The effects of factor taxation in a two-sector model of endogenous growth

MICHAEL B. DEVEREUX  University of British Columbia
DAVID R.F. LOVE  Brock University

Abstract. This paper examines the effects of taxation in a two-sector model of endogenous growth, based on the joint accumulation of physical and human capital. Both transitional dynamics and balanced growth paths are computed, and the response to wage taxes, capital taxes, and consumption taxes is explored. Welfare costs of alternative tax regimes are computed. The capital tax is by far the least efficient method of generating revenue. The differences between taxes with respect to their effects on long-run growth rates are relatively unimportant. The key difference between the capital tax and wage or consumption taxes lies in their different level effects on the permanent paths of output, consumption, and labour supply.

Les effets de la taxation sur les facteurs de production dans un modèle de croissance endogène à deux secteurs. Ce mémoire examine les effets de la fiscalité dans un modèle de croissance endogène à deux secteurs basé sur l'accumulation conjointe de capital physique et humain. On calcule à la fois le sentier de transition dynamique et le sentier de croissance balancée, et on explore la réaction à l'imposition de taxes sur les salaires, sur le capital, et sur la consommation. Les coûts de bien-être des divers régimes de taxation sont calculés. Il semble que la taxe sur le capital est de loin la méthode la moins efficace pour engendrer des revenus. La différence entre la taxe sur le capital et les deux autres vient surtout des effets différents sur les sentiers en régime permanent de la production, de la consommation et de l'offre de travail.

I. INTRODUCTION

Recent macroeconomic theory has made progress in analysing the dynamic effects of taxes, particularly within the framework of the neoclassical growth model (see, e.g., Judd 1987; Auerbach and Kotlikoff 1987). A number of papers have extended

We would like to thank seminar participants at the University of Victoria and Queen's University for comments. The comments of two anonymous referees were very helpful. Any errors are solely due to the authors. Both authors wish to acknowledge gratefully financial assistance from the Social Science and Humanities Research Council of Canada.

This paper analyses the effects of factors income and expenditure taxation in a two-sector endogenous growth model. The model is a straightforward extension of the KR model, allowing for endogenous labour supply. We develop a framework in which the transitional dynamics outside balanced growth paths are explicitly computed. The aims of the paper are (a) to explore the effects of taxation on long-run balanced growth paths as well as on the dynamic adjustment of the economy towards these paths, and (b) to evaluate the welfare costs of alternative taxation policies.¹

Unlike the standard neoclassical growth model, the present environment involves the accumulation of two distinct types of capital. Physical capital is accumulated according to the usual technology whereby delayed consumption is transformed into capital. But human capital can also be augmented in a purposeful manner, by diverting factors away from final goods production into a separate human capital accumulation sector. Along a balanced growth path the ratio of human to physical capital is constant. But because the two sectors will generally have different technologies, this ratio is approached only gradually over time as the economy approaches the balanced growth path. A central feature of the present paper is the explicit computation of these transitional paths.

Three types of taxation are considered: capital income taxes, wage taxes, and consumption taxes. Starting from a calibrated economy as our benchmark, we examine the effects of increases in each of these taxes in turn. For all parameter values, any one of these taxes will reduce the economy’s balanced growth rate.

While all forms of taxation reduce the growth rate, the particular dynamic response to tax increases differs considerably among the different taxes. The response of the economy to a capital tax is almost wholly in the form of substitution of factors out of production of physical capital (or the final good), and into production of human capital. On the other hand, in response to a wage or a consumption tax, most of the adjustment takes place via a fall in total hours worked, in each sector, with no substantial intersectoral reallocation of factors.

In comparing wage taxation with capital income taxation, we show that the negative effect (on the growth rate) of a one per cent rise in the wage tax is always greater than that of a one per cent rise in the capital tax, so long as labour’s share of income in the final product sector exceeds capital’s share. But when we examine the impacts of revenue-equivalent increases in wage and capital income taxes on welfare, we find that the capital tax is at a minimum more than twice as

¹ Fuig (1991) develops an ingenious version of the Lucas (1988) model that allows for explicit diagrammatic solutions for transitional dynamics. He examines the effects of government spending and income taxes, but he does not look at factor taxes in detail.
costly in welfare terms as the wage tax. Thus, the welfare ranking of taxation still accords with estimates based on the neoclassical growth model, such as those in Judd (1987), Lucas (1990), or Cooley and Hansen (1991).

In comparing different (revenue-equivalent) tax regimes, the most important feature is the effect of each regime on the levels of variables rather than on growth rates. In fact, in revenue-equivalent terms, there is only a tiny difference between the wage and the capital tax in their implications for growth. But the capital tax regime generates a large intersectoral reallocation away from the production of physical capital towards human capital. This leaves output on a permanently lower path, since factors are diverted towards the build-up of human capital during the transition. The impact of wage and consumption taxes on the level of output is much less, since there is a much smaller substitution into human capital.

The results underscore the importance of correctly measuring the transitional effects of tax changes when the welfare costs of taxation are assessed. Most of the welfare costs in response to a capital tax are associated with the transition towards a new balanced growth path with a lower ratio of physical to human capital ratio. Ignoring this transition leads one seriously to underestimate the welfare costs of capital taxation.

This paper is part of a recently growing literature on taxation in endogenous growth models. Besides those papers mentioned above, a number of other papers explore the impacts of taxation in endogenous growth settings. Lucas (1990) and Jones, Manuelli, and Rossi (1990) derive optimal Ramsey tax paths in endogenous growth models and give a quantitative estimate of the growth and welfare effects of moving from the current tax system to these paths. Lucas finds significant welfare gains, but very low growth effects of implementing a Ramsey tax policy, based on U.S. data calibration. Jones, Manuelli, and Rossi find both high growth and welfare effects. Stokey and Rebele (1992) survey some of the literature, addressing the sources of the different findings. They identify the share parameters in the capital- and human-capital-producing sectors as key to the growth effects of taxation. In addition, they note that endogenous labour supply makes a substantial difference to the growth effects. Both of these factors are reflected in the results below. The main differences between our paper and those above is that we focus specifically in comparing the different welfare costs of flat-rate factor taxes. In addition, we explicitly investigate the transitional effects of taxes.

The paper is organized as follows. The next section develops the basic model with taxes. Section III defines a competitive equilibrium and explores the implications of a special case of the model that produces closed-form analytical solutions. In section IV the generalized model is numerically solved and empirically calibrated. Quantitative results on growth effects of taxes are reported, transitional dynamics are described, and welfare costs of alternative tax regimes are computed. Some conclusions follow.

2 Lucas (1990) does allow for endogenous growth in his model, but he reports welfare calculations only for the stationary form of the model.

3 Jones, Manuelli, and Rossi (1990) do compute transition effects also.
II. THE MODEL

The specification follows KR quite closely. There are two sectors in which production takes place: final goods and human capital; and two factors of production: physical capital and human capital. Both factors are necessary for production in both sectors. Physical capital is obtained from unconsumed final goods, and human capital is produced in the human capital sector. Human capital is embodied within individuals, so that it is useful only if combined with time spent at work by households. Both human and physical capital are assumed to be able to grow without bound.

1. Households

The economy is populated by a representative household with preferences over consumption and leisure that are time additive and isoelastic,

\[ U = \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - L_t), \]  

where

\[ u(C_t, 1 - L_t) = (C_t^\omega(1 - L_t)^{(1-\omega)})^{(1-\sigma)}/(1 - \sigma) \quad \sigma \geq 0, \quad \sigma \neq 1. \]

\[ u(C_t, 1 - L_t) = \ln(C_t^\omega(1 - L_t)^{(1-\omega)}) \quad \sigma = 1, \quad \omega \in (0, 1). \]

\( C_t \) is consumption expenditure, and \( L_t \) represents hours spent away from leisure, where the available amount of time is normalized to unity. Finally, let \( \tilde{C}_t \equiv C_t^\omega(1 - L_t)^{(1-\omega)}. \)

It is assumed that any taxes are levied on the final goods sector alone. Hours not consumed in leisure may be supplied either directly to working in the final goods (or the market) sector or may be devoted to human capital formation. Hours supplied to the market earn a direct market wage. If hours are devoted to human capital formation, they generate a return in future periods when wages per hour are augmented by a higher productivity of time.

We make the simplifying assumption that agents directly save in terms of capital, renting out capital to firms in each period at competitive interest rates. In choosing among savings, consumption, and hours supplied to the market, households face the constraint

\[ C_t(1 + \tau_{ct}) + K_{t+1} = w_t H_t l_{t+1}(1 - \tau_{wt}) + R_t K_t \phi_t (1 - \tau_{Rt}) + (1 - \delta) K_t + T_t, \]  

where \( \tau_{ct}, \tau_{wt}, \tau_{Rt} \) and \( T_t \) are, respectively, a consumption tax, a wage tax, a tax on capital income, and a lump-sum transfer from the government. When \( \tau_w \) and \( \tau_R \) are the same, we shall refer to it as the income tax. The wage \( w_t \) represents the return to hours measured in efficiency units. \( R_t \) is the rental rate on physical capital. \( K_t \)
is the total stock of physical capital and \( \phi_t \) is the fraction of this stock rented to firms operating in the final-goods sector. Finally, \( l_{1t} \) represents hours supplied to the final-goods sector.

Since the technology for human capital production has constant returns to scale, and since we leave this sector untaxed, without loss of generality we can think of each household individually operating its own human-capital production technology. Human capital is produced as follows:

\[
H_{t+1} = A_H(K_t(1 - \phi_t))^{\eta}(H_t l_{2t})^{(1 - \gamma)} + (1 - \delta_H)H_t.
\]  

For \( \gamma > 0 \), human-capital production requires the use of both physical capital and effective labour. The stock of human capital depreciates at rate \( \delta_H \).

The representative household then maximizes (1) subject to (2), (3), and a boundary condition that will be described below.

2. Firms

Firms in the market sector simply rent capital and employ labour to maximize profits period by period. The technology for final goods production is assumed to be

\[
Y_t = A(K_t \phi_t)^{\alpha}(H_t l_{1t})^{(1 - \alpha)}.
\]

Profit maximization therefore implies that \( w_t = (1 - \alpha)A(K_t \phi_t)^{\alpha}(H_t l_{1t})^{-\alpha} \) and \( R_t = \alpha A(K_t \phi_t)^{(\alpha - 1)}(H_t l_{1t})^{(1 - \alpha)} \).

3. Government

Government is introduced into this economy in a very minimal fashion. We abstract from government spending and simply assume that all tax revenue is rebated in a lump-sum transfer. Letting \( T_t \) denote this transfer, we have

\[
T_t = \tau_{w_l} w_l H_t l_{1t} + \tau_{c_t} C_t + \tau_{R_t} R_t \phi_t K_t.
\]

For simplicity, we have assumed that the government balances its budget in every period, thus avoiding any unnecessary notational burden associated with government debt.

4. Transitional dynamics

The critical determinant of the transitional dynamics in this economy is the relationship between the factor-share parameters in the two sectors. The optimal allocation

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4 We do not include the possibility of directly productive effects of government expenditure. This could easily be done using the model of Barro (1990). Take, for instance, the specification for final-goods production given by \( Y = A K^{\eta}(H_1)^{\eta} G^{\eta - \eta \phi} \). Then, if government spending, \( G \), were set such that \( G = \varphi Y \), the technology could be rewritten as \( Y = \tilde{A} K^{\eta}(H_1)^{\eta} \), where \( \tilde{A} = (A \varphi^{(1 - \eta - \phi)})^{1/(\eta + \phi)} \), and \( \eta' = \eta/(\eta + \phi) \), etc. Under this formulation, there exists an optimal \( \varphi \) to equate the competitive growth rate and the socially optimal rate. Our analysis would be entirely unchanged with this alternative specification, assuming that \( \varphi \) was set at its optimal level.
of factors across the two sectors will determine a certain ratio of human to physical capital that is constant along a balanced growth path. Away from balanced growth paths, however, in general factors will move across the two sectors. The relationship of \( \alpha \) to \( \gamma \) determines how this intersectoral allocation can take place.

When \( \alpha = \gamma \), the sectors have identical factor intensities, and physical and human capital can flow from one sector to another without affecting factor returns. Factor returns are determined by technology alone, independent of demand. In this case there are effectively no transitional dynamics in the model. In response to a tax change, factors are immediately reallocated so as to attain the new optimal ratio of human to physical capital in the space of a single period.

When \( \alpha \neq \gamma \), the sectors have different factor intensities. In this case, intersectoral reallocation does affect factor returns and thus the returns to the accumulation of physical and human capital. Then there are non-trivial transitional dynamics in the model, since an optimal adjustment path dictates that the long-run ratio of human to physical capital is approached over time, rather than a single period.

III. CHARACTERISTICS OF COMPETITIVE EQUILIBRIUM

In this section, we set out the efficiency conditions that determine a competitive equilibrium and derive a special case in which an analytical solution to the model exists.

1. Competitive equilibrium: definition
A competitive equilibrium for the economy outlined above consists of the sequences \( \{C_t, L_t, K_t, \phi_t, H_t, l_{1t}, l_{2t}, w_t, R_t, T_t, \tau_{ct}, \tau_{ht}, \tau_{rt}, G_t\} \) for \( t = 1, 2, \ldots \) that satisfy the following conditions.

a) Household utility maximization

Maximize (1) subject to (2), (3)

\[
\lim_{t \to \infty} K_t \Pi_0 R_t^{-1} = 0, \quad \lim_{t \to \infty} H_t q_t = 0,
\]

\( C_t \geq 0, \quad l_{1t} + l_{2t} = L_t \leq 1. \)

\( H_0 \) and \( K_0 \) given.

b) Profit maximization

c) Government budget constraints

d) Market clearing:

\[ C_t + K_{t+1} - K_t(1 - \delta) = Y_t. \quad (4) \]
The variable $q_t$ represents the shadow price of human capital, or the multiplier on the constraint (3) in the household-maximization decision.

2. Competitive equilibrium characterization
A competitive equilibrium may be characterized by the following series of equations:

\[
\frac{\omega(1 - l_{1t} - l_{2t})}{(1 - \omega)C_t} = \frac{(1 + \tau_{\omega})l_{1t}}{[(1 - \alpha)A(K_t\phi_t)^\alpha(H_t\ell_{1t})^{(1-\alpha)}(1 - \tau_{\omega})]} \tag{5}
\]

\[
\frac{(1 - \tau_{\omega})(1 - \alpha)\phi_t}{(1 - \tau_{Rt})\alpha l_{1t}} = \frac{(1 - \gamma)(1 - \phi_t)}{\gamma l_{2t}} \tag{6}
\]

\[
\bar{C}_t/C_t = \beta(\bar{C}_{t+1}/C_{t+1})(\alpha A(K_{t+1}\phi_{t+1})^{(\alpha-1)}(H_{t+1}\ell_{1t+1})^{(1-\alpha)}(1 - \tau_{R_{t+1}}) + (1 - \delta)) \tag{7}
\]

\[
q_t\bar{C}_t/C_t = \beta q_{t+1}(ar{C}_{t+1}/C_{t+1})[(1 - \gamma)A_H(K_{t+1}(1 - \phi_{t+1}))^\gamma(H_{t+1}\ell_{2t+1})^{-\gamma} \times (l_{1t+1} + l_{2t+1}) + (1 - \delta_H)] \tag{8}
\]

\[
q_t = \frac{(1 - \alpha)A(K_t\phi_t)^\alpha(H_t\ell_{1t})^{-\alpha}(1 - \tau_{\omega})}{(1 - \gamma)A_H(K_t(1 - \phi_t))^\gamma(H_t\ell_{2t})^{-\gamma}} \tag{9}
\]

The system of equations (3) and (4), along with (5)–(9), implicitly determines the solution sequence for the variables $C_t$, $K_t$, $H_t$, $l_{1t}$, $l_{2t}$, $\phi_t$, and $q_t$. Equation (5) represents the consumption-leisure trade-off. The left-hand side is the marginal rate of substitution between consumption and leisure, while the right-hand side is the inverse of the real wage, adjusted for consumption and wage taxes. According to equation (6), for an optimal intersectoral allocation of capital and hours the after-tax marginal rates of technical substitution between factors must be equalized across sectors. This is affected by the wage and capital tax, in opposite ways. Equations (7) and (8) are the Euler conditions determining the optimal accumulation of physical and human capital as a function of their separate returns. The price of human capital, $q_t$, must also satisfy (9), representing the optimal allocation of hours between sectors.

From the system (5)–(10), it is clear that the three types of taxes – consumption taxes, wage taxes, and capital taxes – have independent effects. A consumption tax drives a wedge between the marginal rate of substitution of consumption for leisure and the real wage. A wage tax affects the same margin but also, from equation (6), affects the returns to human capital accumulation by distorting the intersectoral allocation of factors. A tax on capital income affects both the direct intertemporal incentive to invest, as in (7), and the allocation of factors between sectors described in equation (6).

3. Balanced growth paths
It will be instructive for what follows to analyse the properties of balanced growth paths. Similar analysis may be found in Rebele and KR (although they abstract from endogenous leisure demand and do not address the impact of factor taxation).
To illustrate the properties of balanced growth paths in this two-sector economy, we use the system (5)–(9) to derive two relationships between growth and total labour supply. The first relationship arises from the combination of intertemporal efficiency in factor accumulation and atemporal efficiency in factor allocation. Define \( h_t \) as the ratio \( H_t/K_t \). Along a balanced growth path, \( h_t, l_{1t}, l_{2t}, \) and \( \phi_t \) all are constant. We make a further assumption that \( \delta = \delta_H \). By equations (7) and (8), along a balanced growth path it must be the case that

\[
\alpha A(hl_1/\phi)^{(1-\alpha)(1-\tau_R)} + (1-\gamma)A_H(hl_2/(1-\phi))^{-\gamma}(l_1 + l_2)
\]

\[+(1-\delta). \tag{10}\]

The left-hand side is the return on physical capital accumulation, after tax. This is simply the marginal product of capital in the final-goods sector. To satisfy intertemporal efficiency, it must equal the return on the accumulation of human capital, which is given by the right-hand side of (10). That the right-hand side is, in fact, the return to human capital accumulation may be seen intuitively in the following way. The return on human capital is given by the after-tax wage rate, multiplied by total hours worked, plus the value of the undepreciated stock of human capital, all divided by the relative price of human capital. Thus the return may be written as \( ((l_{1t+1} + l_{2t+1})(1-\tau_{wt+1})w_{t+1} + (1-\delta)q_{t+1})/q_t \).

But for atemporal efficiency, it must be that the after-tax wage equals the marginal product of effective labour in the human capital sector. Thus, replacing \( (1-\tau_{wt+1})w_{t+1} \) by \( q_{t+1}A_H(K_{t+1}(1-\phi_{t+1}))^{\gamma}(H_{t+1}l_{2t+1})^{-\gamma} \), and imposing the restrictions that \( q, h, \phi, l_1, \) and \( l_2 \) be constant along a balanced growth path, gives the right-hand side of (10).

While condition (10) gives a relationship between the sectoral physical capital to human capital ratios that satisfies intertemporal efficiency, we also have the atemporal efficiency condition (6), which states that marginal rates of technical substitution between factors should be equated across sectors. Combining (10) with (6) and substituting back into (8), we arrive at the expression for the common rate of return along a balance growth path,

\[
(\Omega(l_1 + l_2)\gamma(1-\eta)^{(1-\alpha)(1-\tau_W)} + (1-\delta),
\]

where \( \Omega \) is a function of \( A, A_H \) and parameters, and \( \eta = (1-\alpha)/(1-\alpha+\gamma) \in (0, 1) \). The expression shows clearly the negative impact of factor taxes on the equilibrium returns, for a given total supply of hours worked. At the same time, a rise in hours worked will raise the equilibrium return.

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5 The reason that only hours worked enter into the returns to human capital is that given specification (1), rewards to human capital accumulation cannot be derived in leisure activities. Thus, for a given wage, the more the individual works habitually, the greater is the return to human capital.

6 An alternative way to derive the condition is to think of the two sectors as being operated by competitive firms. Then efficiency implies that price equals unit cost in both. This implies (a) \( 1 = \Lambda_0(w^{(1-\alpha)}(R))^{\eta} \), and (b) \( q = \Lambda_1(w(1-\mu))^{(1-\eta)}(R(1-\tau_R))^{\eta} \), where the expressions on the right-hand side are unit cost functions, with \( \Lambda_0 \) and \( \Lambda_1 \) being constants. In the stan-
Notice that the relative size of the effect of factor taxes on the rate of return is proportional to the factor share parameter in final-goods production. Thus, if labour has a greater factor share, then a rise in the wage tax causes a greater increase in costs than an equal rise in the capital tax and so requires a greater fall in factor returns.

The balanced growth rate $g$ may now be implicitly defined by the condition

$$
(1 + g)^{(1 - \omega(1 - \sigma))} = \beta((\Omega(l_1 + l_2))^\gamma((1 - \tau_W)^\beta(1 - \tau_R)^{(1 - \alpha)}(1 - \eta) + (1 - \delta)).
$$

(11)

Since $(1 - \omega(1 - \sigma)) \geq 0$, the balanced growth rate is negatively related to the wage tax and the capital income tax and positively related to the fraction of time spent in non-leisure activities. Holding hours supplied constant, the effect of the wage tax on growth is larger than the capital tax when $\alpha < \frac{1}{2}$. Since the $\alpha < \frac{1}{2}$ case is more in accord with observations of factor shares, we conclude that, for equal percentage changes, the wage tax is more important for growth than the capital tax in this model.\(^7\) An appropriate comparison of the growth (and welfare) effects of the two taxes, however, must look at equal-revenue tax changes. This point is pursued in the analysis below.

Expression (11) gives a positive relationship between $g$ and $(l_1 + l_2)$. Another relationship is required to determine each variable in balanced growth. First note that imposing balanced growth on (3) and (8) gives

$$
(1 + g) = A_H(hl_2/(1 - \phi))^{-\gamma}l_2 + (1 - \delta) \quad (3')
$$

$$
(1 + g)^{(1 - \omega(1 - \sigma))} = \beta((1 - \gamma)A_H(hl_2/(1 - \phi))^{-\gamma}(l_1 + l_2) + (1 - \delta)). \quad (8')
$$

Now substitute for the expression $(hl_2/(1 - \phi))^{-\gamma}$ to get the following relationship between $l_2, l_1,$ and $g$:

$$
l_2 = \mu(l_1 + l_2),
$$

(12)

where $\mu = \beta(1 - \gamma)(g + \delta)/[(1 + g)^{(1 - \omega(1 - \sigma))} - \beta(1 - \delta)]$.

The share of total hours worked that is devoted to human capital may be increasing or decreasing in $g$. When $\sigma \leq 1$, $\mu$ is declining in $g$. But for sufficiently high $\sigma$, $\mu$ will be falling in $g$. Intuitively, with a high curvature in preferences, a rise in the growth rate must lead to a greater proportional rise in the real rate of return. From (3') and (8'), this result requires a greater rise in $(l_1 + l_2)$ than in $l_2$.

\(^7\) Note that if $\gamma = 0$, so that capital is unnecessary in the production of human capital, then factor taxes have no direct effects on the balanced-growth rate, holding labour supply constant. This result is due to the fact that, for $\gamma = 0$, the return on both assets must be equal to $A_H(l_1 + l_2)$ and so is unaffected by taxation.
Using equation (12), we can employ the condition for optimal labour supply (5), together with (6) and (7), to describe an implicit relationship between the growth rate and overall hours supplied, \( l_1 + l_2 \). The appendix shows that the following relationship can be derived:

\[
(l_1 + l_2) = \frac{\omega(1 - \alpha)}{\omega(1 - \alpha) + (1 - \omega)[(1 + \tau_c)/(1 - \tau_w)][(1 - \mu)(1 - \mu)(1 - \alpha\mu(1 - \tau_R)/(1 - \gamma)\phi)]}
\]

In this expression \( \phi \) is determined, from (6), as

\[
\phi = \frac{\alpha(1 - \gamma)(1 - \tau_R)(1 - \mu)}{(1 - \alpha)(1 - \tau_w) + \alpha(1 - \gamma)(1 - \mu)(1 - \tau_R)}
\]

Equation (13) describes a complicated non-linear relationship between \( l_1 + l_2 \) and \( g \). Nevertheless, it may be described fairly easily. The balanced growth rate affects both the fraction of working time devoted to final goods, \( 1 - \mu \), as well as the consumption-to-output ratio, which, along a balanced growth path, is \( [1 - \alpha\mu(1 - \tau_R)/(1 - \gamma)\phi)] \). But the second expression depends upon \( g \) only through its influence on \( \mu \). Thus, the relationship between \( l_1 + l_2 \) and \( g \) described by (13) depends only on the sign of \( \mu(g) \) function given above. When \( \sigma \) is very low, \( \mu(g) \) is positively sloped. The appendix then shows that (13) describes a positively sloped function in \( l_1 + l_2 \) and \( g \) space. When \( \sigma \) is much greater than unity, however, (13) is negatively sloped.

We may now describe the joint determination of growth and labour supply by the intersection of the two schedules (11) and (15), describing, respectively, the optimal accumulation decision and the optimal labour supply decision. The \( RR \) schedule represents equation (11), and it is always upward sloping, since higher labour supply raises the rate of return and therefore growth. The \( LL \) schedule may be upward sloping or downward sloping. If it is upward sloping, the schedule must cut the \( RR \) schedule from above, since the left-hand side of (13) is bounded below unity for any feasible growth rate. The two cases are illustrated in figures 1a and 1b corresponding to the upward-sloping and downward-sloping \( LL \), respectively.
We may use the figure to illustrate the impact of each of the three alternative taxes studied. First let us take the capital tax. From (11), it is clear that a rise in the capital tax shifts the RR curve up and to the left. The appendix shows that a rise in \( \tau_R \) will shift the LL curve downward, whatever its slope. Therefore, the growth rate must fall, in response to a rise in the capital tax.\(^8\) A similar analysis can be applied to the case of the wage tax. The RR curve shifts in and the LL curve shifts down, so that growth falls. Finally, a consumption tax leaves the RR curve unaffected but shifts down the LL curve. Thus, growth (as well as employment) again falls.

4. Special case model

Here we briefly explore a special case of the model that allows for a complete derivation of transitional dynamics.\(^9\) To arrive at this special case set \( \delta = \delta_H = 1 \) and \( \sigma = 1 \). Further, assume for now that all tax rates are time invariant.

While the system as defined by (3)--(9) is non-stationary, we may rewrite it in terms of the variables \( h_i \) and \( k_i = K_i / K_{i-1} \). It is easy then to show that the stationary transformed system can be collapsed down to a single first-order difference equation and some auxiliary equations. In this solution both \( l_{1i} \) and \( l_{2i} \) are time invariant. We then have

\[
\begin{align*}
h_{i+1} &= \frac{(1 - \gamma)(l_1 + l_2)}{\alpha(1 - \tau_R)} A_H \frac{A}{1} h_i^{(1 - \alpha)} \left( \frac{\phi}{l_1} \right)^{(1 - \alpha)} \left( \frac{1 - \phi}{l_2} \right)^\gamma \\
k_{i+1} &= \beta \alpha A h_i^{(1 - \alpha)} \left( \frac{l_1}{\phi} \right)^{(1 - \alpha)} (1 - \tau_R) \\
\phi &= \frac{(1 - \beta(1 - \gamma))}{((1 - \alpha)\beta(1 - \tau_w)/(1 - \tau_R) + (1 - \beta(1 - \gamma))\alpha)} \\
l_1 &= \frac{(1 - \alpha)\omega(1 - \tau_w)/(1 + \tau_c)}{[((1 - \beta(1 - \gamma))/(\phi)(1 - \omega) + \omega(1 - \alpha)(1 - \tau_w)/((1 + \tau_c)(1 - \beta(1 - \gamma))))]} \\
l_2 &= \frac{\beta(1 - \gamma)/(1 - \beta(1 - \gamma))}{l_1}.
\end{align*}
\]

Equation (14) captures the fundamental transitional dynamics of the special case model. \( h_i \) converges to the constant \( \bar{h} \), where

\[
\bar{h} = \left[ \frac{(1 - \gamma)(l_1 + l_2)}{\alpha(1 - \tau_R)} A_H \frac{A}{1} \left( \frac{\phi}{l_1} \right)^{(1 - \alpha)} \left( \frac{1 - \phi}{l_2} \right)^\gamma \right]^{1/(1 - \alpha + \gamma)}.
\]

\(^8\) A number of recent papers have shown that it is possible to generate counter-intuitive results from a capital tax in endogenous growth model (see Liu 1992; Uhlig 1992). However, the Liu and Uhlig papers use overlapping generations models. The key differences between their results and those of the present paper are (a) with an infinite horizon model, such as the present one, the distributional effects of taxes wash out, so that a capital tax does not represent a reduction in the income of the old, as in Uhlig; and (b), our model is one in which, in the absence of taxes, competitive equilibrium is Pareto efficient, so that a capital tax, by transferring resources to the human-capital sector, cannot raise the social rate of return, as in Liu.

\(^9\) This case was highlighted by King, Plosser, and Rebelo (1988).
The adjustment behaviour of \( h_t \) depends upon the sign of \((\alpha - \gamma)\). For \((\alpha - \gamma) = 0\), sectoral factor intensities are the same, and there are no adjustment dynamics at all. For \((\alpha - \gamma) > 0\) the system adjusts monotonically to a balanced growth path, while for \((\alpha - \gamma) < 0\), \( h_t \) adjusts in an oscillatory fashion.\(^{10}\) The growth rate of the capital stock, \( k_t \), is monotonically related to \( h_t \) through equation (15).

It is a straightforward task to manipulate the system (15)–(18) to confirm that all three types of taxes also lead to a fall in the balanced growth rate in this special case of the model.

IV. NUMERICAL RESULTS

1. Calibration

In this section we derive a full numerical solution for the model. For this calibration exercise we cannot really hope to be as precise as those who employ the more standard neoclassical growth model can, since strong empirical evidence concerning the nature of the human capital accumulation process is lacking. Nevertheless, to the greatest extent possible we follow the recent literature. Prescott (1986) cites micro evidence for many of the key parameters. Since the parameter values are not as robust as those of the standard model, we vary some parameters around our initial benchmark settings as a check on the sensitivity of the results.

The parameter values we require are (i) preference parameters, \( \beta, \omega, \sigma \), (ii) technology parameters \( A, A_H, \delta, \delta_H, \alpha \), and \( \gamma \), (iii) tax parameters, \( \tau_w, \tau_R, \tau_c \). We proceed by choosing parameters according to the arguments below to pin down a benchmark economy. Balanced-growth results, dynamics, and welfare results are derived for the benchmark economy.

Following Prescott (1986) and others, we let the share of labour in final goods output, \((1 - \alpha)\), be 0.64. In the absence of more precise information about the human capital technology, we initially follow KR in assuming that \((1 - \gamma) = 0.64\) also. Since intersectoral factor intensities are key to the transitional dynamics, we shall explore the effects of deviations from this below. Let depreciation rates be the same across sectors and set equal to 0.1. This leaves us with choices for \( A, A_H, \beta, \omega, \) and \( \sigma \). Since the difference between \( A \) and \( A_H \) affects only the units in which the human to physical capital ratio is measured, we set \( A = A_H \). The level of \( A \) is then set so as to achieve the desired trend growth rate in the benchmark economy. We follow KR in choosing a growth rate of 2 per cent per annum. For tax parameters, we again follow KR in choosing a base income tax of 0.2 and a zero consumption tax. Thus \( \tau_w = \tau_R = 0.2 \) in the benchmark case.

The final parameters, \( \beta, \omega, \) and \( \sigma \) are chosen to satisfy two conditions: (i) given a trend growth rate set equal to 2 per cent per annum, the rate of return on capital

\(^{10}\) Intuitively, when \( \alpha > \gamma \), the human capital sector is the more labour intensive, so that an increase in \( h \) will raise the output of this sector more than that of the final goods sector. Thus, \( h \) will be larger again in the next period and adjustment is monotonic. When \( \alpha < \gamma \), the final-product sector is the more labour intensive, and an increase in \( h \) will raise final production relative to human capital production, implying a lower \( h \) in the next period, and thus adjustment is oscillatory.
(both human and physical) in a balanced growth path is at historical levels for U.S. data; (ii) agents spend about 30 per cent of their available discretionary time working in the final goods sector. Condition (ii) is that employed by Prescott (1986) and Benhabib, Rogerson, and Wright (1991).

Condition (ii) is satisfied by a value of \( \omega = 0.45 \). Condition (i) can be satisfied by any combination of \( \sigma \) and \( \beta \) that, given the growth rate of 2 per cent, leads the Euler equation (11) to deliver an after-tax interest rate equal to the observed average value. Taking \( \beta = 0.998 \) as our candidate discount factor, for an after-tax interest rate of 5.2 per cent, this implies a value of \( \omega(\sigma - 1) \) equal to 1.458. Given \( \omega = 0.45 \), we derive a value of \( \sigma \) equal to 4.24.11

As a by-product of this calibration, it follows that, in a balanced growth path, \( \mu = 0.5 \), so that the agent spends half of total active hours working in the market and half in human capital accumulation. This seems compatible with a life-cycle view that about 30 per cent of agents’ time would be spent in school, college, and other forms of self-improvement.

2. Balanced growth paths

The numerical solution for the balanced growth path is easily derived using a non-linear equations solution procedure for the stationary representation of the system (5)–(9). Table 1 gives the balanced-growth solution values of the variables \( g, h, l_1, l_2, \phi, \) and \( C/Y \) for the benchmark economy. Tables 2 and 3 report the results for different values of the \( \gamma \) parameter, and table 4 examines the effect of eliminating the endogenous leisure choice (setting \( \omega = 1 \)). In each case parameter values are reported below the tables.

Part A of each table compares the balanced-growth solutions for the benchmark economy with the solutions in a number of alternative tax regimes. The first row (LST) illustrates the effect of eliminating all distortional taxation. As in the model of KR, taxation has a significant effect on growth rates. Without distortional taxation, the growth rate is almost 1 per cent higher. Labour supply is 8 per cent higher, although all of this increase is channelled into the final-product sector. For all the experiments, taxation has very little effect on the number of hours worked in the human capital sector. The share of capital devoted to the final-product sector is 6 per cent higher without distortional taxation. Note that as a consequence the ratio \( h \) falls by almost 20 per cent. These intersectoral substitution effects are further highlighted below.

Table 1 also reports the impact of a rise in the income tax to 30 per cent as well as of separate rises in wage and capital taxes. Note that the wage tax reduces growth by more than the capital tax. In this economy a 50 per cent rise in the capital tax has only a slight effect on the growth rate. The table indicates that the abolition of the wage tax would increase growth to 2.7 per cent, while abolishing the capital tax would raise growth only to 2.3 per cent.

11 Note that the intertemporal elasticity of substitution in consumption is \((1 - \omega(1 - \sigma))^{-1}\), and not \(1/\sigma\), in this formulation. Hence a value of \( \sigma = 4.24 \) does not require an extremely low substitution elasticity.
TABLE 1

<table>
<thead>
<tr>
<th>TAX</th>
<th>g</th>
<th>C/Y</th>
<th>l₁</th>
<th>l₂</th>
<th>φ</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>1.02</td>
<td>0.54</td>
<td>0.30</td>
<td>0.31</td>
<td>0.5</td>
<td>2.2</td>
</tr>
<tr>
<td>LST</td>
<td>1.029</td>
<td>0.50</td>
<td>0.35</td>
<td>0.31</td>
<td>0.53</td>
<td>1.8</td>
</tr>
<tr>
<td>τ_R = 0.0</td>
<td>τ_w = 0.2</td>
<td>1.023</td>
<td>0.53</td>
<td>0.31</td>
<td>0.31</td>
<td>0.56</td>
</tr>
<tr>
<td>τ_R = 0.2</td>
<td>τ_w = 0.0</td>
<td>1.027</td>
<td>0.53</td>
<td>0.34</td>
<td>0.31</td>
<td>0.46</td>
</tr>
<tr>
<td>τ_R = 0.3</td>
<td>τ_w = 0.2</td>
<td>1.019</td>
<td>0.56</td>
<td>0.30</td>
<td>0.30</td>
<td>0.46</td>
</tr>
<tr>
<td>τ_R = 0.2</td>
<td>τ_w = 0.3</td>
<td>1.017</td>
<td>0.55</td>
<td>0.28</td>
<td>0.30</td>
<td>0.51</td>
</tr>
<tr>
<td>τ_R = τ_w = 0.3</td>
<td>1.016</td>
<td>0.57</td>
<td>0.27</td>
<td>0.31</td>
<td>0.48</td>
<td>2.5</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>τ_R = τ_w = 0.25</td>
<td>1.018</td>
<td>0.55</td>
<td>0.288</td>
<td>0.30</td>
<td>0.49</td>
<td>2.4</td>
</tr>
<tr>
<td>τ_R(0.378)</td>
<td>1.018</td>
<td>0.60</td>
<td>0.29</td>
<td>0.31</td>
<td>0.42</td>
<td>2.7</td>
</tr>
<tr>
<td>τ_R(0.267)</td>
<td>1.018</td>
<td>0.55</td>
<td>0.28</td>
<td>0.30</td>
<td>0.51</td>
<td>2.3</td>
</tr>
<tr>
<td>τ_R(0.115)</td>
<td>1.019</td>
<td>0.53</td>
<td>0.29</td>
<td>0.30</td>
<td>0.49</td>
<td>2.2</td>
</tr>
</tbody>
</table>

NOTES: The TAX column represents the tax regime. The benchmark case, corresponding to the first row, represents the economy subject to a 20 per cent flat-rate tax on both capital and labour income. The LST row represents the economy without distortionary taxes. The other rows in Panel A represent departures from the benchmark case as indicated. In panel B, each change represents a move, from the benchmark case, that provides a present value of tax revenue equivalent to that provided by an increase in the income tax from 0.2 to 0.25 β = 0.998, σ = 4.24, ω = 0.45, γ = 0.36, α = 0.36, and δ = δ_H = 0.1.

TABLE 2

<table>
<thead>
<tr>
<th>TAX</th>
<th>g</th>
<th>C/Y</th>
<th>l₁</th>
<th>l₂</th>
<th>φ</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>1.02</td>
<td>0.53</td>
<td>0.27</td>
<td>0.37</td>
<td>0.53</td>
<td>3.0</td>
</tr>
<tr>
<td>LST</td>
<td>1.028</td>
<td>0.53</td>
<td>0.31</td>
<td>0.38</td>
<td>0.57</td>
<td>2.3</td>
</tr>
<tr>
<td>τ_R = 0.0</td>
<td>τ_w = 0.2</td>
<td>1.022</td>
<td>0.53</td>
<td>0.28</td>
<td>0.36</td>
<td>0.60</td>
</tr>
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<td>τ_w = 0.0</td>
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<td>1.019</td>
<td>0.60</td>
<td>0.26</td>
<td>0.36</td>
<td>0.49</td>
</tr>
<tr>
<td>τ_R = 0.2</td>
<td>τ_w = 0.3</td>
<td>1.017</td>
<td>0.58</td>
<td>0.25</td>
<td>0.35</td>
<td>0.55</td>
</tr>
<tr>
<td>τ_R = τ_w = 0.3</td>
<td>1.016</td>
<td>0.60</td>
<td>0.24</td>
<td>0.37</td>
<td>0.51</td>
<td>3.5</td>
</tr>
<tr>
<td>B</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>τ_R = τ_w = 0.25</td>
<td>1.018</td>
<td>0.59</td>
<td>0.25</td>
<td>0.37</td>
<td>0.53</td>
<td>3.3</td>
</tr>
<tr>
<td>τ_R(0.378)</td>
<td>1.018</td>
<td>0.61</td>
<td>0.26</td>
<td>0.37</td>
<td>0.46</td>
<td>3.8</td>
</tr>
<tr>
<td>τ_R(0.267)</td>
<td>1.018</td>
<td>0.55</td>
<td>0.25</td>
<td>0.37</td>
<td>0.55</td>
<td>3.1</td>
</tr>
<tr>
<td>τ_R(0.115)</td>
<td>1.019</td>
<td>0.53</td>
<td>0.25</td>
<td>0.37</td>
<td>0.53</td>
<td>3.0</td>
</tr>
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</table>

NOTES: Same as table 1 except that γ = 0.26.
### TABLE 3

<table>
<thead>
<tr>
<th>TAX</th>
<th>( g )</th>
<th>( C/Y )</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( \phi )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>1.02</td>
<td>0.7</td>
<td>0.18</td>
<td>0.52</td>
<td>0.78</td>
<td>12.6</td>
</tr>
<tr>
<td>LST</td>
<td>1.024</td>
<td>0.66</td>
<td>0.2</td>
<td>0.54</td>
<td>0.8</td>
<td>8.8</td>
</tr>
<tr>
<td>( \tau_R = 0.0 ) ( \tau_w = 0.2 ) &amp;</td>
<td>1.021</td>
<td>0.66</td>
<td>0.18</td>
<td>0.53</td>
<td>0.82</td>
<td>9.3</td>
</tr>
<tr>
<td>( \tau_R = 0.2 ) ( \tau_w = 0.0 ) &amp;</td>
<td>1.023</td>
<td>0.71</td>
<td>0.2</td>
<td>0.53</td>
<td>0.76</td>
<td>11.9</td>
</tr>
<tr>
<td>( \tau_R = 0.3 ) ( \tau_w = 0.2 ) &amp;</td>
<td>1.02</td>
<td>0.75</td>
<td>0.17</td>
<td>0.52</td>
<td>0.75</td>
<td>15.12</td>
</tr>
<tr>
<td>( \tau_R = 0.2 ) ( \tau_w = 0.3 ) &amp;</td>
<td>1.018</td>
<td>0.7</td>
<td>0.16</td>
<td>0.52</td>
<td>0.79</td>
<td>13.17</td>
</tr>
<tr>
<td>( \tau_R = \tau_w = 0.3 ) &amp;</td>
<td>1.018</td>
<td>0.7</td>
<td>0.16</td>
<td>0.51</td>
<td>0.79</td>
<td>15.81</td>
</tr>
<tr>
<td><strong>B</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( \tau_R = \tau_w = 0.25 ) &amp;</td>
<td>1.019</td>
<td>0.72</td>
<td>0.17</td>
<td>0.52</td>
<td>0.77</td>
<td>14.1</td>
</tr>
<tr>
<td>( \tau_R(0.378) ) &amp;</td>
<td>1.019</td>
<td>0.75</td>
<td>0.17</td>
<td>0.52</td>
<td>0.74</td>
<td>16.6</td>
</tr>
<tr>
<td>( \tau_w(0.267) ) &amp;</td>
<td>1.019</td>
<td>0.71</td>
<td>0.17</td>
<td>0.51</td>
<td>0.79</td>
<td>13.0</td>
</tr>
<tr>
<td>( \tau_c(0.115) ) &amp;</td>
<td>1.019</td>
<td>0.71</td>
<td>0.17</td>
<td>0.51</td>
<td>0.78</td>
<td>12.7</td>
</tr>
</tbody>
</table>

**NOTES:** Same as table 1 except that \( \gamma = 0.05 \).

### TABLE 4

<table>
<thead>
<tr>
<th>TAX</th>
<th>( g )</th>
<th>( C/Y )</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( \phi )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Benchmark</td>
<td>1.02</td>
<td>0.7</td>
<td>0.6</td>
<td>0.4</td>
<td>0.6</td>
<td>2.2</td>
</tr>
<tr>
<td>LST</td>
<td>1.024</td>
<td>0.65</td>
<td>0.62</td>
<td>0.38</td>
<td>0.6</td>
<td>1.8</td>
</tr>
<tr>
<td>( \tau_R = 0.0 ) ( \tau_w = 0.2 ) &amp;</td>
<td>1.021</td>
<td>0.66</td>
<td>0.6</td>
<td>0.4</td>
<td>0.66</td>
<td>1.8</td>
</tr>
<tr>
<td>( \tau_R = 0.2 ) ( \tau_w = 0.0 ) &amp;</td>
<td>1.022</td>
<td>0.68</td>
<td>0.61</td>
<td>0.39</td>
<td>0.55</td>
<td>2.2</td>
</tr>
<tr>
<td>( \tau_R = 0.3 ) ( \tau_w = 0.2 ) &amp;</td>
<td>1.0193</td>
<td>0.71</td>
<td>0.59</td>
<td>0.41</td>
<td>0.5</td>
<td>2.5</td>
</tr>
<tr>
<td>( \tau_R = 0.2 ) ( \tau_w = 0.3 ) &amp;</td>
<td>1.0187</td>
<td>0.70</td>
<td>0.59</td>
<td>0.41</td>
<td>0.62</td>
<td>2.2</td>
</tr>
<tr>
<td>( \tau_R = \tau_w = 0.3 ) &amp;</td>
<td>1.018</td>
<td>0.72</td>
<td>0.58</td>
<td>0.42</td>
<td>0.58</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau_w = \tau_R = 0.25 ) &amp;</td>
<td>1.019</td>
<td>0.70</td>
<td>0.59</td>
<td>0.41</td>
<td>0.59</td>
<td>2.4</td>
</tr>
<tr>
<td>( \tau_R(0.439) ) &amp;</td>
<td>1.018</td>
<td>0.73</td>
<td>0.58</td>
<td>0.42</td>
<td>0.5</td>
<td>3.1</td>
</tr>
<tr>
<td>( \tau_w(0.264) ) &amp;</td>
<td>1.019</td>
<td>0.69</td>
<td>0.59</td>
<td>0.41</td>
<td>0.61</td>
<td>2.2</td>
</tr>
</tbody>
</table>

**NOTES:** Same as table 1 except \( \omega = 1 \).

The second part of table 1 reports the impact of three different tax-regime changes that provide an equivalent present value of revenue to a rise in the income tax from the benchmark case from 0.2 to 0.25.\(^2\) Thus, the second row (of part B of table 1) indicates that beginning with a 20 per cent income tax, the capital tax would

\(^2\)To calculate these revenue equivalent taxes it is necessary to compute the full transition paths. Thus, the information in this table comes from the dynamic experiments reported below.
have to be raised to 0.378 to provide as much overall revenue as the 0.25 income tax. The wage tax need be raised only to 0.267, however, while the consumption tax would need to be 0.115. As expressed in revenue-equivalent terms, the differences in growth rates for different taxes are negligible, as are differences in total labour supply. But there are some major differences in intersectoral factor allocations. Capital-tax financing leads to a reduction in \( \phi \) by 20 per cent relative to wage-tax financing, and the balanced growth ratio \( h \) is 20 per cent higher. Associated with the substitution towards human capital, the ratio of consumption to output is 10 per cent higher with capital tax financing.\(^{13}\) The balanced-growth effects of revenue-equivalent increases in the wage tax and the consumption tax are quite similar to one another.

Tables 2 and 3 contain the results for alternative assumptions with respect to \( \gamma \). Reducing \( \gamma \) has a relatively small impact on the effects of tax changes. The overall growth effects are lower for \( \gamma = 0.26 \) and lower still for a value of \( \gamma = 0.05 \). As is to be expected, the intersectoral substitution effects are smaller, since, when \( \gamma \neq \alpha \), substitution among sectors becomes more difficult.

To get an estimate of the importance of the consumption-leisure trade-off for growth effects of taxes, we eliminate this trade-off completely by setting \( \omega = 1 \). Thus, the response of total hours supplied is eliminated from the model. Table 4 reports the steady-state results for this case. With no variation in total hours supplied, the impact of taxes on growth rates is substantially smaller. The income tax of 20 per cent now biases the growth rate down by less than half of 1 per cent, relative to the economy without distortional taxes of any kind.

3. Dynamic adjustment

In order to solve for the behaviour of the models’ variables away from balanced-growth paths, additional numerical techniques are required. The procedure used here implemented a two-point boundary value problem solution algorithm, described in Press et al. (1990).\(^{14}\)

An important characteristic of the benchmark case is that transitional dynamics in adjusting to alterations in taxes are completed within a single period. Figures 2a–2f illustrate the response of the benchmark economy to the tax increases defined in part B of table 1. All tax changes are measured in revenue-equivalent magnitudes. Thus, all taxes are adjusted so that the increase in each tax generates the same revenue, in present value, as that generated by an increase in any other tax. The economy attains its new balanced-growth path in one period, when \( \gamma = \alpha \). The most striking aspect of figures 2 pertain to the transitory effects of the capital income tax relative to the other taxes. The one-period fall in the output of the final

---

13 An easy way to see how this effect operates is to take the feasibility constraint (4) in a balanced-growth path, and divide across by \( Y_t \). This gives the relationship: \( C_t/Y_t + (g + \delta)K_t/Y_t = 1 \). The substitution towards human capital reduces the physical-capital output ratio and so must raise the consumption output ratio for a given growth rate. The fall in the growth rate exacerbates this effect.

14 All programs are available from the authors upon request.
Effects of factor taxation

Good is ten times that arising from the wage tax. Figures 2b, 2c, and 2d show that this result is due to a large temporary shift in factor supply into the human capital sector. This shift generates a sharp fall in output and investment in the final-goods sector. Overall hours worked falls and remains at a lower balanced-growth level. Consumption also falls, but by a smaller amount. The substitution towards human capital investment allows the economy to attain the new balanced-growth ratio of human to physical capital within a single period. The growth rate (not shown) first falls, and then it turns sharply positive, as the increased human capital is applied to the final-goods sector. Notice, however, that the transitory shift out of the final-goods sector leaves the path of consumption permanently lower than it would be in the absence of the tax.

The immediate response to the wage tax and the consumption tax is much smaller, since these taxes require only very slight changes in h in the long run. Therefore the required intersectoral substitution effects are much less. The growth rate falls slightly in the first period in response to wage and consumption taxes. Consumption falls immediately, as does overall hours supplied, more so for the consumption tax. Note that wage and consumption taxes have a greater negative effect on overall labour supply than the capital tax has.

Figures 3a–3f illustrate the dynamic adjustment for the case \( \gamma = 0.26 \). In this case there are real-time transitional dynamics. Output, the share of capital employed in the final goods sector, as well as employment in the final goods sector, all fall sharply in response to the capital tax increase and then rise over time to attain their new long-run levels.\(^{15}\) As is clear, the transitional responses to a capital-tax change are again much greater than those for a change in the wage tax or a consumption tax.

Figures 4a–4f illustrate the adjustment path in the very low \( \gamma \) case. Although the results are generally the same, there are some qualitative differences. Note that labour supply now initially rises in response to the capital tax. This rise is due to the fact that there is less ability to substitute capital used in final goods towards human-capital accumulation. Thus, increased demand for labour in the human-capital sector generates an increase in hours worked. In addition, since capital cannot easily be drawn out of the final-goods sector, the decline in investment associated with the capital tax generates a much smaller initial fall in consumption. Another obvious feature of this case is that the length of the transitional dynamics is increased considerably.

The clear message of these results is that transition effects represent an important element in the evaluation of the overall impact of taxes. Although there is almost no difference between the tax regimes in their effect on long-run growth, and only a small difference in intersectoral factor allocations and overall employment, there is a dramatic difference in the transition. This is the case even with equal factor intensities across sectors, where the transition occurs within one period. Wage and

\(^{15}\) This contrasts with the special case model above, where employment by sector and \( \phi \) underwent no transitional dynamics at all.
FIGURE 2
FIGURE 2 (concluded)
FIGURE 3 (concluded)
Employment in Final Goods

Share of Capital in Final Goods
Effects of factor taxation

**Figure 4 (concluded)**

- **Figure d**: Total Employment
- **Figure e**: Consumption
- **Figure f**: Output
consumption taxes have a negligible effect on intersectoral allocation, while capital taxes lead to a sharp reallocation of factors away from current investment in physical capital and towards investment in human capital. The temporary reallocation effects of capital taxes put output on a permanently lower path relative to the effects of the other taxes. In the evaluation of the welfare costs carried out below, these transition effects are crucial.

4. Welfare costs of taxes
We now calculate the welfare costs of various tax policies. Taking the benchmark economy with a 20 per cent income tax as our initial balanced-growth path, we ask what permanent fraction of consumption would agents in this economy be willing to forgo so as to avoid an immediate and permanent rise in some tax rates?

To answer this question the procedure is as follows. First, compute the value of utility in the benchmark model. Then, beginning at the benchmark parameterization, compute the value of utility for the economy following a permanent, unanticipated increase in the tax rate chosen. Then, choose the fraction of permanent consumption that would have to be forgone in the benchmark economy such that the two utility numbers are equal. This fraction is interpreted as the welfare cost of the tax.\textsuperscript{16} In order to calculate this cost, we must explicitly compute the transitional paths for consumption and hours worked in the post-tax-increase economy, and calculate the present discounted value of utility in this economy.

Table 5 gives a number of welfare cost figures. We look at the welfare costs of revenue-equivalent increases in the income tax, the wage tax, the capital tax, and the consumption tax. In each case the particular tax increase is revenue equivalent to an increase from the benchmark 0.2 income tax to a 0.25 income tax. The second column of the table gives the cost of the tax change in terms of percentage of base consumption, as described above. The table also reports the welfare results for lower values of $\gamma$, as well as the case of inelastic labour supply.

The rankings of welfare costs are similar to those in the standard neoclassical model (e.g., Chamley 1981; Lucas 1990; Cooley and Hansen 1991). The capital tax carries the highest welfare cost, followed by the income tax, the wage tax, and the consumption tax. In the benchmark case the welfare costs of the tax changes are substantial.\textsuperscript{17} Even to avoid a consumption tax increase, the representative household would forgo 3 per cent of base consumption. The welfare cost of the capital tax is twice that of the wage tax for the benchmark case. For lower values of $\gamma$, the overall welfare costs fall by almost half when $\gamma = 0.05$. However, the welfare rankings remain the same.

\textsuperscript{16} We can go on to state this as a fraction of GNP by simply taking the ratio of consumption to GNP in the benchmark economy. As noted by King and Rebelo (1990), however, the magnitude of welfare costs computed using this method are very sensitive to the curvature of the utility function. Thus, more emphasis is placed here on the relative welfare costs of alternative taxes than on the absolute number.

\textsuperscript{17} The numbers in KR are somewhat higher, for two reasons. They use a lower value of $\sigma$. They also examine a rise in the income tax to 0.3. We look at a rise to 0.25 principally because the non-linear solution procedure is more difficult to operate for large tax changes.
<table>
<thead>
<tr>
<th>Experiment</th>
<th>Growth rate</th>
<th>Consumption cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income tax increase</td>
<td>1.018</td>
<td>4.9</td>
</tr>
<tr>
<td>0.2 to 0.25</td>
<td></td>
<td></td>
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<tr>
<td>Capital tax increase</td>
<td>1.018</td>
<td>8.6</td>
</tr>
<tr>
<td>0.2 to 0.378</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage tax increase</td>
<td>1.018</td>
<td>4.1</td>
</tr>
<tr>
<td>0.2 to 0.267</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption tax increase</td>
<td>1.019</td>
<td>2.9</td>
</tr>
<tr>
<td>0.0 to 0.115</td>
<td></td>
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</tbody>
</table>

(ii) $\gamma = 0.26$ case

<table>
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<th>Consumption cost</th>
</tr>
</thead>
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<tr>
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<tr>
<td>Wage tax increase</td>
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<td>3.8</td>
</tr>
<tr>
<td>0.2 to 0.27</td>
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<tr>
<td>Consumption tax increase</td>
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<td>2.8</td>
</tr>
<tr>
<td>0.0 to 0.114</td>
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</tr>
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</table>

(iii) $\gamma = 0.05$ case

<table>
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<th>Consumption cost</th>
</tr>
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<tbody>
<tr>
<td>Income tax increase</td>
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<td>3.1</td>
</tr>
<tr>
<td>0.2 to 0.25</td>
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<td>2.7</td>
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<tr>
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<td>Consumption tax increase</td>
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<td>1.7</td>
</tr>
<tr>
<td>0.0 to 0.119</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(iv) Benchmark case with inelastic labour supply

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Growth rate</th>
<th>Consumption cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income tax increase</td>
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<td>2.4</td>
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<tr>
<td>0.2 to 0.25</td>
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<td>1.018</td>
<td>13.5</td>
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<tr>
<td>0.2 to 0.378</td>
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</tr>
<tr>
<td>Wage tax increase</td>
<td>1.018</td>
<td>0.6</td>
</tr>
<tr>
<td>0.2 to 0.267</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In the case of inelastic labour supply the relative welfare costs of the capital tax rise drastically. Now the wage tax has a consumption cost of only 0.6 per cent, while the capital tax cost is 13 per cent. With inelastic labour supply, the direct effect of the wage tax is muted, since there is no response of hours worked. But the welfare impact of the capital tax is exacerbated because there can be no compensating fall in leisure to offset the fall in the path of consumption.

The analysis of the previous section would suggest that the explicit calculation of the adjustment path is a crucial factor in evaluating welfare costs. In particular, the adjustment to a capital tax involves a large immediate reduction in physical investment, leaving the path of the capital stock permanently lower than the no-change path. Ignoring the costs associated with the build-up of \( h \) during the transition phase would obviously lead to an incorrect measure of post-tax welfare.

These welfare results provide some interesting insight into the debate over tax policy and growth. It is often said that if government taxes were to affect long-run growth, the result would be deadweight costs that might far exceed the conventional costs of tax distortions. Although we have allowed for taxes to affect growth, the growth effects are not at all quantitatively important here. Indeed, although the wage tax has the greatest effect on long-run growth, the conventional rankings between capital and wage taxation still apply in this model. The most important factor in measuring welfare costs is the transition effect, rather than the effect on the long-term growth path. For capital taxation, the extent of the transition effects, involving the economy investing heavily in building up a higher stock of human capital, is much larger than it is for the other types of taxes.

V. CONCLUSIONS

This paper has explored the qualitative and quantitative effects of tax changes on growth and welfare in an endogenous growth model with growth arising from the joint accumulation of human and physical capital. We showed that capital income taxes, wage taxes, and consumption taxes all reduce growth rates. For equal percentage tax changes, wage taxes tend to have a larger effect on growth rates than either capital taxes or consumption taxes. When measured in revenue-equivalent terms, however, the difference in growth rates generated by different tax regimes tends to be quite small. In comparing capital taxes with either wage or consumption taxes, it is the transition effects, rather than the long-run growth effects, that are paramount. The model supports the results of standard neoclassical growth models that suggest that the capital tax is much more inefficient than other forms of taxation.

APPENDIX

Equation (13) is derived in the following way. Define output in the final goods sector as \( Y \). Then, from equation (7) of the text along a balanced growth path,
\[ \frac{Y_t}{K_t} = \frac{\phi}{(\beta \alpha(1 - \tau_R))(1 + g)^{(1 - \beta)(1 - \sigma)} - \beta(1 - \delta)).} \]

The income expenditure identity implies that

\[ C_t/Y_t + (g + \delta)K_t/Y_t = 1. \]

Putting this result together with the definition of \( \mu \), following equation (12), we have

\[ C_t/Y_t = 1 - \alpha(1 - \tau_R)\mu/((1 - \gamma)\phi). \]

Substituting this expression into the balanced growth version of equation (5) of the text gives equation (13).

The slope to of the LL function in figure 1 depends upon the effect of changes in \( g \) on the expression

\[ (1 - \mu)[1 - \alpha\mu(1 - \tau_R)/(1 - \gamma)\phi]], \]

where \( \mu \) depends upon \( g \) through (12). Differentiating this expression with respect to \( g \) gives

\[ \{-1 - \alpha(1 - \mu)(1 - \tau_R)/\phi - \mu(1 - \alpha)\gamma(1 - \tau_w)/(1 - \gamma)^2 \]
\[ + \alpha\mu(1 - \tau_w)/(1 - \gamma)\} \mu'(g). \]

The expression inside the braces is negative (since it may easily be shown that \( \alpha\mu(1 - \tau_w)/(1 - \gamma) < 1 \), given the balanced growth value for \( \mu \) and the requirement that \( R > g + \delta \)). It follows that the slope of the LL function is the same as the slope of the \( \mu(g) \) function.

The LL function depends upon all three taxes. Holding \( g \) given, it is obvious that a rise in \( \tau_c \) reduces \( l_1 + l_2 \). The effect of a rise in \( \tau_w \) or \( \tau_R \) on the LL schedule depends upon both the direct effects and the indirect effects through a change in \( \phi \).

It is easy to substitute in for \( \phi \) to demonstrate that a rise in either tax will reduce \( l_1 + l_2 \), holding \( g \) given.

REFERENCES


