State Dependent Pricing and Business Cycle Asymmetries*

Michael B. Devereux and Henry E. Siu†

Abstract

We present a tractable, dynamic general equilibrium model of state-dependent pricing and study the response of output and prices to monetary policy shocks. We find important non-linearities in these responses. For empirically relevant shocks, this generates substantially different predictions from time-dependent pricing. We also find a distinct asymmetry with state-dependent pricing: prices respond more to positive shocks than they do to negative shocks. This is due to a strategic linkage between firms in the incentive for price adjustment. Our state-dependent model can account for business cycle asymmetries in output of the magnitude found in empirical studies.

Keywords: State dependent pricing; sticky prices; endogenous price rigidity; menu costs; monetary business cycles; asymmetric business cycles

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1. Introduction

A large literature in macroeconomics studies the role of nominal rigidities in dynamic general equilibrium settings. In these models, nominal prices adjust slowly in response to shocks. According to the usual argument, the presence of small fixed costs of changing prices makes it unprofitable for firms to adjust prices frequently. Firms adjust prices only when the benefits outweigh the fixed costs. The degree of price flexibility at any point in time – the fraction of price-adjusting firms – depends on the state of the economy. That is, price adjustment is state-dependent.

In contrast, most models of price rigidity employ time-dependent rules for adjustment. In these models, the frequency of a firm’s price adjustment does not depend on its current revenue or cost conditions. Classic contributions include Taylor (1980) and Calvo (1983).1 The argument for this approach is that for small shocks, the gain from changing price is less than the explicit cost of adjustment. Hence, variation in the degree of price rigidity is unlikely to be quantitatively important for monetary business cycle analysis. As such, most of the analysis in this literature is confined to studying local, linearized model dynamics about a steady state. This means that one cannot distinguish between the effects of small and large business cycle shocks on output and inflation; nor between the effects of positive and negative shocks.

Many empirical studies have found evidence of non-linear and asymmetric effects of monetary policy shocks on inflation and output (see Cover, 1992; Macklem et al., 1996; Ravn and Sola, 2004; and the references therein). Large shocks lead to smaller output multipliers than shocks of smaller magnitude; that is, the effect of monetary policy shocks are non-linear. In addition, positive shocks have smaller expansionary effects on output than the contractions associated with negative shocks of the same magnitude; the effect of monetary policy shocks are asymmetric.

Models of state-dependent pricing represent a potential explanation for these non-linearities and asymmetries. However, the analysis is technically challenging due to the induced heterogeneity in prices charged across firms. Study of these models has usually made progress in specialized environments not amenable to quantitative business cycle analysis (see, for instance, Ball and Romer, 1990; Caplin and Leahy, 1991; Ball and Mankiw, 1994; and Conlon and Liu, 1997). By

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1See, also, Rotemberg and Woodford (1997), Chari et al. (2000), and Christiano et al. (2005). This represents an extremely small subset of the relevant research. For good surveys, see Goodfriend and King (1997) and Gali (2002).
contrast, Dotsey et al. (1999) construct a model of state-dependent pricing within a more familiar dynamic general equilibrium environment. Due to the large state space however, their analysis requires local linearization. This does not lend itself to the study of non-linear and asymmetric effects of monetary policy shocks.

In this paper, we study a simple ‘hybrid’ time- and state-dependent pricing model. We follow a suggestion of Ball and Mankiw (1994) and assume that firms specify prices in contracts of fixed duration. However, during the life of a given contract, a firm can ‘opt out’ and revise its price by incurring a fixed cost. The model is made dynamic by assuming that price contracts are staggered as in Taylor (1980). The fixed duration of contracts admits model solutions with a limited state space; this is the role of the model’s time-dependent feature. However, the state-dependent nature of price determination is preserved since a firm’s decision to opt out is determined by the state of nature. This hybrid form of price-setting allows us to completely characterize the non-linear dynamics of our stochastic model.

The analysis starts with a static model, to understand the nature of the firm’s pricing decision. We find a distinct asymmetry in a firm’s state-dependent price adjustment in response to shocks. Positive shocks to marginal cost generate greater price flexibility than negative shocks of the same size. This asymmetry arises due to a strategic linkage between firms in the incentive to change prices. With a positive marginal cost shock, prices are strategic complements: a firm has more incentive to increase its price when other firms increase theirs. But for a negative shock, prices are strategic substitutes: a firm have less incentive to lower its price when other firms lower theirs.

We then integrate state-dependent pricing into a dynamic general equilibrium model, and examine the response of aggregate prices and output to monetary policy shocks. We stress two results. First, there is a substantial difference between time-dependent and state-dependent pricing models for empirically relevant shocks. In our state-dependent model, responses to monetary policy shocks are highly non-linear. When we calibrate our model to postwar US data and empirically plausible estimates for fixed costs, the predicted output response is smaller (and the price response is larger) than that suggested by time-dependent models. For example, with trend inflation and persistent

\footnote{Hence, our work can be seen as an extension of the work of Dotsey et al. (1999) in a more tractable framework. See also Ireland (1997) for an application of a hybrid price-setting model to the study of disinflation.}
shocks, +1% and +2% monetary policy shocks generate output responses under state-dependent pricing that are 23% and 41% smaller than in the analogous time-dependent model.

Secondly, the calibrated model inherits the asymmetry in price change incentives facing the individual firm. In the example discussed above, a +2% money shock generates 32 times greater ex-post price change than a −2% shock. Consequently, aggregate prices respond much more (and output much less) to positive money shocks than to negative shocks of the same magnitude. This confirms the results found in the empirical literature. We update this literature by estimating the asymmetric response of postwar US output to the new monetary policy shock measures of Romer and Romer (2004). Performing simulations with our model, we find that it can broadly account for these business cycle asymmetries.

In addition, we explore the sensitivity of our results to the degree of ‘real rigidity,’ as defined by Ball and Romer (1990). To do this we dampen the model’s responsiveness of real marginal cost to output fluctuations. With ‘flat’ marginal cost, there is little incentive for price adjustment for either positive or negative monetary policy shocks. In this sense, one might argue that consideration of state-dependent pricing is of little relevance.

However, we show that this argument is fragile. With real rigidity, there is a more traditional strategic complementarity present: a firm’s price change incentive is increasing in the number of price-adjusting firms. With flat real marginal costs, nominal marginal costs are highly sensitive to the degree of price flexibility. Hence, state-dependent pricing models are likely to admit two locally isolated equilibria: one in which few firms adjust if other firms are not adjusting, and one in which all firms adjust if other firms are adjusting.\(^3\) With real rigidity our model exhibits multiple equilibria for moderate sized shocks. But this also interacts with our asymmetry result: the model is much more likely to display multiple equilibria for positive shocks than for negative shocks.

Before proceeding, we note that Burstein (2005) also documents asymmetric responses in a model with state-dependent pricing. However, the tractability of our hybrid model allows us a detailed discussion, and to isolate the source of the asymmetry. We find that the asymmetry is due to: (i) the strategic linkage between firms’ pricing decisions, and (ii) the positive covariance of

\(^3\)For the initial exposition of this in a static model, see Ball and Romer (1991). Our results show that this complementarity extends to a dynamic model with staggered price-setting.
aggregate prices and marginal cost in equilibrium. We show that this asymmetry is operational for positive and negative shocks of all magnitudes. Also, it is operational in the model both with and without real rigidities.

In Section 2, we present a simple static version of our state-dependent pricing model, to highlight the key features generating asymmetries. In Section 3, we outline the dynamic version of our model and show how the modelling of state-dependent pricing is made tractable with price contracts of limited duration. Sections 4 and 5 present our quantitative results regarding non-linearities and asymmetries in the dynamic model. Section 4 also provides new estimates of the presence of asymmetries in the postwar US data. Section 6 provides additional discussion. Section 7 concludes.

2. A Static Economy

Here we study a one-period model which is a simplified version of the dynamic model presented in Section 3. We use this example to illustrate the firm’s decision problem when price adjustment is state-dependent, and to determine the degree of equilibrium price flexibility.

2.1. Firms

Firms produce differentiated goods. Let \( MC \) denote the constant nominal marginal cost of production. A firm sets its output price before the state of the world is known, but may then adjust its price ex-post by paying a firm-specific fixed cost.

Following Dotsey et al. (1999), let the fixed cost be stochastic. Its realization is unknown to the firm when it sets its price. Hence, firms are ex-ante identical and set the same price. Let \( \varphi_i \) denote firm \( i \)'s (ex-post) fixed cost realization expressed in real units; firm \( i \)'s nominal cost of price adjustment is given by \( MC \cdot \varphi_i \). For now, we assume that the distribution of real fixed costs is uniform on \([0, \varphi_{\text{max}}]\). Ex-post, firms are randomly assigned to the unit interval, so that for firm \( i \):

\[
\varphi_i = \varphi_{\text{max}} \cdot i, \quad i \in [0,1].
\]

At this point we can think of these as either ‘menu’ costs, or more broadly as costs associated with ex-post price change.
Given the distribution of fixed costs, a measure $z \in (0, 1]$ of firms will choose to adjust their prices ex-post. Prior to the realization of the state of the world, the firm’s problem is:

$$\max_P E \left\{ \alpha \left[ (1 - z) d \left( \frac{P}{\bar{P}} - MC \right) + z \hat{\Pi} \right] \right\},$$

where

$$d = \left( \frac{\bar{P}}{P} \right)^{-\lambda} X,$$

is the familiar demand relation for a Dixit-Stiglitz intermediate good firm, $\lambda > 1$ is the price elasticity of demand, $X$ is aggregate output (or aggregate demand), $P$ is the aggregate price level, and $\alpha$ is the firm’s state-contingent discount factor. Finally, $\hat{\Pi}$ is the profit (gross of fixed cost) that the firm earns if it adjusts its price ex-post. The price that solves this problem is given by:

$$\bar{P} = \lambda \frac{E \left[ (1 - z) \alpha dMC \right]}{E \left[ (1 - z) \alpha d \right]},$$

where $\lambda = \lambda / (\lambda - 1)$ is the markup factor.

After observing the realized state of the world, the firm decides whether to adjust its price. Firms that adjust ex-post set the optimal price given by the markup rule, $\bar{P} = \lambda MC$, and receive profits:

$$\hat{\Pi} = \frac{\lambda^{-\lambda}}{\lambda - 1} MC^{1-\lambda} P^\lambda X.$$

If the firm maintains its pre-set price, its profits are given by:

$$\Pi = d \left( \frac{\bar{P}}{P} - MC \right).$$

The real gross gain from price adjustment is then:

$$\Delta (MC, X, P) = \frac{P^\lambda \bar{P}^{1-\lambda} X}{MC} \left[ \frac{\lambda^{-\lambda}}{\lambda - 1} \left( \frac{MC}{P} \right)^{1-\lambda} + \frac{MC}{P} - 1 \right].$$

This is simply the difference in profits, $\hat{\Pi} - \bar{\Pi}$, normalized by $MC$. Hence, a firm will choose to re-set its price if $\Delta (MC, X, P)$ is greater than its fixed cost, $\phi_i$. 
2.2. Determinants of the Gain to Price Adjustment

To explore the factors that affect the firm’s adjustment decision, we take a third-order approximation of (2) around a non-stochastic steady-state in which \( P^* = \bar{P}^* = \lambda MC^* \); here the asterisk (*) denotes a steady-state value. This approximation is given by:

\[
\Delta (mc, x, p) \approx K \left[ \lambda mc^2 - \frac{\lambda (\lambda + 1)}{3} mc^3 + \lambda^2 mc^2 p + \lambda mc^2 x \right],
\]

where \( K = X^*/2 \) is a constant, and lowercase letters denote log deviations from steady-state. This measures the incentive for the firm to alter its price in the face of shocks to marginal cost \( (mc) \), the prices of other firms \( (p) \), and aggregate demand \( (x) \).

Up to a first-order, this expression must be zero since the firm maximizes expected discounted profits. Up to a second-order, the incentive to adjust depends only on deviations in marginal cost from steady-state. The critical parameter determining this incentive is the elasticity, \( \lambda \). The higher is \( \lambda \), the greater the gain to adjusting. Clearly, this second-order term is greater for larger marginal cost shocks, implying non-linearities in the incentive for price adjustment. Expression (3) also contains third-order terms. These imply asymmetries in adjustment incentives whenever \( p \) and \( x \) covary with \( mc \). The importance of these effects depends on the size of \( \lambda \).

To see this, consider the following example, which is obtained as a special case of the dynamic model we consider in Section 3. Let: (a) \( mc = m \), so that the marginal cost shock is driven solely by an exogenous shock to money (or nominal aggregate demand), and (b) \( x = m - p \), so that real aggregate demand is determined by the quantity equation. Given this, (3) is rewritten as:

\[
\Delta (m, p) \approx K \left[ \lambda m^2 - \frac{\lambda (\lambda - 2)}{3} m^3 + \lambda (\lambda - 1) m^2 p \right].
\]

The second-order effect of marginal cost shocks is the same as before. Now there are two

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4 We are making a simplification here by assuming that the pre-set price, \( \bar{P} \), is equal to the expected value of marginal cost times the markup. The exact price is given by (1). It is easily shown that when no firms adjust, \( P \) is fixed and this simplification is exactly correct. Generally, whether the firm prices above or below expected marginal cost (adjusted by the markup) depends upon the sign of \( cov((1 - z)sd, MC) \). In our quantitative analysis of the dynamic model – where we do not make this simplification – we find that this covariance effect is negligible, so that there is only a trivial difference between the approximation here and the true solution.
third-order terms. The first concerns changes in marginal costs that are of third-order magnitude. Ignoring \( p \), so long as \( \lambda > 2 \), the firm actually has a greater incentive to adjust its price in response to negative shocks than to positive shocks. The intuition comes from (2). Optimal profits, \( \bar{\Pi} \), are decreasing and convex in \( MC \), while fixed-price profits, \( \Pi \), are linearly decreasing in \( MC \). Hence, a fall in \( MC \) generates a larger gain to price adjustment than does a rise in \( MC \).

Now consider the term involving \( m^2p \). This captures the effect of a simultaneous change in the price of other firms on a given firm’s incentive to adjust its price after a marginal cost shock. Suppose that \( p \) responds in the same direction as \( m \). Then when \( m > 0 \), \( \lambda (\lambda - 1) m^2p \) is positive, increasing the firm’s incentive to adjust its price. But when \( m < 0 \), \( \lambda (\lambda - 1) m^2p < 0 \), so this term reduces the firm’s incentive to adjust. For large \( \lambda \), this third order effect is large, as shown below.

Hence, this ‘relative demand’ effect provides a greater incentive for price change in response to positive than to negative shocks. Because it is due to the responses of other firms, it represents a strategic interaction between firms’ pricing decisions. But it is not the standard strategic complementarity that arises in general equilibrium models via the effect of price changes on marginal cost. Indeed, this is obvious here, since the marginal cost shock is given by the exogenous money shock. Rather, the link is due to the impact of changes in other firms’ prices on an individual firm’s demand. Moreover, the nature of the strategic interaction depends on the sign of the shock. For a positive marginal cost shock, prices are strategic complements: a firm has a greater incentive to raise its price if other firms raise theirs. But for a negative shock, prices are strategic substitutes: a firm has less incentive to reduce its price the more other firms reduce theirs.

[FIGURE 1 GOES ABOUT HERE.]

Figure 1 plots the gain to price adjustment, as a function of the size and sign of the shock. Here we consider the common calibration of a 10% steady-state markup, so that \( \lambda = 11 \). The dashed line corresponds to the case when other firms do not adjust, so that \( p = 0 \); the solid line is the case when all other firms do adjust, setting \( p = m \). The former indicates that the incentive to adjust prices is approximately symmetric when we ignore the relative demand effect (when \( p = 0 \)). But if \( p = m \), the asymmetry term becomes \( \lambda (2\lambda - 1) m^3/3 \). As is clear from the solid line, positive shocks generate a greater adjustment incentive if all other firms are adjusting. For negative shocks there is less incentive to adjust when other firms adjust.
The strength of the asymmetry is tied critically to the price elasticity of demand. When goods markets are highly competitive the incentive to adjust becomes highly asymmetric. Take the case where \( \lambda \to \infty \), so that the market structure approaches perfect competition. In steady-state, optimized profits are (approximately) zero. In response to a fall in marginal cost, if other firms lower prices, demand for the non-adjusting firm falls to essentially zero, since its relative price is high and goods are near perfect substitutes. Nonetheless, the incentive for the firm to adjust is small: if the firm does not adjust, it has zero sales and earns zero profits; if it lowers its price, its optimized profits are again near zero.

This is not true for a rise in marginal cost. Near perfect competition, a positive shock drives the non-adjusting firm’s mark-up of price over marginal cost negative. If other firms raise prices in response to this shock, the non-adjusting firm’s demand becomes the market demand, leading it to large negative profits. The firm has a large incentive to adjust its price: it goes from earning large negative profits to near zero profits. Hence, a firm faces greater incentive to adjust to positive marginal cost shocks than to negative ones.

These examples make clear that the asymmetry depends critically on the following two features: (i) the effect of other firms’ pricing decisions on an individual firm’s demand, and (ii) a positive covariance between prices and marginal cost. As we see below, the asymmetry is quantitatively important for a high price elasticity of demand.

2.3. Equilibrium in the Static Model

This model can be extended to illustrate how equilibrium price flexibility is determined. We close the model with the following aggregate price level relation, derived from a Dixit-Stiglitz aggregator over intermediate good prices:

\[
P = \left[ (1 - z) \tilde{P}^{1-\lambda} + z \tilde{P}^{1-\lambda} \right]^{\frac{1}{1-\lambda}}.
\]

Since we are interested only in qualitative implications in this subsection, we consider a first-order approximation to the price level, so that \( p = zm \). The measure, \( z \), is then determined by the condition that the gross gain, \( \Delta (m,p) \), equals the real fixed cost of price adjustment for the
marginal firm:

\[ K \left[ \lambda m^2 - \frac{\lambda(\lambda - 2)}{3} m^3 + \lambda (\lambda - 1) m^3 z \right] - z\varphi_{\text{max}} = 0. \]

In Figure 2, we graph the left-hand side of (5) – the net gain to price adjustment – as a function of \( z \). The solid line illustrates the net gain for a positive money shock, and the dashed line for a negative shock of the same magnitude. As \( z \) rises, the fixed cost of the marginal firm, \( z\varphi_{\text{max}} \), rises. Hence, both net gain loci are downward sloping. However, the net gain for a positive shock is flatter than for a negative shock. This reflects the strategic interaction in pricing. As \( z \) increases, the gross gain to adjustment (the first term on the left-hand side of (5)) increases for positive shocks (as \( \lambda (\lambda - 1) m^3 z > 0 \)), and decreases for negative shocks (as \( \lambda (\lambda - 1) m^3 z < 0 \)). At the equilibrium \( z \), the net gain to price adjustment is zero for the marginal firm.

Panel A of Figure 2 illustrates these net gain loci for a small money shock. For sufficiently small shocks of either sign, the two schedules are effectively coincident, and the equilibrium \( z \) is small. As the absolute size of the shock rises, \( z \) rises. For sufficiently large shocks of either sign, we get \( z = 1 \). However, due to the relative price effect, full price flexibility occurs for smaller positive shocks than for negative shocks. This is illustrated by the unhatched solid and dashed lines in Panel B.

2.4. Implications of Real Rigidity

Until now we have assumed that the marginal cost shock is equal to the aggregate demand shock, independent of firms’ pricing decisions. But recent debate in the literature on sticky prices has emphasized the importance of marginal costs being unresponsive to aggregate demand. How would this affect our results? Take the example:

\[ mc = p + \phi x = (1 - \phi) p + \phi m, \quad \phi \in (0, 1), \]

where, as above, we have used the quantity relation, \( x = m - p \). For low values of \( \phi \), we obtain significant real rigidity in marginal cost (in the next section, the parameter \( \phi \) is derived from an underlying structural model). With small \( \phi \), there is little response of marginal cost to a monetary
shock, therefore little incentive for an individual firm to adjust its price.

In terms of Figure 2, the net gain loci shift down, and are effectively coincident at low values of price adjustment. We illustrate this for the large shock case, as the hatched solid and dashed lines in Panel B. In this sense, real rigidity gives rise to equilibria with low degrees of price adjustment; the net gain loci cross zero at small values of $z$.

However, the net gain rises sharply as $z$ approaches unity. This reflects the fact that if all firms adjust, marginal cost rises by the full amount of the money shock, regardless of the degree of real rigidity. For a given shock, if $z = 1$ is the unique equilibrium in the model without real rigidities (i.e. when $\phi = 1$), then it continues to be an equilibrium with real rigidities, for all $\phi \in (0, 1)$. Conversely, if there does not exist an equilibrium with $z = 1$ in the model with $\phi = 1$, then this cannot be an equilibrium outcome with $\phi < 1$.

Figure 2, Panel B gives an example where for the negative shock, there is less than full price flexibility without real rigidities. Hence, the single crossing of the hatched, dashed locus represents the unique equilibrium with real rigidities. But for the positive shock, the equilibrium without real rigidities features $z = 1$. Hence, with real rigidities there is one equilibrium with low $z$, and another with $z = 1$.5 In summary, asymmetries in price change continue to play a role when $\phi < 1$. Since, in the model with $\phi = 1$, we get full flexibility for smaller positive shocks than negative shocks, with $\phi < 1$ we are more likely to have multiple equilibria for positive than for negative shocks.

3. A Dynamic Economy

Here we extend the static model of Section 2 to a dynamic general equilibrium environment. The price-setting description of the model is made tractable by specifying that firms follow a hybrid time- and state-dependent pricing rule.

3.1. Pricing

Consider a two-period version of Taylor’s (1980) staggered pricing model. Half of the unit measure of firms set (two-period) contract prices in odd periods, half in even periods. As in Ball

5By convention, we disregard the ‘unstable’ intermediate equilibrium where the net gain crosses from negative to positive. Again, this multiplicity stems from the strategic complementarity noted by Ball and Romer (1991).
and Mankiw (1994), we add to the basic Taylor model the possibility that in the second half of their contracts, firms can choose to ‘opt out’ and re-set their price after observing the current state of the world. Doing so requires paying a fixed cost (in terms of labour hours).\(^6\) We discuss the nature of these costs below.

The fixed cost of opting-out is a random draw which is i.i.d. across firms. In addition, any single firm faces draws that are i.i.d. over time. Thus, the firm’s current draw contains no information in predicting its future cost.

We assume that a firm choosing to opt out in the current period simply goes back to staggering (i.e. setting its two-period price) next period.\(^7\) When a firm chooses its two-period price, it takes into account this possibility of opt-out in the second period. Since fixed costs are i.i.d. across firms and over time, all firms face the same probability of opting-out in the second period. Hence, all firms set the same two-period price, allowing us to minimize heterogeneity within pricing cohorts.\(^8\)

This allows us to write the aggregate price index at time \(t\) as:

\[
P_t = \left[ \frac{1}{2} \bar{P}_t^{1-\lambda} + \frac{1}{2} \frac{z_t}{\bar{P}_{t-1}^{1-\lambda}} + \frac{z_t}{2} \frac{\tilde{P}_t^{1-\lambda}}{\bar{P}_t^{1-\lambda}} \right]^{\frac{1}{1-\lambda}}, \quad \lambda > 1.
\]

Here: \(\bar{P}_t\) is the price set by all firms currently entering into new contracts for periods \(t\) and \(t+1\); \(\bar{P}_{t-1}\) is the second period price inherited by firms who entered into contracts at date \(t-1\); and \(z_t\) is the measure of firms in the second period of their contract who opt out and choose the ex-post optimal price, \(\tilde{P}_t\). As will be made clear in Subsection 3.3, the functional form for (7) derives naturally from the specification of household preferences.

\(^6\)Note, of course, that this can be derived from an alternative interpretation in which all price changes involve a fixed cost. However, firms must incur the fixed cost every second period. This results in the same model, except for a minor change to the labour market clearing condition.

\(^7\)We do this so that the firm’s opt-out decision is static, which makes a detailed characterization of the degree of price flexibility intuitive. It is possible, however, to modify the model so that opt-out firms set a new, two-period contract price. This would induce one additional state variable (the measure of firms with inherited contract prices), and using the terminology of Dotsey et al. (1999), adds to the ‘history dependence’ in model dynamics. See Section 6 for more discussion.

\(^8\)As in Dotsey et al. (1999), this greatly simplifies the analysis. Assuming that firm-specific fixed costs were persistent over time would lead to an expansion in the state-space of the model such that our solution method would no longer be feasible.
3.2. Firms

Each firm $i$ is a monopolist and produces a differentiated good using labour alone:

$$Y_{it} = H_{it}.$$  

Labour is hired in a competitive, economy-wide market. It follows that the firm’s nominal marginal cost is given by the nominal wage, $W_t$.

After observing the current state, a firm setting a new contract price at time $t$ solves:

$$\max_{\bar{P}_t} \Pi (\bar{P}_t, P_t, W_t) + E_t \left\{ \alpha_{t+1} \max \left[ \Pi (\bar{P}_t, P_{t+1}, W_{t+1}), \Pi (\bar{P}_{t+1}, P_{t+1}, W_{t+1}) - W_{t+1}\varphi_{it+1} \right] \right\}.$$  

Here, the static profit function for the firm is:

$$\Pi (P_t, P, W) = (P_t - W) \left( \frac{P_1}{P} \right)^{-\lambda} Y,$$

where $Y$ is aggregate output, $\alpha_{t+1}$ represents the state-contingent nominal discount factor used by the firm in maximizing profits,\(^9\) and $\varphi_{it}$ denotes firm $i$’s realized price adjustment cost at date $t$ in units of labour. Draws of $\varphi$ are distributed according to the CDF, $\mathcal{F}(\varphi)$. The firm evaluates future profits taking into account that it may choose to opt out at date $t+1$ with probability $z_{t+1}$.

The first order condition for $\bar{P}_t$ is given by:

$$\left( \frac{\bar{P}_t}{P_t} \right)^{-\lambda} Y_t \left( 1 - \lambda + \lambda \frac{W_t}{P_t} \right) + E_t \left[ \alpha_{t+1} (1 - z_{t+1}) \left( \frac{\bar{P}_t}{P_{t+1}} \right)^{-\lambda} Y_{t+1} \left( 1 - \lambda + \lambda \frac{W_{t+1}}{P_t} \right) \right] = 0.$$  

At any date $t$, after observing the current state, a measure of firms will choose to opt out of their prices inherited from date $t - 1$. A firm that opts-out sets an ex-post profit maximizing price:

$$\bar{P}_t = \hat{\lambda} W_t.$$  

Ranking these firms in order of their fixed cost, we can determine the threshold $\bar{\varphi}_t$ such that all firms with $\varphi_{it} > \bar{\varphi}_t$ stick with their inherited price, while firms with $\varphi_{it} \leq \bar{\varphi}_t$ opt out. Given

\(^9\) Since the firm is owned by the representative household, it discounts profits in the same way as the household.
this, \( z_t = \mathcal{F}(\hat{\varphi}_t) \) satisfies the marginal condition:

\[
(10) \quad (\bar{P}_{t-1} - W_t) \left( \frac{\bar{P}_{t-1}}{P_t} \right)^{\frac{\lambda - \lambda}{\lambda - 1}} Y_t = \frac{\hat{\lambda} - \lambda}{\lambda - 1} P_t^\lambda Y_t W_t^{1-\lambda} - W_t \mathcal{F}^{-1}(z_t).
\]

This condition equates the profits from remaining with the inherited price from \( t - 1 \), with the profits from opting-out (net of the fixed cost) for the marginal firm, \( z_t \). If the right-hand side of equation (10) is greater than the left at the maximal fixed cost, then \( z_t = 1 \).

3.3. Households

The representative household has utility over consumption and labour supply given by:

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, H_t), \quad 0 < \beta < 1,
\]

where \( U = \ln(C) - \eta H \). Composite consumption is a CES aggregator over the unit measure of differentiated goods:

\[
C = \left[ \int_0^1 C_i^{\frac{\lambda - 1}{\lambda}} \, di \right]^{\frac{1}{\lambda}},
\]

where the elasticity of substitution is \( \lambda > 1 \). Given this, (7) is the natural measure of the aggregate price level.

At the beginning of each period, after observing the current state of the world, the household receives wage and profit income from the previous period, returns on nominal bonds, and a lump-sum transfer from the monetary authority. Using this and unspent cash balances, it obtains money and bonds for the current period. Hence, the household’s budget constraint is:

\[
M_t + B_t = (1 + s_{t-1}) \left( W_{t-1} H_{t-1} + \int_0^1 \Pi_{it-1} \, di \right) + M_{t-1} - P_{t-1} C_{t-1} + (1 + r_{t-1}) B_{t-1} + T_t.
\]

Here: \( M_t \) denotes money holdings; \( B_t \) are purchases of nominal one-period bonds which pay an interest rate of \( r_t \); \( \Pi_{it} \) represents the profits of firm \( i \); \( T_t \) is the transfer; and \( s_t \) is an income subsidy (discussed below). Households hold money in order to satisfy a cash-in-advance constraint
on consumption purchases:

\[(11) \quad P_tC_t \leq M_t.\]

The household’s choices are characterized in the standard way. Demand for good \(i\) is given by:

\[(12) \quad C_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\lambda} C_t,\]

where \(P_{it} \in \{ \tilde{P}_t, \tilde{P}_{t-1}, \tilde{P}_t \}\) is firm \(i\)’s price. The household’s intertemporal Euler equation is written as:

\[(13) \quad \frac{1}{1 + r_t} = \beta E_t \left[ \frac{P_tC_t}{P_{t+1}C_{t+1}} \right],\]

so that \(\alpha_{t+1} = \beta P_tC_t/P_{t+1}C_{t+1}.\) Finally, the implicit labour supply function is:

\[W_t (1 + s_t) = \eta P_tC_t (1 + r_t).\]

This displays the familiar distortionary impact of nominal interest rates on labour supply in a cash-in-advance model. To focus exclusively on the effects of monetary policy due to price stickiness, we assume that an optimal wage subsidy is chosen so as to negate this distortion. Hence, we set \(s_t = r_t\) so that the labour supply function becomes:\(^{10}\)

\[(14) \quad W_t = \eta P_tC_t.\]

### 3.4. Monetary Policy Rule

The monetary authority follows a money growth rule of the form:

\[M_t = M_{t-1} \exp (\mu + v_t),\]

\(^{10}\)This is used in the firm’s intertemporal discount factor as well. Since profits at time \(t\) cannot be used until time \(t + 1\), the firm discounts current profits by the nominal interest rate. In (8), this has been eliminated by the subsidy to profit income equal to the nominal interest rate.
where \( v_t = v_{t-1} + \varepsilon_t \), and \( \varepsilon_t \) is mean zero and i.i.d. Money growth finances transfers and the income subsidy, so that in the aggregate we have:

\[
M_t = M_{t-1} + T_t + s_{t-1} \left( W_{t-1} H_{t-1} + \int_0^1 \Pi_{t-1} di \right).
\]

### 3.5. Equilibrium

A symmetric equilibrium in this economy is defined in the usual way. In particular, equilibrium must satisfy equations (7) – (14), \( Y_t = C_t \), in addition to the labour market clearing condition:

\[
H_t = \frac{1}{2} \left[ Y_{1t} + (1 - z_t) Y_{2t} + z_t \tilde{Y}_t + \int_0^{\tilde{\varphi}_t} \varphi dF(\varphi) \right].
\]

Absent government bond issue, it must be that \( B_t = 0 \). In (15), \( Y_{1t} \), \( Y_{2t} \), and \( \tilde{Y}_t \) denote, respectively, time \( t \) output of firms that set contract prices in period \( t \), in period \( t - 1 \), and firms that opt out of their price contracts in period \( t \). Product market clearing implies:

\[
Y_{1t} = \left( \frac{\tilde{P}_t}{P_t} \right)^{-\lambda} C_t, \quad Y_{2t} = \left( \frac{P_{t-1}}{P_t} \right)^{-\lambda} C_t, \quad \tilde{Y}_t = \left( \frac{\tilde{P}_t}{P_t} \right)^{-\lambda} C_t.
\]

To render the model’s equilibrium stationary, we scale all date \( t \) nominal variables by the date \( t \) money stock. Given this, the model’s natural state space is simply \((\tilde{p}_{t-1}, v_t)\) where \( \tilde{p}_{t-1} \) is the scaled contract price inherited at date \( t \). Because of this small dimensionality, we are able to fully characterize the model’s non-linear dynamics. Given the presence of an occasionally binding constraint \((z \leq 1)\), we solve the model by approximating the expectation terms in (8) and (13) with linear combinations of Chebychev polynomials. This is implemented via an iterative algorithm which adapts the projection methods of Judd (1992) (details are available upon request). With these expectation functions in hand, we derive exact, non-linear solutions for decision variables from the model’s equilibrium conditions.

### 3.6. Introducing Real Rigidity

We also consider an amended version of the model which reduces the elasticity of marginal cost to aggregate demand. As is well known, this is important in generating persistent effects of money
shocks on real economic activity. Although there are a number of approaches to generating real rigidity in dynamic general equilibrium models, we follow Dotsey and King (2005) in choosing an especially simple approach that requires no fundamental alteration to our model.\footnote{In Section 6, we discuss the relationship of our results generated in this parsimonious manner to that which would be obtained from more detailed models.}

We amend preferences so that:

\begin{equation}
U = \frac{C^{1-\phi}}{1-\phi} - \eta H.
\end{equation}

In contrast to (14), the amended labour supply condition is:

\begin{equation}
W_t = \eta P_t C_t^\phi.
\end{equation}

For values of $\phi < 1$ this model displays an elasticity of the real wage with respect to consumption (output) that is less than unity.\footnote{This specification also alters the intertemporal discount factor, $\alpha_{t+1}$, which affects the pricing decision of firms.} Combining this with the cash-in-advance constraint, we see that in equilibrium:

\begin{equation}
W_t = \eta P_t^{1-\phi} M_t^\phi.
\end{equation}

For small $\phi$, the nominal wage is determined largely by $P_t$.

4. Quantitative Results

To investigate the quantitative importance of the non-linearities and asymmetries discussed above, we study a calibrated version of our dynamic model. We set $\beta = 0.98$ to correspond to a time period of 6-months; firms set new contract prices once a year. In line with many studies, the elasticity of substitution between goods is set at $\lambda = 11$, corresponding to a steady state price-cost markup of 10%.$^{13}$ We experiment with lower markups below. Steady-state hours are set at 30% of the household’s time endowment. To gain intuition, the baseline monetary policy rule has $\mu = 0$ and $\rho = 0$, implying the money stock follows a random walk without drift. For all experiments, the $\mu$ and $\rho$ are varied to achieve the desired steady-state values.

\footnote{See, for instance, Chari et al. (2000). The results of Basu and Fernald (1997) indicate mark-ups of similar magnitude or slightly lower.}
standard deviation of money growth is set to 1.7%, which corresponds to that of postwar US data on M2.\footnote{This is very close to the estimates of Bouakez et al. (2003) for the standard deviation of monetary policy shocks, when money is measured as M2.} We consider more realistic cases with trend inflation and autocorrelated money growth shocks below. We begin with a model without real rigidity, so that the labour supply condition is given by (14).

A key aspect of the model is the distribution of costs that firms incur in opting-out of their price contracts. There is very little evidence to guide the specification of this distribution. We begin with the simplest possible, namely that the distribution is uniform over \([0, \varphi_{\text{max}}]\), so that the CDF is \(F(\varphi) = \varphi/\varphi_{\text{max}}\). We set \(\varphi_{\text{max}}\) at 2.85% of per period steady-state firm revenue.

The calibration of this last value is based on Zbaracki et al. (2004), the most comprehensive study to date. They study the price-setting process of a multi-product US manufacturing firm and classify the fixed costs associated with the annual issuance of the firm’s list prices. These fall into three categories. The first are ‘managerial’ costs associated with the information-gathering and decision-making process in which new prices are determined. The second are ‘customer’ costs of communicating and negotiating the price change with its consumers. The final category are the ‘menu’ costs of actually issuing the new prices. These comprise respectively, 0.28%, 0.91%, and 0.04% of the firm’s annual revenue.\footnote{For econometric estimates, see Slade (1998) and Willis (2000). For instance, Slade estimates a total fixed cost per-price-change for saltine crackers that, adjusted for inflation, is smaller than the per-product value of Zbaracki et al. Further direct evidence on the size of strict menu costs is provided by Levy et al. (1997). For typical large US supermarket chains, per-store menu costs total about 0.7% of annual revenue. Given that these stores change prices on a weekly basis, this figure corresponds closely to the 0.04% figure above.} In the model, a firm decides whether to change its price after evaluating the difference between optimized and sticky price profits, i.e. having already ‘spent’ the managerial cost of determining its ex-post optimal price. As such, we identify the model’s fixed cost with the customer and menu costs of implementing a price change. According to Zbaracki et al., and since our model frequency is 6-months, this adds up to 1.9% of semi-annual revenue for a one-time price change. To be conservative, our calibration of \(\varphi_{\text{max}}\) at 2.85% of revenue sets the maximal fixed cost at 1.5 times this value. We also experiment with smaller values of \(\varphi_{\text{max}}\) and alternative specifications for \(F(\varphi)\) below.\footnote{In terms of calibration studies, \(\varphi_{\text{max}}\) corresponds to 1.9% of semi-annual revenue in Dotsey et al. (1999). Golosov...}
4.1. The Baseline Model and Extensions

In Figure 3 we present for the baseline calibration, the impact responses of key model variables to monetary policy shocks between $+10\%$ and $-10\%$. The solid line describes the response for the state-dependent pricing model. The dashed line describes the equivalent response with time-dependent pricing, i.e. for the standard two-period Taylor model; this is obtained from our state-dependent model by setting the fixed cost sufficiently high that no firms choose to opt out.

[FIGURE 3 GOES ABOUT HERE.]

Figure 3 addresses our primary question concerning the accuracy of the time-dependent pricing model as an approximation to the true model with fixed costs. This can be assessed by the degree to which the dashed line in each panel departs from the solid one, or equivalently, by the total measure of firms who choose to opt out (Panel A). For the baseline calibration, the bottom panels reveal that the time-dependent model is a reasonably accurate characterization of the response of aggregate prices and output (consumption) within the range of $\pm 3.5\%$ shocks; within this range, no more than 20\% of firms choose to opt out of their second period prices. Outside this range, however, the time-dependent model becomes increasingly inaccurate. For shocks greater than $\pm 3.5\%$, prices respond by much more, and output by much less, than occurs in the time-dependent pricing model. For example, for a $+5\%$ shock, the time-dependent model overstates the response of output by 85\% relative to the state-dependent model ($+2.77\%$ as opposed to $+1.50\%$); for a $-5\%$ shock, the response of output is overstated by 71\% ($-2.21\%$ versus $-1.29\%$). The reason for the difference is clear from the first panel: for large shocks a substantial measure of sticky price firms choose to opt out. Hence the state-dependent pricing model displays distinct non-linearities in response to relatively large monetary shocks (for the current calibration).

Note that the accuracy of the time-dependent pricing model depends not just on the size, but also the sign of the money shock. State-dependent price adjustment is also asymmetric. In Figure 3, positive shocks greater than 6.5\% induce all firms to adjust their prices, so there is full neutrality of money (apart from the adjustment costs). But even for a $-8.5\%$ money shock, only 80\% of firms in the second period of their contracts will have adjusted. The source of this asymmetry is the same as in Section 2: for positive money shocks, non-adjusting firms find their profits fall quickly and Lucas (2003) consider a degenerate distribution, with a fixed cost of 0.5\% of semi-annual revenue.
when other firms adjust, while for negative shocks, the loss in profits is much less.\footnote{As discussed in Section 2, the degree of asymmetry is sensitive to the value of \( \lambda \). When calibrated to a 2\% steady state markup (\( \lambda = 51 \)), we find a dramatic asymmetry in the adjustment process. For positive shocks of +2.5\%, all firms have adjusted their prices; but even for a −10\% shock, only 75\% of firms in the second period of their contract have adjusted their price. See an earlier version of this paper, Devereux and Siu (2004), for details. Note also that the time-dependent model generates an asymmetry in the response of prices and output: positive money shocks generate smaller price responses (and, counterfactually, larger output responses) relative to negative shocks. This can be seen in Figure 3, and more clearly in Figures 5 and 6. This is due to the fact that the aggregate price index (7) is a concave function of first-period contract prices. Again, see Devereux and Siu (2004) for discussion.}

How sensitive are these findings to the specification of \( \mathcal{F}(\varphi) \)? In the top row of Figure 4, we illustrate the effects of reducing \( \varphi_{\text{max}} \) to 1.9\% of steady state revenue, the value measured by Zbaracki et al. (2004). The accuracy of the time-dependent model diminishes still further. For money shocks of ±3\%, over 20\% of firms opt out of their second period price, and for shocks of +5.5\% and −8\%, all firms adjust.

\[ \text{[FIGURE 4 GOES ABOUT HERE.]} \]

A second aspect is the functional form for \( \mathcal{F}(\varphi) \). We use a uniform distribution in the baseline case. Following Dotsey et al. (1999), we also consider a density function which places greater probability mass near 0 and \( \varphi_{\text{max}} \); in particular, we consider a beta distribution with shape parameters (0.4, 0.15). Relative to the uniform, this distribution is relatively flat over a wide part of its support. From the bottom row of Figure 4, we see that for moderate valued shocks (between +3\% and +6.5\%, and between −3\% and −10\%), this leads to less responsive price adjustment, so that the time-dependent model is a more accurate approximation to the true model. The state-dependent model still implies that all firms adjust for positive shocks greater than +6.5\%. The difference between this and the uniform distribution is that a large group of firms opt out simultaneously as the money shock reaches this threshold. Again, we see that the degree of price adjustment is asymmetric. Even for −8\% shocks, only 40\% of firms opt out.

We now add to the baseline model a positive rate of trend money growth/inflation. A value of \( \mu = 0.03 \) matches the semi-annual growth rate of M2 in the data. On top of the effect of strategic interaction in firms’ pricing decisions, trend inflation adds to the asymmetric response of price change. This is through the mechanism highlighted by Ball and Mankiw (1994).
positive inflation, firms setting two-period prices on average have second-period prices below their static profit maximizing levels. A small positive shock to marginal cost exacerbates this difference, further encouraging them to adjust prices. A small negative shock reduces the difference between price and marginal cost, diminishing the incentive to adjust.

[FIGURE 5 GOES ABOUT HERE.]

Figure 5 illustrates this case. With trend inflation, the price adjustment schedule is essentially shifted uniformly to the left.\textsuperscript{18} When there is no shock at all, a small measure of firms still opt out of their contracts so as to adjust their prices for trend inflation. The range of monetary policy shocks for which the time-dependent model is a close approximation to the state-dependent model is clearly asymmetric, as seen in the output and inflation responses. At least 20\% of firms opt out for positive shocks 2\% and greater, whereas the equivalent cut-off for negative shocks is −5\%. For shocks greater than +5\%, all firms opt out; but for even a −10\% shock, only 85\% of firms opt out. In terms of the output response, this ‘trend inflation’ asymmetry reinforces the relative demand asymmetry that we have already documented. The peak positive response of output is at a +2.5\% shock, while the peak negative response is at a −6\% shock.

The experiments displayed in Figures 3 through 5 assume i.i.d. shocks. But empirically, monetary policy shocks are persistent. We now consider a value of $\rho = 0.6$, which corresponds to the autocorrelation in postwar US M2 growth.\textsuperscript{19} In the top row of Figure 6, we examine the case with $\rho = 0.6$ and $\mu = 0.03$. The key effect of persistent shocks is to further diminish the overlap between the time- and state-dependent models. At the same time, it exacerbates the asymmetric response of prices and output, relative to Figure 5. This is due to the ‘front-loading’ behaviour of firms setting new contract prices. With autocorrelated shocks, a positive shock today signals greater than trend money growth in the future as well. Hence, firms respond by setting two-period prices greater than that dictated solely by trend money growth. The same is true for negative

\textsuperscript{18}Note that we are assuming that firms do not automatically update their scheduled price for trend inflation as part of the Taylor contract. If automatic updating took place, then Figure 5 would look very similar to Figure 3. See Devereux and Siu (2004). Also, note that Figure 5 incorporates a range over which a negative money shock leads to a reduction in $z$, as in Dotsey et al. (1999).

\textsuperscript{19}This is very close to the estimate of $\rho = 0.5$ found by Christiano et al. (1998) as the persistence of monetary policy shocks identified in quarterly M2 data. See also Bouakez et al. (2003).
shocks: new contract prices fall by more since the current shock signals slower money growth in the future. This front-loading of prices strengthens the asymmetry due to strategic linkages across pricing decisions. The largest positive response of output in our model is now +0.50% for a +2% shock, while the largest negative response is −1.25% for a −5% shock.

Our baseline model is built around a cash-in-advance model of money demand. As pointed out by Dotsey et al. (1999), with interest elastic money demand, persistent money growth shocks have much greater effects on aggregate demand than i.i.d. shocks. With persistent shocks, the anticipation of higher future inflation drives up nominal interest rates and reduces current demand for money. The bottom row of Figure 6 illustrates the same case as the top row, but for a money-in-the-utility-function specification of the model. This exacerbates even further the non-linearity (or inaccuracy of time-dependent pricing) and asymmetry in the model. Positive money growth shocks of +2% lead to full price adjustment, while shocks of −6% are necessary for full adjustment on the negative side. The asymmetry also highlights a substantial difference in the maximal output response to positive and negative shocks (+0.60% for a +1% shock, −2.25% for a −2.5% shock).

Finally, note that in Figures 5 and 6, when all firms adjust their price to a positive shock, the output response is negative. This is due to the behaviour of firms setting price contracts facing trend inflation. With trend inflation, firms set two-period prices that are high relative to demand and marginal cost in the first period, but low relative to the second period. For sufficiently large positive shocks, \( z = 1 \) applies, and firms opting-out of their second period prices set ex-post prices in proportion to current marginal cost. At the same time, new contract price-setters continue to set high first period prices. Since in total, the aggregate price level is rising by more than the current shock, output is lower, relative to the case of a zero shock.

4.2. Empirical Estimates of Asymmetry

In this subsection, we provide new estimates of the asymmetric effect of monetary policy shocks on output, and assess the extent to which our state-dependent pricing model can account for these

\(^{20}\)See Devereux and Siu (2004) for details of this model. With money-in-utility, the interest elasticity of money demand is unity; this contrasts with the cash-in-advance model which has elasticity of zero.
estimates. Cover (1992) represents an early example of this sizeable literature. Cover’s analysis can be roughly summarized as a two-step procedure. First, identify monetary policy shocks as the residuals from a linear monetary policy rule for the growth rate of M1; denote the series of estimated shocks as \( \{ \omega_t \} \). Second, run a linear regression of output growth on a constant, positive shocks, negative shocks, and other controls. Positive shocks are constructed as
\[
pos_t = \max[\omega_t, 0]
\]
and negative shocks as
\[
neg_t = \text{abs}(\min[\omega_t, 0]).
\]

Column 1 of Table 1 presents the coefficient estimates on the response of output growth to positive and negative shocks for Cover’s benchmark regression. These results suggest that both positive and negative shocks are contractionary; however, while negative shocks generate a large, statistically significant fall in output growth, positive shocks are only mildly contractionary and this is imprecisely estimated.

A central critique of this analysis – and of much of the literature – is the identification of exogenous shocks from first stage regressions treating endogenous monetary variables such as M1 or the federal funds rate as the policy instrument. To address this critique, and the critique of anticipatory changes in the policy instrument to future conditions, we use the new measure of monetary policy shocks due to Romer and Romer (2004). These are constructed from regressions of intended (as opposed to actual) federal funds rate changes at FOMC meetings to forecasts prepared for those meetings, and represent desirable improvements over existing measures. From this series we construct positive and negative shocks as described above, except that we ‘switch the sign’ on the shocks (so that negative, or ‘expansionary,’ shocks to the intended fed funds rate are labelled as positive shocks, and vice-versa) for the purpose of interpretation.

**[TABLE 1 GOES ABOUT HERE.]**

The second and third columns of Table 1 present results when we run second stage regressions in the spirit of Cover (1992). We first regress RGDP growth, 1969:I – 1996:IV, on a constant, lagged output growth, the current and lagged change in the 3-month T-bill rate, and positive and negative shocks; this is Cover’s benchmark specification. Column 2 reports the coefficient estimates on the shocks. Positive shocks generate small and statistically insignificant output expansions, while negative shocks generate large contractions that are estimated precisely.\(^{21}\)

\(^{21}\)As discussed in Romer and Romer (2004) page 1071, April 1980 represents a potentially important outlier since
Table 1: Asymmetric Effects of Monetary Policy Shocks on Output. Notes: Coefficient estimates of positive and negative shocks on output growth, with standard errors in parentheses. Column 1 is taken from Cover (1992), Table 2, column 3. Columns 2 and 3 are updated estimates using Romer and Romer (2004) shocks. Significance at 5 percent and 1 percent levels denoted by \( a \) and \( b \) respectively. Columns 4 and 5 use simulated data from the cash-in-advance and money-in-utility models.

<table>
<thead>
<tr>
<th></th>
<th>Cover</th>
<th>Version 1</th>
<th>Version 2</th>
<th>CIA</th>
<th>MIU</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>−0.143</td>
<td>+0.064</td>
<td>+0.113</td>
<td>+0.152</td>
<td>−0.227</td>
</tr>
<tr>
<td>(0.135)</td>
<td>(0.336)</td>
<td>(0.325)</td>
<td>(0.121)</td>
<td>(0.253)</td>
<td>(0.246)</td>
</tr>
<tr>
<td>negative</td>
<td>−0.960(^b)</td>
<td>−0.595(^a)</td>
<td>−0.539(^a)</td>
<td>−0.387</td>
<td>−1.061</td>
</tr>
</tbody>
</table>

In the third column, we report results from a regression that adds the current and lagged growth rate of sensitive commodity prices as regressors to the previous specification. This is intended as a check for omitted variable bias; this may occur if the Romer-Romer shocks incorporate endogenous responses to supply shocks that also affect output. From the table we see that the results are substantively unchanged. A +1% monetary policy shock generates a 0.11% rise in output growth with a P-value of 0.73, while a −1% shock causes output growth to fall by 0.54% with P-value 0.03. As a final check, we consider a specification that includes the set of regressors Christiano et al. (1999) consider in their VAR analysis. Relative to Column 3, we replace the T-bill rate with the actual fed funds rate, and add the lagged growth rates of the aggregate price level, M1, and the non-borrowed reserves to total reserves ratio. Though not reported in the table, we estimate a coefficient on positive shocks of 0.18 with a P-value of 0.60, and a coefficient on negative shocks of −0.43 with P-value 0.12. Taken together, we interpret these results as strong evidence for asymmetric responses of output to monetary policy shocks in the postwar US data.

We now investigate the ability of our model to match this range of estimates. We focus on the specification with trend inflation (\( \mu = 0.03 \)) and autocorrelated shocks (\( \rho = 0.6 \)). We simulate the this month experienced a fall in output and is attributed a large positive shock (unpredicted fall in the intended fed funds rate). To ensure that our results are not driven by this one observation, we replace this period’s shock with a zero. If instead, the April 1980 shock is left in, the coefficient for positive shocks is −0.26 (‘expansionary’ shocks cause output growth to fall) with a P-value of 0.24, and the coefficient on negative shocks remains −0.59 with a P-value of 0.02. We also performed all of the analysis replacing the T-bill rate with the actual fed funds rate; none of the results were changed.
model and run a regression of output growth on a constant, lagged output growth, and positive and negative monetary policy shocks. Because there is no other shock in the model, we do not include other regressors to avoid collinearity problems. In columns 4 and 5 we present, respectively, the results for the benchmark cash-in-advance model, and the money-in-the-utility-function model discussed in Subsection 4.1.

Because the definition of monetary policy shocks differs across Cover’s analysis, our empirical analysis, and the simulation exercise, the absolute value of coefficient estimates across columns are not comparable. However, the ratio of the positive and negative coefficient estimates are. From this, we conclude that the state-dependent pricing model that we study is able to generate significant asymmetry in the output response to shocks. For instance, in the money-in-utility case, the model displays the extreme form of asymmetries obtained in Cover (1992), where both positive and negative shocks generate output contractions.

5. Implications of Real Rigidity

Our analysis indicates that there is a substantial difference in the response to monetary policy shocks, between pure time-dependent pricing models and our hybrid time- and state-dependent model. Prices adjust endogenously so as to make the time-dependent model inaccurate for moderate valued shocks. Moreover, the discrepancy is asymmetric: inaccuracy of the time-dependent model is more pronounced for positive than for negative shocks.

One objection to our analysis is that we use a specification in which the elasticity of marginal cost to output is relatively high (although smaller than in the baseline specification of Chari et al., 2000). If on the other hand we assume substantial real rigidity, as implied by the amended model’s labor supply condition, (17), the difference between the models with endogenous and exogenous price adjustment might be considerably reduced.

At one level, this reasoning is correct. Here, we recompute the results for the model with trend inflation of 3% and i.i.d. money growth shocks ($\mu = 0.03$ and $\rho = 0$), but now using the amended model of Subsection 3.6 with $\phi = 0.1$. The real wage is insensitive to fluctuations in output, so that movements in nominal marginal costs are determined largely by those of aggregate prices.

The solid line in Figure 7 displays the impact response of the switching fraction within the
range of ±10% money growth shocks. The locus shows a large range of shocks for which there is very little opt-out by sticky price firms. Since marginal costs are largely unresponsive, most firms are content to leave their second period prices unchanged. Nevertheless, there is still substantial asymmetry in price adjustment. For positive shocks greater than +6.75%, all prices adjust; but even for −10% shocks, only 8% of firms opt out.

[FIGURE 7 GOES ABOUT HERE.]

But the presence of real rigidity also raises the possibility of multiple equilibria, since it introduces a strategic complementarity in pricing: because nominal marginal cost depends largely on the aggregate price level, an individual firm’s incentive to adjust its price is increasing in the price adjustment of other firms (see Ball and Romer, 1991). If all firms in the second period of their contract opt out, then all prices in the economy are revised in response to a monetary shock. From (17), this generates a significant response in marginal cost. For sufficiently large shocks, this validates the opt-out firms’ decision to adjust. In Figure 5, we saw that without real rigidities, prices are fully flexible for shocks greater than +5%. This suggests that for the same sized shocks, there is an equilibrium with full switching with real rigidities, since marginal cost would be at least as responsive as in the baseline model.22

There is an additional element that is omitted from this argument, however, when real rigidity is introduced via the preference specification of (16). This specification changes not only the marginal cost schedule (17), but also the intertemporal Euler equation, and thus, the nominal discount factor used by two-period price-setting firms. The nominal discount factor becomes:

\[ \alpha_{t+1} = \beta \left( \frac{C_t}{C_{t+1}} \right)^\phi \frac{P_t}{P_{t+1}}. \]

When \( \phi = 1 \), \( \alpha_{t+1} = \beta M_t/M_{t+1} \) in equilibrium. But as \( \phi \) falls below unity, \( \alpha_{t+1} \) becomes less sensitive to consumption growth, and more sensitive to anticipated inflation. This introduces a

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22 In fact, when \( \phi < 1 \), marginal cost is even more responsive than in the baseline model, if all firms adjust their price. This is because in the baseline case, marginal cost responds in exact proportion to the shock itself. But when \( \phi < 1 \), marginal cost is more sensitive to the aggregate price level, and less so to the money supply. Since with trend inflation, new two-period price-setters set prices higher than current marginal cost, the aggregate price level responds by more than the money shock. Hence, marginal cost does also when all sticky price firms adjust.
new channel through which a full price adjustment equilibrium becomes more likely. The dashed line in Figure 7 illustrates that, for positive money shocks above +2.25%, there is a (second) equilibrium with full price adjustment in our amended model. But in the analogous case without real rigidity (i.e. $\phi = 1$), the threshold shock that induces full adjustment is +5%.

The explanation for the greater sensitivity to positive money growth shocks is tied directly to the change in the nominal discount factor. In the case of trend money growth, a new price-setting firm sets a two-period price that represents a weighted average between the optimal price for dates $t$ and $t+1$ (which is higher, due to trend money growth). The discount factor determines the weight that the firm places on each period. From (18), when $\phi < 1$, a rise in $P_t$, holding expected $P_{t+1}$ constant, leads to an immediate rise in the discount factor (a fall in the nominal interest rate). This increases the weight that the price-setting firm places on future profits relative to current profits, leading it to set a higher two-period price. This ‘discount factor’ mechanism then gives rise to an enhanced possibility for multiple equilibrium.

To see how this works, consider the following argument. Suppose that for a given positive shock, all firms in the second period of their price contract are expected to opt out and adjust to the ex-post optimal level. When $\phi < 1$, this leads to a rise in the discount factor, since $P_t$ rises for a given expectation of $P_{t+1}$. This leads newly price-setting firms to set higher two-period prices. Given the discussion of the previous footnote, we get a larger increase in current marginal cost, which in turn increases the incentive for sticky price firms to opt out. In this way, the original expectation of full price adjustment becomes self-fulfilling.

On the other hand, when $\phi = 1$, the equilibrium discount factor is pinned down exogenously by the money growth process. As a result, there is no additional feedback of the decision of sticky price firms to opt out on their incentive to opt out, via changes in the discount factor.

Hence, the threshold value of a positive money growth shock that sustains an equilibrium with full price adjustment is actually lower in the presence of real rigidity (when introduced as in (16)), than in its absence. The degree of price flexibility remains distinctly asymmetric in the presence of real rigidity. Prices are very unresponsive to even large negative monetary shocks. But, for relatively small positive shocks, although there is an equilibrium with low price adjustment and large output responses, there is also an equilibrium in which all prices fully adjust.
6. Discussion of Potential Extensions

We have employed a relatively parsimonious model of the monetary business cycle. In this section, we discuss the robustness of our results to several possible modifications. First, one could consider a more ‘neoclassical’ version allowing for physical capital accumulation. This would enrich the dynamic response of the model to monetary shocks. However, because of the short-run inelasticity of capital supply, the primary effect would be greater diminishing returns to variable inputs, and with economy-wide factor markets, greater sensitivity of real marginal cost to aggregate demand shocks. This would result in an even greater deviation of time-dependent pricing from state-dependent pricing, in a similar fashion to the experiments presented in Figure 6. An obvious further modification to counteract this would be the inclusion of variable capacity utilization, as in Dotsey and King (2001) and Christiano et al. (2005).\(^{23}\)

Many macro models are designed to generate persistent responses in output and inflation to monetary shocks. A number of studies have incorporated features that generate ‘flatter’ marginal costs curves. For instance, Christiano et al. (2005) include sticky wage-setting on the part of households, so that a key component of nominal marginal cost responds sluggishly to shocks. Dotsey and King (2001) include produced intermediate goods as a factor of production. Hence, if intermediate good prices are sticky, this introduces nominal rigidity into the costs of other price-setting firms using those goods as inputs. However, as long as these wages and input prices are set in a state-dependent manner, we retain the key strategic complementarity that is present in our more reduced-form analysis of real rigidities.\(^{24}\)

In addition, a number of papers introduce features that dampen the responsiveness of price change to nominal marginal cost. We have in mind models with variable elasticity of demand as in Kimball (1995), so that a firm’s demand is highly sensitive to changes in relative price (see for instance, Chari et al., 2000; and Dotsey and King, 2005). But again, this modification displays

\(^{23}\)See also Dotsey and King (2005) for evidence that under state-dependent pricing, the inclusion of specific factor markets actually increases price flexibility and decreases persistence relative to models with economy-wide factor markets.

\(^{24}\)We view as an interesting open question the extent to which nominal wages are in fact sticky, and measuring the magnitude of fixed costs associated with wage change. Note that in a model with state-dependent wage-setting, it is unclear how the incentives facing households would affect asymmetry in wage change decisions.
the type of strategic complementarity that we stress above. If few firms adjust their price, there is little incentive for an individual firm to do so, while if many firms adjust prices, there is a large incentive for price change. As a result, we view the existence of multiple equilibria due to such complementarities to be a salient feature of state-dependent pricing models. Hence, it is unclear that models designed to generate realistic, persistent fluctuations are immune from potentially large degrees of endogenous price adjustment.

Finally, we note that a number of researchers have suggested that hybrid time-and-state-dependent models might provide for the most realistic approach to the study of price rigidity (see for example, Ball and Mankiw, 1994; and Taylor, 1999). However, we view as an interesting open question the quantitative difference in the degree of non-linearity and asymmetry derived from our hybrid model, and a pure state-dependent pricing model. Answering this question would entail extending our simple two-period model, first, by having opt-out firms set new two-period prices, and second, by increasing the exogenous length of price contracts. Our conjecture is that as long as the frequency of price change is sufficiently high, as is true in current calibration studies, our results will not be significantly altered. Moreover, a model with multi-period price contracts and shorter time periods might shed light on evidence for asymmetric price adjustment presented in Peltzman (2000); in particular, the result that over a horizon of several quarters, output prices respond more strongly to positive cost shocks than they do to negative shocks.

7. Conclusions

In this paper we study the business cycle properties of a model with endogenous price adjustment; in particular, we study the impact of monetary policy shocks on inflation and output when firms face fixed costs of price adjustment. We make this analysis tractable by considering a hybrid model of time- and state-dependent price setting. State-dependency is modelled by allowing firms to opt out of price contracts in response to the state of nature. However, because these contracts are of fixed duration, the state space maintains a manageable size. This allows us to characterize the complete non-linear dynamics generated by the model's endogenous degree of price fixity.

In calibrated versions of our model, monetary policy shocks of moderate size generate a significant degree of endogenous price change. Hence in these cases, time-dependent pricing provides a
poor approximation to responses generated by a model with fixed costs of price change.

We also find that the model’s output and inflation responses are asymmetric across positive and negative monetary shocks. For positive shocks, prices are strategic complements so that firms have greater incentive to increase their price when other firms increase theirs. For negative shocks, prices are strategic substitutes, and firms have less incentive to lower prices when other firms lower theirs. This asymmetry is due to two crucial features: (i) the effect of other firms’ pricing decisions on an individual firm’s demand, and (ii) the positive equilibrium covariance of aggregate prices and marginal cost.

In equilibrium, this results in smaller output expansions associated with positive monetary shocks relative to output contractions due to negative shocks of the same size. In simulating our state-dependent pricing model, we find that it accounts for the magnitude of output asymmetries found empirically in the US data. Finally, in the model with real rigidities, we find asymmetries in the presence of multiple equilibria across positive and negative shocks.

To conclude, we note that the quantitative importance of these non-linearities and asymmetries depends on the calibration of several key parameters. Hence, we believe that it is particularly important that more evidence be brought to bear on the size and distribution of fixed costs of price adjustment. Such evidence would allow researchers to better distinguish between the predictions and plausibility of competing sticky price models.

University of British Columbia, Canada and CEPR

University of British Columbia, Canada
References


Figure 1: Price adjustment incentives for the individual firm. Solid line: when other firms adjust; dashed line: when other firms do not adjust.
Figure 2: Equilibrium price adjustment to small and large money shocks.
Solid line: positive shock; dashed line: negative shock; hatched (*) line: with real rigidity.
Figure 3: Impact response to $x\%$ monetary policy shock, baseline case.
Solid line: state-dependent model; dashed line: time-dependent model.
Figure 4: Impact response to x% monetary policy shock, alternative specifications of fixed cost distribution (see text). Solid line: state-dependent model; dashed line: time-dependent model.
Figure 5: Impact response to x% monetary policy shock, with trend inflation ($\mu = 0.03$).
Solid line: state-dependent model; dashed line: time-dependent model.
Figure 6: Impact response to x% monetary policy shock, $\mu = 0.03$ and $\rho = 0.6$, alternative specifications of money demand (see text). Solid line: state-dependent model; dashed line: time-dependent model.
Figure 7: Impact response of switching fraction in real rigidities model.
Solid line: low switching equilibrium; dashed line: high switching equilibrium.