Money and the real exchange rate with sticky prices and increasing returns*

Paul Beaudry†

University of British Columbia

Boston University, Dept. of Economics, Boston, MA 02215

and

Michael B. Devereux

University of British Columbia, Dept. of Economics, Vancouver, BC V6T 1W5, Canada

Abstract

This paper constructs a dynamic general equilibrium model aimed at exploring the link between money shocks and the real exchange rate. The key ingredient of the model is the presence of increasing returns to scale in technology. This allows for an equilibrium in a monopolistic competitive price setting in which prices are endogenously sticky. The combination of increasing returns and very short term price stickiness leads to significant monetary non-neutralities. Under a floating exchange rate, a monetary shock leads to an immediate and persistent real and nominal depreciation. A monetary shock generates persistent positive effects on output, consumption, and investment. Finally, the behavior of the real exchange rate in the model is consistent with the broad empirical characteristics of real exchange rates.

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†Correspondence to: Michael B. Devereux, Department of Economics, University of British Columbia, Vancouver, B.C. Canada, V6T 1Z1.
1 Introduction

A central task of open economy macroeconomics is to explain the behavior of the real exchange rate. The observed pattern of the real exchange rate movements displays a high degree of exchange rate volatility, a close correlation between real and nominal rates, and differential behavior of the real exchange rate across exchange rate regimes. Many economists have argued that this is best explained by models in which nominal price levels are sticky (e.g., Mussa 1986). The traditional open economy model used price stickiness as a central assumption. This emphasized the importance of monetary shocks for real and nominal exchange rate movements. According to that model, a monetary expansion should generate a protracted real and nominal exchange rate depreciation (e.g., Dornbusch 1976). In fact recent evidence from structural VAR estimates of monetary policy shocks in Eichenbaum and Evans (1993) and Clarida and Gali (1994) offers support for this traditional view of the money-real exchange rate linkage.

The present paper develops a dynamic general equilibrium model in the tradition of recent international real business-cycle models, primarily focused on exploring the link between monetary policy and the real exchange rate. International real business-cycle (RBC) models (e.g., Backus, Kehoe, and Kydland 1992, Ahmed et al. 1993, Stockman and Tesar 1991, Head 1991) have proved a useful framework for understanding international comovements of macro aggregates. But they have not been so successful at explaining the behavior of relative prices (Backus, Kehoe, and Kydland (1993)). Since these models usually allow no channel for monetary disturbances to affect the real exchange rate, it therefore seems unlikely that they could account for the stylized facts of real exchange rate behavior. In the international RBC framework, business cycles are driven principally by stochastic technology shocks. As constructed they cannot produce an international monetary business cycle. Although money shocks will affect the dynamics of the nominal exchange rate, they will have negligible effects on the real exchange rate.

It may be argued, of course, that real exchange rate behavior is still consistent with flexible price models. In particular Stockman (1988) puts forward this viewpoint. He argues that the degree of persistence in shocks to the real exchange rate is hard to reconcile with sticky price models in which monetary disturbances generate only temporary deviations from purchasing power parity. By contrast, flexible price models with very persistent real disturbances may be more consistent with the evidence.

Our model is based upon the existence of very short-term price stickiness. In particular, our assumption is that, in response to unanticipated monetary disturbances, prices cannot respond immediately. Prices are predetermined for the duration of one period. In contrast to the traditional price-setting
framework in macro models, some of which have been extended to more rigorous real business-cycle environments (e.g., Cho and Cooley (1992)), price stickiness in our model is consistent with optimizing behavior. That is, there is an equilibrium in which firms find it in their interest to set prices one period in advance. Moreover, our price stickiness is small. We calibrate the model so that firms find it optimal to preset prices for the duration of only one month.

In our model, under a floating exchange rate, a monetary shock leads to an immediate and persistent real and nominal depreciation. In addition, as in the Dornbusch (1976) model, a monetary shock generates persistent positive effect on output, consumption, and investment.

The key ingredient in the model is the presence of increasing returns to scale in technology. As has been shown in Beaudry and Devereux (1993), the presence of increasing returns to scale of sufficient magnitude allows for sticky prices to arise as an equilibrium in a monopolistic competitive price-setting game. The argument is simple. In the presence of increasing returns to scale that cross a certain threshold level, it is possible that the equilibrium of a dynamic general equilibrium model becomes indeterminate (Benhabib and Farmer (1994)). On the one hand, this opens up the possibility of sunspots and self-fulfilling expectations as causes of business-cycle fluctuations. An alternative strategy that can be pursued within this modelling framework is to focus on a particular equilibrium in which the initial response of prices to money shocks is zero (Benhabib and Farmer (1992)). In this case, the impact of monetary shocks on the economy becomes determinate. We focus on this latter approach, and argue that we can think of this as an economy in which monopolistic competitive firms set prices one period in advance.

Since the price level is a predetermined variable, in any period, unanticipated money shocks can generate real effects. Because of the presence of increasing returns, which gives rise to a complementarity between consumption and investment activities, these effects display considerable persistence.

Our results present an interesting contrast with traditional models of real exchange rate dynamics. Our model predicts that a monetary expansion generates an immediate depreciation of the real and nominal exchange rate. But unlike the Dornbusch (1976) model, both the real and nominal rates continue to depreciate for a considerable time after the initial shock. The shape of the real exchange rate response to a money shock in our model is very similar to the response estimated by Eichenbaum and Evans (1993).

A second issue that comes out of the results relates to the argument of Stockman (1988). In the absence of permanent real disturbances, our model does indeed contain the prediction of long-run PPP. But the quantitative results imply that the real exchange rate may take a long time to converge with its initial level following a monetary-induced shock. Thus, empirically,
it might be hard to discriminate between this model and one in which the exchange rate contained a unit root. This difficulty is well-known in the empirical literature on exchange rates (e.g., Adler and Lehman (1983), Hakkio (1986) and Huizinga (1987)).

Since we focus on an environment with monetary shocks only, our purpose is not to try to match the unconditional correlations measured in data. We do, however, document the properties of the model when subjected to persistent monetary shocks. As in the data, real and nominal exchange rate volatility are considerably greater than the volatility in prices. This finding is not implied by our assumptions. Prices are set one period ahead in our model, where a period is one month, but may respond in any way following this. Moreover, our results are all reported in quarterly frequencies. The implication of our model is that there is a considerable degree of endogenous sluggishness of prices in response to monetary disturbances.

The model also has implications for the pattern of cross-country correlations of macroeconomic aggregates for an environment with money shocks only. One key implication we note is that, in the presence of increasing returns, there is a positive cross-country correlation of output levels. This is interesting in view of the discussion of Backus, Kehoe and Kydland (1993), who identify the tendency to generate low or even negative cross-country output correlations as one of the key anomalies in international business-cycle models.

This paper is not the first to introduce price frictions in dynamic general equilibrium open economy macro models. Innovative papers by Svensson (1986) and Svensson and Van Wijnbergen (1986) introduce monopolistic competitive price setting in a cash-in-advance setting. They do not model the production or employment decision of firms directly. More recent papers by Obstfeld and Rogoff (1995), and Stockman and Ohanian (1993) also develop dynamic general equilibrium models of the international economy with price stickiness and real effects of monetary policy. Obstfeld and Rogoff are more closely related to the present paper. They construct a two-country model in which agents are imperfectly competitive suppliers and set their prices one period in advance. The paper differs from ours in some important respects, however. The key difference is in the fact that the endogenous price stickiness in our model relies on the indeterminacy of equilibrium without price setting. In addition, we focus more on the dynamic propagation of monetary shocks through changes in the capital stocks of each country. Stockman and Ohanian (1993) develop a two-country model in which, in each country, some prices are sticky while some remain flexible. They focus on two main issues: the ‘liquidity effects’ of shocks, and the possibility of monetary independence under pegged exchange rates.

The rest of the paper is structured as follows. Section 2 develops the
model. Section 3 discusses calibration and presents the numerical results. Section 4 presents some conclusions.

2 The model

There are two countries in the world economy: Home and Foreign. Home produces a single final good, $x$, while Foreign produces only the final good, $m$. We follow the convention of denoting foreign variables with an asterisk. In each country there are two types of production firms: final goods firms, who produce with the use of intermediate inputs; and intermediate producers, who produce with the use of capital and labor inputs. The employment of factors is subject to a transactions cost which is described more fully below. Home final goods require only home-produced intermediate goods, and the analogous condition holds for the production of foreign final goods. We maintain complete symmetry throughout the analysis, so the countries are assumed to be of equal size, have identical preferences, and their technologies are mirror images of one another.

Final goods technologies exhibit constant returns to scale, and final goods firms are price-takers in both output and input markets. An intermediate good of any type is produced by a single monopolist. Consequently, the market structure in the intermediate sector is one of monopolistic competition.

2.1 Technologies

The final goods production technology for the home country is written as (ignoring time subscripts)

$$ X = \left[ \int_0^1 x^\rho_i \, di \right]^{1/\rho} $$

where $X$ is total output of good $x$, and $x_i$ is the amount of intermediate input $i$ used in the production of final goods. The measure of intermediate goods available is 1, and this is held fixed throughout the analysis. The final goods technology for the foreign country is entirely analogous.

The intermediate good production technology, for any firm $i \in (0, 1)$ in the home country, is

$$ x_i = F(k_{ix}, k_{im}, l_i, \Lambda) - \Gamma $$

Here $k_{ix}$ is the employment of home capital (i.e., capital of type $x$) by firm $i$, and $k_{im}$ is the employment of foreign capital (capital of type $m$), by the home intermediate firm. $l_i$ is the employment of labor by firm $i$ in the home country. $\Gamma$ is a fixed cost of production, which must be borne by the intermediate producer in each period.
The function $F$ is given by $F(k_{ix}, k_{im}, l_i) = k_{ix}^{\alpha_1} k_{im}^{\alpha_2} l_i^{(1-\alpha_1-\alpha_2)} \Lambda$, and

$$\Lambda = (k_{ix}^{\alpha_1} k_{im}^{\alpha_2} l_i^{(1-\alpha_1-\alpha_2)})^{\gamma-1}$$

with $\gamma > 1$. The expression $\Lambda$ is taken to represent the presence of external economies of scale in the production of intermediate goods. Thus, $k_x$ represents the economy-wide use of home-produced capital, $k_m$ the economywide use of foreign-produced capital, etc. While each monopolistic competitive firm operates a constant returns to scale technology, in the aggregate, there are increasing returns to scale in intermediate goods production if $\gamma > 1$. The specification of external economies of scale is consistent with evidence for U.S. and European manufacturing data reported in Caballero and Lyons (1992a, b) and Bartlesman, Caballero, and Lyons (1994).

The corresponding production technology for the foreign intermediate good is symmetrically described as

$$m_i = F(k_{im}^{*}, k_{ix}^{*}, l_i^{*}, \Lambda^{*}) - \Gamma^{*}$$

where $F(k_{im}^{*}, k_{ix}^{*}, l_i^{*}) = k_{im}^{*\alpha_1} k_{ix}^{*\alpha_2} l_i^{*(1-\alpha_1-\alpha_2)} \Lambda^{*}$, and

$$\Lambda^{*} = (k_{im}^{*\alpha_1} k_{ix}^{*\alpha_2} l_i^{*(1-\alpha_1-\alpha_2)})^{\gamma-1}$$

Capital is accumulated in each country through investment of that country's output. Thus, for the home country, the accumulation technology for capital is

$$K_{xt+1} = I_t + (1 - \delta)K_{xt}$$

where $I_t$ is investment of the home good, and $\delta$ is the depreciation rate. An analogous expression must hold for the foreign country.

### 2.2 Households

Households in each country have preferences over consumption of each good and leisure. Let lower-case letters denote household decision variables. Households preferences in the home country are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_{xt}, c_{mt}, 1 - h_t)$$

where $U(c_{xt}, c_{mt}, 1 - h_t) = log c_{xt} + log c_{mt} + \eta(1 - h_t)$, and $c_{xt}, c_{mt}$ and $h_t$ represent individual household choices over consumption of the two goods and hours worked. Under specification (4) preferences are linear in leisure $(1 - h)$. This indicates that we employ the Hansen (1985) and Rogerson (1988) specification for time spent at work. Individuals either work a fixed
number of hours or none at all, and a lottery is run to decide those individuals who work.

Households choose consumption of each good, saving, and labor supply (through the lottery) to maximize (4). We follow the Lucas (1982) tradition of focusing on a perfect pooling equilibrium for the world economy. That is, markets are complete, and households in each country begin with identical fractions of world wealth. Given market completeness, this pooling equilibrium will be sustained over time. It will then follow that households in each country will consume equal amounts of each commodity.¹

Households receive wage income, profit income from intermediate goods firms, rental return on their capital from intermediate firms, cash flow from financial intermediation firms, and a return on an internationally traded state-contingent asset. As we see below, money is introduced into the model through a transactions cost technology facing firms.

Let \( S_t \) denote the state of the world. Assume this is a Markov process, to be specified below. We let \( S_t \) be the set of all possible states at time \( t \). \( q(S_{t+1}, S_t) \) represents the price of a state-contingent bond that repays one unit of good \( x \), in state \( S_{t+1} \), period \( t + 1 \), given state \( S_t \). Then let \( b(S_{t+1}) \) represent the number of units of the asset purchased. An analogous market could be opened for the state-contingent bonds repaying in terms of foreign good \( m \), but it would be redundant so long as there is free ex post trade in commodities. Then we may write the budget constraint of the home household as

\[
P_t c_{xt} + e_t P^*_t c_{mt} + P_t \int_{S_{t+1} \in S_t} q(S_{t+1}, S_t)b(S_{t+1})dS + P_t K_{xt+1}
\]

\[
= W_t h_t + R_t K_{xt} + (1 - \delta) P_t K_{xt} + \pi_t + CF_t + P_t T_t + b(S_t)
\]

Here \( P_t \) represents the price of the home good, \( P^*_t \) the price of the foreign good, and \( e_t \) is the nominal exchange rate. Home capital \( K_{xt+1} \) is accumulated by domestic households. \( W_t \) and \( R_t \) represent, respectively, the nominal wage and nominal return on capital. \( \pi_t \) represents the profit income from the intermediate goods sector, and \( CF_t \) represents the cash flow from intermediation firms (these firms are described below). Finally, \( T_t \) represents a money transfer from the government.

Households maximize (4) subject to (5) and the appropriate terminal condition. Take it that \( S_t \) is represented by the Markov distribution \( G(S_{t+1}, S_t) \)

¹Clearly this model does not address the issue of low observed cross-country consumption correlations (see Stockman and Tesar (1991)). It seems desirable at a first pass to address the issue of international monetary non-neutralities in a more standard setup, however, without introducing other complicating features that might account for these observations.
Necessary conditions for the household decision problem are:

\[ q(S_{t+1}, S_t) c_{xt}(S_t)^{-1} = \beta c_{xt+1}(S_{t+1})^{-1} dG(S_{t+1}, S_t) \]  
(6)

\[ q(S_{t+1}, S_t) c_{mt}(S_t) = \beta \frac{\kappa(S_t)}{\kappa(S_{t+1})} c_{mt+1}(S_{t+1})^{-1} dG(S_{t+1}, S_t) \]  
(7)

\[ c_{xt}(S_t) = \beta \int_{S_{t+1} \in S_t} (R(S_{t+1}, S_t) + 1 - \delta) c_{xt+1}(S_{t+1})^{-1} dG(S_{t+1}, S_t) \]  
(8)

\[ \eta c_{xt}(S_t) = \frac{W_t(S_t)}{P_t(S_t)} \]  
(9)

Condition (6) and (7) arise from the households optimal state-contingent consumption of each good, where the real exchange rate is defined by

\[ \kappa_t = \frac{\kappa}{P_t} \]  

Condition (8) comes from the optimal choice of home capital by the home household. Finally, condition (9) represents the households consumption-leisure trade off.

Again, the corresponding conditions for the foreign country are entirely symmetric and are omitted to shorten the exposition.

2.3 Final goods firms

Final goods firms in each country are price-takers. Equilibrium in the final goods industry can be described by the conditions of production efficiency and zero profits. Let the price charged to a final good firm in the home country by intermediate producer \( i \) be \( p_{it} \). Cost minimization by the final good firm implies

\[ C(X_t, p) = X_t p_t \]  
(10)

where \( p_t = [\int_0^1 p_{it}^{\rho-1} dt]^{\frac{\rho}{\rho-1}} \) is the input price index for final goods firms.

Zero profits in the final goods industry is therefore achieved by the condition that price equals unit cost,

\[ P_t = p_t \]  
(11)

2.4 Intermediation firms

In this economy households earn income in part from their labor and capital income. But there are frictions involved in allocating labor and capital to production in intermediate goods firms. This could be explained by the difficulties of matching, adjustment costs, or other transactions costs in factor
markets. We model this friction by the assumption that if a home country firm employs factors $k_x, k_m,$ and $l$, then there are transaction costs equal to
\[
\phi\left(\frac{n}{P}, F(k_x, k_m, l, \Gamma)\right)
\]
cURRED, where $n$ denotes nominal balances, and
\[
\phi = B\left((\frac{n}{P})^{\lambda}(1 - v) + F^\lambda v\right)^{\frac{\lambda}{1 - \lambda}}
\]
for $\lambda < 1, \lambda \neq 0$

when $\lambda = 0$. Thus, $\phi$ is homogenous of degree one in $\frac{n}{P}$ and $F$. In addition, assume that $v > 1$, so that $\phi_1 < 0$, and $\phi_2 > 0$. Transactions costs are increasing in total activity, but may be reduced by the holding of real balances in domestic currency. The more output is produced by a home firm, the greater are the costs of allocating factors and fitting them into production. But the real allocational costs are lower, the greater the holdings of real money balances. One way to think about this is as a shorthand way of describing the use of money balances in order to repay factors in advance of production.

The parameter $\lambda$ governs the degree of complementarity between real balances and the value of real output in the transactions cost function. As $\lambda$ approaches unity, real balances and output become perfect complements. In that case, holding transactions costs constant, real balances must increase proportionally with output. As $\lambda$ falls, the degree of complementarity falls. As $\lambda$ gets more and more negative, the degree of complementarity goes to zero. In that case, the constant transactions cost isolines become inverse L shaped in $\frac{n}{P}, F$.

Financial intermediation firms are set up in order to alleviate this trading friction. They hire labor and capital at the beginning of a period and rent them to firms, but incur the transactions costs of allocating factors. In order to do this, they must hold money. If they are to recover costs, they must charge a premium on factors rented to intermediate good-producing firms. Let $\hat{W}, \hat{R},$ and $\hat{R}^*$ represent the rental prices of labor, home capital, and foreign capital that are charged to domestic intermediate good producers, respectively. Then the cash flow of an intermediation firm at any period may be written as
\[
CF(S_t) = \hat{W}(S_t)l(S_t) - W(S_t)l(S_t) + \hat{R}(S_t)k_x(S_t) - R(S_t)k_x(S_t) + \\
+n_{t-1} - n_t\hat{R}(S_t)^*k_m(S_t) - c_t R(S_t)^* k_m(S_t) - P_t \phi\left(\frac{n_t}{P_t}, F_t\right)
\]  
(12)

Since firms are also hiring foreign capital, we might also include a role for foreign real balances in the transactions cost function. Doing this would complicate the analysis without changing any of the general conclusions.
Intermediation firms choose contingency plans for labor, capital, and money holdings to maximize the present value of cash flow, evaluated at state-contingent prices. In equilibrium, intermediation firms will be the only holders of money in this economy. Money transfers given to households will be deposited to accounts in the intermediation firms.

The necessary conditions for the optimal choice of labor, capital, and money holdings for an intermediation firm may be written as

\begin{align}
\dot{W}(S_t) &= W(S_t) + ph_1(t, F_t)F_3 \\
\dot{R}(S_t) &= R(S_t) + \phi_2(\frac{n_t}{P_t}, F_t)F_1 \\
e_t \dot{R}(S_t)^* &= e_tR(S_t)^* + \phi_2(\frac{n_t}{P_t}, F_t)F_2 \\
1 + \phi_1(\frac{n_t}{P_t}, F_t) &= \int_{s_{t+1} \in S_{t+1}} \frac{P(S_{t+1})}{P(S_t)} q(S_{t+1}, S_t)
\end{align}

Thus, the factor prices faced by firms and households will differ by an amount proportional to the marginal transactions cost. Equation (14) represents the implicit money demand schedule of an intermediation firm. The right-hand side has the natural interpretation as the home nominal discount factor, or the inverse of one plus the home nominal interest rate, which is the current value of a dollar tomorrow, while the left-hand side is the current cost of a dollar-one minus the reduction in transactions costs that an extra dollar generates.

### 2.5 Monopolistic competitors

Intermediate goods firms hire labor and rent both home and foreign capital directly from financial intermediation firms. We wish to focus on a set-up in which these firms set their prices one period in advance. Initially, however, we abstract from this and look at the ex post profit-maximizing price-setting problem. In that case, each intermediate good-producing firm faces a simple static decision problem, choosing its price to maximize profits subject to the product-specific demand function of the final goods firms.

In general shocks to the money supply will continually shift the demand schedules facing intermediate good producers. One would then conjecture that their prices must continually respond to these. It is shown below, however, that there is an equilibrium of this economy in which intermediate producers choose not to adjust to the ex post state of the world. Prices are predetermined one period in advance. How can this be the case? The answer is that the economy with increasing returns to scale of a sufficient
magnitude will display multiple stationary, stochastic, rational expectations equilibria. But there is a single equilibrium in which the price level is predetermined. Our interest is to focus on this particular equilibrium. It will then follow (see below) that intermediate goods prices are predetermined in this equilibrium. We can then reinterpret the ex post price-setting decisions of intermediate goods producing firms as one-step-ahead price-setting, since, in an equilibrium with an aggregate predetermined price level, their prices will be predetermined.

From (10), the demand function facing each intermediate producer $i$, in the home country is

$$m_{it}(p_{it}, p_t, X_t) = X_t(p_{it}/p_t)^{(1-\rho)}$$

(15)

An intermediate good producer $i$ faces the following cost function

$$Z(\hat{W}_t, \hat{R}_t, e_t \hat{R}_t^*, m_{it}(p_{it}, p_t, X_t)) = A \hat{R}_{t}^{\alpha_1} (e_t \hat{R}_t^*)^{\alpha_2} \hat{W}_t^{(1-\alpha_1-\alpha_2)} \left( m_{it}(p_{it}, p_t, X_t) \right) / \Lambda_t$$

where $A$ represents a constant function of parameters.

An intermediate good firm $i$ in the home country, chooses $p_{it}$ to maximize total profits, given by

$$\hat{\pi}(S_t) = (p_{it}m_{it}(p_{it}, p_t, X_t) - Z(\hat{W}_t, \hat{R}_t, e_t \hat{R}_t^* m_{it}(p_{it}, p_t, X_t)).$$

(17)

The solution to this problem gives the optimal markup rule

$$p_{it} = \frac{A}{\rho \Lambda_t \hat{R}_{t}^{\alpha_1} e_t \hat{R}_t^* \hat{W}_t^{1-\alpha_1-\alpha_2}}$$

(18)

Thus, the optimal price for each intermediate producer is a constant markup over marginal cost. Now focus on a symmetric equilibrium where all intermediate firms choose identical prices and produce identical amounts. Then

$$p_t = \frac{A}{\rho} \hat{R}_{t}^{\alpha_1} (e_t \hat{R}_t^*)^{\alpha_2} \hat{W}_t^{1-\alpha_1-\alpha_2} (k_{x_t}^{\alpha_1} k_{m_t}^{\alpha_2} (1-\alpha_1-\alpha_2)) (1-\gamma)$$

Moreover, from (11),

$$P_t = p_t$$

(19)

where $p_t = p_{it}$ for each $i$.

In an equilibrium in this economy, intermediate goods prices and final goods prices must be proportional to one another. Note in particular that intermediate goods prices are state dependent (i.e., contingent on $S_t$) if and only if the nominal price level is state dependent. In the analysis below, we show that for our parameterization of the model, a state invariant nominal
price level is a rational expectations equilibrium, for the reason explained above. In that case the pricing rule (18) can be interpreted as an outcome of an ex ante price-setting game being followed by monopolistic competitors. In effect, the one-step-ahead pricing policy serves to isolate a particular equilibrium, which is one that is associated with a predetermined aggregate price level.

2.6 Factor prices

If intermediate goods firms in the foreign country behave in a manner as described in the previous section, then we can derive the factor prices that must hold in any equilibrium, conditional on a given total stock of home and foreign capital, and on total home and foreign employment. To keep the analysis as simple as possible, we make the assumption that physical capital can be relocated across countries freely within the period. Thus, intermediation firms that hire home capital can rent it not just to home intermediate producers but also to foreign producers. Therefore, the rental rate paid to capital of each type must be equated across countries.

Let $K_t, K^*_t, H_t$ and $H^*_t$ represent the total supply of home and foreign capital, and home and foreign labor, respectively, in any period. Also let $K_{xt}, K_{mt}, K^*_{xt}$ and $K^*_{mt}$ stand for the allocation of home and foreign capital to the production of home and foreign intermediate goods, respectively. Profit maximization in the home economy will imply that factor prices are given by

$$W_t = \frac{\Omega \rho P_t (1 - \alpha_1 - \alpha_2) (K_{xt}^\alpha K_{mt}^\alpha H_t^{(1-\alpha_1-\alpha_2)} H_t)}{(1 - \phi_2)}$$ (20)

$$R_t = \frac{\rho P_t \alpha_1 (K_{xt}^\alpha K_{mt}^\alpha H_t^{(1-\alpha_1-\alpha_2)} H_t)}{(1 - \phi_2)}$$ (21)

$$e_t R_t^* = \frac{\Omega \rho P_t \alpha_2 (K_{xt}^\alpha K_{mt}^\alpha H_t^{(1-\alpha_1-\alpha_2)} H_t)}{(1 - \phi_2)}$$ (22)

where $\Omega$ is a constant function of underlying parameters.

Three similar conditions must hold for the foreign country. Given these, and the value of the real exchange rate, the ex post allocation of capital of each type to each country can be computed.

2.7 Exchange rate regimes

In this model, unlike for example, that of Helpman (1982) or Lucas (1982), the probability distribution of real variables will critically depend upon the international monetary regime being followed. We distinguish between a regime of free-floating exchange rates and that of managed floating. The
difference between monetary regimes may be fully characterized by the properties of the joint distribution of national monetary aggregates. Under a free float, the money supplies are independent of one another. They may then be written as

\[ N_{t+1} = N_t(1 + \mu + \epsilon_t) \] (23)

\[ N_{t+1}^* = N_t^*(1 + \mu^* + \epsilon_t^*) \] (24)

where \( \epsilon_t \) and \( \epsilon_t^* \) are independently distributed shocks with mean zero and constant variance, and \( N_t(N_t^*) \) represents the economy-wide home (foreign) money stock. The values of \( \mu \) and \( \mu^* \) determine the trend rate of inflation for the domestic and foreign economies.

A managed floating regime entails some coordination of international monetary policies. One type of managed float would be where the foreign country adjusted its money supply immediately in response to money innovations in the home country, in order to keep nominal exchange rate variation within a certain band. This might be alternatively thought of as a target zone regime.

Empirically, we do not observe this type of instantaneous response of monetary policies, however. One way to see this is to look at the reaction of foreign money supplies to U.S. money shocks in the data. The estimates (discussed below) tend to support the view that, in response to U.S. monetary shocks, foreign countries gradually adjust their money supplies so as to reduce the movement in the nominal exchange rate. We investigate the characteristics of our simulated model in a managed floating regime with this type of gradual adjustment.

We might also look at the characteristics of a regime of fully fixed exchange rates. In this model, we will see that these can be seen as a particularly simple extension of a managed float.

2.8 Equilibrium

We define an equilibrium in the world economy conditional upon the assumption that the nominal price level of each country is sticky. To be specific, we conjecture that there is an equilibrium of the model where the equilibrium solutions \( P_t \) and \( P_t^* \) do not depend upon the ex post state \( S_t \). In this environment, price levels are predetermined variables, being known one period ahead. It follows from (19) that intermediate goods prices, in equilibrium, are also predetermined. This then can be interpreted as an equilibrium in which intermediate producers choose prices in advance, and have no incentive to adjust to the ex post state of the world.

In general, this conjecture will be incorrect. In particular, in a deterministic, constant returns to scale economy in which the money stock remains appropriately bounded, there is a unique rational expectations equilibrium
path in which the price level, consumption, and capital stocks all converge to a steady state. Along this equilibrium path, the initial price and consumption must be set by a transversality condition to ensure convergence to steady state. The price is a nonpredetermined variable. Extending this reasoning to a stochastic environment, we can say that in a constant returns-to-scale environment, there is a unique, rational expectations, equilibrium distribution in which the price level, consumption, and capital stock converge to bounded stationary distributions. Again, the price level must be a nonpredetermined variable.

In order to introduce price stickiness in this standard environment, it would be necessary to abandon the assumption of market-clearing in some markets, and to introduce some exogenous price-setting rule by which prices adjust towards their market-clearing levels. This is the manner in which typical dynamic macro models with sticky prices, such as Gray (1976), Fischer (1977), or Dornbusch (1976) behave.

In a infinite horizon economy with a sufficient degree of increasing returns, however, it has been shown by Benhabib and Farmer (1994) that an equilibrium can be indeterminate. In the deterministic context, this means that there is a multiplicity of rational expectations equilibria, all of which converge to steady state. In the stochastic environment, there are multiple stationary stochastic rational expectations distributions.

One way of eliminating this indeterminacy is to select a particular equilibrium path in which the price level is predetermined (see Benhabib and Farmer 1992b). In effect, this is an equilibrium in which all agents believe (correctly) that the price level does not respond contemporaneously to new information. Thus, the property that prices are predetermined along an equilibrium path is equivalent to the finding that there are multiple convergent rational expectations equilibria in the economy without predetermined prices.

It follows that, in this economy, sticky prices do not imply non-market clearing, since by definition, in any equilibrium all agents are choosing optimal policies, and we do not require an exogenous price-setting rule. Rather, prices are set by firms exactly as we would expect- in order to maximize profits. The key point is that there exists an equilibrium in which profit maximizing prices are set one period in advance. Each monopolistic competitive firm conjectures that the price level is predetermined and sets its price on the basis of that conjecture. In equilibrium this conjecture is confirmed.

Effectively, we are using as an equilibrium selection device the constraint that prices cannot instantaneously move in response to contemporaneous shocks. Why do we choose to focus on this rather than any other equilibrium? First, it seems inherently reasonable; there is overwhelming empirical evidence that the price level does not display much high frequency variation.
Prices do seem to be sticky, at least for short periods. We will calibrate such that prices are predetermined for a one-month period alone. This in itself imposes hardly any price rigidity in the economy. Any unanticipated nominal shocks could in principle be undone after one month. But an important result of this model is that a slow price response occurs endogenously. In fact, as shown below, the equilibrium response to monetary shocks may display price sluggishness for a considerable time. Thus, in a sense, the model accords with the low response of prices to money shocks that is seen in data, without in fact imposing this behavior ex ante.

Ultimately, we must argue that the merit of focusing on this equilibrium must be based on the results. Benhabib and Farmer (1992b) argue that, in models with multiple equilibria, focusing on the equilibrium in which prices are preset provides insight into the empirical behavior of prices and money in U.S. data. Analogously, we will argue that in an international setting, an equilibrium with predetermined prices produces results that are consistent with a number of empirical findings regarding the behavior of real and nominal exchange rates.

2.9 Characterization of the equilibrium path

Let the state $S_t$ be defined as the vector of money shocks and aggregate predetermined variables. Thus, $S_t = N_t, N_t^*, K_t^*, P_t, P_t^*$. Then an equilibrium is a set of functions $(C_{xt}, C_{mt}, K_{t+1}^*, H_t^*, P_{t+1}, P_{t+1}^*, W_t^*, W_t^*, R_t, R_t^*)$ that depend on $S_t$ and that satisfy the conditions of (i) from profit maximization, (ii) utility maximization, (iii) satisfaction of monetary authority budget constraints, and (iv) market-clearing. The equilibrium may be characterized from (14), the implicit money demand function, (6), and (7), the utility maximization conditions, (18), (19), the markup rule, and (20)-(22), the factor-pricing conditions. Imposing market clearing implies that $h_t = H_t, h_t^* = H_t^*, c_{jt} = C_{jt},$ for $j = x, m, n_t = N_t, n_t^* = N_t^*, T_t = N_t - N_{t-1}$, and finally $T_t^* = N_t^* - N_{t-1}^*$.

An equilibrium can then be characterized by the solution to the following equations;

\[
\frac{1}{C_{xt}} = \beta E_t C_{xt+1} \Omega \alpha_1 \rho \left( K_{zt+1}^{\alpha_1} K_{mt+1}^{\alpha_2} H_{t+1}^{1-\alpha_1-\alpha_2} \right)^{\gamma} \\
\left( 1 - \phi_2 \left( \frac{N_{t+1}}{P_{t+1}}, F_{t+1} \right) \right) + (1 - \delta) \tag{25}
\]

\[
C_{xt} \eta = \Omega (1 - \alpha_1 - \alpha_2) \rho \frac{K_{zt}^{\alpha_1} K_{mt}^{\alpha_2} H_{t}^{1-\alpha_1-\alpha_2}}{H_{t}} \left( 1 - \phi_2 \left( \frac{N_t}{P_t}, F_t \right) \right) \tag{26}
\]

\[
2C_{xt} + K_{t+1} - K_t (1 - \delta) + \phi \left( \frac{N_t}{P_t}, F_t \right) + \Lambda_t = \Omega (K_{zt}^{\alpha_1} K_{mt}^{\alpha_2} H_{t}^{1-\alpha_1-\alpha_2})^{\gamma} - \Gamma \tag{27}
\]
Equation (25) is just the Euler equation for the optimal choice of consumption of the home good, where the expected marginal utility of future consumption must be adjusted to take account of the transactions costs of intermediation. Equation (26) represents the home country consumption-leisure tradeoff. Equation (27) describes aggregate feasibility for home economy output, taking into account real intermediation costs. Finally, equation (28) describes implicitly the money-market equilibrium relationship. The right-hand side of this equation represents the nominal interest factor. Equations (30)-(32) represent the equivalent conditions for the foreign economy. Finally, equations (33) describe the equilibrium allocation of capital stocks across countries in any period.

In the absence of money shocks, there is a unique steady state of the system (25)-(33). Given the assumed functional forms, the steady state can
be explicitly computed. Because the model is entirely symmetric, each country’s capital stock, hours worked, consumption of each good, and level of real balances is the same. The implicit conditions for a steady state are

\[ 1 = \beta(\alpha_1 + \alpha_2)\rho_\gamma K^{\gamma(\alpha_1 + \alpha_2) - 1}\tilde{H}^{\gamma(1 - \alpha_1 - \alpha_2)}(1 - \phi_2(\tilde{N}, \tilde{F})) + (1 - \delta) \]  

\[ \tilde{C}_n = \Omega_1(1 - \alpha_1 - \alpha_2)\rho_\gamma \frac{(\tilde{K}^{\alpha_1 + \alpha_2}\tilde{H}^{(1 - \alpha_1 - \alpha_2)})^\gamma}{\tilde{H}}(1 - \phi_2(\tilde{N}, \tilde{F})) \]

\[ 2\tilde{C} + \tilde{K} \delta + \phi(\frac{\tilde{N}}{\tilde{F}}) + \Lambda = \Omega_1^{(1 - \rho)}(\tilde{K}^{(\alpha_1 + \alpha_2)}\tilde{H}^{(1 - \alpha_1 - \alpha_2)})^\gamma - \Gamma \]

\[ 1 + \phi_1(\frac{\tilde{N}}{\tilde{F}}, \tilde{F}) = \frac{\beta}{(1 + \mu)} \]

In the steady state of this economy, money growth has real affects solely through the distortions introduced by the transactions cost technology (3). Money is neutral, but not superneutral, in the steady state. An increase in the rate of money growth will raise the steady state nominal interest rate, implicitly defined by \( l_\mu \). Through (36), this will lead to a substitution away from real balances. A reduction in real balances will reduce factor returns, and this will reduce the steady state \( \tilde{K} \) and \( \tilde{H} \). Therefore, the steady state of the model behaves qualitatively exactly like that of the standard general equilibrium transactions-based model of money, such as Den Haan (1990), and Marshall (1990). This means that predeterminateness of the nominal price level is irrelevant, in the steady state.

2.10 The labor market and indeterminacy

What is the intuition for the indeterminacy of equilibria? It may be worthwhile to give a brief intuitive explanation, based on the characteristics of labor market equilibrium. This explanation is for the labor market of either country and is meant to complement that found in Benhabib and Farmer (1994).

Consider Figure 1 where the \( L^* \) curve represents an implicit labor supply curve indexed by investment and capital. We derive this curve from the consumption-constant labor supply curve, but replacing consumption by output minus investment. The shape of the curve is explained as follows. A rise in employment will raise output and consumption, and therefore requires a rise in the real wage so as to satisfy condition (9). There are two important characteristics to note about this curve. First, a movement up the curve is associated with an increase in consumption since, as defined, investment is held constant. Second, an increase in investment causes the curve to shift right.
The downward sloping $L^d$ curve in Figure 1 represents the implicit labor demand schedule for the individual firm, holding the external factor $\Lambda$ constant. For the individual firm, an increase in employment must reduce the marginal produce of labor. In a constant returns-to-scale economy ($\Lambda = 1$), the same property carries over to the aggregate since the aggregate labor demand curve coincides with that perceived by the representative firm. In the presence of an external effect, however, this need not carry over to the aggregate economy. When all firms together expand employment, there is an external effect on the productivity of each individual firm through an increase in $\Lambda$. If the parameter restriction $(1 - \alpha_1 - \alpha_2)\gamma > 1$ is satisfied, then in aggregate this external effect is so strong that an increase in aggregate hours raises marginal productivity. This relationship, or aggregate expansion path for the economy, is represented by the positively sloped curve $NN$.

A simple and heuristic way to describe why indeterminacy can arise in this setting is to consider the following conjecture. Beginning in the steady state, imagine that firms contemplate increasing investment temporarily, with investment returning to its steady state level after one period. Can such a coordinated action constitute a profitable deviation by firms?

Take first the case of a constant returns-to-scale economy. A temporary increase in investment will cause the $L^s$ curve to move out along the $L^D$ curve. The result must be an immediate fall in consumption. But, note that a temporary fall in consumption must imply that the real interest rate rises, since we are considering a deviation that leaves next period values unchanged.

If the market interest rate rises, however, the initial increase in investment cannot be rationalized by firms. Thus, there is not another rational expectations equilibrium with higher investment.

Think of the same experiment in the presence of aggregate externalities, however. An increase in investment is not only associated with the $L^s$ curve shifting along the $L^D$ curve. Instead, the initial shift in the $L^s$ curve induces the $L^D$ curve to shift out as increased employment raises the productivity of all firms. The new temporary equilibrium is along the $NN$ curve. But this new point is above and to the right of the initial equilibrium. That means that consumption must be higher at this point. The final consumption level must be temporarily above its steady state level. However, if consumption is now temporarily above its steady state level, then the market interest rate must be below its steady state level. The low interest rate then rationalizes

---

3This is easy to see. Since the new equilibrium point in the labor market is below and to the right of the initial point, the consumption-constant labor supply curve must have moved down, so that consumption must have fallen. Note that the constant consumption labor supply curve in the case of Hansen Rogerson preferences is horizontal. Alternatively, the fall in consumption can be inferred directly from the investment-constant labor supply curve.
the increase in investment contemplated by firms and therefore explains the possibility of indeterminate transition paths in such an economy. One interesting implication of this argument is that consumption and hours worked may move in the same direction even in the absence of any shocks to the production function. This feature of the model will be emphasized below.

3 Solving the model

There is no analytical solution for the dynamic system described by (25)-(33). But we may use standard techniques to compute a numerical solution. We follow the method of King, Plosser, and Rebelo (1988a) in linearizing the system around the deterministic steady state. In order to do this, we first need to take a stand on parameter values.

3.1 Calibration

There are four classes of parameter values in the model: (a) those pertaining to preferences and ; (b) those pertaining to the production technologies and ; (c) those pertaining to the transactions cost technology and ; and finally, (d) those pertaining to the money growth process . Table 1 gives a full list of the parameter values used in the benchmark model.

The choice of and determine the time period length over which the model is calibrated. The model is constructed so that the response of the price level to a money shock is inflexible for the duration of one period. Thus, a shorter time period involves less price stickiness. To minimize the degree of price stickiness implied by the model, we calibrate so that a time period corresponds to one month. That is, if we take the annual time discount factor to be 0.96, following Prescott (1986), we set , and if we take annual depreciation rate to be 0.1, then we set .

The most important parameter for our overall results is the magnitude of , the degree of increasing returns to scale in the production technology for intermediate goods firms. Estimates of this parameter have varied widely. Hall’s (1990) estimate of increasing returns to scale for one-digit U.S. manufacturing industries ranges from to , with six out of seven industries having a value higher than 1.7. Baxter and King (1991) directly estimate the degree of increasing returns at the aggregate level, using an instrumental variables procedure, and find estimates of equal between 1.81 and 1.1. Card (1990) reports estimates of inferred using firm microdata. His estimates are generally above 2. Finally, Caballero and Lyons (1992) in estimates for U.S. manufacturing, find increasing returns as high as 1.56. Their study is also relevant to the present paper because they find increasing returns at the aggregate manufacturing level that are significantly higher than those found
Table 1: Parameter Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
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</tr>
<tr>
<td>$\eta$</td>
<td>5</td>
</tr>
<tr>
<td>$\gamma$</td>
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</tr>
<tr>
<td>$\rho$</td>
<td>1.15</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.29</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\delta$</td>
<td>$0.1\frac{1}{12}$</td>
</tr>
<tr>
<td>$B$</td>
<td>0.015</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1.675</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.15</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$0.04\frac{1}{12}$</td>
</tr>
</tbody>
</table>
at the two-digit level. We take $\gamma - 1.56$, as our benchmark. This is on the conservative side of both Card (1990) and Hall (1990) findings and in the range found by Caballero and Lyons (1992) and by Baxter and King (1991). We report some experiments using different numbers.\footnote{Recently Basu and Fernald (1994a, b) have criticized both Hall (1990) and Caballero and Lyons (1992) for their use of value-added data to estimate the degree of increasing returns to scale. They find that estimates based on the more theoretically desirable gross output data produce much lower returns to scale. Basu and Fernald results may be still somewhat questionable, though. In particular, when estimating by instrumental variables, estimates of returns to scale should be independent of whether inputs are regressed on outputs or the inverse. However, the evidence against high degrees of increasing returns are only found when output is regressed on inputs, and not when the inverse regression is run.

This is clearly a controversial area. Our aim in this paper, however, is to explore the qualitative features of economies with increasing returns to scale rather than to be quantitatively exact.}

The parameter $\rho$ determines the markup over marginal cost that is set by monopolistic competitive firms. This parameter has no important effects on the dynamic behavior of the model. It serves to determine the level of hours worked and the capital stock in the deterministic steady state, however, which are necessary to construct welfare results. Empirical estimates of markups in U.S. data are presented by Hall (1990), who estimates markups of 1.5 and above. Morrison (1990) derives smaller estimates in the range of 1.2 to 1.3. Finally Basu and Fernald (1994a) re-estimate markups in Hall, using gross output data, and find significantly smaller markups than Hall. The average markup they report is 1.15. We choose this number as our benchmark.

The fixed costs per firm are set so as to achieve zero profits in the deterministic steady state. This matches the observation of Hall (1990) that average profits in U.S. data are very low.

With zero profits on average, the share of labor in GDP for each country is given by $(1 - \alpha_1 - \alpha_2)$. We set this to .71, as reported in Greenwood, Hercowitz, and Huffman (1988). We do not have direct evidence on the individual parameters $\alpha_1$ and $\alpha_2$, so we simply set $\alpha_1 = 0.29$, and $\alpha_2 = .01$.

The parameter $\eta$ is set to lead the model to predict a steady state fraction of time spent at work equal to the sample average in U.S. data, which is about thirty percent. A value of $\eta = 5$ achieves this.

The financial intermediation cost function contains three parameters. These are chosen to set (a) the total share of transactions costs in GDP equal to 1 percent, (b) the gap between borrowing and lending rates of interest equal to 2 percent, and (c), the interest elasticity of money demand equal to .7. Criterion (a) is rationalized by the attempt to reduce the importance of the transactions cost role of money in the model and to highlight the effects of price stickiness. A similar number is used by Marshall (1990) in calibrating transactions costs in a monetary general equilibrium model. As a
reference, we might note that in the cash-in-advance model, such as Cooley and Hansen (1989), for instance, the analogous number is zero.

A 2-percent gap between borrowing and lending rates of interest implies that at an annualized rate, the real rate of return is about 30 basis points lower than it would be in a frictionless general equilibrium economy. Finally, an interest elasticity of money of .7 is one of the numbers considered by Lucas (1993). An elasticity of between .3 and .7 can be used to match time series observations on M1. We report results based on alternative elasticities below.

The variable \( \mu \) is chosen to produce an average annual inflation rate of 4 percent, roughly the average in post-1960 U.S. data.

### 3.2 Model solution

To solve the model the dynamic system (27)–(30) is first written in terms of the six state variables \( C_{xt}, C_{mt}, K_t, K_t^*, \hat{P}_t, \) and \( \hat{P}_t^* \), where \( \hat{P}_t = P_t / N_t \) and \( \hat{P}_t^* = P_t^* / N_t^* \) are stochastically detrended prices. The system is solved by a linear approximation in the neighborhood of the steady state values; \( K, \hat{C}, \) and \( \hat{P} \). Let \( \hat{X}_t = \log \hat{X}_t^j \). Then the linear system may be written as

\[
E_t X_{t+1} = M Z_t + RV_t
\]

where \( Z_t = [\hat{K}_t, \hat{K}_t^*, \hat{C}_{xt}, \hat{C}_{mt}, \hat{P}_t, \hat{P}_t^*] \), \( M \) is a 6 by 6 matrix of coefficients, \( R \) is a 6 by 2 matrix of coefficients, and \( V_t = [\epsilon_t, \xi_t^*] \).

The system described in (37) has four predetermined variables: \( \hat{K}_t, \hat{K}_t^*, \hat{P}_t, \) and \( \hat{P}_t^* \), and two nonpredetermined variables: \( C_{xt} \) and \( C_{mt} \). Therefore, there is a unique rational expectations solution to (37) as long as the transition matrix \( M \) has four stable roots and two unstable roots. In contrast to the benchmark RBC model, the equilibrium in this economy cannot be represented by the solution to a social planning problem. Therefore, we are no longer guaranteed that the solution to the linear approximation (37) will be a saddlepath. This must be ascertained by computation for each calibration of the economy. For the calibration described above, the roots to \( M \) are calculated as \( [1.01, 1.01, 0.98 \pm .053i, .95 \pm .008i] \). Since this satisfies the stability criterion, it follows that the solution is a saddlepath.\(^5\)

### 3.3 Monetary shocks under alternative exchange rate regimes

We now examine the impact of shocks to the home money supply under the different exchange rate regimes discussed above. Any unanticipated shock to

\(^5\)Note that if prices could instantaneously jump at any moment in time, the equilibrium here would be non-unique. In that case, we would have the indeterminacy discussed by Benhabib and Farmer (1994).
the money supply will generate non-neutral responses in the economy, since the price level cannot immediately adjust. But the price is predetermined only for a one-month period. Once the shock is revealed, the whole sequence of future prices can adjust, beginning one month later.

### 3.4 Floating exchange rates

Under pure floating exchange rates the two money supplies are independent of one another. We explore the models' properties by looking at the impact of a shock to the domestic money supply of one percent. Thus, beginning at a steady state we set \( ε_1 = 1 \) for \( t = 0 \), and \( ε_t = 0 \) for \( t > 0 \), holding \( ε^*_t = 0 \), for all \( t \). This represents a once-off permanent increase in the money supply.

Figure 2a illustrates the impact of this shock on the real and nominal exchange rate. All the scaling on the figures is in terms of proportional rates of change. Periods are in quarters. The figure indicates that there is an immediate depreciation of the nominal exchange rate by about 0.8 percent in response to the shock. Thereafter, the exchange rate gradually depreciates further, in fact overshooting its long-run response (unity). The maximal response of the nominal rate is felt after about 8 quarters. Thereafter, the rate falls to its long-run level. But note also that the real exchange rate depreciates significantly immediately following the shock. By the structure of the model, it has to be the case that in the first month of the shock, the behavior of the real and nominal exchange rate is identical. Thus, at time \( t = 0 \), nominal depreciation implies real depreciation. The figure, however, indicates that the movement in both the real and nominal exchange rate is essentially identical for the first three quarters. Figure 2b explains this. The figure shows that prices show essentially no response to the money shock for the first three quarters. Thus, the movement in the nominal exchange rate and real exchange rate is the same during this time.

The impact of the money shock on the real exchange rate must be transitory. Figure 2 shows, however, that the initial real depreciation is extremely persistent. Even after 20 quarters, the real rate is .4 percent higher than its initial steady state. In addition, as in the case of the nominal exchange rate, the real depreciation builds up over time. The maximal effect is not felt for 6 quarters. Note also that the maximal effect is a 1.3-percent real depreciation. Thus, the real exchange rate response is a multiple of the initial monetary injection.

The persistent real depreciation reflects the fact that the money shock has differential real effects on the home and foreign economy. Under the

\[^{6}\text{Note that the data in the figures are aggregated from the monthly simulations, so the equivalence of real and nominal exchange rates may not hold exactly in the figures.}\]
Figure 2a Nominal and Real Exchange Rates
Figure 2b: Real/Nominal E. Rates; Relative Prices
hypothesis of the model the equilibrium real exchange rate can be written as

\[ \kappa_t = \frac{C_{xt}}{C_{mt}} \]

Figure 3 illustrates the effect of the money shock on consumption in the domestic and foreign economy. The money shock raises consumption of the home good relative to consumption of the foreign good. Thus, the real depreciation reflects the fall in the equilibrium relative to price of the home good. The persistent impact of the shock indicates that the wedge between the consumption streams persists for a considerable time.

The movement of consumption also explains the hump-shaped response of both the real and nominal exchange rates. Since consumption of the home good relative to the foreign good rises gradually, the real exchange rate inherits the same property. The fact that prices rise only gradually explains the similarity in the behavior of the real and nominal exchange rates in the aftermath of the shock.

Figure 4 reports the impact of the money shock for different elasticities of money demand. Figure 4a represents an elasticity of 0.5, while Figure 4b represents an elasticity of 0.3. As the interest elasticity of money demand falls, the effect of the shock on the real exchange rate diminishes, while the response of the nominal rate remains almost the same.

There is an interesting contrast between the behavior of the real and nominal exchange rates here and that of Dornbusch (1976). In the Dornbusch sticky price model, monetary expansion typically generates exchange rate ‘overshooting.’ The impact of the money shock on the real and nominal rates is at its maximal level at time zero. By contrast, this model implies that both real and nominal rates continue to depreciate following the shock. There is overshooting, however, in that the proportional change in the nominal exchange rate rises above unity during the transition.

The estimates of monetary policy shocks on the real exchange rate given in Eichenbaum and Evans (1993) and Clarida and Gali (1994), using different identification schemes, both tend to support the gradual overshooting response captured in Figure 2 rather than the instantaneous overshooting implied by the Dornbusch model. Eichenbaum and Evans (1993) estimate the effect of shocks to nonborrowed reserves on the real and nominal exchange rates for the bilateral U.S. dollar rate against a number of other countries. Figure 5 gives the response of the nominal and real exchange rates to a monetary expansion from a VAR model using Eichenbaum and Evans’ procedure.7

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7We estimated a separate 5 variable VAR for Germany, France, and Japan, where the variables included in the VAR are U.S. nonborrowed reserves, U.S. M1, foreign M1, the nominal exchange rate, and the real exchange rate. Six lags of each variable were included in estimating the VARs over the period 1974:1-1992:4. The results in Figure 5 are for the US$ D-Mark rate, and are very typical.
Fig. 4a Nominal and Real Exchange Rates

Fig. 4b Nominal and Real Exchange Rates
As in Figure 2, the real exchange rate shows continued depreciation following the shock.

3.5 International transmission

The effect of the money shock on real variables is illustrated in Figure 6. The shock generates a jump in employment and output in both countries, but the effects are significantly greater in the home country.\(^8\)

Prices in both countries begin to rise with a lag. The price rise in the foreign country dies away over time, while the rise in the home country is permanent. The capital stock in both countries rises, with the increase being greater in the domestic country. Again, there is considerable persistence in all these effects.

The presence of price stickiness is central to the effects of the money shock. The impact of the shock is felt initially in the intermediation sector in the home economy. With predetermined prices, the increase in the money supply leads to an immediate increase in the real balances of intermediation firms. From (28), this must lead to a rise in output, or a decline in the nominal interest rate. Since the price level remains essentially unchanged for the first few quarters and consumption growth is positive, the nominal rate will in fact rise, although by a small amount. Thus, output must rise immediately in response to the monetary shock.

Why does foreign output and employment rise? To see this, recall our discussion of the labor market mechanism in the increasing returns economy. The domestic money shock raises output in the home economy. This increases home demand for capital. Some of both types of capital are then shifted from the foreign economy to the domestic economy. The fall in foreign capital leads to a shift to the right in the \(NN\) curve (see Figure 1) of the foreign economy, increasing foreign output and employment.\(^9\)

In the traditional textbook open economy macro model, based on Mundell (1968), the international transmission of monetary policy is negative. Here we derive the opposite result. Monetary expansion in one country leads to a temporary expansion in all countries, even with sticky prices \(and\) under flexible exchange rates. In a world economy driven by monetary shocks alone, we would expect to see positive cross-country correlation of output.

The response of output to the monetary expansion in both countries is cyclical. Output initially rises and then falls below its steady state value after

---

\(^8\)The rise in output may seem somewhat excessive. Adding adjustment costs to the model might help to reduce the magnitude of the response in output to a money shock.

\(^9\)Benhabib and Farmer (1994) have noted this property of an economy with increasing returns and indeterminacy. An increase in the capital stock, ceteris paribus, leads to a fall in employment.

84
Figure 5 Sample
about two years. This property of business-cycle models with indeterminacy has also been noted by Farmer and Guo (1994).

3.6 Standard deviations

We now discuss results derived from simulations of the model in which both countries' money stocks are given by the processes given above. For each experiment, we subject the model to 150 simulations of 200 quarters each.\footnote{Since the model is entirely symmetric, we report only one country’s standard deviations. The number for the other country are the same.}

An immediate question that arises is how to scale the money shocks. One way to do this would be to directly calibrate the standard deviation of \( N \) and \( N^* \) from data. But this would open up the difficult question of the appropriate monetary aggregate to use. Rather than take a stand on the fraction of the business cycle accounted for by money shocks, we simply calibrate the size of the shocks so that the standard deviation of output is equal to 1 in each country.

Table 2 reports the results for the real and nominal exchange rate. Since the nominal exchange rate is nonstationary, we report standard deviations of first differences of nominal and real exchange rates and national price levels (the standard deviations are the same for each country). The results are quite striking. Nominal and real exchange rate variability are about the same, and both are four times as high as nominal price variability. The model thus accords well with the stylized facts of real and nominal exchange rate variability relative to price variability. Both nominal and real exchange rate variability greatly exceed price variability.

Figure 7 confirms this statement. It illustrates the movement of the nominal and real exchange rates relative to price levels for a sample period. It makes clear that real and nominal exchange rates move very closely together in the model and that they fluctuate much more than prices.

Considerable evidence suggests that real exchange rate movements are highly persistent (e.g., Adler and Lehman 1982, Hakkio 1986). In fact, the stochastic behavior of the real exchange rate may best be thought of as containing a unit root (Huizinga 1987). Stockman (1988) argues that the persistence property of real exchange rate movements supports the flexible-price equilibrium view of real exchange rate determination, as opposed to the sticky-price Dornbusch-type models. This is because the Dornbusch model driven by monetary shocks predicts a rapid return to purchasing power parity, a prediction hard to reconcile with the evidence. In contrast, our model is consistent with very persistent movements in exchange rates even when the source of fluctuations is purely monetary, and the real exchange rate is a stationary process. The magnitude of persistence in the model suggests that
Table 2:
Standard Deviations

<table>
<thead>
<tr>
<th>RER</th>
<th>NER</th>
<th>( \gamma = 1.56 )</th>
<th>IE = 0.7</th>
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<tr>
<td>.099</td>
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<tr>
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<td>.03</td>
<td>( \gamma = 1.5 )</td>
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<tr>
<td>.097</td>
<td>.092</td>
<td>.024</td>
<td>( \gamma = 1.6 )</td>
</tr>
<tr>
<td>.087</td>
<td>.09</td>
<td>.026</td>
<td>( \gamma = 1.56 )</td>
</tr>
<tr>
<td>.067</td>
<td>.086</td>
<td>.036</td>
<td>( \gamma = 1.56 )</td>
</tr>
</tbody>
</table>

Numbers refer to standard deviations of log first-differences.
RER: Real Exchange Rate, NER: Nominal Exchange Rate, P: Price Level
IE: Interest Elasticity of Money Demand

it might be hard to statistically reject the hypothesis that the real exchange rate contained a unit root using standard tests. For instance, the AR(1) coefficient on the real exchange rate in the simulations with monetary shocks is 0.98.

Table 2 also reports the results of alternative parameterizations. Increasing the degree of growing returns actually reduces the magnitude of real exchange rate variability, and vice versa.

The standard deviations of other macro aggregates relative to output are reported in Table. The rank ordering of these numbers is in accord with those observed in U.S. and other data. The standard deviation of investment exceeds that of output; the standard deviation of hours is almost the same as that of output, and the standard deviation of consumption is less than output. In terms of scale, consumption variability is considerably less than seen in the data, but this is to be expected, since the model is being driven by temporary shocks. The variability of the real exchange rate is a little less than half that of output. Again, this is considerably less than seen in the data. Backus, Kehoe, and Kydland (1993) show that terms of trade volatility (in our example the same thing as real exchange rate volatility) is considerably higher than output in postwar data for most countries. International real business-cycle models driven by productivity shocks alone typically predict variability of the terms of trade that is much smaller than that of output. Backus, Kehoe and Kydland refer to this as the price anomaly in international RBC models. It seems from these results that our model has difficulty in
explaining the price anomaly, at least in the context of monetary shocks alone. But a more complete exploration of this question still needs to be done within this model.

Table 3 also reports first-order autocorrelations and cross-country correlations from the model. All variables display considerable persistence. The cross-country correlation of output is 0.44. This accords with the intuition from the impulse response functions. This evidence suggests that the current model has some potential for explaining a second anomaly noted by Backus, Kehoe and Kydland (1993). That is the discrepancy between theory and data with respect to the cross-country correlations of consumption aggregates relative to output aggregates. A constant returns to scale multi-country theoretical real business-cycle model driven by aggregate productivity disturbances implies that cross-country consumption correlations are very high as a basic property of risk-sharing, while cross-country output correlations are much lower or even negative, due to the high substitutability between foreign and domestic locations of production. In the increasing returns-to-scale model we see a tendency towards positive cross-country output correlations. Our complete markets setup implies a perfect correlation of consumption across countries. An extension of our model to incomplete international assets markets along the lines of Baxter and Crucini (1992), however, holds the potential of explaining simultaneously low cross-country consumption correlations with high cross-country output correlations.11

3.7 Managed float

The exchange rate response shown in Figure 2 indicates that the model is consistent with the traditional view that monetary expansion produces a significant and persistent real depreciation. But the precise pattern of the response is somewhat different from that reported in Eichenbaum and Evans (1993) and shown in Figure 5. The key discrepancy is that according to the sample estimates, the nominal exchange rate tends to display considerable mean reversion, not just the real exchange rate as shown in the figure.

What might account for this discrepancy? A strong possibility is that our assumption of a pure floating exchange rate is off base. Rather than fully floating exchange rates, the current worldwide system is effectively a managed float. Under managed floating, monetary authorities react to changes in foreign money supply by changing their own money supply. In the impulse response experiments of the previous section, however, foreign money did not respond at all to domestic money shocks.

In the VAR system estimated above (see footnote 7), we found evidence

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11 Preliminary results in Beaudry and Devereux (1994) indicate that this intuition is correct.
Table 3:

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<tr>
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</thead>
<tbody>
<tr>
<td>Y</td>
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<td>0.89</td>
<td>0.44</td>
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<tr>
<td>I</td>
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<td>0.12</td>
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<tr>
<td>C</td>
<td>0.3</td>
<td>0.98</td>
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Case $\gamma = 1.5$

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<tbody>
<tr>
<td>Y</td>
<td>1.47</td>
<td>0.87</td>
<td>0.68</td>
</tr>
<tr>
<td>I</td>
<td>4.95</td>
<td>0.86</td>
<td>0.25</td>
</tr>
<tr>
<td>H</td>
<td>1.06</td>
<td>0.86</td>
<td>0.7</td>
</tr>
<tr>
<td>C</td>
<td>0.25</td>
<td>0.97</td>
<td></td>
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</table>

Case $\gamma = 1.6$

<table>
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</thead>
<tbody>
<tr>
<td>Y</td>
<td>0.81</td>
<td>0.9</td>
<td>0.36</td>
</tr>
<tr>
<td>I</td>
<td>4.62</td>
<td>0.89</td>
<td>0.08</td>
</tr>
<tr>
<td>H</td>
<td>0.82</td>
<td>0.89</td>
<td>0.4</td>
</tr>
<tr>
<td>C</td>
<td>0.35</td>
<td>0.98</td>
<td></td>
</tr>
</tbody>
</table>

As discussed in the text, the benchmark case is set so that the standard deviation of output is unity. For deviations from the benchmark case, we report how the standard deviations of output changes for the same magnitude of shocks. All other standard deviations reported for case $\gamma = 1.5$ and $\gamma = 1.6$ are relative to the standard deviation of output for that particular shock.
that foreign money does indeed catch up gradually with a U.S. money shock. Following an increase in U.S. nonborrowed reserves, we found that the money supply in each of the sample countries (as measured by M1) increased until it caught up to the induced increase in U.S. M1. The estimates suggest that within 3 to 5 years the initial differential in domestic versus foreign M1 induced by an innovation in nonborrowed reserves is completely eliminated.

A simple way to model this adjustment process in foreign money is to let foreign and domestic money shocks be related in the following way

\[ \ln N_t^* = \zeta_1 \ln N_{t-1}^* + (1 - \zeta_1) \ln N_{t-1} + \zeta_2 \epsilon_t. \]

This money supply rule states that, when \( \zeta_1 > 0 \) and \( 0 < \zeta_2 < 1 \), the foreign monetary authority gradually accommodates an increase in domestic money supply by letting their own money supply increase. We set \( \zeta_1 = .96 \) and \( \zeta_2 = .00 \). This implies that there is a zero initial response of the foreign money supply to the domestic money shock, while the foreign money supply gradually catches up to the domestic money supply.

Figure 8 illustrates the impact on real and nominal exchange rates of a one-time shocks to \( \epsilon_t \) in this environment. Again, we report results for three different values of the interest elasticity of money (Figures 8a, 8b, and 8c represent the elasticities 0.7, 0.5, and 0.3, respectively). As could be expected, both the real and nominal exchange rates are close to their initial values after 20 quarters. However, the notable observation from the figure is the degree to which the real and nominal rate track each other throughout the transition period. Both rates depreciate on impact and continue to depreciate for several quarters. Following this, both appreciate with the shock, again generating great persistence in the real exchange rate. This pattern is especially notable given that the two money supplies are not tracking each other closely: the foreign money supply is only catching up to the domestic money supply very slowly. The pattern differs from different elasticities of money demand. An elasticity of 0.3 leads to a closer relationship between the real and nominal exchange rate in this case. Note, however, that in this case the magnitude of the response of the real and nominal exchange rate is considerably less than in the floating exchange rates regime.

Figure 9 shows the response of output in each country to the domestic money shock. Output rises sharply now in both countries, despite the fact that the foreign money stock is accommodating the domestic shock only very slowly. From these figures, we would anticipate that the cross-country correlation of output would be very high in a world with monetary shocks alone. Table 4 reports the standard deviations and cross-country correlations computed in the same way as in Table 2 above. The striking fact is that the cross-country output correlations are raised dramatically under this type of exchange rate regime. The model clearly implies that the nominal exchange
rate regime makes a very big difference to the international business cycle.

Table 4:

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<tbody>
<tr>
<td>Y</td>
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<td>0.9</td>
<td>0.97</td>
</tr>
<tr>
<td>I</td>
<td>4.8</td>
<td>0.89</td>
<td>0.94</td>
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<tr>
<td>H</td>
<td>0.86</td>
<td>0.89</td>
<td>0.97</td>
</tr>
<tr>
<td>C</td>
<td>0.26</td>
<td>0.97</td>
<td></td>
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</tbody>
</table>

Case $\gamma = 1.5$

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>Y</td>
<td>1.69</td>
<td>0.88</td>
<td>0.98</td>
</tr>
<tr>
<td>I</td>
<td>5.03</td>
<td>0.88</td>
<td>0.94</td>
</tr>
<tr>
<td>H</td>
<td>0.92</td>
<td>0.88</td>
<td>0.98</td>
</tr>
<tr>
<td>C</td>
<td>0.16</td>
<td>0.96</td>
<td></td>
</tr>
</tbody>
</table>

Case $\gamma = 1.6$

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>0.87</td>
<td>0.91</td>
<td>0.97</td>
</tr>
<tr>
<td>I</td>
<td>4.68</td>
<td>0.9</td>
<td>0.94</td>
</tr>
<tr>
<td>H</td>
<td>0.83</td>
<td>0.9</td>
<td>0.97</td>
</tr>
<tr>
<td>C</td>
<td>0.32</td>
<td>0.98</td>
<td></td>
</tr>
</tbody>
</table>

As discussed in the text, the benchmark case is set so that the standard deviation of output is unity. For deviations from the benchmark case, we report how the standard deviations of output changes for the same magnitude of shocks. All other standard deviations reported for case $\gamma = 1.5$ and $\gamma = 1.6$ are relative to the standard deviation of output for that particular shock.

The pattern of exchange rate movements observed in Figure 9 suggests that in looking at the real and nominal exchange rate correlation in bilateral data, it may be important to take into account the endogenous response of foreign country monetary policy in face of domestic country shocks. With free floating, as described in Figure 2, the responses of real and nominal exchange rates diverge from one another after 4 quarters. But the shape of responses in Figure 9 is much closer to that of empirical estimates (e.g., Figure 5). Thus, an accommodating foreign monetary policy can allow for both very similar and highly persistent movements in the real and nominal exchange rate in response to a money shock.

We could easily extend the model to examine the properties of a fully fixed exchange rate; for instance, a one-sided peg as in Helpman (1982). Due to the symmetry of the model, this would require that the foreign country react immediately and in proportion to any domestic monetary shocks. In that case, both real and nominal exchange rate variability would be eliminated,
and monetary business cycles across countries would be identical.

4 Conclusions

In this paper we have shown that the combination of increasing returns with endogenous price-setting behavior can have a dramatic effect on the predictions of an international business-cycle model for the domestic and international impact of monetary disturbances. Under a floating exchange rate regime, a domestic money shock was shown to generate both real and nominal depreciation and extremely prolonged deviations from PPP. Our results indicate that a real business-cycle model can be extended in such a way that, for the effects of monetary shocks, they deliver predictions that are in accord with many economists' intuition. In particular, for those who interpret the data as indicating that money shocks tend to generate significant real and nominal exchange rate depreciation, this paper offers a model which is consistent with this observation and in which meaningful policy experiments can be conducted. Nonetheless, we believe that the current model needs to be extended in many directions before a reasonable assessment of its value can be made.
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