



## Aggregate fluctuations with increasing returns to specialization and scale

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### Abstract

The effects of technology shocks in a real business cycle model with monopolistic competition and increasing returns to both specialization and scale are considered. Market power *per se* and increasing returns due to fixed costs have no effect on the responses of aggregate variables to a technology shock relative to those exhibited by a standard, perfectly competitive real business cycle model. In contrast, the responses of aggregates to technology shocks are reduced by returns to scale in variable factors and increased by returns to specialization. Returns to specialization and scale also affect the measurement of technology shocks. Increasing returns to scale generally cause the variance of the Solow residual to undermeasure the variance of the 'true' technology shock, while returns to specialization result in the opposite bias. With both types of increasing returns present, the variance of output is increased relative to a standard competitive model despite a significant reduction of the variance of technology shocks.

*Key words:* Monopolistic competition; Increasing returns; Aggregate fluctuations

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### 1. Introduction

The measurement of fluctuations in total factor productivity is central to real business cycle theory, since cycles in these models are typically driven by

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aggregate technology shocks. Under the joint hypotheses of constant returns to scale and perfect competition, productivity shocks can be measured by fluctuations in the Solow residual.<sup>1</sup> Many authors, however, have questioned the use of the Solow residual as a measure of technology shocks. Hall (1990) notes that if markets are imperfectly competitive and/or technologies exhibit increasing returns, the Solow residual yields a biased measure of true shifts in the production function. He also argues that in these circumstances Solow residuals may change endogenously in response to other macroeconomic shocks which do not affect technology directly. Empirical evidence for endogeneity of Solow residuals is presented in Evans (1992) and Finn (1992). This evidence may be seen as calling into question the extent to which technology shocks can be said to ‘account’ for aggregate fluctuations within the standard real business cycle framework.

Hall argues that at least two factors lie behind the failure of Solow residuals to accurately measure technology shocks. These are the presence of monopolistic competition and the existence of external economies or *thick market* effects. The first factor is consistent with the observations that firms tend to price above marginal cost and that average profits are low for U.S. industry.<sup>2</sup> This requires that there be increasing returns at the level of the *firm*. The second factor is based on the idea that a firm’s productivity may increase when the output of other firms increases. When aggregate activity rises, all firms may become more efficient, as suggested by the work of Diamond (1982) and Cooper and John (1988). This explanation requires increasing returns at the *aggregate* level. Caballero and Lyons (1992) present empirical evidence consistent with this view, suggesting that increasing returns in U.S. manufacturing are strongest at the aggregate level.

We develop a business cycle model of monopolistic competition along the lines of the international trade model of Ethier (1982) and the growth models of Romer (1987) and Grossman and Helpman (1991). In our environment, the two factors put forward by Hall are important *both* for the measurement of technology shocks and for the effects of those shocks on the variability of aggregates. The economy is one in which monopolistically competitive producers supply intermediate inputs to a competitive final goods sector. Individual intermediate goods producers experience increasing returns to scale at the firm level due both to fixed costs of production and declining marginal costs. In addition there may be increasing returns at the aggregate level. Such increasing returns, which we

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<sup>1</sup>The Solow residual generally refers to the difference between the growth rate of aggregate output and the sum of the growth rates of factors, weighted by their compensation shares in total output.

<sup>2</sup>For the former see Morrison (1990) or Rotemberg and Woodford (1991). As stated by Hall (1990) with regard to the latter: ‘With free entry and fixed costs, firms will reach a zero-profit equilibrium in which both the original and cost-based Solow residuals will fail invariance.’

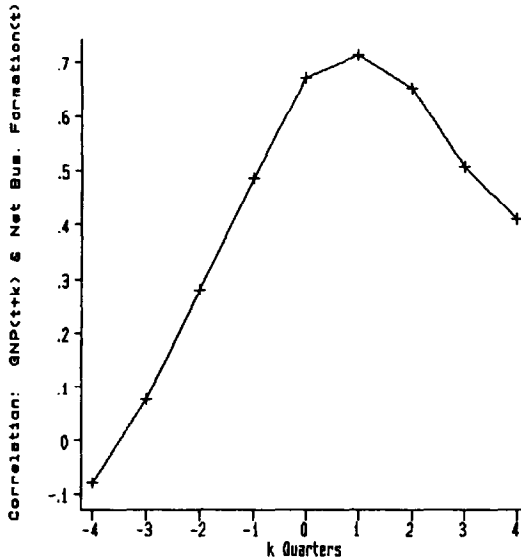
refer to as returns to specialization, exist if, for a given stock of primary factors, final goods output is higher the greater the *variety* of specialized intermediate goods produced. We interpret this increasing return to specialization as a simple formalization of the notion of a thick market effect.

In our model, technology shocks generate entry and exit of firms over the cycle. It is through the mechanism of entry and exit that returns to specialization have implications for aggregate fluctuations. The use of this mechanism is motivated by the observation that net business formation is a strongly cyclical activity. Audretsch and Acs (1992) present evidence that net business formation is strongest during a macroeconomic expansion. Evidence that entry and exit of firms has implications for employment is provided by Davis and Haltiwanger (1990). Based on their analysis of U.S. manufacturing over the period 1972–86, they estimate that 25% of annual gross job destruction can be attributed to establishment deaths and 20% of annual gross job creation to establishment births. Fig. 1 illustrates the cyclical nature of entry and exit. Fig. 1A shows that net business formation is strongly procyclical. This results from the combined effects of both strongly procyclical incorporation of new firms and strongly countercyclical business failures, as shown in Figs. 1B and 1C. An interesting fact conveyed by Fig. 1A is that net entry takes place either contemporaneously with or slightly *prior* to an increase in aggregate output. This suggests that firm entry may play a significant role in producing an expansion. In this study we do not attempt to account empirically for cyclical industry evolution. Rather, we use a very simple model of cyclical net business formation as a device for capturing the idea of a thick market externality.

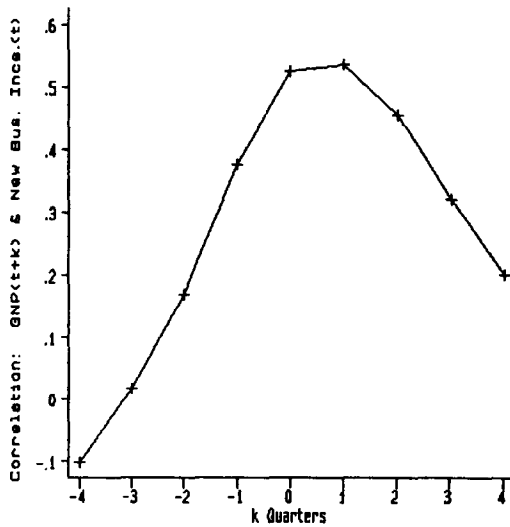
We begin our analysis by first characterizing the effects of both increasing returns to scale and specialization on the responses of aggregates to a given technology shock. In particular, we refer to the quantitative response of aggregate output to a technology shock as the multiplier and consider the effects of the various degrees and types of increasing returns on its size. Secondly, we demonstrate that the introduction of monopolistic competition and increasing returns has implications for the *measurement* of technology shocks. Our model yields a measure of true shifts in technology which enables us to correct for the bias inherent in using the Solow residual as a measure of technology shocks when the assumptions of perfect competition and constant returns are not met. Finally, we use the corrected measure of technology to address the question of the extent to which technology shocks can account for aggregate fluctuations.

The multiplier implied by our model economy depends upon the degrees of increasing returns to scale and specialization. The presence of imperfect competition, in and of itself, has no effect on the multiplier when there is endogenous entry and exit. Increasing returns to scale due to declining marginal costs at the firm level, however, *reduce* the multiplier in our framework. This finding contrasts with those of Hornstein (1993) and Ambler and Cardia (1992), who report

that increasing returns to scale, in the absence of entry and exit, accentuate the response of the economy to technology shocks. With increasing returns to specialization, a technology shock which precipitates firm entry results in an *endogenous* increase in total factor productivity at the aggregate level,



1A: Net Business Formation



1B: New Business Incorporations

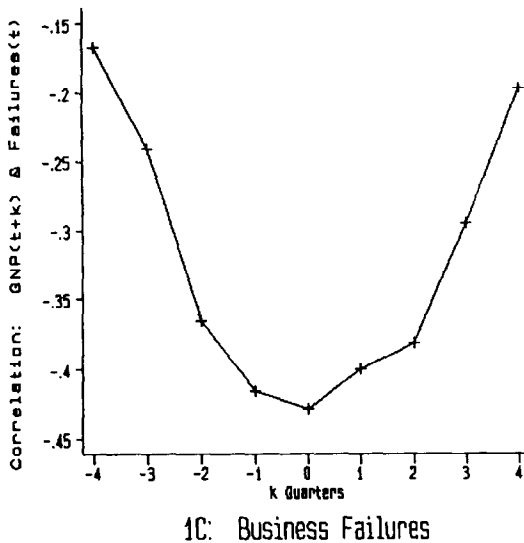


Fig. 1. Cyclical properties of entry and exit.

U.S. quarterly data from 1960.1–1989.4, detrended with the Hodrick–Prescott filter. Net Business Formation, New Business Incorporations, and GNP (1982 dollars) were taken from *Citibase*. Net Business Formation is an index of new business incorporations and business failures, seasonally adjusted. Business Failures refers to the number of bankruptcies as reported in Dun & Bradstreet's Record of Business Closings.

amplifying the effect of the original shock. Hence, increasing returns to specialization lead to an increase in the multiplier.

With regard to the measurement of technology shocks, both increasing returns to scale and specialization may cause the Solow residual to mismeasure exogenous changes in technology. In the presence of entry and exit, imperfect competition alone does not impart a bias; the Solow residual remains a valid measure of exogenous shifts in technology. In the presence of increasing returns to scale in variable factors at the firm level, however, the variance of the Solow residual generally *understates* that of the true technology shock. With returns to specialization, in contrast, the Solow residual measures the endogenous change in total factor productivity attributable to changes in the number of firms as well as the exogenous effect of the actual technology shock. Therefore, the variance of the Solow residual generally *overstates* that of the technology shock.

By combining the two facets of our analysis, we are able to address the issue of the share of the variance of aggregate output accounted for by technology shocks, following the line of inquiry pursued by Kydland and Prescott (1982) and Hansen (1985). Using the correct measure of technology shocks implied by

the model, we find that in the presence of increasing returns to scale alone, technology shocks account for the same share of aggregate fluctuations in our economy as in a benchmark competitive real business cycle model. When increasing returns to specialization are present, however, the share of aggregate fluctuations accounted for by technology shocks is generally *increased*. The variance of technology shocks is smaller when computed using our measure rather than the Solow residual. Due to the endogenous increase in total factor productivity, however, the amplification of the technology shock multiplier dominates, and the variances of aggregates are increased relative to the standard real business cycle model.

We also demonstrate that the endogeneity of total factor productivity in the presence of increasing returns to specialization will cause other types of expansionary shocks to produce fluctuations in total factor productivity, even if they do not affect technology directly. We illustrate this by showing that shocks to government spending will produce a Solow residual that is positively correlated with aggregate output. In principle, returns to specialization could account for failures of the Solow residual to be invariant to a wide variety of factors that are typically thought of as non-technological. Any impulse which causes the equilibrium number of firms to fluctuate will generate endogenous cycles in total factor productivity.

This paper is related to several others in the recent literature that consider business cycles in the presence of increasing returns and imperfect competition. Hornstein (1993) shows that monopolistic competition and increasing returns to scale result in only a minor reduction of the importance of technology shocks when the number of firms is fixed over the cycle. Our research may be seen as complementary by suggesting that if the constraint on firm entry is relaxed, even the modest reduction in the impact of these shocks may be eliminated. Furthermore, if there are returns to specialization, these results may actually be reversed. In another related paper, Baxter and King (1991) study a productive externality that is similar in effect to ours, but that arises in a competitive environment and is not related to entry and exit of firms, nor is it contrasted with returns to scale at the firm level. Chatterjee and Cooper (1993) also examine the impact of entry and exit on the properties of a real business cycle model. They are mainly concerned with the impact of entry and exit on the persistence of responses to shocks and do not consider the combined effects of changes to both the multiplier and the measurement of shocks on the variance of output.

The paper is organized as follows. The economy and its equilibrium are described in Section 2. The analysis of the effects of increasing returns to scale and specialization on both technology shock multipliers and the measurement of shocks is contained in Section 3. Section 4 illustrates the effect of a government spending shock on the Solow residual in the presence of returns to specialization. Section 5 concludes.

## 2. The economy

There is a single final good which is used for both consumption and investment and a continuum of potential intermediate goods which are used solely in production of the final good. Let  $N_t$  denote the measure of intermediate goods produced at time  $t$ . Each intermediate good is produced by a monopolist using an increasing returns to scale technology:  $\forall t, \forall i \in [0, N_t]$ ,

$$m_{it} = z_t [k_{it}^\alpha h_{it}^{1-\alpha}]^\gamma - \phi, \tag{1}$$

where  $\alpha \in (0, 1)$ ,  $\phi > 0$ , and  $\gamma \geq 1$ . Here  $k_{it}$  and  $h_{it}$  denote respectively the quantities of capital and labor employed in production of intermediate good  $i$  and  $z_t$  is a technology shock affecting symmetrically the technologies for producing all intermediate goods. The technology shock,  $z_t$ , evolves according to

$$\ln z_t = \omega_z \ln z_{t-1} + \varepsilon_t, \tag{2}$$

where  $\omega_z \in (0, 1)$  and  $\varepsilon_t$  is a mean zero, i.i.d. process with variance  $\sigma_\varepsilon^2$ . The intermediate good technologies (1) exhibit increasing returns to scale emanating from two sources: fixed costs governed by  $\phi > 0$  and increasing marginal productivity for  $\gamma > 1$ .

The technology for producing the final good depends upon both the quantity and measure of intermediate goods employed:  $\forall t$ ,

$$y_t = F(N_t, m_{it}; i \in [0, N_t]). \tag{3}$$

This technology may exhibit either constant or increasing returns in the measure of intermediate goods:

$$y_t = N_t^\lambda \left[ \int_0^{N_t} m_{it}^\rho di \right]^{1/\rho}, \quad \rho \in (0, 1), \tag{4a}$$

where these returns are governed by  $\lambda$ . Suppose that an equal quantity of each intermediate good is employed (i.e.,  $m_{it} = m_t \forall i \in [0, N_t]$ ). Then final goods output can be written:

$$y_t = N_t^{\lambda+1/\rho} m_t. \tag{4b}$$

We define *returns to specialization* as the change in  $y_t$  for a given change in the measure of products,  $N_t$ , holding quantity employed of each product,  $m_t$ , constant. Two cases are considered:  $\lambda = 0$  in which case the degree of returns to

specialization is  $1/\rho > 1$ , and  $\lambda = 1 - 1/\rho$ , in which case there are constant returns to specialization. In the increasing returns to specialization case, the technology is analogous to that of Ethier (1982).

Capital is produced by means of a standard intertemporal technology. The law of motion for the aggregate capital stock,  $K$ , is given by

$$K_{t+1} = (1 - \delta)K_t + X_t, \quad (5)$$

where  $\delta$  is the depreciation rate and investment,  $X_t$ , is foregone consumption of the final good at time  $t$ .

A representative household has preferences over consumption of the final good and leisure at time  $t$  represented by the period utility function:

$$U(c_t, h_t) = \ln c_t + \eta \ln(L - h_t), \quad (6)$$

where  $L$  is per period time endowment and  $h_t$  is the time spent working. Throughout, lower-case letters will denote *per capita* quantities and upper-case letters aggregates. The lifetime utility function of the representative household is then given by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \right],$$

where  $0 < \beta < 1$  is the discount factor. The measure of households is normalized to one. Therefore, feasibility requires

$$K_t = k_t = \int_0^{N_t} k_{it} di \quad \text{and} \quad H_t = h_t = \int_0^{N_t} h_{it} di. \quad (7)$$

We consider a symmetric recursive equilibrium. Consumers and final good producers behave competitively in all markets. Consumers own the capital stock and rent both it and labor services to intermediate goods producers. Intermediate goods producers are monopolistically competitive and can enter and exit the industry freely. We begin by solving the static profit maximization problems of both the final and intermediate goods producers and derive the equilibrium process for the number of firms/varieties as a function of the aggregate state and decision variables. We then solve the utility maximization problem of a representative household which takes as given factor prices and the number of firms.

The final goods technology gives rise to the following cost function:

$$C^Y(p, y) = yN^{-\lambda} \left[ \int_0^N p_i^{\rho/(\rho-1)} di \right]^{(\rho-1)/\rho}, \quad (8)$$

where  $p_i$  is the price of intermediate good  $i$  (time subscripts have been suppressed). The conditional demand function for intermediate good  $j$  is given by

$$m_j(p, y) = (yN^{-\lambda}) \frac{p_j^{1/(\rho-1)}}{\left[ \int_0^N p_i^{\rho/(\rho-1)} di \right]^{1/\rho}}. \tag{9}$$

This demand function is characterized by a constant elasticity of demand,  $1/(\rho - 1)$ , which is independent of  $\lambda$ . Returns to specialization are *external* in the sense that final goods producers choose the quantity of each intermediate good to hire, taking the number of intermediate goods as given.

A representative intermediate good producer (the subscript  $j$  is suppressed) faces the following profit maximization problem in each period:

$$\max_p pm(p, y) - C^M(w, r, m; z),$$

where

$$C^M(w, r, m; z) = Ar^\alpha w^{1-\alpha} \left[ \frac{m + \phi}{z} \right]^{1/\gamma}, \tag{10}$$

$$A = (1 - \alpha)^{-(1-\alpha)} \alpha^{-\alpha}.$$

Note that here  $\gamma > 1$  implies decreasing marginal costs. The conditional demand functions for labor and capital are given by

$$h(w, r, m; z) = \left( \frac{r}{w} \right)^\alpha \left( \frac{1 - \alpha}{\alpha} \right)^\alpha \left[ \frac{m + \phi}{z} \right]^{1/\gamma}, \tag{11}$$

$$k(w, r, m; z) = \left( \frac{r}{w} \right)^{\alpha-1} \left( \frac{1 - \alpha}{\alpha} \right)^{\alpha-1} \left[ \frac{m + \phi}{z} \right]^{1/\gamma}. \tag{12}$$

Since there are a continuum of intermediate good producers, each producer takes the term in the denominator of the demand function, (9), as given. Therefore, solution of the profit maximization problem yields the familiar (Dixit and Stiglitz, 1977) constant mark-up pricing rule:

$$p = \left( \frac{1}{\rho} \right) \left[ \frac{\partial C^M(w, r, m; z)}{\partial m} \right]. \tag{13}$$

In equilibrium, the price of the final good must equal its unit cost. Normalizing the price of the final good to 1, this condition is written:

$$1 = N^{-\lambda} \left[ \int_0^N p_i^{\rho/(\rho-1)} di \right]^{(\rho-1)/\rho}. \quad (14)$$

In a symmetric equilibrium, prices and quantities of intermediate goods and factor employment per intermediate good producer are equal across firms (i.e.,  $p_i = p$ ,  $m_i = m$ ,  $k_i = K/N$ , and  $h_i = H/N$ ,  $\forall i \in [0, N]$ ). The solutions to the firms' static maximization problems, together with feasibility and factor market clearing, yield the following conditions that must hold period by period in such an equilibrium:

$$p = N^{(\lambda+1/\rho-1)}, \quad (15a)$$

$$\left[ \frac{m + \phi}{z} \right]^{1/\gamma} = \frac{K^\alpha H^{1-\alpha}}{N}, \quad (15b)$$

$$K/N = \left( \frac{w}{r} \right)^{1-\alpha} \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} \left[ \frac{m + \phi}{z} \right]^{1/\gamma}, \quad (15c)$$

$$p = \left( \frac{A}{z\gamma\rho} \right) r^\alpha w^{1-\alpha} \left[ \frac{m + \phi}{z} \right]^{(1-\gamma)/\gamma}, \quad (15d)$$

$$pm - Ar^\alpha w^{1-\alpha} \left[ \frac{m + \phi}{z} \right]^{1/\gamma} = 0. \quad (15e)$$

Eq. (15a) is the symmetric equilibrium equivalent of (14). Eq. (15b) represents technological feasibility in production of intermediate goods. Eq. (15c) represents capital market clearing. Eq. (15d) is the mark-up pricing equation given by (13). Eq. (15e) represents the free entry condition characterized by zero profits in the intermediate good sector.

Eqs. (15a)–(15e) are used to derive expressions for equilibrium factor prices and the measure of intermediate goods as functions of aggregate capital, employment, and the technology shock, all of which are taken as given by households,

$$w(K, H, z) = (1 - \alpha) \Lambda z^{(1+\rho\lambda)/\rho\gamma} \left[ \frac{(K^\alpha H^{1-\alpha})^{(\lambda+1/\rho)}}{H} \right], \quad (16a)$$

$$r(K, H, z) = \alpha \Lambda z^{(1+\rho\lambda)/\rho\gamma} \left[ \frac{(K^\alpha H^{1-\alpha})^{(\lambda+1/\rho)}}{K} \right], \quad (16b)$$

$$N(K, H, z) = \left[ \frac{z(1-\rho\gamma)}{\phi} \right]^{1/\gamma} (K^\alpha H^{1-\alpha}), \quad (16c)$$

where

$$A = \rho\gamma \left[ \frac{1 - \rho\gamma}{\phi} \right]^{(\lambda + 1/\rho - \gamma)/\gamma}.$$

Inspection of (16c) reveals that the model will deliver a positive measure of intermediates if and only if  $\rho\gamma < 1$ . This is implied by the second-order conditions for profit maximization by individual monopolists and reflects the requirement that, for a given degree of increasing returns to scale, firms must have sufficient market power to earn nonnegative profits. Also note that combining (16c) and (15b), the quantity of each intermediate good produced is constant and independent of the technology shock:<sup>3</sup>

$$m = \frac{\rho\gamma\phi}{1 - \rho\gamma}. \tag{17}$$

Therefore, a technology shock will affect equilibrium variables only through its effect on factor demands and the equilibrium *measure* of intermediate goods. From the perspective of each firm, a technology shock will lead to a downward shift in the cost function as well as an outward shift in the demand function (9). But for either case of  $\lambda$ , a rise in factor prices will ensure that the optimal scale of operation for the individual firm remains unchanged.

A household chooses individual consumption, leisure, and investment to maximize expected discounted lifetime utility taking aggregate variables, factor prices [(16a) and (16b)], and the number of firms (16c) as given. A representative household's optimization problem can be written recursively as

(P.1)

$$V(k, K, z) = \max_{c, h, x, k'} \{U(c, h) + \beta EV(k', K', z' | k, K, z)\},$$

subject to

$$c + x = w(K, H, z)h + r(K, H, z)k,$$

$$k' = (1 - \delta)k + x,$$

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<sup>3</sup>This is a consequence of the assumption that fixed costs are unaffected by the technology shock, implicit in the intermediate goods technology, (1). An alternative specification is:  $m_{it} = z_t [k_{it}^\alpha h_{it}^{1-\alpha}]^\nu - z_t^\nu \phi$ , where  $\nu \in [-1, 1]$ . In this case (17) is replaced by  $m_t = z_t^\nu [\rho\gamma\phi/(1 - \rho\gamma)]$ . Clearly, depending on the sign of  $\nu$ ,  $m$  could be either pro- or counter-cyclical. We have chosen to focus on the case where *average* firm size,  $m$ , is constant ( $\nu = 0$ ) to isolate the effect of entry and exit over the cycle. This also provides a sharp contrast to Hornstein (1993), where all variation is in output per firm and  $N$  is constant over the cycle.

$$K' = (1 - \delta)K + X,$$

$$C = C(K, z),$$

$$H = H(K, z),$$

$$X = X(K, z),$$

$$\ln z' = \omega_z \ln z + \varepsilon.$$

A symmetric recursive equilibrium is a collection of individual decision rules,  $\{c, h, x, k'\}$ , aggregate decision rules,  $\{C, H, X, K'\}$ , and a value function,  $V(k, K, z)$ , such that:

- (i)  $\{c, h, x, k'\}$  solve (P.1) given  $\{C, H, X, K'\}$  and  $V(k, K, z)$ .
- (ii)
  - a.  $C(K, z) = c(K, K, z)$ ,
  - b.  $H(K, z) = h(K, K, z)$ ,
  - c.  $X(K, z) = x(K, K, z)$ ,
  - d.  $K'(K, z) = k'(K, K, z)$ .

We apply the procedures for computing approximations to the equilibrium stochastic process of a homogeneous agent economy as described in Hansen and Prescott (1995). The first step is to compute the economy's deterministic steady state. We then quadratically approximate (P.1) in a neighborhood of this allocation and solve the resulting linear-quadratic dynamic programming problem using standard methods, imposing equilibrium conditions (ii.a)–(ii.d) at each iteration.<sup>4</sup> The decision rules, together with the state laws of motion given by (2) and (5), can then be used to compute realizations of the equilibrium stochastic processes.

### 3. Multipliers and measurement in the theoretical economy

We now consider the effects of increasing returns to scale and specialization on the equilibrium responses of aggregate variables to technology shocks. We

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<sup>4</sup>The departures from an efficient environment introduced with monopolistic competition and increasing returns create the possibility of multiple equilibria (see for example Chatterjee, Cooper, and Ravikumar, 1993). We focus only on situations in which the deterministic steady state exhibits the 'saddle path' property. For the range of parameters considered in this study, the eigenvalues of the linearized system satisfy the necessary restrictions.

begin by holding the characteristics of the technology shock process constant to isolate the effects of different dimensions of increasing returns on these responses. We will refer to the quantitative response of aggregate output (GNP) to a technology shock as the ‘multiplier’ and consider the effect of increasing returns on its magnitude. We then examine the effects of returns to scale and specialization on the measurement of technology shocks in U.S. data. Finally, we consider the share of output fluctuations that can be accounted for by these measured technology shocks in a calibrated version of our artificial economy.

### 3.1. Multipliers

In a standard real business cycle model with perfect competition and constant returns, equilibrium aggregate output,  $Y$ , is given by the aggregate production function:

$$Y = z(K^\alpha H^{1-\alpha}). \tag{18}$$

For the economies with imperfect competition and increasing returns, analogous functions which relate aggregate output in a symmetric equilibrium to the technology shock and aggregate factor employment can be derived. The effects of increasing returns to scale and specialization on the multiplier can be made explicit by evaluating these functions.

A positive productivity shock creates profit opportunities that induce firm entry. As there is a unique optimal firm size that is independent of the technology shock, fluctuations in final goods output are driven by fluctuations in the measure of intermediate goods alone. Aggregate output in this economy is the value of final goods produced:

$$Y = pNm. \tag{19a}$$

Using (15a) we have

$$Y = N^{(\lambda+1/\rho)}m. \tag{19b}$$

Consider first the case of constant returns to specialization, i.e.,  $\lambda = 1 - 1/\rho$ . In this case, (19b) collapses to

$$Y = Nm. \tag{20}$$

Thus, without increasing returns to specialization, aggregate output is proportional to the number of firms. Using (20) and the expression for the equilibrium number of firms, (16c), we derive:

$$Y = A_1 z^{1/\gamma} (K^\alpha H^{1-\alpha}), \tag{21}$$

where  $A_1$  is a constant. Comparing (18) and (21), we see that differences between the standard real business cycle model and the economy with increasing returns to scale arise when  $\gamma > 1$ , i.e., when there are increasing returns to scale in production of intermediate goods. In particular, the elasticity of aggregate output to a technology shock is inversely proportional to the degree of increasing returns to scale. Hence, with constant returns to specialization and increasing returns to scale, the response of output to a technology shock, holding factor inputs constant, will be *less* than that of a standard real business cycle model.

The intuition for this result lies in the mechanism of entry and exit. A positive technology shock has the effect of increasing the number of firms, holding output per firm constant. This necessarily implies a reduction in the employment of capital and labor for each existing firm. The factors released by these firms are then reallocated to new entrants. The higher the degree of returns to scale, the steeper the marginal product schedules for existing firms, the lower the quantity of factors released and made available for new entrants, and the less entry will take place. Since output responds in exactly the same proportion as the number of firms, this implies that the response of output will be smaller for a larger  $\gamma$ . We should thus expect that the variability of output relative to that of a technology shock will be less in this case.

Note that neither the markup ratio ( $1/\rho$ ) nor the level of fixed costs ( $\phi$ ) have any effect on the relationships between total output and either factor employment or technology shocks in equilibrium. Endogenous entry and exit effectively negates the importance (for cyclical fluctuations) of market power and fixed costs. This makes our model notably different from other general equilibrium models of imperfect competition, such as Blanchard and Kiyotaki (1988), Hall (1990), Hornstein (1993), Rotemberg and Woodford (1992), and others. Those studies consider economies in which the number of firms is *fixed* over the cycle. They generally find that increasing returns due to fixed costs magnify the output response to demand or supply shocks. Intuitively, with a fixed number of firms, an expansionary shock causes firms to move down their average cost curves, magnifying the response of output to a given change in marginal cost or price. In our environment, however, a positive shock induces entry of new firms rather than an expansion of output by existing ones. Thus, no movement along the average cost curve takes place.

Now consider the introduction of increasing returns to specialization. With  $\lambda = 0$ , (19b) becomes

$$Y = N^{1/\rho} m. \quad (22)$$

Combining (22) with the equilibrium number of firms given by (16c) we derive the following relationship between aggregate output, the technology shock, and

aggregate factors:

$$Y = A_2 z^{1/\rho\gamma} (K^\alpha H^{1-\alpha})^{1/\rho}, \quad (23)$$

where  $A_2$  is a constant. In comparing (18) with (23), we see that differences between a standard real business cycle model and the economy with increasing returns to specialization and scale arise for two reasons. First, the elasticity of output with respect to the technology shock is increasing in the degree of returns to specialization,  $1/\rho$ , and decreasing in the degree of returns to scale,  $\gamma$ . Since  $\rho\gamma < 1$ , this elasticity necessarily exceeds that of a standard real business cycle model (unity). Second, the elasticity of output with respect to the composite of total factor inputs is increasing in the degree of returns to specialization. The technology shock multiplier encompasses both of these effects. Given these effects, we would expect the multiplier to exceed that of a standard real business cycle model.

Intuition for these relationships can again be found in the mechanism of entry and exit. As before, a positive technology shock leads to profit opportunities and entry. The dampening effect of increasing returns to scale again arises, for the same reasons as in the constant returns to specialization economy. In the presence of increasing returns to specialization, however, as the number of firms increases, the quantity of capital and labor required to produce a unit of the final good falls. This effect can be thought of as an endogenous increase in total factor productivity. Hence, output will increase by more in response to a technology shock than if there were no increasing returns to specialization.

We now quantify the effects of returns to scale and specialization in a series of computational experiments. To perform this exercise, it is necessary to choose specific values for parameters. We can then consider how the multiplier and the volatility of aggregates change as the degrees of increasing returns are varied. Given that our model nests a standard real business cycle model, we are able to take many of the values of these parameters from elsewhere in the literature. Loosely speaking, as we drive  $\rho \rightarrow 1$ ,  $\gamma \rightarrow 1$ , and  $\phi \rightarrow 0$ , our economy converges to the ‘divisible labor’ model of Hansen (1985). Therefore, we set the subjective rate of time preference,  $\beta = 0.99$ , the share parameter in the period utility function,  $\eta = 2$ , the capital share parameter,<sup>5</sup>  $\alpha = 0.36$ , period time endowment,  $L = 1$ , and the rate of capital depreciation,  $\delta = 0.025$ , to correspond to the parameters used in Hansen (1985). The two parameters of the technology shock process given by Eq. (2) are set to  $\omega_z = 0.95$  and  $\sigma_\varepsilon^2 = (0.007)^2$ . Following

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<sup>5</sup>Regardless of the degrees of returns to scale and specialization, the labor and capital shares are given by  $(1 - \alpha)$  and  $\alpha$  in our economy, just as in a standard real business cycle model. This is because profits are zero in equilibrium and can be seen by combining Eqs. (15a), (16a)–(16c), and (17).

Hansen and Wright (1992) these are chosen to be approximately equal to the values used by Prescott (1986).<sup>6</sup> The only parameter that cannot be taken directly from the literature is the level of fixed costs,  $\phi$ . Since this parameter has no effect on cyclical fluctuations in our economy, we set it to one.<sup>7</sup>

For different degrees of returns to specialization and scale we compute 1000 realizations of the economy's equilibrium stochastic process, each 620 periods in length. The first 500 periods of each realization are discarded, and the remaining are detrended using the Hodrick–Prescott (1980) (HP) filter with smoothing parameter 1600. Using this artificial data, we compute the percent standard deviations of aggregate output, consumption, hours worked, investment, capital, and the number of firms. The means and standard deviations of these second moments over the 1000 realizations are reported in Tables 1 and 2.<sup>8</sup> We define the ratio of the variance of HP filtered output to the variance of the unfiltered technology shocks as our measure of the technology shock multiplier. We choose this as a measure because it relates the statistic of particular interest, the average variance of HP filtered output, to an exogenous parameter, the variance of technology shocks.

Table 1 contains results for the economy with constant returns to specialization and varying degrees of returns to scale,  $\gamma$ . The first column contains the corresponding moments for a standard real business cycle model with perfect competition and no increasing returns. As discussed above, for this economy the parameter  $\rho$  governs only the markup ratio, has no effect on the degree of increasing returns, and has no effect on aggregate fluctuations. Therefore, we hold it constant across these experiments. With  $\gamma = 1$ , the second moments for the economy with imperfect competition and increasing returns are *identical* to those for the perfectly competitive economy. In both cases, our measure of the multiplier is 0.34. Hence, market power *per se* ( $\rho < 1$ ) and increasing returns to scale emanating from *fixed costs* ( $\phi$ ) have no implication for the effects of

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<sup>6</sup>Prescott (1986) obtained these values by analyzing the time series behavior of the Solow residual, given by Eq. (24) below. As will be discussed in detail below, this is a valid measure of changes in technology only under the combined assumptions of perfect competition and constant returns to scale. In this analysis of the effects of different types increasing returns on the technology shock multiplier, we hold the shock process constant so that different economies can be compared.

<sup>7</sup>Because the average firm size is constant over the cycle in our economy, fixed costs affect the number of varieties and the level of output of each intermediate good only in the steady state.

<sup>8</sup>For each economy considered, the same 1000 draws of technology shocks are used. Therefore, the standard deviations of the second moments should not be used to assess the significance of the differences in moments across economies. In fact, the relationships between economies that are discussed below are all significant at extreme levels. This can be seen by considering the differences of the moments across economies for each realization of technology shocks. The results of this exercise are available upon request.

Table 1  
Effects of increasing returns to scale

Moment	Perfect competition	Constant returns to specialization, $\rho = 0.7$				
		$\gamma$				
		1.0	1.1	1.2	1.3	1.4
$\sigma_Y$	1.35 (0.17)	1.35 (0.17)	1.23 (0.16)	1.12 (0.14)	1.04 (0.13)	0.96 (0.12)
$\sigma_C$	0.42 (0.06)	0.42 (0.06)	0.38 (0.06)	0.35 (0.05)	0.32 (0.05)	0.30 (0.05)
$\sigma_H$	0.69 (0.08)	0.69 (0.08)	0.63 (0.08)	0.58 (0.07)	0.53 (0.07)	0.49 (0.06)
$\sigma_X$	4.24 (0.55)	4.24 (0.55)	3.85 (0.49)	3.53 (0.45)	3.25 (0.41)	3.02 (0.38)
$\sigma_K$	0.36 (0.08)	0.36 (0.08)	0.33 (0.07)	0.30 (0.07)	0.28 (0.06)	0.26 (0.06)
$\sigma_N$	— —	1.35 (0.17)	1.23 (0.16)	1.12 (0.14)	1.04 (0.13)	0.96 (0.12)
$\sigma_{SR}$	0.91 (0.12)	0.91 (0.12)	0.82 (0.11)	0.76 (0.10)	0.70 (0.09)	0.65 (0.08)
$\sigma_Y^2/\sigma_z^2$	0.34	0.34	0.28	0.25	0.20	0.18

$\sigma_Y, \sigma_C, \sigma_H, \sigma_X, \sigma_K, \sigma_N,$  and  $\sigma_{SR}$  denote the standard deviations of aggregate output, consumption, labor input, investment, capital, the number of firms, and the Solow residual, respectively. Reported moments are means of statistics computed from 1000 simulations, each 120 periods in length. Standard deviations are in parentheses.  $\sigma_Y^2/\sigma_z^2$  is the ratio of the variance of HP-filtered output to the the raw variance of technology shocks. Perfect competition refers to a benchmark economy with perfect competition and constant returns to scale. In all of these experiments, the markup parameter is held constant at  $\rho = 0.7$ . Note that under constant returns to specialization, the economy's cyclical properties are invariant to changes in this parameter.

technology shocks. In addition, Table 1 verifies the intuition presented above that as the degree of increasing returns to scale associated with declining marginal cost,  $\gamma$ , increases, the technology shock multiplier falls. In particular, the multiplier falls by 17.5% when the degree of increasing returns to scale increases to 1.1 and by 49% when it increases to 1.4.

Table 2 contains results for the economy with both increasing returns to specialization and scale. Again the first column contains corresponding moments for a standard real business cycle model. In the first set of experiments (columns 2–5), the degree of returns to specialization is held constant at  $1/\rho = 1.43$  and the degree of returns to scale,  $\gamma$ , is varied. In the second set (columns 6–9), the degree of returns to scale is held constant at  $\gamma = 1$  and the degree of returns to specialization is varied. Columns 2–5 verify that in this case

Table 2  
Effects of increasing returns to specialization and scale

Moment	Perfect competition	Increasing returns to specialization							
		$\rho = 0.7$				$\gamma = 1$			
		$\gamma$		$\rho$		$\gamma$		$\rho$	
		1.1	1.2	1.3	1.4	0.6	0.7	0.8	0.9
$\sigma_Y$	1.35 (0.17)	2.14 (0.29)	1.96 (0.26)	1.81 (0.24)	1.68 (0.22)	3.08 (0.48)	2.36 (0.33)	1.90 (0.25)	1.58 (0.20)
$\sigma_C$	0.42 (0.06)	0.72 (0.11)	0.66 (0.10)	0.61 (0.09)	0.57 (0.09)	1.11 (0.18)	0.80 (0.12)	0.62 (0.09)	0.50 (0.08)
$\sigma_H$	0.69 (0.08)	1.04 (0.13)	0.96 (0.12)	0.88 (0.11)	0.82 (0.10)	1.43 (0.18)	1.15 (0.14)	0.95 (0.12)	0.80 (0.10)
$\sigma_X$	4.24 (0.55)	6.62 (1.00)	6.04 (0.88)	5.55 (0.79)	5.14 (0.72)	9.56 (2.01)	7.33 (1.16)	5.92 (0.84)	4.95 (0.66)
$\sigma_K$	0.36 (0.08)	0.57 (0.13)	0.52 (0.12)	0.48 (0.11)	0.44 (0.10)	0.80 (0.19)	0.62 (0.14)	0.51 (0.11)	0.42 (0.09)
$\sigma_N$	—	1.49 (0.19)	1.37 (0.17)	1.26 (0.16)	1.17 (0.15)	1.82 (0.23)	1.64 (0.20)	1.51 (0.19)	1.42 (0.18)
$\sigma_{SR}$	0.91 (0.12)	1.47 (0.21)	1.35 (0.19)	1.24 (0.17)	1.15 (0.16)	2.17 (0.39)	1.62 (0.24)	1.29 (0.18)	1.07 (0.14)
$\sigma_Y^2/\sigma_z^2$	0.34	0.86	0.72	0.62	0.53	1.78	1.05	0.68	0.47

$\sigma_Y, \sigma_C, \sigma_H, \sigma_X, \sigma_K, \sigma_N, \sigma_{SR}$ , and  $\sigma_z$  denote the standard deviations of aggregate output, consumption, labor input, investment, technology, capital, the number of firms, the Solow residual, and the technology shock, respectively. Reported moments are means of statistics computed from 1000 simulations, each 120 periods in length. Standard deviations are in parentheses. Perfect competition refers to a benchmark economy with perfect competition and constant returns to scale. In the first set of experiments,  $\rho$  is held constant at 0.7, and the degree of returns to scale ( $\gamma$ ) is varied. In the second set, the degree of returns to specialization ( $1/\rho$ ) is varied with  $\gamma$  held constant at 1.

also increasing returns to scale ( $\gamma > 1$ ) lower the multiplier. The table also confirms that the multiplier is increasing in the degree of increasing returns to specialization ( $1/\rho$ ). Comparing column 3 of Table 2 to column 2 of Table 1, for example, we see that incorporating increasing returns to specialization of degree  $1/0.7 = 1.43$  causes the multiplier to rise by 206%. For lower degrees of returns to specialization, the tables indicate that the effect on the multiplier is also substantial. Furthermore, as noted above, Table 2 verifies that with both increasing returns to specialization and scale the multiplier will *always* exceed that of a standard real business cycle model.

### 3.2. Measurement of technology shocks

The results presented in Tables 1 and 2 illustrate the effects of returns to specialization and scale on aggregate fluctuations and the multiplier, holding the

technology shock process constant. The technology shocks used were generated using the process calibrated by Prescott (1986) to the properties of the Solow residual in U.S. data. The Solow residual, SR, can be computed as follows:

$$\ln SR = \ln Y - \alpha \ln K - (1 - \alpha) \ln H. \quad (24)$$

As Hall (1990) and others have argued, the Solow residual is a valid measure of technology shocks *only* under the combined assumptions of perfect competition and constant returns to scale. To calibrate fully the model with increasing returns, it is necessary to take account of the bias in using the Solow residual as the parameterization of the technology shock process. We now consider measurement of technology shocks in the presence of increasing returns to specialization and scale.

We define the Solow residual as the difference between the log of output and the factor share weighted sum of the logs of the capital and labor inputs. Using the equilibrium relationships between aggregate output, the technology shock, and total factor input, we can obtain measures of technology shocks for our economy that are functions of the Solow residual. Consider first the economy with constant returns to specialization. Taking the log of (21) and solving for the log of the technology parameter, we derive

$$\ln z = \gamma [\ln Y - \alpha \ln K - (1 - \alpha) \ln H] - \gamma \ln A_1. \quad (25)$$

This equation provides an explicit characterization of the bias inherent in using the Solow residual as a measure of technology shocks in the presence of increasing returns to scale. Given this, the expression can be applied to U.S. aggregate data to obtain corrected measures of technology shocks that can be used in the calibration of the theoretical economy.

Using (24) and (25), we have

$$\ln SR = \frac{1}{\gamma} \ln z + \ln A_1. \quad (26)$$

Eq. (26) illustrates that the variance of the Solow residual will be less than that of the true technology shock in the presence of increasing returns to scale ( $\gamma > 1$ ), although the autocorrelation of the two will be identical. Thus, for  $\gamma > 1$ , a given percentage change in the observed Solow residual implies a greater than proportional change in the true technology. Therefore, in contrast to the conjectures of Hall (1990) and others, it is possible that increasing returns might lead us to *underestimate* the size of technology shocks in the economy. Table 1 reports the standard deviation of the Solow residual in the computational experiments with constant returns to specialization and illustrates this bias. As the degree of returns to scale increases, the standard deviation of the Solow residual falls,

although the standard deviation of the actual technology shocks remains the same.

It is worth noting, however, that the bias in using the Solow residual as a measure of technology shocks in this economy arises only when  $\gamma > 1$ . In the case where  $\gamma = 1$ , the Solow residual is a valid measure of technology shocks. Therefore, and again in contrast to previous literature, there is no measurement bias due to imperfect competition *per se* ( $\rho < 1$ ) or fixed costs ( $\phi > 0$ ) alone when there is free entry and exit.

We now turn to the economy with increasing returns to scale and specialization. Taking the log of (23) gives

$$\ln z = \rho\gamma \ln Y - \gamma[\alpha \ln K + (1 - \alpha)\ln H] - \rho\gamma \ln A_2. \quad (27)$$

Hence, the Solow residual can be written in terms of the true technology shock as follows:

$$\ln SR = \frac{1}{\rho\gamma} \ln z + \left( \frac{1 - \rho}{\rho} \right) [\alpha \ln K + (1 - \alpha)\ln H] + \ln A_2. \quad (28)$$

Under this specification, there are two sources of bias in the residual. First, the variance of the Solow residual will tend to be greater than that of the true technology shock since  $\rho\gamma < 1$ . There is an additional bias, however, due to the presence of increasing returns to specialization. Some of the variability in factor inputs is incorrectly attributed to technology fluctuations, rather than to the endogenous movement in total factor productivity. Since these two sources of bias may in principle offset one another, it is not clear whether the variance in the Solow residual will be greater than or less than that of the true technology shocks. But because capital and labor are likely to co-vary positively over the cycle, we might guess that the overall bias would lead the Solow residual to be more variable than the technology shock. Table 2 indicates that this conjecture is correct for our economy. The higher the degree of returns to specialization, the greater is the endogenous response of total factor productivity, which is incorrectly attributed by the Solow residual to an exogenous change in technology. Thus the gap between the variance in the Solow residual and that of the true technology shock is increasing in  $1/\rho$ . Even for the case of  $\rho = 0.9$ , the standard deviation of the Solow residual overstates true technology fluctuations by 18%. It remains true that increasing returns to scale introduce a bias that works in the opposite direction of that associated with returns to specialization. Nevertheless, with  $\rho = 0.7$ , even when  $\gamma = 1.4$  the Solow residual still overstates true technology fluctuations by 27%.

### 3.3. Calibration

To summarize our results so far, we have found that increasing returns to scale at the level of the firm (with entry and exit) reduces the technology shock multiplier, but increases the variance of technology shocks implied by the data. Increasing returns to specialization, on the other hand, raises the technology shock multiplier, but is likely to reduce the implied variance of technology shocks. We now assess the overall quantitative implications of these two effects in a calibrated version of our theoretical economy. To perform this analysis, we must calibrate the degrees of increasing returns.

We begin by associating the parameter  $\rho$  with the markup ratio, which is  $1/\rho$  in our economy. Several studies have estimated markups of price over marginal cost in U.S. manufacturing. The estimates have ranged widely. Using gross output data, Basu and Fernald (1993b) obtain estimates for total U.S. manufacturing ranging from 1.15 to 1.23. Morrison (1990) also estimates markups for total U.S. manufacturing and for 17 two-digit (SIC) manufacturing industries using gross output data for the period 1960–86. Her estimate for total manufacturing is 1.197, and for two-digit industries, her estimates range from 1.186 to 1.695. We use Morrison's estimate for total manufacturing, 1.197 (Morrison's Table 2, p. 25), implying  $\rho = 0.8354$ . This value is at the low end of Morrison's estimates, and falls within the range estimated by Basu and Fernald (1993b). In addition, our value is at the low end of the range of markups estimated for 19 two-digit U.S. manufacturing industries by Domowitz, Hubbard, and Peterson (1988). Their estimates varied from 1.198 to 1.513.

We maintain the assumption that the parameter  $\lambda$  is either  $1 - 1/\rho$  (constant returns to specialization) or zero (increasing returns). Hence the degree of increasing returns to specialization will equal the markup ratio,  $1/\rho$ , when  $\lambda = 0$ . Of course, different degrees of increasing returns to specialization are compatible with a given markup ratio in our environment, if  $\lambda$  is allowed to vary.<sup>9</sup> While our assumption that the markup ratio equals the degree of returns to specialization will have some effect on our computational experiments, it does not affect our overall quantitative findings. This point is made explicit below.

We also take our measure of the degree of increasing returns to scale,  $\gamma$ , from Morrison (1990), in which returns to scale coefficients are estimated jointly with markups. Again we use the coefficient for total manufacturing and set  $\gamma = 1.1416$  (Morrison's Table 3, p. 29). In the case of constant returns to specialization, this parameter alone accounts for differences in both the

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<sup>9</sup>The degree of returns to specialization used in our calibration is somewhat lower than estimated by Caballero and Lyons (1992), who estimated external increasing returns in the range 1.32–1.49. These estimates, however, have been criticized by Basu and Fernald (1993a) because they were constructed using value added data.

Table 3  
Economy parameters

A. Preferences and technologies			
Parameter value	Description		
$\beta = 0.99$	Discount factor		
$\eta = 2$	Leisure time share		
$\alpha = 0.36$	Capital share		
$\delta = 0.025$	Depreciation rate		
$\phi = 1$	Fixed cost		
$1/\rho = 1.197$	Markup ratio/Returns to specialization		
$\gamma = 1.1416$	Returns to scale		
(B) Technology shock processes			
	$\omega_z$	$\sigma_\varepsilon^2$	$\sigma_z^2$
Solow residual	0.917 (0.031)	0.000054	0.000341
Constant returns to specialization	0.917 (0.031)	0.000071	0.000444
Increasing returns to specialization	0.896 (0.035)	0.000048	0.000246

Technology shocks are calculated from (25) and (27) using quarterly data for U.S. real GNP (base 1982), total hours worked in industry (both taken from Citibase), and the gross capital stock in 1982 dollars interpolated using the annual gross stock and quarterly gross capital formation (source: *OECD Flows and Stocks of Fixed Capital*).

calibration of technology shocks and the economy's cyclical properties relative to those of a standard real business cycle model. Our overall findings are not sensitive to the exact choice of the degree of increasing returns to scale. The robustness of our findings in this regard carries over as well to the case in which this dimension of increasing returns is combined with increasing returns to specialization.

Given these values for  $\rho$  and  $\gamma$ , we compute technology residuals for the U.S. using Eqs. (25) and (27). Table 3 contains OLS estimates of the AR(1) coefficients ( $\omega_z$ ) and innovation variances ( $\sigma_\varepsilon^2$ ) for three economies: (i) a standard real business cycle model with perfect competition and no increasing returns [Eq. (24)], (ii) the model with increasing returns to scale and constant returns to specialization [Eq. (25)], and (iii) the model with increasing returns to scale and increasing returns to specialization [Eq. (27)]. The table also includes the unconditional variance of the implied technology shock process. These variances are used in our multiplier calculations for the calibrated economies. The moments of the implied technology processes accord with our intuition from the previous section. Thus, relative to the variance of technology shocks calculated

Table 4  
Computational experiments: Calibrated economies

Moment	U.S. economy	Perfect competition	Constant returns to specialization	Increasing returns to specialization
$\sigma_Y$	1.74	1.43 (0.18)	1.43 (0.18)	1.59 (0.19)
$\sigma_C$	1.29	0.37 (0.06)	0.37 (0.06)	0.39 (0.06)
$\sigma_H$	1.58	0.80 (0.09)	0.80 (0.09)	0.91 (0.11)
$\sigma_X$	5.56	4.75 (0.59)	4.75 (0.59)	5.39 (0.67)
$\sigma_K$	0.55	0.39 (0.08)	0.39 (0.08)	0.43 (0.09)
$\sigma_N$	—	—	1.43 (0.18)	1.33 (0.16)
$\sigma_{SR}$	0.96	0.92 (0.11)	0.92 (0.11)	1.02 (0.12)
$\sigma_Y^2/\sigma_z^2$	0.89	0.60	0.46	1.03

$\sigma_Y$ ,  $\sigma_C$ ,  $\sigma_H$ ,  $\sigma_X$ ,  $\sigma_K$ ,  $\sigma_N$ , and  $\sigma_{SR}$  denote the standard deviations of aggregate output, consumption, labor input, investment, capital, the number of firms, and the Solow residual, respectively. Reported moments are means of statistics computed from 1000 simulations, each 120 periods in length. Standard deviations are in parentheses.  $\sigma_Y^2/\sigma_z^2$  is the ratio of the variance of HP-filtered output to the variance of the technology shock,  $z$  (see Table 3).

The U.S. data is quarterly, 1960.1–1989.4.  $Y$  refers to real GNP (base 1982),  $C$  is total private consumption expenditures,  $X$  is gross capital formation, and  $H$  is total hours worked in industry (source: *Citibase*).  $K$  is the gross capital stock in 1982 dollars and was interpolated using the annual gross stock and quarterly gross capital formation (source: *OECD Flows and Stocks of Fixed Capital*). For the U.S. economy, variance of the Solow residual (Table 3) is used in the computation of the multiplier.

using the Solow residual in the first economy, the variance of technology shocks is 30.34% *larger* with constant returns to specialization and 27.80% *smaller* with increasing returns to specialization. Thus for these parameters increasing returns of either type has a significant effect on the variance of measured technology shocks. Also note that the assumption of increasing returns to specialization has only minor implications for the autocorrelation of the technology parameter.

Table 4 contains the averages and standard deviations over 1000 simulations of 120 periods each of the percent standard deviations of economy aggregates. The table also presents the corresponding second moments for the U.S. economy over the 120 quarters 1960.1–1989.4. These moments are computed from

HP-filtered data. The table also contains the measure of the technology shock multiplier as previously defined. In what follows we focus only on these second moments. The autocorrelations of aggregates in the benchmark competitive economy have been discussed at length elsewhere in the literature. Given the negligible effect of imperfect competition and increasing returns on the autocorrelation of technology shocks, the economies here all exhibit similar autocorrelations. Similarly, the comovements of aggregates over the cycle remain basically identical to those exhibited by a standard real business cycle model. Since the realizations of the innovation process (before scaling by the appropriate variance) are the same for each of the theoretical economies, the differences in moments are not due to sampling variability. Rather they are due to the combined effects of the different dimensions of increasing returns on both the measurement of technology shocks and the technology shock multipliers.

Under constant returns to specialization, the economy with imperfect competition and increasing returns to scale exhibits virtually identical cyclical properties to those of the perfectly competitive benchmark. This occurs in spite of the fact that the variance of the technology shocks experienced by this economy is 30.34% *greater* than that of those experienced by a standard real business cycle model. Because of the effects of increasing returns to scale, the lower technology shock multiplier results in a lesser response of aggregates in this economy to a given technology shock. This lower multiplier is exactly offset by the increase in the variance of technology shocks when the bias inherent in using the Solow residual is taken into account. Although not presented here, other experiments indicate that this finding is not affected by varying the degree of increasing returns to scale. Larger degrees result in larger shocks and smaller multipliers, but the two effects continue to offset one another. This finding may be interpreted as implying that if there is free entry, technology shocks account for the same share of the variance of aggregate output (here that share is 67%) in the presence of monopolistic competition and increasing returns to scale as they do in the standard real business cycle model.

When returns to specialization are introduced, the variance of output is increased by more than 24.5% even though the variance of technology shocks is *reduced* by 27.8% relative to the competitive economy and 44.6% relative to the economy with constant returns to specialization. All aggregates except  $N$ , including the Solow residual, are more volatile and the share of output fluctuations accounted for by technology shocks rises to 85%. This greater aggregate volatility is the combined effect of smaller technology shocks and a larger multiplier. For these parameters, the effect of the increased multiplier clearly dominates in spite of the fact that the variance of the number of firms is reduced.

The specific estimates of the increase in the multiplier and the fall in the variance of technology shocks depend on the the degree of increasing returns to specialization chosen. The overall finding that the effect of the increased multiplier dominates does not. Holding the markup ratio constant, it is possible to

lower the degree of returns to specialization by driving  $\lambda$  from 0 down to  $1 - 1/\rho$ . This exercise breaks the one-to-one correspondence between the markup ratio and the degree of returns to specialization. In this case, it can be shown that the variances of aggregates converge monotonically to those exhibited by the constant returns to specialization economy.<sup>10</sup> Thus, to the extent that technology shocks account for a substantial share of aggregate fluctuations under the assumptions of perfect competition and constant returns, this share is *increased* under monopolistic competition and increasing returns to specialization.

#### 4. Endogenous total factor productivity: Government spending shocks

The previous section showed that returns to specialization increase the multiplier associated with technology shocks. This effect is associated with an endogenous increase in total factor productivity that occurs during an expansion. We now demonstrate that the same phenomenon will arise in response to other types of shocks. With increasing returns to specialization, an expansion in government spending (i.e., a positive demand shock) will also increase productivity as it leads to firm entry. This exercise suggests that an endogenous response of total factor productivity can be associated with *any* expansionary shock, regardless of whether it directly affects technology.

The economy is modified to incorporate stochastic government spending, modeled in a very simple manner. The government consumes a time-varying portion of the final good each period and finances its consumption through a lump-sum tax on households. We assume that government consumption neither enters the representative consumer's utility function nor has any direct effect on aggregate production possibilities (as it would if it were used, for example, to augment a stock of public capital). These assumptions are intended to restrict attention to a simple expansionary shock, one which is well-known to have no effect on productivity under the assumptions of perfect competition and constant returns.

The government consumes  $G_t$  units of the final good in period  $t$ , where

$$\ln G_t = \omega_G \ln G_{t-1} + \theta_t, \quad \forall t, \quad (29)$$

$\omega_G \in (0, 1)$ , and  $\theta_t$  is an iid random variable, distributed normally with mean  $(1 - \omega_G) \ln \bar{G}$  and variance  $\sigma_\theta^2$ . Here  $\bar{G}$  is the steady state level of government spending. The government finances its consumption each period by levying

<sup>10</sup>This can be accomplished by estimating  $\omega_x$  and  $\sigma_\epsilon^2$  and simulating for each choice of  $\lambda$ . These results are not reported here, but are available on request.

a lump sum tax,  $\tau_t$ , to balance its budget:

$$G_t = \tau_t, \quad \forall t. \quad (30)$$

Given that government spending is financed by nondistorting taxation, it has no effect on the producer equilibrium conditions given by (15a)–(15e), the equilibrium factor prices, (16a) and (16b), or the equilibrium measure of firms, (16c). The representative consumer's dynamic problem, (P.1), changes only through the addition of another exogenous state variable,  $G$ , and through the budget constraint, which becomes:

$$c + x = w(K, H, z, G)h + r(K, H, z, G)k - \tau. \quad (31)$$

We again focus on a symmetric recursive equilibrium and solve the model as before. In equilibrium, an increase in government spending causes output to rise as it increases aggregate demand and stimulates labor supply due to the negative wealth effect of increased taxation. Thus government spending creates profit opportunities and provokes entry.

We again measure total factor productivity by the Solow residual. Under constant returns to specialization, total factor productivity is unaffected by the changes in the equilibrium number of firms and, therefore, by changes in government spending. This finding is independent of the degree of market power and the degree of increasing returns to scale; it depends only on the fact that as the number of firms rises, factors become no more productive in the making of final output.

In contrast, total factor productivity is endogenous in the presence of returns to specialization and will respond to changes in government spending. Consider the calibrated economy with increasing returns to specialization presented in Section 3. We add government spending to that economy calibrated as follows. Steady state government expenditure is chosen to reflect the average share of government purchases in quarterly U.S. GNP. For the period 1960.1–1989.4, this average share is 0.2097. Values for the parameters governing the stochastic process for government spending given by Eq. (29) were obtained by estimating that equation using this data. Resulting estimates are  $\omega_G = 0.973$  with a standard error of 0.016 and a point estimate of  $\sigma_\theta^2 = 0.00011$ . The particular calibration, however, has no effect on the qualitative findings that we focus on here.

Fig. 2 illustrates the dynamic responses of aggregates to a one-time government spending shock. In the diagram, aggregate output and the number of firms are labelled 'Y' and 'N' respectively. Total factor productivity is measured by the Solow residual and labelled 'SR'. A shock to government spending causes total factor productivity to rise. While the size of the increase in the Solow residual depends on the degree of returns to specialization, the finding that it will respond to changes in the level of government spending does not. As long as

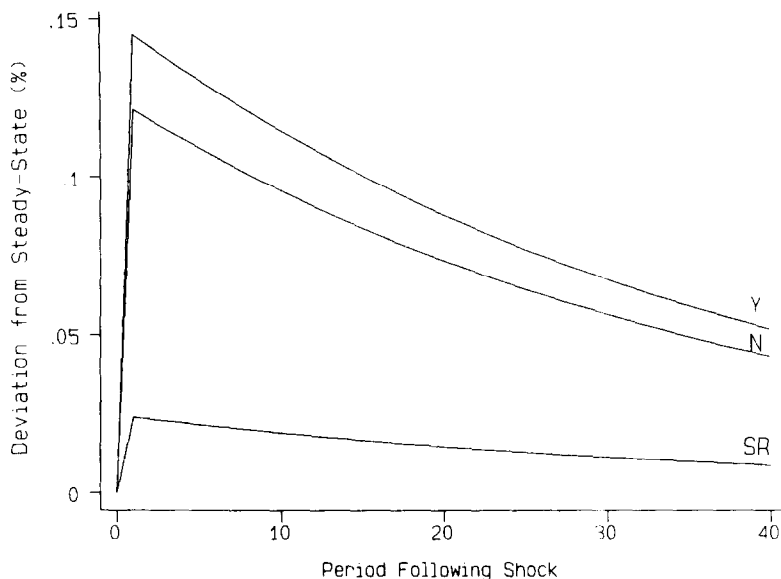


Fig. 2. Response to a government spending shock.

$Y$  denotes output,  $N$  the number of firms, and  $SR$  the Solow residual.

there are returns to specialization of any degree, total factor productivity closely tracks the path of government spending.

The effect depicted in Fig. 2 results not from the particular way in which government spending has been modeled, but only from the fact that an increase in aggregate demand stimulates an increase in the variety of intermediate goods. Therefore, we expect that any sort of aggregate demand disturbance will cause total factor productivity to rise and fall with aggregate output. This suggests that returns to specialization may not only augment the multiplier associated with technology shocks, but also those associated with aggregate demand shocks.

Empirical evidence that the Solow residual responds to changes in government spending has been documented by Evans (1992), Finn (1992), Hall (1990), and others. Their finding that total factor productivity is not exogenous is consistent with the model studied here. If we have correctly modeled the effect of government spending on total factor productivity, however, then our measures of technology should be invariant to nontechnological shocks. The exogeneity tests of Evans (1992) were performed using the corrected technology measure. The findings did not support the hypothesis of invariance for this measure of technology.<sup>11</sup> In fact, our measure of technology is highly correlated with the

<sup>11</sup>The results of these tests are available on request.

Solow residual for the period studied. This suggests that our modeling of the relationship between government spending and total factor productivity may be incomplete. For example, it is possible that government spending itself is directly productive, as in Aschauer (1989) and Braun and McGrattan (1993). We leave an exploration of this issue to future work.

## 5. Conclusions

This paper has considered the effects of technology shocks in a real business cycle model with monopolistic competition and increasing returns to both specialization and scale. In the theoretical economy, technology shocks create profit opportunities, inducing entry of producers of differentiated intermediate products. Market power *per se* and increasing returns due to fixed costs have no effect on the responses of aggregate variables to a technology shock vis-a-vis those exhibited by a standard, perfectly competitive real business cycle model. In contrast, the multiplier associated with technology shocks is reduced by returns to scale in variable factors and increased by returns to specialization. Returns to specialization increase the multiplier because of an endogenous response of total factor productivity which augments the direct effect of the shock on productivity. The multiplier associated with shocks to government spending (and presumably with other types of aggregate demand shocks) is also increased by returns to specialization.

The introduction of returns to scale and specialization also has implications for the measurement of technology shocks. When the technology shock measure is adjusted for the effects of these types of increasing returns, we find that returns to scale generally causes the variance of the Solow residual to under-measure the variance of the 'true' technology shock. Returns to specialization, on the other hand, produces a bias in the Solow residual in the opposite direction. Measured technology shocks account for the same share of U.S. output fluctuations in an economy with increasing returns to scale and constant returns to specialization as in a benchmark competitive economy. In contrast, when increasing returns to specialization are added, the economy exhibits an increase in the variance of output of more than 24% despite a reduction of the variance of technology shocks by almost 28%.

While our quantitative findings are sensitive to the details of the calibration, the qualitative finding that the effect of returns to specialization on the multiplier will exceed that of the reduction in the variance of technology shocks is not. These findings may be seen as complementary to those of Hornstein (1993) who showed that monopolistic competition and increasing returns to scale resulted in a minor reduction of the share of output fluctuations accounted for by technology shocks in an economy with a fixed number of firms. Our results show that if the constraint on firm entry is removed, then even the modest

reduction found by Hornstein may be eliminated, and to the extent that there are returns to specialization, the share of output variability accounted for by technology shocks may be greater than suggested by competitive models.

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