Gamma W \hat{X}^{1/\alpha})^{(1-\lambda)\omega}(E\Gamma \hat{X})^{\lambda\omega},$$  \hfill (8)

where $\Psi = \delta^{1-\lambda} - \delta^{-(\lambda/\alpha)}$. The firm will choose $D_{i} = 1$ whenever the gain in discounted expected profits exceeds the discounted expected fixed costs. That is, $D_{i} = 1$ whenever

$$\tilde{V}(\Theta) - \bar{V}(\Theta) \geq E\Gamma W \Phi_{i}.$$  

Since $\Phi_{i}$ is known to the firm ex-ante, $E\Gamma W \Phi_{i} = \Phi_{i} E\Gamma W$. We can therefore rewrite this condition as

$$\Delta(\Theta) = \frac{[\tilde{V}(\Theta) - \bar{V}(\Theta)]}{E\Gamma W} \geq \Phi_{i},$$  \hfill (9)

where $\Delta(\Theta)$ represents the gain in price flexibility.
2.1 Approximation of equation (9)

To provide analytical results in the next section, we can evaluate the gains in price flexibility by taking a second-order logarithmic approximation to $\Delta(\Theta)$ around the mean value $E \ln(\Theta)$. In the Appendix, it is shown that

$$\Delta(\Theta) \approx \Omega \frac{\alpha^2}{2} \left[ \sigma_w^2 + \left( \frac{1 - \alpha}{\alpha} \right)^2 \sigma_x^2 + 2 \frac{1 - \alpha}{\alpha} \sigma_{wx} \right] > 0,$$

where

$$\Omega = \frac{V(\exp(E \ln \Theta))}{\exp(E(\ln \Gamma + \ln W))} \lambda(\lambda - 1) \omega^2 > 0,$$

$V(\exp(E \ln \Theta))$ represents profits evaluated at the mean $E \ln \Theta$, and $\sigma_w^2$, $\sigma_x^2$, $\sigma_{wx}$ represent the variance of the wage, market demand, and their covariance, respectively.

From (10), we see that, up to second order, the incentive for a firm to incur the costs of price flexibility depends on the variance of the wage, the variance of market demand, and their covariance. Note that if $\alpha = 1$, so that marginal cost is independent of output, then uncertainty in market demand gives no incentive for adjusting prices, and the gains from flexibility depend only on uncertainty in wages. Intuitively, if $\alpha = 1$, then optimal expected profits are linear in market demand, and furthermore, if the wage is known, then the firm’s price is the same whether it is set before or after $\Theta$ is observed. In this case, there is no gain to price flexibility. More generally, however, optimal profits are convex in $W$ when prices
are flexible, but linear in $W$ under a fixed price. Hence, wage volatility raises expected profits when prices are flexible relative to expected profits with preset prices. When $\alpha < 1$, optimal profits are concave in market demand $\hat{X}$, either when prices are flexible or fixed. Intuitively, however, the optimized profit function is more concave in demand when prices are fixed than when they are flexible. Hence, uncertainty in market demand increases the benefits to price flexibility, for $\alpha < 1$.

Finally, we note that (10) does not depend on the properties of the stochastic discount factor $\Gamma$. Up to a second-order approximation, the discount factor affects profits of fixed and flexible price firms in the same way.

2.2 Determination of price flexibility in the aggregate

The left-hand side of (9) is common to all firms. Hence, firms will differ in their choice of price flexibility solely because of differences in their specific fixed costs of flexibility. Without loss of generality, let each firm $i$ draw from a distribution of fixed costs, $\Phi(i)$, described by $\Phi(0) = 0$, $\Phi'(i) > 0$. Hence, firms are ranked according their fixed cost of flexibility. In that case, we may describe the determination of price flexibility in the aggregate as the measure $z$ of firms, $0 \leq z \leq 1$, who choose to incur the fixed cost of
price flexibility. Then $z$ is determined by the following conditions:

$$\Delta(\Theta) = \Phi(z), \quad 0 \leq z \leq 1,$$

(11)

$$\Delta(\Theta) > \Phi(1), \quad z = 1.$$

(12)

These conditions give a link between the underlying uncertainty facing firms and the aggregate degree of flexibility in the economy. So far, however, we have left $\Theta$ unexplained. In the next section we develop a two-country model that identifies the macroeconomic sources of uncertainty facing firms.

## 3 A Two-Country Model

Consider a two-country world economy, where countries are called ‘home’ and ‘foreign’. Foreign variables are denoted with an asterisk. In each country, there are consumers and firms who have a single period horizon. There is a continuum of households in each country along the unit interval, consuming both home and foreign goods. Households receive income from wages and the ownership of firms. Firms have the production technology as described by the previous section, and sort themselves into two categories; those with fixed prices, and those with flexible prices.
3.1 Households

The home country household $i$, $i \in (0, 1)$, has preferences given by

$$\ln C(i) + \chi \ln \frac{M(i)}{P} - \frac{\eta}{1 + \psi} H(i)^{1+\psi},$$

(13)

where $C(i)$ is a composite of the consumption of home and foreign goods, given by

$$C(i) = \left(\frac{C_h(i)}{\gamma}\right)^{\gamma} \left(\frac{C_f(i)}{1 - \gamma}\right)^{1-\gamma},$$

(14)

$P$ is the price index, given by $P = (P_h)^{\gamma} \left(S P_f^*\right)^{1-\gamma}$, and $\chi$, $\eta$, and $\psi$ are exogenous preference parameters. $P_f^*$ is the foreign currency price of foreign goods, $S$ is the exchange rate, and $\gamma$ represents the relative preference for home goods. $M(i)$ is the quantity of domestic money held. We assume $\chi$ is a random variable which captures shocks to the consumption velocity of money. In addition, $\gamma$, the weight of the home goods in composite consumption, is also a random variable, with mean 0.5.

Foreign country preferences are identical to home country preferences, except that foreign households value foreign money, and assume that $\chi^*$ (the foreign velocity shock) and $\chi$ are i.i.d. The random foreigners receive the same composite consumption weight $\gamma$ as that of home residents.

Consumption of home and foreign goods (indexed by $j$) are differ-
entiated, so that, for household $i$, the home goods consumption and price indices are

$$C_h(i) = \left( \int_0^1 C_h(i, j)^{1-(1/\lambda)} \, dj \right)^{1/(1-(1/\lambda))}, \quad P_h = \left( \int_0^1 P_h(j)^{1-\lambda} \, dj \right)^{1/(1-\lambda)},$$  \hspace{1cm} (15)$$

where $\lambda > 1$. The indices for the foreign goods are analogous.

Home household $i$ faces the budget constraint

$$PC(i) + M(i) = WH(i) + M_0(i) + T(i) + \Pi,$$  \hspace{1cm} (16)$$

where $M_0(i)$ is the initial money holdings, $T(i)$ is the transfer from the monetary authority, and $\Pi$ is the total profits of the final goods firms.

Households choose money balances, labor supply, and consumption of each good to maximize utility, subject to their budget constraint$^6$. We find the demand for each good, $C_h(i)$ and $C_f(i)$, that of money balances, and implicit labor supply as

$$C_h(i, j) = \left( \frac{P_h(j)}{P_h} \right)^{-\lambda} C_h(i), \quad C_h(i) = \frac{\gamma PC(i)}{P_h}, \quad C_f(i) = \frac{(1-\gamma)PC(i)}{P_f},$$  \hspace{1cm} (17)$$

$$M(i) = \chi PC(i), \quad W = \eta H(i)^{\psi} PC(i).$$  \hspace{1cm} (18)$$

$^6$Households act after the realizations of the preference shocks are observed.
3.2 Firms

Firms in each country set prices, based on the technologies described in the previous section, and demand coming from home and foreign consumers. In the home country, for instance, a measure $z$ of firms set prices $\tilde{P}_h(j)$ after the state of the world is realized, and $(1 - z)$ set prices $\bar{P}_h(j)$ in advance. The condition given by (11) (or 12) determines the size of the flexible price sector. Total profits of all firms are written as

$$\int_0^z \tilde{P}_h(j)\tilde{Y}(j) dj + \int_z^1 \bar{P}_h(j)\bar{Y}(j) dj - \int_0^1 WH(i) di. \quad (19)$$

3.3 Equilibrium

We focus on symmetric equilibria where all households and firms (of each type) within a country are alike. Equilibrium is defined in the usual way. Given money market clearing, $M = M_0 + T$, the households ex-post budget constraints are given by

$$PC = z\tilde{P}_h\tilde{Y}_h + (1 - z)\bar{P}_h\bar{Y}_h. \quad (20)$$

The goods market for each category of firm implies that

$$\tilde{Y}_h = \left(\frac{\tilde{P}_h}{\bar{P}_h}\right)^{-\lambda} \gamma \left[\frac{PC}{\tilde{P}_h} + \frac{SP^*C^*}{\bar{P}_h}\right], \quad (21)$$

$$\bar{Y}_h = \left(\frac{\tilde{P}_h}{\bar{P}_h}\right)^{-\lambda} \gamma \left[\frac{PC}{\tilde{P}_h} + \frac{SP^*C^*}{\bar{P}_h}\right]. \quad (22)$$
Labor market clearing implies

\[ H = z\bar{Y}^{1/\alpha} + (1 - z)\bar{Y}^{1/\alpha} + \int_0^z \Phi(z)dz, \tag{23} \]

where the last term on the right-hand side denotes the fixed cost incurred by the measure \( z \) of firms that choose price flexibility.

Analogous conditions hold for the foreign economy.

We may define aggregate real GDP by aggregating over fixed and flexible price firms. Thus,

\[ Y = z\bar{P}\bar{Y} + (1 - z)\bar{P}\bar{Y}. \]

3.4 Solving the model for a given price flexibility

For given \( z \) and \( z^* \), the equilibrium is very simple to characterize. From the definition of aggregate GDP and the household budget constraint, \( PC = P_hY \). Hence we may write the money market equilibrium condition as

\[ M = \chi P_hY. \tag{24} \]

Using this in combination with the goods market equilibrium (21) and (22), and aggregating, we get solutions for both the exchange rate and GDP:

\[ S = \frac{1 - \gamma}{\gamma} \frac{M\chi^*}{M^*\chi}, \quad Y = \frac{M}{P_h\chi}. \tag{25} \]

A home country monetary expansion causes an exchange rate depreciation, whereas a positive home country velocity shock causes an appreciation. A
shift in relative world demand towards home goods (rise in \( \gamma \)) causes an appreciation. Home GDP is determined by the value of home real balances, in terms of home goods, relative to the home velocity shock.

Since demand for the individual firm may be defined from (21) and (22), and wage determination is given from (18), we may use (5) and (24) to define the flexible price firm’s price:

\[
\tilde{P}_h = \delta \left[ \eta H^{\psi \alpha} P_h^{(\lambda - 1)(1 - \alpha)} \frac{M}{\chi} \right]^{\omega}. \tag{26}
\]

The appropriate discount factor for firms is given by \( \Gamma = (PC)^{-17} \).

Then we can write \( \bar{P}_h \) as

\[
\bar{P}_h = \delta \frac{E[\eta H^{\psi \alpha} (P_h^{(\lambda - 1)(1 - \alpha)} (M/\chi))^{\omega}]}{E[P_h^{(\lambda - 1)(1 - \alpha)}]} \tag{27}
\]

The domestic goods price index is

\[
P_h = \left[ z \tilde{P}_h^{(\lambda - 1)} + (1 - z) \bar{P}_h^{(\lambda - 1)} \right]^{1/(1 - \lambda)}. \tag{28}
\]

Using this, (21), (22), and (23), we may write employment as

\[
H = \left[ z \left( \frac{\tilde{P}_h}{P_h} \right)^{-\lambda/\alpha} + (1 - z) \left( \frac{\bar{P}_h}{P_h} \right)^{-\lambda/\alpha} \right] \left( \frac{M}{\lambda P_h} \right)^{1/\alpha} + \int_0^z \Phi(z)dz. \tag{29}
\]

3.5 The determination of optimal price flexibility

To determine equilibrium price flexibility, we use condition (11) (or 12) from the previous section, in combination with the values of \( \Gamma, W, \) and \( \hat{X} \)

\(^7\)This is the household’s marginal utility of a dollar of home currency.
implied by the two-country general equilibrium model. From the model equilibrium, market demand and the wage are written as

$$\hat{X} = P_h^{\lambda-1} \frac{M}{\chi}, \quad W = \eta H^{\psi} \frac{M}{\chi}. \quad (30)$$

Equation (30), in combination with equations (26) - (29), and (11) determine the values of $W, \hat{X}, \hat{P}_h, \bar{P}_h, \bar{P}_h, H,$ and $z$ for the home economy.

Notice that the simple structure of the model implies that the two economies dichotomize. The home country wage, demand, prices, employment, and equilibrium price flexibility are determined solely by the behavior of home nominal aggregate demand $M/\chi$. Conditional on the domestic money supply, the equilibrium $z$ is independent of the distribution of foreign shocks and foreign monetary policy, as well as movements in the share parameter $\gamma$. This results from the unit elasticity of substitution between home and foreign goods. For a given $M$, shocks to foreign demand are offset by movements in the exchange rate, so as to leave overall demand for the home country’s goods unchanged.

In general, the model has no analytical solution. In Section 5, we report results from the exact numerical solution of the (stochastic) model, for a given calibration. Here, however, we describe an approximate solution using the second-order approximation used in (10). In order to determine the gains to price flexibility using (10), we must obtain the variance of
\[
\left( \ln(W) + [(1 - \alpha)/\alpha] \ln(\tilde{X}) \right). \quad \text{We may write}
\]
\[
\ln(W) = \psi \ln(H) + \ln(M) - \ln(\chi), \quad (31)
\]
\[
\ln(\tilde{X}) = (\lambda - 1) \ln(P_h) + \ln(M) - \ln(\chi). \quad (32)
\]

For a given \( z \), the model is log-linear, except for the price index equation (28) and the aggregate employment term (29). In the Appendix, it is shown that the second moments of \( \ln P_h \) and \( \ln H \) can be calculated by an approximation of (28) and (29) around the mean values \( E \ln P_h \) and \( E \ln H \), respectively. Using the expressions for \( \text{Var}(\ln P_h) \), \( \text{Var}(\ln H) \), and their covariances reported in the Appendix, we can show that (10) is determined solely by the variance of \( m - \tilde{\chi} \). Here small-case letters represent deviations from the log mean, and \( \tilde{\chi} \) represents the log deviation of the velocity shock from its mean value. The expressions for (11) and (12) then become
\[
\frac{\Omega}{2\alpha} \left[ \frac{(1 + \psi\varsigma)}{1 - (1 - \alpha)(\lambda - 1)\omega\varphi(z) + \psi\varsigma\omega\varphi(z)} \right]^2 \left( \sigma_m^2 + \sigma_{\tilde{\chi}}^2 + 2\sigma_m\tilde{\chi} \right) = \Phi(z),
\]
\[
0 \leq z \leq 1 \quad (33)
\]
\[
\frac{\Omega}{2\alpha} (\alpha + \lambda(1 - \alpha))^2(\sigma_m^2 + \sigma_{\tilde{\chi}}^2 + 2\sigma_m\tilde{\chi}) \geq \Phi(1), \quad z = 1. \quad (34)
\]
Here $\varsigma < 1$ is a constant term given in the Appendix, and $\varphi(z)$ is an increasing function of $z$, which satisfies $\varphi(0) = 0$, $\varphi'(z) > 0$, $\varphi''(z) > 0$, and $\varphi(1) = 1$. Note that, by the definition of $\omega$, we have $(1 - \alpha)\omega(\lambda - 1) < 1$.

Figure 1a illustrates the determination of $z$. The VV locus illustrates the left-hand side of condition (33). This represents the benefit of price flexibility to the marginal price setter, as measured along the horizontal axis. The higher is the variance of nominal aggregate demand $m - \hat{\chi}$ (which also equals $p + c$), the higher this is. The CC locus represents the fixed flexibility cost facing the marginal price setter. The CC locus is upwards sloping, by assumption, marginal firms have higher costs of price flexibility. The VV locus is also upwards sloping, under the condition that $\lambda > 1 + [\psi\varsigma/(1 - \alpha)]$. This is explained by the link between the decisions made by all other firms and the incentive of any one firm to have flexible prices. To see this relationship, focus on the flexible price firm’s optimal pricing policy, obtained from (26), which is written as

$$\tilde{P}_h = \delta \left[ H^{\psi\alpha} M \right]^{\chi} \left( \frac{\alpha}{\chi} \right)^{1 - \alpha} \omega.$$

Say that there is a money shock which gives rise to a desire for the flexible price firm to adjust its price upwards (both because the money shock increases demand directly, and also increases the wage). The extent to which the firm will adjust its price depends on the number of other firms (i.e. $z$) who also adjust. When other firms adjust, this gives rise to two
opposing forces. First, given that other firms raise their prices, market demand for any one firm rises, since $\lambda - 1 > 0$. This leads the firm to raise its price further, so long as $(1 - \alpha) > 0$, as its marginal cost is rising. However, counter to this, the rise in the price of other firms will reduce real balances $M/P_h$, reducing the home demand for labor. This reduces the real wage faced by the firm, and reduces its desired price adjustment. If the first effect dominates, then the price response of the firm to a money shock is an increase in $z$. If the second effect dominates, the price response is a decline in $z$. The first effect will dominate whenever $(\lambda - 1)(1 - \alpha) > \psi\varsigma$.

A given firm’s ex-ante incentive for price flexibility depends on how much it would wish to adjust its price in response to a shock. If $(\lambda - 1)(1 - \alpha) > \psi\varsigma$, then, in response to a shock, the firm will have a greater incentive to adjust its price, and the greater is the measure of other firms adjustments. Hence the VV curve is upwards sloping in $z$. Intuitively, this is more likely: the more elastic the labor supply (lower is $\psi$), the higher the market elasticity of demand $\lambda$, and the more upwards sloping the marginal cost (lower is $\alpha$). If the VV curve is upwards sloping, there is a strategic complementarity in the pricing decisions of firms; the greater the measure of other firms adjusting to a money shock, the greater the incentive of any one firm to adjust its own price. Moreover, VV is also convex in $z$ if $(\lambda - 1)(1 - \alpha) > \psi\varsigma$. In the opposite case, when $(\lambda - 1)(1 - \alpha) < \psi\varsigma$, the VV
curve is actually downwards sloping, and there is a \textit{strategic substitutability} between the pricing decisions of firms. In the discussion below, we find that the conventional calibration suggests that $(\lambda - 1)(1 - \alpha) > \psi \zeta$. In light of this, we focus henceforth on the case where the VV curve is upwards sloping.

It is clear that there is the possibility of multiple equilibrium. While Figure 1a describes the case of a unique equilibrium, Figure 1b characterizes a situation where the VV curve intersects twice with the CC curve. There are three equilibria, corresponding to low $z$, $z = 1$, and an intermediate value of $z$ (unstable based on the usual reasoning). In the low-$z$ equilibrium, a small fraction of firms choose price flexibility, weakening the incentives for other firms to have flexible prices. When $z = 1$, the volatility of demand is so great that all firm’s willingly pay the costs for flexibility, because all others do. Therefore, multiple equilibria are generated by strategic complementarity in price setting. This strategic complementarity, and the possibility of multiple equilibrium\footnote{In general, for different assumptions regarding $\Phi(i)$, there may be multiple crossing points. An equilibrium with high price flexibility is not necessarily associated with full flexibility.}, is greater for lower $\alpha$, lower $\psi$, and higher $\lambda$.

We may state a condition for a unique equilibrium as follows.

\textbf{Condition 1.} If $\Phi(i)$ is uniform, so that $\Phi(i) = \bar{\Phi} i$, for $i > 0$, then there is
a unique equilibrium whenever $(\Omega/2\alpha)(\alpha+\lambda(1-\alpha))^2(\sigma_m^2+\sigma_M^2+2\sigma_{mM}) \leq \bar{\Phi}$.

The left-hand side of this expression gives the value of the VV curve at $z = 1$, while the right-hand side gives the value of the CC curve at $z = 1$. Since in this case the CC curve is a straight line, and VV is convex, as long as VV falls below the CC curve at $z = 1$ then a unique equilibrium is assured.

4 Price Flexibility and the Exchange Rate Regime

We now focus on the impact of monetary policy and the exchange rate regime on the equilibrium degree of price flexibility. We assume henceforth that Condition 1 holds, so that the equilibrium is unique. From (33), it is immediate to see that an increase in the volatility of money or velocity will increase the degree of price flexibility. To see how the exchange rate regime will affect price flexibility, we note that the exchange rate, in log deviation form, may be written as

$$s = m - m^* - (\hat{\chi} - \hat{\chi}^*) - 2\hat{\gamma}. \quad (35)$$

To define an exchange rate regime, we need to specify both the form of the monetary rules as well as the degree to which each country participates in the monetary policy. Since our objective in this section is just to describe the link between exchange rate policy and price flexibility,
we focus on a simple monetary rule where the authorities of one or both countries target the exchange rate directly. This has the advantage that it allows for variation in the importance that exchange rate stability plays in policy\textsuperscript{9}.

With respect to the degree to which each country participates in the exchange rate policy, we describe two alternatives. A unilateral or one-sided policy is a situation where one country alone follows a monetary rule to target the exchange rate. Alternatively, a bilateral (or cooperative) exchange rate policy is one where both monetary authorities target the exchange rate\textsuperscript{10}. In a unilateral policy, the home monetary authority follows the rule $m = -\mu s$, where $\mu$ is the degree of exchange rate intervention, and the foreign country maintains a passive monetary rule, $m^* = 0$. Under a bilateral policy, both home and foreign monetary authorities target the exchange rate, using the rules $m = -(\mu s/2)$ and $m^* = \mu s/2$. In both cases, a value of $\mu = 0$ corresponds to a freely floating exchange rate, and $\mu \to \infty$ corresponds to a fixed (or pegged) exchange rate.

Under these intervention rules (whether unilateral or bilateral), the
\textsuperscript{9}We compare this to a situation where monetary policy can directly target the stochastic disturbances later. Note that the monetary rules here are not chosen optimally, in a welfare sense. We describe the welfare-maximizing monetary rules in Section 6.
\textsuperscript{10}These definitions were first made by Helpman (1981).
exchange rate can be described as

\[ s = \frac{-(\hat{\chi} - \hat{\chi}^*) - 2\hat{\gamma}}{1 + \mu}. \]

Using this, and (33), we may establish the following.

**Proposition 1**

a) The degree of price flexibility \( z \) is higher under a one-sided peg than under a freely floating exchange rate.

b) In the absence of velocity shocks, \( z \) is uniformly increasing in the degree of exchange rate intervention under a unilateral peg.

**Proof:** Under the assumptions made, \( z \) is determined by

\[
\frac{\Omega}{2\alpha}(1 + \psi\chi)^2 \left( \frac{\mu^2 + 1}{(1 + \mu)^2} \sigma^2_{\chi} + \frac{\mu^2}{(1 + \mu)^2} 4 \sigma^2_{\gamma} \right) = \Psi(z), \tag{36}
\]

where \( \Psi(z) = \Phi z (1 + \varphi(z)\psi\omega - (\lambda - 1)(1 - \alpha)\varphi(z)\omega)^2 \).

The first part of the proposition follows because the left-hand side is higher when \( \mu \to \infty \) (fixed exchange rate) than under \( \mu = 0 \) (floating exchange rate). Then, as long as the equilibrium is unique, the right-hand side must be increasing in \( z \). The second part of the proposition follows because, without velocity shocks (i.e. \( \sigma^2_{\chi} = 0 \)), the left-hand side of the above condition is always increasing in \( \mu \).

To see the result more intuitively, note that equilibrium price flexibility will be higher, whenever the variance of \( m - \hat{\chi} \) is higher. However, in order to keep the exchange rate from changing in the face of relative
demand shocks, the variance of $m$ must rise. Thus, in the face of relative demand shocks, a one-sided peg tends to increase $z$. Without relative demand shocks, $\text{Var}(m - \hat{\chi})$ is equal to $\text{Var}(\hat{\chi})$, both under a floating exchange rate and under a unilateral peg. Although the peg offsets $\hat{\chi}$ shocks, it must adjust the money supply to prevent $\chi^*$ from affecting the exchange rate, so as to leave $\text{Var}(m - \hat{\chi})$ unchanged. Hence, when relative demand and velocity shocks are put together (and velocity shocks have equal variance), $\text{Var}(m - \hat{\chi})$ must be higher in a one-sided peg than under a floating exchange rate.

How does this compare to a bilateral pegged exchange rate? In this case, we have the following.

**Proposition 2**

a) The degree of price flexibility $z$ may be higher or lower with a bilateral exchange rate peg than a freely floating exchange rate, depending on the size of relative demand shocks and velocity shocks. In the absence of relative demand shocks, $z$ is lower under a bilateral peg.

b) There is always more price flexibility with a one-sided peg than with a bilateral fixed exchange rate.

**Proof:** Part a). In this case, $z$ is determined by

$$
\frac{\Omega}{2\alpha}(1 + \psi\varsigma)^2 \left( \frac{\mu^2/2}{(1 + \mu)^2} + \frac{\mu + 1}{(1 + \mu)^2} \sigma^2_\chi + \frac{\mu^2}{(1 + \mu)^2} \sigma^2_\gamma \right) = \Psi(z). \quad (37)
$$

When $\mu \to \infty$, the variance terms inside expression (37) become
\sigma^2 + (1/2)\sigma^2_\chi. From this condition we see that, without relative demand shocks, the volatility of nominal aggregate demand is strictly lower under a bilateral peg than under a freely floating exchange rate. Hence, if the variance of relative demand shocks is small, price flexibility will be lower in a bilateral peg than under a floating exchange rate.

Part b). Because each monetary authority cooperates in offsetting demand shocks, the volatility of aggregate demand specifically due to relative demand shocks is reduced, relative to the analogous volatility in a one-sided peg. Hence, in combination with part a), this means price flexibility is higher under a one-sided peg. □

Intuitively, under a bilateral peg, in the case of velocity shocks alone, then \text{Var}(m - \hat{\chi}) is lower, because countries cooperate in eliminating these shocks, rather than putting the onus on the pegging country alone. As a result, the volatility of aggregate demand due to velocity shocks in both countries is reduced, whereas with a one-sided peg the pegging country has to absorb the full effect of the velocity shock in the foreign country. Likewise, a bilateral peg also reduces the volatility of aggregate demand due to relative demand shocks, as compared with a one-sided peg, again because both countries respond to relative demand shocks.

Note that in the cooperative peg, \( z \) and \( z^* \) are equal. The cooperative peg affects price flexibility in both countries, whereas the one-sided peg
affects price flexibility in the pegging country alone.

From Propositions 1 and 2, we see that the question of whether a pegged exchange rate enhances price flexibility depends on the nature of the shocks as well as the nature of the peg. When relative demand shocks are the principal source of exchange rate fluctuations, then an exchange rate peg will enhance price flexibility, and this is more so in a country that adopts a one-sided peg. In order to stabilize the exchange rate following a relative demand shock, countries must follow a pro-cyclical monetary policy, which increases the variance of nominal aggregate demand, and hence encourages more price flexibility on the part of firms. However, when all exchange rate volatility is caused by velocity disturbances, an exchange rate peg will either leave price flexibility unchanged (in a one-sided peg), or actually reduce overall price flexibility (in a bilateral peg).

How would these results be altered if we instead made the assumption that countries could fix the exchange rate by directly reacting to the shocks themselves, rather than by indirectly doing so by way of an exchange rate intervention rule? Essentially the same conclusions apply. In the case of a one-sided peg, the monetary rule given by \( m = \hat{\chi} - \hat{\chi}^* + 2\hat{\gamma} \) keeps the exchange rate fixed. This would increase the fraction of \( Var(m - \hat{\chi}) \) due to relative demand shocks, while leaving the variance due to velocity shocks unchanged, and hence would increase the degree of price flexibility. In a
bilateral peg, the rules $m = \hat{\chi} - \hat{\gamma}$ and $m^* = \hat{\chi}^* + \hat{\gamma}$ ensure a fixed exchange rate. They eliminate the component of $\text{Var}(m - \hat{\chi})$ due to velocity shocks, but increase the component of this variance due to relative demand shocks.

We have assumed that the variance of velocity shocks is equal in the two countries, but imagine that $\sigma^2_{\chi} > \sigma^2_{\chi^*}$. We might think of this as a case where overall monetary/financial stability is higher in the foreign country, and the home country chooses a pegged exchange rate in order to ‘import’ stability from abroad. This has been a common rationale for fixed exchange rates in countries with a history of monetary instability, such as in Latin America. Under this assumption, $z$ is determined by the condition

$$\frac{\Omega}{2\alpha} (1 + \psi) \left( \frac{\mu^2}{(1 + \mu)^2} \sigma^2_{\chi} + \frac{1}{(1 + \mu)^2} \sigma^2_{\chi^*} + \frac{\mu^2}{(1 + \mu)^2} 4\sigma^2_{\gamma} \right) = \Psi(z). \quad (38)$$

Now it is no longer necessarily true that price flexibility is higher as $\mu \to \infty$. If $\sigma^2_{\chi}$ is sufficiently greater than $\sigma^2_{\chi^*}$, and relative demand shocks are unimportant, then even a one-sided peg can reduce overall aggregate demand volatility, and reduce the equilibrium degree of price flexibility.
5 Output and Relative Price Stability

The traditional view of floating exchange rates (e.g. Friedman 1953) argues that the exchange rate acts as a ‘shock absorber’. A freely floating exchange rate helps to stabilize output in response to relative demand shocks, because it allows for a greater adjustment of relative prices. This suggests that the volatility of output should be higher under an exchange rate peg than under a float, while the volatility of the terms of trade should be lower. In our model, home country output is given by $Y = M/(P_h \chi)$. Hence, output volatility may be expressed as

$$\text{Var}(y) = \text{Var}(m - \hat{\chi} - p_h)$$

$$= \frac{(1 - \varphi(z))^2 \text{Var}(m - \hat{\chi})}{(1 - (1 - \alpha) \omega(\lambda - 1)\varphi(z) + \psi \omega \varphi(z))^2}. \quad (39)$$

From this expression, we may establish the following.

**Proposition 3.** If an exchange rate peg increases the volatility of output for a given degree of price flexibility, then it will also increase the degree of price flexibility.

**Proof:** Expression (39) makes it clear that output volatility will rise, for a given $z$, whenever the volatility of $m - \hat{\chi}$ rises. However, this is exactly the same condition for an increase in price flexibility under Propositions 1 and 2. □

Thus, holding $z$ constant, a unilaterally pegged exchange rate will
always increase output volatility, when the volatility of velocity shocks is equal across countries. More generally, output volatility is higher (for fixed $z$) under a fixed exchange rate when the variance of $m - \hat{\chi}$ is dominated by relative demand shocks.

With endogenous movements in $z$, however, there is a countervailing force. As a higher fraction of firms choose to adjust prices ex-post, output is stabilized. Hence, the indirect effects of an exchange rate peg, through endogenous price flexibility, run counter to the direct effects, through increasing volatility of aggregate demand.

A similar conclusion may be obtained by looking at the terms of trade. We define the terms of trade as $(SP^*_f)/P_h$. When $z$ and $z^*$ are close to zero, so that most prices are sticky, a fixed exchange rate prevents any terms of trade adjustment at all. However, allowing price flexibility to respond to a peg creates terms of trade volatility through nominal price adjustment, even if the exchange rate is fixed.

Is it possible that, taking both the direct effect on volatility and the indirect effect through increased price flexibility, the overall macroeconomic volatility is similar across exchange rate regimes? Although it is to be expected that endogenous price flexibility would lessen the impact of an exchange rate peg on output volatility, it would seem unlikely that this indirect response to the policy change would reverse the effects of the
change itself. In the presence of strategic complementarities in price setting behavior, however, even small policy changes might have substantial effects. Because both the benefits and costs of flexibility are increasing in the number of firms that choose flexibility, the indirect effect of policy changes, through movements in equilibrium price flexibility, may be of the same order as the direct effects. Intuitively, from Figure 1a, both the VV and CC curves are upwards sloping, so that a small rise in VV might lead to a substantial rise in $z$.

Table 1 provides a quantitative illustration of this result. We calibrate the model so that the standard deviations of relative demand shocks and velocity shocks are both set at 2%. The elasticity of substitution between categories of goods is set at 11, corresponding to the standard 10% markup of price over marginal cost reported in Basu and Fernald (1997). The consumption constant elasticity of labor supply $\psi$ is set to unity, following Christiano, Eichenbaum and Evans (1998). We assume that the distribution $\Phi(i)$ is uniform. In the calibration, we choose the cost function so that if all firms were choosing ex-post price flexibility, the total cost of this would be only 3% of GDP. This corresponds to the quantification of costs of price change as measured by Zbaracki et al. (2000), and the calibration used in Dotsey et al. (1999).

Table 1 illustrates the implications of alternative exchange rate regimes
with and without endogenous price flexibility\textsuperscript{11}. Under a floating exchange rate, output volatility is 1.7\%, while terms of trade volatility is 6.2\%. The fraction of firms choosing ex-post price flexibility is only 10\%. Now if we impose a one-sided pegged exchange rate, but hold the degree of price flexibility unchanged, the second column of the table shows that output volatility more than doubles to 4.2\%, whereas terms of trade volatility falls to 0.2\%. By contrast, allowing for endogenous price flexibility shows a dramatic rise in the fraction of firms choosing ex-post price flexibility - $z$ rises from 10\% to 68\%. As a result, output volatility is stabilized - the standard deviation of output falls to 1.8\% - effectively the same as under floating exchange rates. On the other hand, the volatility of the terms of trade is now only 2.2\%, rather than 6.2\% under the floating exchange rate. Hence, comparing floating and fixed exchange rates in the presence of endogenous price flexibility, we see that the fixed exchange rate regime leads to a large drop in terms of trade volatility, but effectively no movement in output volatility. The increased flexibility of nominal prices tends to offset the direct increase in macroeconomic volatility introduced by a fixed exchange rate.

\textsuperscript{11}Table 1 is constructed by exact numerical solution of the stochastic non-linear model, using a discrete state space for $\chi$, $\chi^*$, and relative demand shocks, assuming four states of the world. The procedure uses a standard non-linear equation solver.
leaving output volatility essentially unchanged? Holding price flexibility constant, the peg leads output volatility to rise substantially, and eliminates almost all terms of trade volatility. As price flexibility increases, output volatility declines, since nominal prices adjust to offset relative demand shocks. The rise in price flexibility also raises terms of trade volatility, but this effect is limited to that coming from the increased domestic nominal price adjustment, which contributes less to terms of trade variability than does the nominal exchange rate under floating exchange rates.

<table>
<thead>
<tr>
<th>Table 1.</th>
<th>Real Effects of an Exchange Rate Peg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>St. dev. GDP</td>
</tr>
<tr>
<td>Flexible</td>
<td>1.7</td>
</tr>
<tr>
<td>Fixed (Exogenous z)</td>
<td>4.2</td>
</tr>
<tr>
<td>Fixed (Endogenous z)</td>
<td>1.8</td>
</tr>
</tbody>
</table>

These results may help to throw light on the well-known puzzle raised by Mussa (1986), Baxter and Stockman (1989), and Flood and Rose (1995), concerning the relationship between exchange rate regimes and macroeconomic volatility. Standard theory implies that, holding the distribution of underlying fundamentals constant, a move from a fixed exchange rate to a floating exchange rate should have substantial implications for the volatility of both exchange rates and real GDP, as well as other macroeconomic aggregates.
The evidence clearly shows that floating exchange rate regimes are associated with much higher real exchange rate volatility. However, as argued by Baxter and Stockman (1989), there is little evidence that other macroeconomic aggregates, such as the volatility of real GDP, changed substantially after economies changed from fixed to floating exchange rate regimes. Flood and Rose (1995) further show that there is little change in the underlying exchange rate fundamentals across fixed and floating exchange rate regimes. So floating exchange rates appear to cause a large increase in nominal and real exchange rate volatility, but have little affect on any other macroeconomic variables\textsuperscript{12}. The quantitative results of our model are consistent with the observation that by holding the distribution of economic fundamentals constant, following a move from fixed to floating exchange rates, a small economy may experience very little change in the volatility of GDP, but a substantial rise in relative price variability\textsuperscript{13}.

\textsuperscript{12}This conclusion has recently been challenged for developing economies by Broda (2001, 2004). He shows that, in response to terms of trade shocks, floating exchange rates tend to substantially cushion the impact on GDP, relative to fixed exchange rates, for developing economies. For other shocks, however, floating exchange rates tend to have little affect on the response.

\textsuperscript{13}Note that in our model the consumption-based real exchange rate is always constant, as PPP holds. In a more general model with systematic home bias in preferences, the real exchange rate and the terms of trade would be positively correlated. Duarte (2003) and Dedola and Leduc (2000) propose a different explanation of the puzzle of small
6 Optimal Monetary Policy

So far we have simply compared arbitrary monetary rules that target the exchange rate. In this section, we discuss properties of the optimal monetary policy, taking into account the endogenous nature of price rigidity. The typical result in this literature is that the monetary authority chooses a rule so as to replicate the flexible-price equilibrium. As shown by King and Wolman (1999) and Woodford (2003), this naturally implies that an optimal monetary rule ensures price stability, since a rule which obviates the necessity for firms to change prices ensures that a sticky price equilibrium coincides with the flexible price equilibrium.

In a two-country setting, the environment becomes more complicated, because there are two new sources of market failure. First, there is a problem of strategic interaction between monetary authorities and the potential welfare losses from the absence of coordination (see, for instance, Benigno and Benigno 2003, and Sutherland 2002). Second, markets for international risk-sharing may be incomplete, so that even the flexible price equilibrium of the world economy is inefficient. Obstfeld and Rogoff (2002), and Benigno (2001) show that, in the absence of full international differences in macroeconomic volatility across exchange rate regimes. In their models, the presence of ‘local currency pricing’ causes output and consumption volatility to be similar across fixed and flexible exchange rate systems.
risk sharing, an optimal monetary policy rule may not target the flexible price equilibrium allocation and, moreover, there may be gains to international policy coordination.

The two-country model of our paper falls into the second category; international risk sharing is incomplete, and an optimal monetary policy will not in general try to replicate the flexible price allocation\textsuperscript{14}.

How does the possibility of endogenous price rigidity affect these conclusions? Dotsey, King and Wolman (1999) note that allowing for state-dependent pricing does not alter the main implication for optimal monetary policy in King and Wolman (1999). If the monetary authority wishes to replicate the flexible price equilibrium, then a monetary policy ensuring price stability will achieve this. Firms will not wish to change their prices, even when they can do so in a state contingent manner. When the monetary authority wishes to achieve an allocation other than the flexible price allocation, however, the state-dependent pricing technology may alter the optimal monetary policy problem.

We first characterize the optimal monetary rule in the case where all prices are set in advance. We may state the results in the following form.

**Proposition 4.** When $\Phi(i) \to \infty$, for all $i \in (0, 1)$:

\textsuperscript{14}Risk sharing is incomplete in this model because, even though countries experience the same $\gamma$ shocks, the terms of trade and wealth effects of this shock are different for the two countries. This point is explored in Devereux (2004).
a) the optimal monetary policy rule for the home and foreign country is written as

\[ M = \chi \gamma^{\alpha/(1+\psi)} \Lambda, \quad M^* = \chi^* (1 - \gamma)^{\alpha/(1+\psi)} \Lambda \]

(40)

where \( \Lambda \) is a constant function of parameters;

b) there are no gains to international monetary policy coordination.

Proof: See the Appendix.

Hence, in response to a positive relative demand shock for the country’s output, the monetary authority should respond in the proportion \( \alpha/(1 + \psi) \). The lower the elasticity of labor supply and the smaller the share of labor in production, the weaker should be the response of the monetary policy.

Note, however, that this policy will not sustain the flexible price equilibrium allocation. The flexible price equilibrium allocation\(^{15}\) is achieved using the rules \( M = \chi \Lambda \) and \( M^* = \chi^* \Lambda \), the optimal monetary policy should be expansionary in the face of positive relative demand shocks for a country. Without monetary policy reaction, the domestic output response to relative demand shocks is smaller than efficient, due to the absence of markets for international risk sharing (as in Obstfeld and Rogoff 2002, and Benigno 2001)\(^{16}\). The intuition for part b) of the proposition is that

\(^{15}\)It is easy to see from (26) and (29) that if \( M\chi^{-1} \) is constant, then the ex-post optimal price is constant.

\(^{16}\)Some more intuition for this can be given. In the flexible price economy, a relative
the expected disutility of employment is constant, so that effectively both monetary authorities have the same objective function.

Now allowing for endogenous price flexibility, we note that, without relative demand shocks, the rule (40) would sustain the flexible price allocation, so firms would have no incentive to invest in flexibility\textsuperscript{17}. However, when relative demand shocks are present, (40) would add monetary volatility to the firms environment, and give them an incentive to invest in price flexibility. Of course, with endogenous price flexibility, (40) is no longer necessarily an optimal monetary rule. In general, there will be no simple closed-form representation of optimal policy comparable to (40). We can, however, establish the following.

**Proposition 5.** The optimal monetary rule takes the form \( M = \chi \Upsilon(\gamma) \).

demand shock raises a country’s terms of trade and income. This generates a wealth effect which reduces optimal labor supply. On the other hand, the terms of trade increase itself tends to increase optimal labor supply, as the return to work increases. In equilibrium these effects exactly offset each other, and output is constant. With full international risk sharing, however, the first effect - the wealth effect - is mitigated, because part of it goes to foreign households. As a result, the second effect dominates, and equilibrium domestic output increases (as is efficient).

\textsuperscript{17}It is easy to extend the model to allow for country-specific productivity shocks, and to show that this conclusion is unaffected by such an extension. That is, the optimal monetary rule with velocity and productivity shocks would still target the flexible price allocation, and so therefore endogenous price rigidity would not affect the optimal rule.
\textbf{Proof:} See the Appendix.

The monetary authority should exactly accommodate domestic velocity shocks in the same way as before, since with only velocity shocks the optimal rule would sustain the flexible price allocation.

In order to determine the form of $\Upsilon(\gamma)$, we can numerically compute optimal monetary rules where the monetary authorities take account of the impact of their policy rules on equilibrium price flexibility. We assume that $\Upsilon$ takes the form $\Upsilon = \gamma^a$, and solve for $a$ by maximizing expected utility, for the home and foreign countries. We use the same calibrated economy as that of Table 1.

Table 2 shows the results. The first column shows the optimal policy with exogenous (and zero) price flexibility, and confirms Proposition 4 (here $\hat{a} \equiv (aa)/(1 + \psi)$, where $a$ is defined in the rule $\Upsilon = \gamma^a$, so $\hat{a} = 1$ corresponds to Proposition 4). We find that the optimal monetary rule when the monetary authorities take account of endogenous price flexibility calls for a slightly less pro-cyclical stance ($\hat{a} = 0.89$). That is, the authorities in each country place slightly less weight on the relative demand shock than they would in the pure sticky price economy. Intuitively, they take into account the fact that the pro-cyclical stance of monetary policy leads more firms to incur the costs of price flexibility. This has welfare costs both directly, in terms of the fixed labor cost incurred by flexible price
firms, and indirectly, because the closer that the firms get to the flexible price equilibrium, the farther away will be the allocation from that desired by the monetary authorities. On both counts, monetary policy becomes less activist. Quantitatively, we find that, under the optimal monetary policy rule, price flexibility is very low under the benchmark calibration - effectively the presence of endogenous price rigidity has little impact on the optimal monetary policy.

Table 2. Optimal Policy with Endogenous Flexibility.

<table>
<thead>
<tr>
<th>Regime</th>
<th>$\Phi \to \infty$</th>
<th>Endogenous</th>
<th>Endogenous, high volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Passive</td>
<td>Optimal</td>
<td>Passive</td>
</tr>
<tr>
<td>$\tilde{a}$</td>
<td>1</td>
<td>1</td>
<td>0.89</td>
</tr>
<tr>
<td>$z$</td>
<td>0</td>
<td>0.013</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 2 also illustrates the degree of price flexibility that would occur if the monetary authorities were ‘passive’ in the sense that they maintained $\tilde{a} = 1$. In the benchmark case, this would lead to a $z$ of 1.3%, which is only slightly different from the optimal rule. If relative demand shocks were much more volatile (columns 5 and 6 in Table 2, where the standard deviation of relative demand shocks is set at 8%), then a passive policy

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18We can also look at an environment where the optimal monetary rule is determined after firms have chosen the degree of price flexibility. We find that the quantitative results differ only slightly.
would lead \( z \) to jump to 19\. By contrast, in this case, the optimal policy would be much less activist, setting \( \tilde{a} \) equal to 0\.58, and ensuring a lower \( z \) of 6\. Interestingly, the more volatile the relative demand shock, the less activist the optimal policy under endogenous price flexibility (and the more closely targeted the flexible price equilibrium).

What do these results imply for the monetary rules of Section 3? First, we can note that in the sticky price economy of Proposition 4 neither a fixed exchange rate (whether one-sided or cooperative) nor a flexible exchange as defined above is an optimal rule. The exchange rate in the pure sticky price economy, under the optimal monetary policy rule, will be:

\[
S = \left( \frac{1 - \gamma}{\gamma} \right)^{(1-\alpha+\psi)/(1+\psi)}.
\]

So, with all prices sticky, the exchange rate will still respond to relative demand shocks under the optimal rule (but by less than it would under the flexible exchange rate rule). By extension, a fixed exchange rate is then not an optimal monetary policy rule in the economy with endogenous price flexibility, because, as we found in that case, optimal monetary policy is even less responsive to relative demand shocks. As an additional implication, a one-sided peg is not optimal for a second reason, because it requires that the monetary authority responding to foreign as well as domestic velocity shocks.

Hence, optimal monetary policies are likely to take on quite differ-
ent characteristics than the simple fixed and floating exchange rate rules described in the previous section. Moreover, it is possible, for different parameter values governing the size of the relative demand shock, the fixed cost of price flexibility, and the elasticity of labor supply, for either a fixed or flexible exchange rate regime (as defined) to be welfare superior. Nevertheless, from a positive viewpoint, our results do emphasize the sensitivity of the standard predictions about fixed and floating exchange rates to the possibility of endogenous price flexibility.

7 Conclusions

Theoretical discussion of the merits of exchange rate flexibility almost always takes the structural characteristics of wage and price determination as being independent of the exchange rate regime choice. However, in policy circles, it is often emphasized that exchange rate commitments may help to affect private-sector expectations and alter the institutional structure of wage contracting and price setting. For instance, many Latin American countries pursued exchange-rate-based stabilizations in the 1990s in the hope that the exchange rate commitment would feed directly into private-sector expectations and price setting behavior. Likewise, economies such as Hong Kong that operate on Currency Board arrangements emphasize that a pre-requisite for success is the flexibility of internal prices (Latter
In a different context, there has been speculation that the introduction of the euro might encourage price flexibility within Europe, and limit the cost of giving up on the exchange rate as a response to national shocks within the euro area.

This paper has developed a theory underpinning the link between exchange rate regimes and nominal price flexibility. Using the basic microeconomics of a firm’s decision to invest in price flexibility, and then integrating this into a two-country macroeconomic model, we show that, for the link between exchange rate regimes and price flexibility, the form of the exchange rate rule is critical. A unilateral peg will increase price flexibility, perhaps by a large amount, but in a cooperative peg, price flexibility is unlikely to be substantially affected, and might even be reduced.

There are a number of ways in which the analysis could be extended. In the model of the paper, we assume that prices are set in the seller’s currency (producer currency pricing), so that exchange rate pass-through into import prices is immediate. An alternative assumption would be to have prices set in the buyer’s currency (local currency pricing). In this case, the trade-off between fixed and flexible prices for an exporting firm would depend on exchange rate volatility. In this sense, higher exchange rate volatility would give rise to more flexibility in the setting of export
prices, and a higher degree of exchange rate pass-through\textsuperscript{19}.

\section{Appendix}

\subsection{Obtaining the approximation (10)}

We first describe how the approximation given in (10) is obtained. Note that

\[ \Delta(\Theta) = \frac{\Psi}{ETW} \left[ E \Gamma(W^{\alpha(1-\lambda)} \hat{X})^{\omega} - (ETW \hat{X}^{1/\alpha})^{(1-\lambda)\omega} (E \Gamma \hat{X})^{\lambda \omega} \right]. \quad (A.1) \]

This may be written in the form

\[ \Delta(\Theta) = \frac{\Psi}{E \exp(\ln \Gamma + \ln W)} \left[ E \exp(\ln \Gamma + \omega(\alpha(1-\lambda) \ln(W) + \ln \hat{X}))^{(1-\lambda)\omega} (E \exp(\ln \Gamma + \ln \hat{X}))^{\lambda \omega} \right]. \quad (A.2) \]

\textsuperscript{19}A more general issue would be to allow the currency of price setting to be endogenous itself. This problem is examined in Devereux, Engel and Storgaard (2004), and Bacchetta and van Wincoop (2003). In these papers, there may also be a strategic complementarity in price setting.
Now take a second-order logarithmic approximation of $\Delta(\Theta)$ around the mean $E\ln \Theta$. This gives

$$\Delta(\Theta) \approx \Delta(\exp(E\ln \Theta)) \quad (A.3)$$

$$+ \Xi E(g + \omega(\alpha(1 - \lambda)w + x)$$

$$- \Xi E(((1 - \lambda)\alpha + \lambda)\omega g + (1 - \lambda)\alpha \omega w + ((1 - \lambda)\omega + \lambda\omega)x)$$

$$+ \Delta(\exp(E\ln \Theta))E(g + w)$$

$$+ \frac{1}{2} \Xi E(g^2 + (\omega(\alpha(1 - \lambda)))^2 w^2 + \omega^2 x^2 + 2\omega\alpha(1 - \lambda)gw + 2\omega gx + 2\omega^2 \alpha(1 - \lambda)wx)$$

Note that this approximation is around the mean of the stochastic equilibrium of the model, not a non-stochastic equilibrium. Although approximation (10) has exactly the same form if we approximate around the non-stochastic equilibrium, in order to obtain the variance of $\ln \Theta$ we must approximate $P_h$ and $H$ around the stochastic equilibrium of the model (see later), since in a non-stochastic equilibrium $z = 0$ would always hold (with no uncertainty no firms would pay a fixed cost of flexibility). The reason that we can approximate around the stochastic equilibrium is that $\Delta(\exp(E\ln \Theta)) = 0$ by the definition of the $\Delta$ function, evaluated at the mean of the stochastic equilibrium. Hence, we can derive the approximation (10) without having to solve for $E\ln \Theta$ itself, as is required in the standard second-order solution methods for sticky price models (e.g. Sutherland 2002, Benigno and Woodford 2004, Schmitt-Grohe and Uribe 2004). For welfare evaluation, it is necessary to take account of the effects of monetary policy on $E\ln \Theta$ (see equation (A.15)).
\[-\frac{1}{2} \Xi[(1 - \lambda)\alpha \omega E(g^2 + w^2 + \alpha^{-2}x^2 + 2gw + 2\alpha^{-1}gx + 2\alpha^{-1}wx) \\
+ \lambda \omega E(g^2 + x^2 + 2gx)]
\]

\[-\frac{1}{2} \Xi(1 - ((1 - \lambda)\alpha + \lambda)\omega)Eg(g + w)
\]

\[-\frac{1}{2} \Xi(\omega\alpha(1 - \lambda) - (1 - \lambda)\alpha\omega)Ew(g + w)
\]

\[-\frac{1}{2} \Xi(1 - ((1 - \lambda)\alpha + \lambda)\omega)Ex(g + w),
\]

where small-case letters denote logarithmic deviations from their mean levels, \(g = \ln \Gamma - E \ln \Gamma\), \(w = \ln W - E \ln W\), \(x = \ln \hat{X} - E \ln \hat{X}\), and \(\Xi \equiv [V(\exp(E \ln \Theta))]/[\exp(E \ln \Gamma + E \ln W)].\)

Using the definition of \(\Delta(\Theta)\) in (9), it must be that \(\Delta(\exp E \ln \Theta) = 0\), since profits for fixed or flexible price firms are equal, evaluated at the constant value of \(E \ln \Theta\). Hence the first term on the right-hand side of (A.3) must be zero.

The second, third and fourth terms represent first-order effects, evaluated around \(E \ln \Theta\). The second and third terms are also zero by definition, since \(Eg = Ew = Ex = 0\). The fourth term, capturing the first-order effects of the terms in the denominator of (A.2), is zero for the same reason. The fifth and sixth terms capture the second-order effects coming from the numerator of (A.2), while the seventh, eighth, and ninth terms capture the second-order effects coming from the denominator of (A.2). These last three terms are zero, given the definition \(\omega = 1/[(\alpha + \lambda(1 - \alpha))].\)
Intuitively, the second-order effects coming from the denominator of (A.2) are zero because they interact with expected profits in the flexible and fixed price case in ways which exactly cancel out.

Then, defining $Ew^2$ as $\sigma_w^2$, etc, the fourth and fifth terms in expression (A.3) may be rearranged, after cancelling out terms in $\sigma_g^2$, $\sigma_{gx}$, and $\sigma_{gw}$, to give

$$
\frac{\Omega\alpha}{2} \left[ \sigma_w^2 + \frac{(1 - \alpha)^2}{\alpha^2} \sigma_x^2 + 2 \frac{(1 - \alpha)}{\alpha} \sigma_{wx} \right],
$$

(A.4)

which is just expression (10) in Section 2.

### A.2 Approximating $P_h$ and $H$

We first take a second-order approximation of $P_h$ around the mean $E \ln P_h$.

From (28), we have

$$
\ln P_h = \ln \left[ z \exp((1 - \lambda) \ln \tilde{P}_h) + (1 - z) \exp((1 - \lambda) \ln \bar{P}_h) \right]^{1/(1 - \lambda)}.
$$

Since $\tilde{P}_h$ is predetermined we can ignore this term, so that we have

$$
\ln P_h \approx \left[ z \exp((1 - \lambda)E \ln \tilde{P}_h) + (1 - z) \exp((1 - \lambda) \ln \tilde{P}_h) \right]^{1/(1 - \lambda)} + \varphi(z)\tilde{p}_h
$$

$$
+ \frac{1}{2} \varphi(z)^2(1 - \lambda) \left[ \frac{z \exp((1 - \lambda) \ln \tilde{P}_h)}{z \exp((1 - \lambda)E \ln \tilde{P}_h)} \right] \tilde{p}_h^2
$$

(A.5)

where $\tilde{p}_h \equiv \ln \tilde{P}_h - E \ln \tilde{P}_h$, and

$$
\varphi(z) = \frac{z \exp(E \ln \tilde{P}_h(1 - \lambda))}{z \exp(E \ln \tilde{P}_h(1 - \lambda)) + (1 - z) \exp(E \ln \tilde{P}_h(1 - \lambda))}
$$
is an increasing function of $z$, which satisfies the properties $\varphi(0) = 0$, $\varphi(1) = 1$, as well as $\varphi''(z) > 0$ (This latter property holds because the pre-set prices are higher in mean than the mean of the flexible prices.). This approximation allows for the fact that the mean values $E \ln \tilde{P}_h$ and $E \ln \bar{P}_h$ will not in general be the same.

To derive an approximation to $\ln H$, we use an identical procedure. We approximate (29) around the mean level of $E \ln H$, up to second order. This gives

$$\ln H = \Gamma_0 + \frac{s}{\alpha} (m - p_h - \hat{\chi}) + \frac{s}{\alpha} \varphi p_h + \frac{1}{2} \left( \Gamma_1 (m - \hat{\chi})^2 + 2 \Gamma_2 (m - \hat{\chi}) p_h + \Gamma_3 p_h^2 \right).$$

(A.6)

Here we define the terms

$$\Gamma_0 \equiv \left[ z \exp \left( -\frac{\lambda}{\alpha} E \ln \left( \frac{\tilde{P}_h}{P_h} \right) \right) + (1 - z) \exp \left( -\frac{\lambda}{\alpha} E \ln \left( \frac{\bar{P}_h}{P_h} \right) \right) \right] \exp \left( -\frac{1}{\alpha} E \ln \left( \frac{M \chi P_h}{P_h} \right) \right) + \int_0^z \Phi(i) di,$$

$$s \equiv \frac{\Gamma_0 - \int_0^z \Phi(i) di}{\Gamma_0},$$

$$\varphi \equiv -\lambda [\rho(z)(1 - \varphi(z)) - (1 - \varphi(z))\varphi(z)] \varphi(z)^{-1},$$

and

$$\rho(z) \equiv \frac{z \exp(E \ln \tilde{P}_h)^{-\lambda/\alpha}}{z \exp(E \ln \tilde{P}_h)^{-\lambda/\alpha} + (1 - z) \exp(E \ln \bar{P}_h)^{-\lambda/\alpha}}.$$

$\Gamma_1$, $\Gamma_2$, and $\Gamma_3$ are constant coefficients, the second-order derivatives of the $\ln H$ equation.

Why do we take second-order approximations to $\ln P_h$ and $\ln H$?

This is because, when we approximate around the stochastic means $E \ln P_h$
and \( E \ln H \), the functions (28) and (29) are not log-linear at the point of approximation. This introduces an approximation error (due to Jensen’s inequality) at the first order, which is corrected by the second-order terms.

More specifically, from (A.5), we note that

\[
\ln P(E \ln \tilde{P}_h) \equiv \ln \left[ z \exp((1 - \lambda) E \ln \tilde{P}_h) + (1 - z) \exp((1 - \lambda) \ln \tilde{P}_h) \right]^{1/(1-\lambda)} \\
\neq E \ln P(\ln \tilde{P}_h) \equiv E \ln \left[ z \exp((1 - \lambda) \ln \tilde{P}_h) + (1 - z) \exp((1 - \lambda) \ln \tilde{P}_h) \right]^{1/(1-\lambda)}.
\]

Then, taking away \( E \ln P_h \) from each side of (A.5), we have

\[
p_h = \ln P_h - E \ln P_h \approx \left[ z \exp((1 - \lambda) E \ln \tilde{P}_h) + (1 - z) \exp((1 - \lambda) \ln \tilde{P}_h) \right]^{1/(1-\lambda)} \\
- E \ln \left[ z \exp((1 - \lambda) \ln \tilde{P}_h) + (1 - z) \exp((1 - \lambda) \ln \tilde{P}_h) \right]^{1/(1-\lambda)} + \varphi(z) \tilde{p}_h
\]

\[
+ \frac{1}{2} \varphi(z)^2 (1 - \lambda) \left[ \frac{(1 - z) \exp((1 - \lambda) \ln \tilde{P}_h)}{z \exp((1 - \lambda) E \ln \tilde{P}_h)} \right] \tilde{p}_h^2.
\]

In order to ensure that \( E p_h = E(\ln P_h - E \ln P_h) = 0 \), so that the approximation for \( \ln P_h \) is accurate, the second-order term must satisfy

\[
[z \exp((1 - \lambda) E \ln \tilde{P}_h) + (1 - z) \exp((1 - \lambda) \ln \tilde{P}_h)]^{1/(1-\lambda)} \\
- E \ln [z \exp((1 - \lambda) \ln \tilde{P}_h) + (1 - z) \exp((1 - \lambda) \ln \tilde{P}_h)]^{1/(1-\lambda)}
\]
\[
\frac{1}{2} \varphi(z)^2 (1 - \lambda) \left[ \frac{(1 - z) \exp((1 - \lambda) \ln \tilde{P}_h)}{z \exp((1 - \lambda) E \ln \bar{P}_h)} \right] \text{Var}(\tilde{P}_h) = 0.
\]

A similar property holds for (A.6). Finally, note that in the (second-order) approximation done here we have ignored the notation accounting for third and higher-order terms, on the assumption that they are small enough to be neglected. Taking account of these, it would be more accurate to say that the right-hand side in the last equation is equal to terms of third order and higher rather than equal to zero.

The first-order terms in the approximation (A.6) can be explained as follows. A rise in real aggregate demand \( m - p_h - \hat{\chi} \) raises demand for labor by a factor of \( \varsigma/\alpha \), where \( \varsigma < 1 \) reflects the fact that employment rises in proportion to aggregate demand by less than \( 1/\alpha \), due to the fixed costs of price flexibility. The term \( \kappa \) in (A.6) reflects the fact that changes in the flexible price index cause movements in aggregate demand between the fixed price and flexible price sectors, which do not cancel out when \( E \ln \tilde{P}_h \neq E \ln \bar{P}_h \). In the exact numerical solution of the model we find the term \( \kappa \) to be second order relative to the first terms in (A.6). Given this, and in order to simplify the exposition of the model, we ignore these compositional effects in the discussion in Section 4. Incorporating these terms would not change any of the propositions in Section 3, but would add considerably to the notation. Moreover, since the quantitative results of
Section 5 solve the exact model, the compositional effects are automatically included in Table 1.

Although an accurate calculation of $\ln P_h$ and $\ln H$ requires a second-order approximation, in fact the analysis of the text uses only the *second moments* of these variables. That is, to derive the approximation (10), using the equations of the open economy model (31) and (32), we only need to compute the second moments of $\ln P_h$ and $\ln H$. These may be obtained by using the first-order terms in (A.5) and (A.6), since when using these approximations to compute second moments the second-order terms become third order or higher, and therefore beyond second-order accuracy.

In order to compute the second moments, however, we further need to evaluate $\ln \tilde{P}_h$. Since (26) is log-linear, it may be written as

$$\tilde{p}_h = \omega \alpha \psi h + \omega (\lambda - 1)(1 - \alpha) p_h + \omega (m - \tilde{\chi}). \quad (\text{A.7})$$

Then, using (A.5), (A.6), and (A.7), we may establish that

$$\text{Var}(p_h) = \frac{[\varphi(z) \omega (1 + \psi \varsigma)]^2}{\Delta^2} \text{Var}(m - \tilde{\chi}), \quad (\text{A.8})$$

$$\text{Var}(h) = \frac{\varsigma^2}{\alpha^2} \frac{[1 - \varphi(z) \omega - (\lambda - 1)(1 - \alpha) \omega \varphi(z)]^2}{\Delta^2} \text{Var}(m - \tilde{\chi}). \quad (\text{A.9})$$
\[
\text{Covar}(p_h, h) = \frac{\varsigma}{\alpha} \frac{[1 - \varphi(z)\omega - (\lambda - 1)(1 - \alpha)\omega\varphi(z)][\varphi(z)\omega(1 + \psi\varsigma)]}{\Delta^2} \text{Var}(m - \check{\chi}),
\]

\[
\text{Covar}(p_h, m - \check{\chi}) = \frac{\varphi(z)\omega(1 + \psi\varsigma)}{\Delta} \text{Var}(m - \check{\chi}), \quad (A.10)
\]

\[
\text{Covar}(h, m - \check{\chi}) = \frac{\varsigma}{\alpha} \frac{(1 - \varphi(z)\omega - (\lambda - 1)(1 - \alpha)\omega\varphi(z))}{\Delta} \text{Var}(m - \check{\chi}), \quad (A.11)
\]

where \(\Delta = 1 - (\lambda - 1)(1 - \alpha)\omega\varphi(z) + \varphi(z)\omega\psi\varsigma\).

Now, substituting these expressions into (31) and (32), and using (10), gives (33) and (34).

### A.3 Optimal Monetary Rules

Here we derive the results of Section 6 of the paper. Following most of the literature (e.g. Obstfeld and Rogoff 2002), we assume that the optimal monetary rules maximize expected utility net of the utility of real balances.

If all prices are sticky, then the home country price may be written as

\[
P_h = \eta^{\alpha/(1 + \psi)} \delta^{1/(\omega(1 + \psi))} \left( E \left( \frac{M}{\chi} \right)^{(1 + \psi)/\alpha} \right)^{\alpha/(1 + \psi)}. \quad (A.12)
\]
It is straightforward to show that equilibrium consumption and employment in the home country are given by

\[ C = \frac{M}{\chi P_h^\gamma (SP_f^*)^{1-\gamma}} = \gamma \left( \frac{M}{\chi P_h^\gamma (SP_f^*)^{1-\gamma}} \right) \left( \frac{M^*}{\chi^* P_f^* (1 - \gamma)} \right)^{(1-\gamma)}, \quad (A.13) \]

\[ H = \left( \frac{M}{\chi P_h^\gamma} \right)^{1/\alpha}. \quad (A.14) \]

From (A.14) and (A.12), we can show that

\[ \frac{\eta}{1 + \psi} EH^{1+\psi} = \frac{\eta}{1 + \psi} E\frac{(M/\chi)^{(1+\psi)/\alpha}}{P_h^{(1+\psi)/\alpha}} = \Omega_1 E\frac{(M/\chi)^{(1+\psi)/\alpha}}{E(M/\chi)^{(1+\psi)/\alpha}}, \]

which is a constant ($\Omega_1$ is a constant function of the parameters). Hence, for monetary policy evaluation, expected utility in this case depends only on the expected value of log composite consumption.

We may then write out the expected utility objective function for the home country monetary authority as (ignoring constants)

\[ E \ln C = E\gamma \left( \ln \frac{M}{\chi} - \ln P_h \right) + E(1 - \gamma) \left( \ln \frac{M^*}{\chi^*} - \ln P_f^* \right). \quad (A.15) \]

**Proof of Proposition 4:** Without loss of generality, assume that there is a finite number of possible states of the world $\Sigma$ and let any state be denoted $\epsilon \in \Sigma$. Let the money supply of each country be state contingent.
Choosing \( M(\epsilon) \) to maximize (A.15) gives us the first-order condition

\[
\gamma(\epsilon) \frac{1}{M(\epsilon)} - \frac{1}{M(\epsilon)} \left( \frac{M(\epsilon)}{\chi(\epsilon)} \right)^{(1+\psi)/\alpha} (E\gamma) \left( \frac{\delta^{\alpha/(1+\psi)}}{E(M/\chi)^{(1+\psi)/\alpha}} \right) = 0.
\]

For a solution to exist, it is necessary to normalize the money supply in some way (since systematic money is neutral) - any normalization will do. For instance, if we fix \( E(M/\chi)^{(1+\psi/\alpha)} \), it is straightforward to re-arrange this first-order condition to establish the monetary rules given in Proposition 4.

The second part of the Proposition follows due to the fact that, given constant expected utility of employment for each country, the monetary objective function (A.15) will be the same for each country.

**Proof of Proposition 5:** When some prices are endogenous, the expected utility of employment is no longer independent of the distribution of money. However, the expected utility of log consumption (A.15) is written in the same way. Thus, the objective function for the monetary authority may be described as

\[
E\gamma \left( \ln \frac{M}{\chi} - \ln P_h \right) + E(1 - \gamma) \left( \ln \frac{M^*}{\chi^*} - \ln P^*_f \right) - \frac{1}{1 + \psi} EH^{1+\psi}. \quad (A.16)
\]

The monetary authorities choose a state contingent monetary rule to maximize (A.16) subject to the conditions on prices, employment, and the determination of \( z \). These are
\[ \hat{P}_h = \delta \left[ \eta H^\psi \alpha \hat{P}_h \right]^{\omega}, \quad (A.17) \]

\[ \hat{P}_h = \delta \frac{E[\eta H^\psi (P_h^{\lambda-1}M/\chi)^{1/\alpha}]^{\omega}}{E[P_h^{\lambda-1}]^{\omega}}, \quad (A.18) \]

\[ \hat{P}_h = [z\hat{P}_h^{1-\lambda} + (1-z)\hat{P}_h^{1-\lambda}]^{1/(1-\lambda)}, \quad (A.19) \]

\[ H = \left[ z \left( \frac{\hat{P}_h}{\tilde{P}_h} \right)^{-\lambda/\alpha} + (1 - z) \left( \frac{\hat{P}_h}{\tilde{P}_h} \right)^{-\lambda/\alpha} \right] \left( \frac{M}{\chi \tilde{P}_h} \right)^{1/\alpha} + \int_0^z \Phi(z) dz, \quad (A.20) \]

\[ \Delta(\Theta(z)) = \Phi(z). \quad (A.21) \]

From inspection of these equations, we can check that there is still no strategic interaction between the home and foreign monetary authorities. Also, since the state contingent money supply enters in the form \([M(\epsilon)]/\chi\), it is still optimal to exactly accommodate velocity shocks. Maximizing (A.16) with respect to \(M(\epsilon)\), subject to (A.17)-(A.21) gives an implicit function \(M(\epsilon) = \chi(\epsilon) \Upsilon(\gamma(\epsilon))\), as described in Proposition 5. Although the form of \(\Upsilon(\cdot)\) cannot be characterized analytically, the procedure underlying the numerical solution of Table 2 is used to describe this function.
In this case, it is no longer necessarily true that there are no gains to international monetary policy coordination, since it is clear from (A.16) that monetary authorities no longer maximize the same objective function. Nevertheless, since there is no strategic interaction between policies, Table 2 describes an optimal monetary policy for each monetary authority, independent of the actions of the other authority.
References


Figure 1a
The determination of price flexibility
Figure 1b
Multiple equilibrium in price flexibility