Exchange rate dynamics in a model of pricing-to-market

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Abstract

This paper develops a general equilibrium exchange rate model consistent with the weak empirical evidence supporting the law of one price. Some firms segment markets by country, and set prices in local currency of sale, a practice we refer to as pricing-to-market (PTM). The presence of PTM increases exchange rate volatility, relative to a situation where the law of one price holds. PTM also affects the international transmission of monetary and fiscal policy. The higher is the degree of PTM, the lower is the comovement in consumption across countries, but the higher is the comovement in output. In terms of welfare, monetary policy is a “beggar-thy-neighbor” instrument in the presence of a high degree of PTM. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

This paper develops a simple exchange rate model which combines international market segmentation by imperfectly competitive firms and local currency price-setting. We refer to this as a situation of “pricing-to-market” (PTM). We find that

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PTM plays a central role in exchange rate determination and in international macroeconomic fluctuations. It acts to limit the pass-through from exchange rate changes to prices, and reduces the traditional “expenditure switching” role of exchange rate changes. This leads to potentially much greater exchange rate variability than in an economy without PTM. Nominal price stickiness associated with PTM magnifies the response of the exchange rate to shocks to fundamentals. In addition, because PTM generates departures from purchasing power parity, it tends to reduce the comovement in consumption across countries, while increasing the comovement of output. Finally, the presence of PTM has important welfare implications for the transmission of monetary policy shocks.

While many international economists have argued that price stickiness is important for understanding real exchange rate fluctuations\(^1\), most exchange rate models are based on the principle of the law-of-one-price in internationally traded goods. Deviations from purchasing power parity arise only if there are movements in the relative price of nontradables to tradables across countries. However there is now a lot of evidence that, at least at high frequencies, real exchange rate fluctuations are mainly attributable to failures of the law of one price among traded goods. Engel (1993), and Engel and Rogers (1996), show that for many commodities the deviation from the law-of-one-price across international borders is far greater than can be explained by geographical distance or the costs of transportation\(^2\).

These results are consistent with separate evidence since the mid-1980’s of limited pass-through from exchange rate movements to import prices\(^3\). Marston (1990) estimates an average degree of exchange rate pass-through for industrialized countries of about 50%. In a series of subsequent papers, Knetter (1989), (1993), and Gagnon and Knetter (1995) find pass-through estimates at an even lower levels\(^4\).

A simple characterization of these findings would be that prices of many goods are set in the local currency of the buyer, and do not adjust at high frequencies, so that real exchange rate movements are driven primarily by fluctuations in the nominal exchange rate. This characterization implies that price/cost markups

\(^1\)See, for instance, Dornbusch, 1987; Mussa, 1986; Svensson and van Wijnbergen, 1989; Stockman and Ohanian, 1993, and Obstfeld and Rogoff, 1995.

\(^2\)Engel (1993) presents evidence that relative price variability of identical goods across international borders is far greater than that between differentiated goods within a single country, using disaggregated Canadian and US price data. From disaggregated consumer good price series from a set of Canadian and US cities, Engel and Rogers (1996) show that deviations from the law-of-one-price are far greater between cross-border cities at a given geographical distance than for within-country cities at the same distance.

\(^3\)See for instance Mann (1986); Krugman (1987); Giovannini (1988); Marston (1990); Knetter (1989), (1993), among many others.

\(^4\)Goldberg and Knetter (1997) provide a comprehensive review of the empirical evidence on market segmentation, exchange rate pass-through, and pricing-to-market.
fluctuate endogenously in response to exchange rate movements, rather than nominal prices themselves. Knetter (1993) refers to the implied price behaviour as “local currency price stability”. Overall, the evidence from a large number of studies supports the presence of local currency price stability, as well as the idea that segmented markets and local currency invoicing support this form of pricing.

This paper develops a model that is consistent with these observations, and explores its implications for real and nominal exchange rate variability and the international transmission of monetary and fiscal policy shocks. There are two important features of the model. The first is that (traded) goods are characterized by a significant degree of national market segmentation. For goods that are sold in segmented markets, trade is carried out by firms alone, and households cannot arbitrage away price differences across countries. The second feature is that firms engage in short-term nominal price-setting. For firms that segment domestic from foreign markets, and engage in local currency pricing, prices are sticky in terms of the local currency of the markets in which they are selling.

There is a substantial literature on the strategic determinants of international pricing-to-market behaviour, beginning with the well-known contributions of Dornbusch (1987) and Krugman (1987). Most of this takes a partial equilibrium perspective, however. Taking the stochastic process for the exchange rate as given, the consequences of market segmentation for the pass-through of nominal exchange rate changes to prices is explored.

By contrast, our analysis takes the converse approach. We employ a general equilibrium framework, in which the exchange rate is endogenous, while the structure of national market segmentation and local-currency pricing is exogenous. In the absence of sticky prices, there would be no consequence of pricing-to-market in our model, since there are no strategic reasons for having price-cost markups varying across countries. In fact, without nominal rigidities, the law-of-one-price would obtain for all goods, and PPP would hold in the aggregate, despite the presence of international market segmentation. But when local-currency prices are sticky, unanticipated shocks to the exchange rate will lead to deviations from the law-of-one-price. In effect, the relationship of our model with the previous

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1 Page (1981) presents direct evidence that many export goods for a number of industrialized countries are invoiced in the local currency of the buyer rather than in the currency of the seller. For Japan, firms invoice as much as 62% of all exports in dollars for example.

2 Dixit (1989); Froot and Klemperer (1989); Giovannini (1988); Kasa (1992) and Krugman (1987) present partial equilibrium analyses which can rationalize pricing-to-market and imperfect pass-through of nominal exchange rate movements to import prices under certain industry level conditions.

3 Goldberg and Knetter (1997) describe this phenomenon as “short-run PTM”. Our approach is consistent with the evidence presented above that nominal exchange rate volatility produces short-run deviations in common currency prices. This is also argued in Engel and Rogers (1995) and Wei and Parsley (1995), while evidence presented in Verboven (1996) suggests that absolute or long-run deviations from the law of one price are associated with various types of trade barriers.
pricing-to-market literature lies in the fact that international market segmentation is a necessary condition for local-currency price setting, in the face of unanticipated shocks to the exchange rate.

The model is built around an economy with differentiated products. A fraction of all goods are sold in segmented markets where firms can price-discriminate across countries. In keeping with the evidence, we assume that these firms engage in local currency pricing. We can vary the fraction of firms that engage in PTM, and explore the consequence for the exchange rate and other variables. With a high degree of PTM, an exchange rate depreciation has little effect on the relative price of imported goods facing domestic consumers. This weakens the allocative effects of exchange rate changes, compared with a situation where prices are set in the seller’s currency (where pass-through of exchange rates to prices would be immediate). PTM therefore reduces the classic “expenditure switching” effects of exchange rate depreciation, which conventionally induce a shift in world demand towards exports of the country whose currency is weakening.

Because with PTM, domestic prices show little response to a depreciation, the equilibrium exchange rate response is magnified. A simple quantitative exercise shows that the increase in exchange rate variability may be quite large. With PTM, exchange rates may vary by more than “fundamentals”. Thus, the model is consistent with the Dornbusch (1987) argument that price stickiness magnifies the response of exchange rates to fundamentals.

PTM also affects the international transmission of macroeconomic shocks. Without PTM, monetary disturbances will tend to generate high positive comovements of consumption across countries, but large negative comovements of output. With PTM, this ordering is reversed. The deviations from PPP generated by PTM lead consumption comovements to fall. At the same time, the elimination of the expenditure switching effects of the exchange rate under PTM enhances the comovement of outputs across countries.

Finally, PTM also has quite important welfare implications. In the PPP-based environment, an unanticipated monetary expansion raises welfare of all agents, both domestic and foreign. But with PTM, a domestic monetary expansion tends to raise home welfare, while at the same time reducing foreign welfare. Monetary policy is a “beggar-thy-neighbor” instrument in the presence of PTM. This implies that the strategic implications for international monetary policy coordination are therefore very different than in a PPP environment.

Our paper is quite related to a number of recent papers which explore the consequences of price stickiness in dynamic general equilibrium models of the

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8The expenditure switching effect operates in both the traditional Mundell-Fleming model, as well as the newer model of Obstfeld and Rogoff (1995).
9Backus et al. (1992) point out that observed low cross country consumption correlation and high cross country output correlations are anomalous when compared to the predictions of models based on PPP.
open economy. In particular, we build directly on the paper of Obstfeld and Rogoff (1995), who construct a very similar model, except with the assumption that the law-of-one-price holds in traded goods at all time. Svensson and van Wijnbergen (1989) is an early contribution which looks at monopolistic competitive price setting in a cash-in-advance economy. They do not model the production or employment decisions of firms directly however. Stockman and Ohanian (1993) develop a two country model in which some prices are sticky while some remain flexible. In all of these papers, the law-of-one-price remains true for traded goods. In Betts and Devereux (1996), a static model based on segmented markets and pricing-to-market is introduced and applied simply to the issue of nominal exchange rate volatility. The present paper extends that approach to a dynamic context, provides a much more comprehensive analysis of the features of the model, examines international macroeconomic transmission effects, and focuses on welfare issues.

The rest of the paper is organized as follows. The next section develops the model. Section 3 examines the impact of PTM on exchange rate variability, while Section 4 examines in detail the impact of monetary and government spending shocks in the presence of PTM. Section 5 reports the results of a brief calibration exercise. Section 6 discusses welfare issues. Conclusions are contained in Section 7.

2. The model

The model is structured around that of Obstfeld and Rogoff (1995). There are two countries; home and foreign. Foreign country variables are represented with an asterisk. Households in each country consume a group of differentiated goods of total measure unity. Of these goods a fraction \( n \) are produced by the home country. Since the first draft of this paper, we have received three papers which address similar issues. Kollman (1997) examines the impact of monetary policy shocks in a sticky price model of a semismall economy, where all prices are set in the local currency. He develops a quantitative version of the model with more persistence in price stickiness than we have here. He shows the possibility of exchange rate overshooting and high correlations between real and nominal exchange rates. Chari et al. (1997) also develop a model with complete local currency price-setting and persistent sticky prices. They introduce endogenous investment, and investigate the model’s potential for explaining real exchange rate variability and persistence. Their model is also solved quantitatively. Hau (1998) explores the consequences of monetary shocks in a version of the Obstfeld and Rogoff (1995) model, with nominal wage rigidities in labor markets rather than nominal price rigidities in goods markets. His focus is on the role of nontradables for real exchange rate determination. There is full pass-through of exchange rate changes to prices however, and PPP deviations result from monetary shocks only to the extent that the relative price of nontradables is affected. Thus, the connection between our paper and his is quite limited, despite the similarity of the framework. By contrast to the first two of these papers, our paper uses analytical solution techniques, subsumes the PPP model as a special case, undertakes a welfare-based analysis and, in addition, investigates the impact of government spending shocks.
country, and $1 - n$ are produced in the foreign country. In addition, $n$ and $1 - n$
also represent the home and foreign country population, respectively.

Each good is sold exclusively by a price-setting firm. A fraction $s$ of firms in
each country can segment their markets by country. These firms can then in
principle “price-to-market” (PTM). They set prices separately for the home and
the foreign market, since by assumption only firms can trade these goods. Prices
for these goods will be set in local currency. The remaining $1 - s$ goods can be
freely traded by consumers, so that firms must set a unified price across the two
countries. Prices for these goods are set in the currency of the seller.

2.1. Households

Preferences are identical across countries. Home economy residents have
preferences over consumption given by:

$$U = \sum_{i=0}^{\infty} \beta^i \left( \log C_t + \frac{\gamma}{1 - \epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} + \eta \log(1 - h_t) \right)$$

Where $C_t = (f^i c(i, t)^{\rho(1-\rho) \epsilon} d_i)^{\rho/(1-\rho)}$ ($\rho > 1$). Here $c(i, t)$ is the consumption of
good $i$ at time $t$. $h_t$ represents total hours worked by the domestic household.
Households also value real money balances ($M_t/P_t$), where $M_t$ are nominal
balances and $P_t$ is the home country CPI, defined as

$$P_t = \left[ \int_0^n (p(i, t))^{1-\rho} \, di + \int_n^{n+(1-n)s} (p^*(i, t))^{1-\rho} \, di \right]^{1/(1-\rho)}$$

We employ the convention of letting prices denoted $p_i$ represent home currency
prices, while prices denoted $q_i$ represent foreign currency prices. Here $p(i, t)$ is the
home currency price of the home-produced good, $p^*(i, t)$ is the home currency
price of a foreign PTM good $i$, while $q^*(i, t)$ is the foreign currency price of a
foreign non-PTM good. The exchange rate (home unit cost of foreign currency) is
given by $e_t$.

Home country households receive income from wages, $W_i h_t$, and profits on their
ownership of domestic firms, $\pi_t$. They also receive transfers $TR_t$ from the
government. Their budget constraint is

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1In Appendix B, we discuss the implication of more general preferences.
2Most of the results would be unaffected if money were to enter the model due to a cash-in-advance
or transactions cost motive.
\[ P_t C_t + M_t + d_t F_t = W_t h_t + \pi_t + M_{t-1} + \text{TR}_t + F_{t-1} \]  
\( (1) \)

where \( M_t \) is the money holding at the beginning of the period, \( F_{t-1} \) represents holdings of home currency denominated nominal bonds, and \( d_t \) is price of a bond (the inverse of one plus the nominal interest rate). There is free trade between countries in a nominal bond. We let this bond be denominated in domestic currency units\(^{13}\).

Households maximize utility subject to (1) and optimally allocate consumption between each of the differentiated goods as follows

\[ c(i, t) = \left( \frac{v(i, t)}{P_t} \right)^{-\rho} C_t \]  
\( (2) \)

where \( v(i, t) \) is equal to either \( p(i, t) \), \( p^*(i, t) \), or \( eq^*(i, t) \), depending upon which category good \( i \) falls within.

In addition, the household’s optimal money demand schedule can be written as

\[ \frac{M_t}{P_t} = \left( \frac{\gamma C_t}{1 - d_t} \right)^{\frac{1}{\rho}} \]  
\( (3) \)

The optimal labour supply decision is characterized by

\[ \frac{\eta}{1 - h_t} = \frac{W_t}{P_t C_t} \]  
\( (4) \)

Finally, the household’s intertemporal consumption stream is chosen such that

\[ d P_{t+1} C_{t+1} = \beta P_t C_t \]

The situation of foreign households is entirely analogous.

2.2. Government

Governments in each country print domestic currency and impose taxes/transfers to finance government consumption. Assume that the composite government consumption good in each country is defined in the same way as the private composite consumption. Then, letting \( G_t \) represent per capita home country composite government consumption, the home country government’s demand for good \( i \) is

\[ g(i, t) = \left( \frac{v(i, t)}{P_t} \right)^{-\rho} G_t \]

The home country government budget constraint is

\(^{13}\)The absence of a real, indexed bond is important. In the presence of PTM among a fraction of commodities, prices are not necessarily equated across countries. This leads to a failure of real interest rate equalization. With a consumption indexed bond, this could not happen.
The foreign country’s government composite consumption and budget constraint is analogously defined.

2.3. Firms

A home firm $i$ has production function

$$y(i, t) = h(i, t)$$

where $y(i, t)$ is total output of the firm, and $h(i, t)$ is employment. For firms that segment markets, let total output be divided between output sold at home, given by $x(i, t)$, and output sold abroad, given by $z(i, t)$.

The firm hires labour domestically and chooses $p(i, t)$ and $q(i, t)$, the nominal prices in the home and foreign market, respectively. Profits are given by

$$\pi_i = p(i, t)x(i, t) + e_iq(i, t)z(i, t) - W_i(x(i, t) + z(i, t))$$

Imagine that firms set prices after the exchange rate and costs are known; i.e. nominal prices are flexible. Since a PTM firm can price-discriminate across countries, it can set $p(i, t)$ and $q(i, t)$ separately to maximize profits. It faces the demand schedule given by (2) and the (essentially identical) domestic government’s demand schedule, and analogous schedules for the foreign consumer and government. The price markups will be chosen as

$$p(i, t) = e_iq(i, t) = \frac{\rho}{\rho - 1} W_i \quad \text{(5)}$$

Since elasticities of demand are the same in each market, the law-of-one-price must hold even for goods where firms can “price-to-market”. It is immediate that for firms that do not segment markets, the optimal price-wage markup will be identical.

The foreign firm’s pricing policy is described analogously, so that, for the foreign firm $i$, $p(i, t)^* = e_iq(i, t)^*$. It follows that, with flexible prices, PPP must hold, since the prices of all goods are equalized across countries, so that $P = e_iP^*$. We may define the home country export price index as

$$P_i = \int_{0}^{(1-s)n} p(i, t)^{1-\rho} \, di + \int_{(1-s)n}^{n} (e_iq(i, t))^{1-\rho} \, di \left(\frac{1}{1-\rho}\right)$$

Of the $n$ goods that the home country produces, $(1-s)$ are priced in home currency and $s$ in foreign currency. Defining the foreign country export price index in the same way, the terms of trade $\tau_i$ may be written as
\[ \tau_t = \frac{r_t^*}{e_t F_t^*} \]

3. Exchange rate variability with PTM

3.1. Flexible prices

With flexible prices, world general equilibrium is simple to construct. To simplify notation, focus on an initial equilibrium where \( G = G^* = 0 \) and initial international debt claims are zero. The world market for each good and the world capital market must clear, labour supply must equal labour demand for each country, and money market equilibrium must hold in each country also. A stationary rational expectations equilibrium will be attained immediately in this environment. It is straightforward to show that the equilibrium values for employment, consumption, and the exchange rate, are given by

\[ h = h^* = \frac{\rho}{\left( \frac{\rho - 1}{\rho} + \eta \right)} \]  

(6)

\[ C = \frac{P}{P^*} h = \bar{h} \]  

(7)

\[ C^* = h^* \]  

(8)

\[ e = \frac{M}{M^*} \left( \frac{C}{C^*} \right)^{\frac{1}{\epsilon}} \]  

(9)

In addition, given the symmetry of the model, \( F_t = 0 \) must hold for all \( t \), and the terms of trade \( \tau_t \) must equal unity. Note that by (6), employment is inefficiently low, due to the monopoly price distortions. The socially efficient level of employment is \( 1/(1 + \eta) \) in each country.

In this flexible-price economy, money is completely neutral. More generally, due to the flexibility of prices and equal markups across countries, the presence of segmented international markets has no aggregate implications at all.

3.2. Sticky prices

Now assume that prices are set in advance and cannot respond to shocks within the period. Assume that firms set prices at a level so as to achieve the optimal markup in the absence of shocks. Given preset prices, firms then produce so as to meet ex-post market demand.

For PTM firms, nominal prices are set in the local currency of sale. For other
firms, the nominal price will be set in the domestic currency. Therefore, for the first category of goods, unanticipated changes in the exchange rate leads to deviations from the law-of-one-price. For the second category, the law-of-one-price holds.

With this new constraint, we explore the response of the exchange rate to an unanticipated (monetary or fiscal) shock at some time $t$.

Prices will adjust fully after a single period. Therefore, since there is no capital in the model, the long-run response to a shock is attained at time $t+1$. To derive the impact of the shock on the exchange rate, it is useful to proceed by first deriving the time $t+1$ response of the economy to a shock to money, government spending, and international bonds $F$, under flexible prices. Then the response to shocks with fixed prices (the period $t$ response) can be derived with the aid of these long-run responses.

Assume that, at time $t+1$, all changes in $M, M^*_s, G, G^*_s$ are permanent. It is assumed that the world economy is in an initial steady state with no government spending, identical output and consumption levels, and no international asset claims outstanding. Then we solve the system by linear approximation around the initial, zero-shock equilibrium with $F = G = G^* = 0$, as defined previously. For any variable $U$, let $\dot{U} = (U - \bar{U})/\bar{U}$, where $\bar{U}$ is the zero-shock equilibrium value. Thus $\dot{U}$ represents the proportional deviation from the initial equilibrium defined by (6)–(9).

Appendix A shows that the long-run (i.e. period $t+1$ onwards) response of the exchange rate and relative consumption levels is given by

$$\dot{e}_{t+1} = \dot{M}_{t+1} - \dot{M}_s^{*} - \frac{1}{\varepsilon} (\dot{C}_{t+1} - \dot{C}_s^{*})$$  \hspace{1cm} (10)$$

$$\dot{C}_{t+1} - \dot{C}_s^{*} = (1/\sigma) \frac{dF(1-\beta)}{\bar{P}\bar{C}^*(1-n)} - (1/\sigma) \frac{(dG_{t+1} - dG^*_s)}{\bar{C}_w}$$  \hspace{1cm} (11)$$

where $\bar{P}$ represents the initial price level, $\sigma = (\rho - 1 + \rho \eta)/(\rho - 1 + \eta)$ and $\bar{C}_w$ represents the initial value of world consumption; i.e. $C_w^* = nC_t + (1-n)C^*_t$. Eq. (10) comes from money market clearing conditions. Eq. (11) comes from goods market clearing, labour market clearing, and balance of payments equilibrium conditions. It says that a home country trade balance surplus in period $t$, implying $dF_t > 0$, will permanently raise home consumption relative to foreign consumption, for a given pattern of government spending.

In period $t$, in which prices are preset, we may describe an equilibrium by the following equations

$$\frac{M_t}{P_t} = \left( \frac{\gamma C_t}{1 - d_t} \right)^{\frac{1}{\varepsilon}}$$  \hspace{1cm} (12)$$
Eqs. (12) and (13) are just money market clearing conditions. Eqs. (14) and (15) are the national budget constraints (or balance of payments equations). In particular (14) says that per capita domestic nominal expenditure must equal per capita nominal income from non-PTM firms $(1 - s)p_t x_t + s(p_t x_t + e_t q_t z_t)$, plus nominal income from PTM firms $(d_t / e_t) F_t^* = (1 - s) q_t^* y_t^* + s \left( \frac{p_t^*}{e_t^*} x_t^* + q_t^* z_t^* \right)$.

Eq. (15) has a similar interpretation for the foreign country. Here $F_t^*$ represents the purchase of home currency denominated bonds by foreign residents, which, in terms of foreign currency, have price $(d_t / e_t)$. Eqs. (16)–(21) represent goods market clearing equations, for home and foreign non-PTM firms, and for home and foreign PTM firms in both home and foreign markets, respectively. Finally, Eqs. (22) and (23) represent the Euler equations for optimal home and foreign consumption growth.

For given nominal prices, and given period $t + 1$ values, the system (12)–(23), along with the definitions of $P_t$ and $P_t^*$, gives twelve equations in the eleven

\[
\frac{M_t^*}{P_t^*} = \left( \frac{\gamma C_t^*}{1 - d_t^* e_{t+1}^*} \right)^{\frac{1}{\delta}}
\]

\[
P_t^* C_t + P_t^* G_t + d_t^* F_t = (1 - s)p_t y_t + s(p_t x_t + e_t q_t z_t)
\]

\[
P_t^* C_t^* + P_t^* G_t^* + (d_t / e_t) F_t^* = (1 - s) q_t^* y_t^* + s \left( \frac{p_t^*}{e_t^*} x_t^* + q_t^* z_t^* \right)
\]

\[
y_t = \left( \frac{P_t}{P_t^*} \right)^{-\rho} n(C_t + G_t) + \left( \frac{p_t}{e_t P_t^*} \right)^{-\rho} (1 - n)(C_t^* + G_t^*)
\]

\[
y_t^* = \left( \frac{e_t q_t}{P_t^*} \right)^{-\rho} n(C_t + G_t) + \left( \frac{q_t}{P_t^*} \right)^{-\rho} (1 - n)(C_t^* + G_t^*)
\]

\[
x_t = \left( \frac{P_t}{P_t^*} \right)^{-\rho} n(C_t + G_t)
\]

\[
z_t = \left( \frac{q_t}{P_t^*} \right)^{-\rho} (1 - n)(C_t^* + G_t^*)
\]

\[
x_t^* = \left( \frac{q_t^*}{P_t^*} \right)^{-\rho} n(C_t + G_t)
\]

\[
z_t^* = \left( \frac{q_t^*}{P_t^*} \right)^{-\rho} (1 - n)(C_t^* + G_t^*)
\]

\[
d_t P_{t+1} C_{t+1} = \beta P_t C_t
\]

\[
d_t \frac{e_{t+1}}{e_t} P_{t+1} C_{t+1}^* = \beta P_t^* C_t^*
\]
unknowns \{C_t^*, C^*, e_t, y_t^*, x_t, z_t^*, y_t, d_t, F_t\}. One equation is redundant by Walras’ Law.

First, from the definitions of the price indices, \(P_t\) and \(P_t^*\), it is easy to verify that

\[
\hat{P}_t = (1 - n)(1 - s)\hat{e}_t
\]

(24)

and

\[
\hat{P}_t^* = -n(1 - s)\hat{e}_t
\]

(25)

With sticky prices, the response of aggregate price indices to an exchange rate depreciation is lower, the greater the share of goods subject to PTM. In the limit, as \(s \to 1\), \(P_t\) and \(P_t^*\) are entirely unaffected by an exchange rate depreciation. In this case, exchange rate depreciation has no impact at all on the relative prices facing households in either country since no firm adjusts nominal sale prices.

### 3.3. The response of the exchange rate and relative consumption

We can now derive the models’ implications for shocks to two key variables; the exchange rate, \(\hat{e}_t\), and relative consumption movements, \(\hat{C}_t - \hat{C}_t^*\). The response of all other variables can be directly computed from these two solutions.

Using (22), (23), (24) and (25), we calculate linear approximations to the money market clearing Eqs. (12) and (13). This gives

\[
\hat{e}_t (1 - s) = (\hat{m}_t - \bar{M}_t^*) - \frac{1}{\epsilon} (\hat{C}_t - \hat{C}_t^*) - \frac{1}{\epsilon r} (\hat{e}_t - \hat{e}_{t+1})
\]

(26)

where \(r = ((1 - \beta)/\beta)\) is the steady state real interest rate.

From (22) and (23), noting that PPP will obtain in period \(t + 1\), we may deduce that

\[
\hat{C}_{t+1} - \hat{C}_{t+1}^* = \hat{C}_t - \hat{C}_t^* - s\hat{e}_t
\]

(27)

The presence of PTM, by generating a departure from PPP, leads ex-post real interest rates to diverge across countries. This means that consumption growth will not be equated, despite the presence of free trade in nominal bonds.

Now, using (10) and (27), in (26), noting that since money shocks are permanent, we may describe the movement of the exchange rate as

\[
\hat{e}_t \left(1 - s + \frac{(e - s)}{\epsilon r}\right) = (\hat{M}_t - \bar{M}_t^*) \left(1 + \frac{1}{\epsilon r}\right) - \frac{1}{\epsilon} \left(1 + \frac{1}{\epsilon r}\right) (\hat{C}_t - \hat{C}_t^*)
\]

(28)

This corresponds to the fundamental exchange rate equation common to the standard monetary approach to the exchange rate. The term \((1 - s)\) enters on the left hand side of this equation due to the fact that the size of \(s\) determines the
magnitude of the departure from PPP. When \( s = 1 \), the exchange rate does not affect the price level at all. Moreover, when, in addition, \( \epsilon = 1 \), the nominal interest rate differential is unaffected by movements in the exchange rate, and the exchange rate drops out of this equation altogether.

Now take a linear approximation of (14) and (15), using (16)–(21), and subtracting, we arrive at

\[
\hat{e}_t = \frac{\hat{C}_t - \hat{C}^*_t}{(1 - s)(\rho - 1) + s} + \frac{\beta dF_t}{\hat{P}_C^*(1 - n)} + \frac{(dG_t - dG^*_t)}{\hat{C}^w(1 - s) + s} \tag{29}
\]

To see the intuition behind (29), hold \( F_t \) and \( dG_t - dG^*_t \) constant, and ask what effect an exchange rate depreciation would have on home and foreign consumption. When \( s = 0 \), an exchange rate depreciation causes a rise in the relative price of foreign goods facing households and governments in both countries. World demand shifts towards the home country, leading to an increase in production, income, and therefore consumption in the home country, relative to the foreign country. The increase in domestic relative to foreign consumption is proportional to the reallocation of world demand, which is determined by the magnitude of \( \rho - 1 \), i.e. the elasticity of substitution between goods.

Alternatively, take instead the case of an exchange rate depreciation under complete PTM (\( s = 1 \)). Relative prices facing households and governments in both countries remain unaffected by this. However, relative incomes are affected. When export prices are set in foreign currency, a depreciation raises the home currency earnings of home firms, and reduces the foreign currency earnings of foreign firms, at given production levels. Thus a depreciation generates a world redistribution of income towards the home country, which raises home consumption relative to foreign consumption. This occurs without the influence of relative price changes.

Now substituting from (11) and (27) into (29) to get a relationship between the endogenous variables \( \hat{e}_t \) and \( \hat{C}_t - \hat{C}^*_t \) given by (30).

\[
\hat{e}_t = \frac{(1 + \sigma/r)(\hat{C}_t - \hat{C}^*_t) + (dG_t - dG^*_t)}{(1 - s)(\rho - 1) + s(1 + \sigma/r)} \left( \frac{(1 + \sigma/r)}{1 + \sigma/r} \right) \tag{30}
\]

Then (28) and (30) may be combined to compute the impact of unanticipated monetary and fiscal policy shocks on the period \( t \) value of the exchange rate. This is given by

\[
\hat{e}_t = \frac{(1 + \sigma/r)(M_t - M^*_t) + (dG_t - dG^*_t)}{(1 + \sigma/r)(1 + \sigma/r) + \epsilon(1 + \sigma/r) + \rho - 1 + s(1 + \sigma/r) + (1 + \sigma/r)/(1 + \sigma/r)} \tag{31}
\]
If government spending shocks are purely transitory, then $dG_{t+1} = 0$, in this equation, while for permanent spending shocks, $dG_{t+1} = dG_t$. Then the relative consumption response is

$$
\hat{C}_t - \hat{C}^*_t = \left( \frac{(1-s)(\rho - 1)}{1 + \sigma/r} + s \right) \hat{e}_t
$$

$$
\frac{(dG_t - dG^*_t)}{C^*} + (1/r) \frac{(dG_{t+1} - dG^*_{t+1})}{C^*}
$$

$$
1 + \sigma/r
$$

where $e_t$ is given in (31).

From (31) we see that unanticipated domestic money or government spending shocks generate exchange rate depreciation, whatever the size of $s$. The logic behind the depreciation with full PTM is quite different than in the PPP case however.

Take the case of a monetary shock. With full PTM, both countries’ price levels are unaffected by the shock. Therefore, through the money market equilibrium, (26), the impact of a surprise home country monetary expansion must be felt through either a rise in $\hat{C}_t - \hat{C}^*_t$ or a fall in the home (relative to the foreign) nominal interest rate. From (10), (11), and (28), it is evident that in the simple case $e = 1$, the nominal interest differential rate is unchanged, and a home country monetary expansion raises $\hat{C}_t - \hat{C}^*_t$ proportionally. Then the rise in $\hat{C}_t - \hat{C}^*_t$ precipitates exchange rate depreciation through (30). When $e \neq 1$ the situation is slightly more complicated, but both $\hat{C}_t - \hat{C}^*_t$ and $\hat{e}_t$ must still be positive.

How does the presence of PTM affect the response of the exchange rate to policy shocks? From (31), it can be seen that with PPP (i.e. $s = 0$), the extent of depreciation is lower, the higher is the elasticity of demand $\rho$. The reason is as follows. Exchange rate depreciation leads to an expenditure switching effect; world demand shifts away from foreign goods and towards home goods. The higher is $\rho$, the greater is the substitutability between home and foreign goods, and the smaller is the depreciation itself. But with full PTM, the substitutability between goods across countries is immaterial, since the relative price of home to foreign goods faced by consumers does not change.

To be more precise, it can be seen from (31) that a rise in $s$ will magnify the response of the exchange rate to both monetary and government spending shocks so long as

$$
\rho - 1 + (\epsilon - 1) \frac{(1 + \sigma/r)}{(1 + 1/\epsilon r)} > 0
$$

Since $\rho > 1$, this says that a sufficient condition for PTM to magnify the response of the exchange rate to monetary and fiscal shocks is that $\epsilon \geq 1$, i.e. the consumption elasticity of money demand is no greater than unity. Empirical estimates of this elasticity are consistently at unity or below (See Mankiw and Summers (1986) and Helliwell et al. (1990)). We may conclude therefore that
PTM is likely to raise the volatility of exchange rates, for a given pattern of fundamental shocks.

4. Analysis of monetary and government spending shocks

We now examine in detail the movement in exchange rates and other macroeconomic aggregates for monetary and government spending shocks. We begin with the analysis of monetary shocks.

4.1. Monetary shocks

Is it possible for a monetary expansion to generate “overshooting” of the nominal exchange rate? That is, can the initial (time \( t \)) response of the exchange rate exceed the long-run (time \( t+1 \)) response?

From (10), (27) and (29), we have

\[
\hat{e}_{t+1} = \hat{M}_t - \hat{M}^*_t - \frac{(\rho - 1)(1-s)}{(1+\sigma/r)} \hat{e}_t.
\]

(32)

Note, from (32), that for \( s<1 \), the long run rise in the nominal exchange rate is less than the movement in relative money supplies. As explained in Obstfeld and Rogoff (1995), an unanticipated home monetary expansion will generate a home country trade surplus in time \( t \), leading to a rise in \( F \) and a permanent rise (fall) in home country (foreign country) consumption. This mitigates the depreciation of the home currency (c.f. Eq. (10)).

When \( s=1 \), however, \( \hat{e}_{t+1} \) is proportional to the movement in relative money supplies. This is because, as we demonstrate below, with \( s=1 \) there are no trade balance implications of monetary shocks at all. In that case, future consumption is unaffected by current money shocks, and the response of the long-run exchange rate is exactly as in the flexible-price economy.

From (32) we may derive

\[
\hat{e}_{t+1} - \hat{e}_t = -s \frac{(\epsilon - 1)}{\epsilon(1+\sigma/r)} \hat{e}_t.
\]

In the absence of PTM (when \( s=0 \)), the response of the exchange rate to a money shock cannot display overshooting. But when \( s>0 \), the exchange rate overshoots, so long as the consumption elasticity of money demand is below unity. Take the limiting case where \( s=1 \). Then, while the long run exchange rate response is proportional to relative money shocks, the first period response, from (31) is

\[
(\hat{M}_t - \hat{M}^*_t) \frac{\epsilon + 1/r}{1+1/r}
\]

which exceeds the long run response when \( \epsilon>1 \).
Since uncovered interest rate parity must hold in this environment, overshooting of the nominal exchange rate must imply that a monetary expansion reduces the home currency nominal interest rate relative to the foreign nominal interest rate; i.e. there must be a “liquidity effect” in relative short term interest rates.

How does the real exchange rate respond to money shocks? From (24) and (25) above, the response of the real exchange rate to a money shock is

\[ \hat{P}_t^* + \hat{\varepsilon}_t - \hat{P}_t = s \hat{\varepsilon}_t, \]

Then, as \( s \to 1 \), the response of the real and nominal exchange rate become identical.

We now wish to explore the impact of monetary disturbances on consumption and output for each country, as well as the current account. To do this, it is first helpful to derive the response of the terms of trade and real interest rates. The response of the terms of trade is

\[ \hat{r}_t = (2s - 1) \hat{\varepsilon}_t, \]

The direction of terms of trade movement depend crucially on the degree of pricing-to-market. When \( s = 0 \), the terms of trade will deteriorate. Since prices are set in the exporters currency, the exchange rate depreciation raises the home currency price of imports, but leaves export prices unchanged. When \( s = 1 \), the terms of trade must improve, since when prices are set in the importers currency, a depreciation will raise the home currency price of exports, but leave import prices unchanged. When \( s = \frac{1}{2} \), the rise in export prices just cancels out with the rise in import prices, and the terms of trade is unchanged. Therefore, the terms of trade will improve, following a depreciation, for all \( s > \frac{1}{2} \).

Real interest rates may differ across countries due to the failure of PPP. From (27),

\[ \hat{C}_{t+1} - \hat{C}_t = \hat{C}_{t+1}^* - \hat{C}_t^* - s \hat{\varepsilon}_t, \]  \hspace{1cm} (33)

Using this and (4)–(12), we may derive the response of individual real interest rates as

\[ \hat{r}_t = - \frac{(1 + r)}{r} ((1 - n)s \hat{\varepsilon}_t + \hat{C}_t^*) \]  \hspace{1cm} (34)

\[ \hat{r}_t^w = \frac{(1 + r)}{r} (ns \hat{\varepsilon}_t - \hat{C}_t^w) \]  \hspace{1cm} (35)

where \( \hat{C}_t^w \) denotes the response of world consumption, which from the linear approximation to Eqs. (12)–(15), is
\[
\hat{C}_t^w = \frac{e + \frac{1}{r}}{1 + \frac{1}{r}} (n\hat{M}_t + (1 - n)\hat{M}_t^*) 
\]

(36)

Since real interest rates are directly proportional to consumption growth, (33) makes clear that an expansionary home country disturbance of either type will reduce the home real interest rate relative to the foreign real interest rate, as long as \( s > 0 \). Eqs. (34) and (35) illustrate that for a home country money shock, the home real interest rate will always decline, whereas the direction of movement in the foreign real interest rate is ambiguous. If \( e = 1 \) and \( s = 1 \), then the foreign real interest rate is actually unchanged by a home country money shock.

From the definition of \( C^w \), using the linear approximation to (14)–(21), it can be shown that

\[
\hat{C}_t = \hat{C}_t^w + (1 - n)(\hat{C}_t - \hat{C}_t^*) \\
= \hat{C}_t^w + (1 - n) \left( \frac{(1 - s)(\rho - 1)}{1 + \sigma/r} + s \right) \hat{c}_t 
\]

(34a)

\[
\hat{C}_t^* = \hat{C}_t^w - n(\hat{C}_t - \hat{C}_t^*) = \hat{C}_t^w - n \left( \frac{(1 - s)(\rho - 1)}{1 + \sigma/r} + s \right) \hat{c}_t 
\]

(34b)

Using these results, it is straightforward to derive the impact of unanticipated money shocks on home and foreign consumption. As \( s \to 0 \), consumption tends to rise in each country. Intuitively, with a greater pass-through of a home currency depreciation, the foreign CPI tends to fall, allowing for an increase in foreign consumption via the money market equilibrium. For instance, in the special case where \( e = 1 \), the response of home consumption when \( s = 0 \) is

\[
\hat{C}_t = \frac{\rho - 1 + n(1 + \sigma/r)}{\rho + \sigma/r} \hat{M}_t + \frac{(1 + \sigma/r)(1 - n)}{\rho + \sigma/r} \hat{M}_t^* 
\]

Thus, consumption rises in both countries following a surprise monetary expansion in any one country.\(^{14}\) As \( s \) rises however, the extent of pass-through is less, and the impact of the money shock on foreign consumption is mitigated, while the impact on domestic consumption is exacerbated. In the limit, as \( s \to 1 \), the response of home consumption is

\[
\hat{C}_t = \hat{M}_t 
\]

Thus, an increasing degree of PTM reduces the international transmission of money shocks on consumption. This is reflected in the differential movement in

---

\(^{14}\)When \( e > 1 \) it is no longer always the case that foreign monetary expansion raises domestic consumption. But for almost all reasonable parameter values, this occurs. See also Obstfeld and Rogoff (1995).
real interest rates across the two countries. In the limit, as $s \to 1$, $C_t$ rises, but $C_{t+1}$ is unchanged, so the domestic real interest rate falls, but the foreign real interest rate is unchanged, since foreign consumption is unaffected in either period.

Noting that employment is determined by the conditions $h = (1 - s)y + s(x + z)$, and $h^* = (1 - s)y^* + s(x^* + z^*)$ for the home and foreign countries, we may show that the response of output in each country is given by

$$\hat{h}_t = \hat{C}_t + (1 - n)(1 - s)p\hat{e}_t,$$

$$\hat{h}_t^* = \hat{C}_t^* - n(1 - s)p\hat{e}_t.$$

As $s \to 0$, the effects of money shocks on foreign output tend to be smaller. Again, in the special case where $\epsilon = 1$, with $s = 0$ we have

$$\hat{h}_t = \frac{\rho + (n + (1 - n)\rho)\sigma/r}{\rho + \sigma/r} \hat{M}_t + \frac{\sigma/r(1 - \rho)(1 - n)}{\rho + \sigma/r} \hat{M}_t^*.$$

Thus, a home money shock raises home output. But a foreign money shock reduces home output (since $\rho > 1$). For a home money shock, the home currency depreciation generates an expenditure switching away from foreign goods, which raises domestic output, but reduces foreign output. Thus, while there is a positive international consumption transmission of money shocks when $s = 0$, there is a negative international output transmission.\(^{15}\)

But as $s \to 1$, the effect of home money shocks on foreign output rises. With a higher $s$, there is a smaller pass-through of exchange rates to relative prices facing consumers, in either country. The impact effect on output, in both countries, is then determined primarily by the direct demand increase coming from home consumers. When $s = 1$, the demand increase is identical for both home and foreign goods. Then real output rises by the same amount $C_t$ in both economies.

Thus, the results imply that, in an environment of international monetary disturbances, the presence of PTM in international pricing should reduce the observed cross country correlations in consumption, but raise the cross country correlations in output.\(^{16}\)

The movements of output and consumption can also be interpreted in the light of the terms of trade adjustment. In the extreme case, when $s = 1$ (and also $\epsilon = 1$), a domestic money shock raises foreign output by $n\hat{M}_t$, but generates a terms of trade deterioration equal to $\hat{M}_t$, which is weighted by the share of exports in GDP, $n$.

\(^{15}\)Again, when $\epsilon > 1$ the same caveat as footnote (14) applies.

\(^{16}\)The presence of PTM then, relates to the “quantity anomaly” identified by Backus et al. (1992). They note that in a world economy with unrestricted trade and full risk-sharing, one would anticipate cross country consumption correlations would be high, while the movement of physical capital across borders would imply a low or even negative cross country correlation in output. In the data, however, cross country correlations of output are typically higher than cross country consumption correlations.
Thus, the terms of trade deterioration offsets the increase in foreign production, leaving foreign income unchanged. In the home economy, the combination of a rise in output by \( nM_t \) and the terms of trade appreciation (weighted by \( 1 - n \)) causes income to rise by \( M_t \).

To derive the effect of monetary disturbances on long-run consumption and output, we need to compute the response of the trade balance. Using (11) and (27), the trade balance response is

\[
\frac{\beta dF_t}{\bar{\rho} \bar{C}^w} = (1 - n)\sigma r - \left( \frac{\rho - 1)(1 - s)}{1 + \sigma r} \right) \hat{e}_t
\]

A domestic monetary expansion will in general improve the home country trade balance. The effect is diminishing in \( s \), however. In the limit, as \( s \to 1 \), the trade balance is unchanged. In this case, domestic income and consumption rise by equal proportions (i.e. \( M_t \)). Although home country residents would like to spread out some of their higher income into future consumption, the fall in the real interest rate encourages them to consume the full amount in the present. Due to the segmentation of national markets, ex post real interest rates are not equalized across countries, so the foreign real interest rate is unchanged. Therefore, the lower home country real interest rate does not encourage the foreign country to borrow more. Consequently, the monetary disturbance generates no trade balance effects when there is full pricing-to-market.

It follows that the impact of a money shock on future consumption and output will be diminished in importance, as \( s \to 1 \). When the home country trade balance improves, the effect of a money shock is to permanently raise home consumption, and permanently reduce foreign consumption, as illustrated by Eq. (11). This permanent legacy of a monetary surprise in an open economy is highlighted by Obstfeld and Rogoff (1995). But with complete pricing-to-market, the full impact of a monetary disturbance is felt within the period. After the first period, there is full money neutrality.

4.2. Government spending shocks

We now focus on the implications of government spending shocks. Note from (31) that in the case of government spending shocks,

\[
\hat{e}_t = \frac{(dG_t - dG^*_t)}{C^w} + \frac{(dG_{t+1} - dG^*_t)}{C^w} \frac{(1 + 1/r)}{(1 + 1/er)} \frac{(1 + 1/r)}{(1 + 1/er)}
\]

(37)

Thus, a home country government spending shock will generate a nominal exchange rate depreciation, which translates into a real depreciation when \( s \geq 0 \).

To compute the impact of government spending shocks on individual country
consumption, we first solve for the response of world consumption. From (35) and (36), we may derive

\[
\dot{C}_{w}^* = -\frac{(\epsilon - 1)}{\epsilon(1 + r)} \frac{\rho - 1}{\rho \eta + \rho - 1} \frac{n dG_{t+1} + (1 - n) dG_{t+1}^*}{C_{w}} \tag{38}
\]

Initial world consumption is affected by a government spending shock only if the shock is persistent, and only if \(\epsilon \neq 1\). Eq. (38) indicates that an anticipated future increase in government spending will reduce current world consumption, if \(\epsilon > 1\). But a temporary government spending disturbance has no impact on world consumption.

Then solving (28) and (29) for the endogenous variable \(\dot{C}_i - \dot{C}_i^*\), the response of home country consumption is given by

\[
\dot{C}_i = \dot{C}_i^w + (1 - n)(\dot{C}_i - \dot{C}_i^*)
\]

\[
= \dot{C}_i^w + (1 - n) \left( \frac{(1 - s)(\rho - 1)}{1 + \sigma/r} + s \right) \hat{e}_i
- \left( \frac{dG_t - dG_t^*}{C_w^*} \right) - (1/r) \left( \frac{dG_{t+1} - dG_{t+1}^*}{C_w^*} \right)
\tag{39}
\]

Using (37), (38) and (39), we may discuss the impact of a home country government spending shock. First, take a temporary government spending increase. In the case \(s = 0\), home consumption will fall, while foreign consumption will rise. Intuitively, this is due to the fact that the exchange rate depreciation at home will raise the home price level, and reduce the foreign price level, which by the money market conditions, requires a fall in home consumption and rise in foreign consumption. When \(s = 1\), a temporary home country government will have similar effects, so long as \(\epsilon > 1\) (if \(\epsilon = 1\) then both home and foreign consumption are independent of spending shocks). In this case, while there are no price level effects, the exchange rate depreciation leads to a fall in the home (relative to foreign) nominal interest rate. Through money market equilibrium, this must imply a fall in home consumption and a rise in foreign consumption. Thus, government spending shocks again tend to produce a negative cross country comovement of consumption. With permanent government spending shocks, the situation is more complicated, due to the fact that world consumption will fall. But nevertheless, under the same conditions as in the previous paragraph, home country consumption will fall by more than foreign consumption.

Using the same approach as before, the output responses to a government spending shock may be shown to be
\[ \hat{h}_t = \hat{C}_i^w + (1 - n)\rho(1 - s)\hat{e}_t + n \frac{dG_i}{\hat{C}_i^w} + (1 - n) \frac{dG^*_i}{\hat{C}_i^w} \]

\[ \hat{h}^*_t = \hat{C}_i^w - n\rho(1 - s)\hat{e}_t + n \frac{dG_i}{\hat{C}_i^w} + (1 - n) \frac{dG^*_i}{\hat{C}_i^w} \]

A temporary government spending shock raises home country output whatever the value of \( s \). When \( s = 0 \) however, foreign output tends to fall, as demand is reallocated towards home goods. Thus, without PTM, the cross country comovement of output levels is reduced by currency depreciation. But when \( s = 1 \), both home and foreign output must rise by equal amounts, so that the cross-country comovement of output levels is positive.

Again, we note that the case of permanent government spending shocks differs to the extent that both home and foreign output must rise by less, due to the fall in world consumption.

Finally, we compute the trade balance effects of government spending shocks. The trade balance response may be written as

\[ \frac{\beta dF_i}{P\hat{C}_i^w} = (1 - n) \left( \hat{e}_t [(1 - s)(\rho - 1) + s] - [\hat{C}_i - \hat{C}_i^*] - \left[ \frac{dG_i - dG^*_i}{\hat{C}_i^w} \right] \right) \]

Using this, (11) and (27) gives

\[ \frac{\beta dF_i}{P\hat{C}_i^w} = (1 - n) \frac{(\sigma/r)(1 - s)(\rho - 1)}{(1 + \sigma/r)} \hat{e}_t \]

\[ - \frac{(\sigma/r)}{1 + \sigma/r} \left[ \left( \frac{dG_i - dG^*_i}{\hat{C}_i^w} - (1/r) \frac{dG_{i+1} - dG^*_{i+1}}{\hat{C}_i^w} \right) \right] \]

The impact of a government spending shock on the trade balance depends on the degree of PTM, the duration of the shock, and the magnitude of the elasticity of labour supply. The first expression on the right hand side of (40) indicates that without PTM, the exchange rate depreciation mitigates the trade balance deterioration induced by the fiscal spending shock. This is due to the expenditure switching effect of depreciation, and is therefore determined by the size of \( \rho \). But with PTM, this effect is eliminated, and when \( s = 1 \), the compensating depreciation has no impact on the trade balance. It is easy to show from (40) that the trade balance always deteriorates in response to a temporary home government spending increase. For the case of a permanent spending increase, the effect depends upon the elasticity of labour supply.
5. Quantitative evaluation of the model

Two findings from our discussion above are that the presence of pricing-to-market increases the variability of exchange rates in face of monetary disturbances, and tends to reverse the international transmission effects of monetary shocks on consumption and output levels.

We can use the model to provide a rough quantitative characterization of these two features. To construct the quantitative implications of the model, we require five parameters; the elasticity of demand for consumption goods $\rho$, the consumption elasticity of demand for money $1/e$, the share of goods subject to PTM, $s$, the elasticity of labour supply (determined by $\eta$ and $\rho$), and the steady state interest rate $r$ (determined by $\beta$). The first parameter can be derived from the empirical estimates of markups. Basu and Fernald (1994) suggest a relatively low markup of about 1.1 in US data. In our model the markup is $\rho/(\rho-1)$. A value of $\rho=1.1$ gives a markup of 1.1. An estimate of the consumption elasticity of the demand for money (equal to $1/e$ in the model) very close to unity is reported by Mankiw and Summers (1986). A comparison of estimates of money demand elasticities by Helliwell et al. (1990) suggest that $e \approx 1$. We choose a value $e = 1$. Knetter (1993) reports estimates of the proportion of industries that are subject to “local-currency price stability”, by country. This is essentially equivalent to PTM in our model. His point estimates differ across countries. We report results for a range of values of $s$.

The consumption-constant elasticity of labour supply in this economy is $(\rho - 1/\eta \rho)$. We choose a value of unity for this. This is low by the standards of real business cycle models, but higher than most micro estimates. Finally, we choose $\beta = 0.94$, which produces a steady-state real interest rate of six percent, about the average long run real return on stocks.

Although this economy has no uncertainty, it can easily be extended to an environment where monetary policies are stochastic. Then, using the same linearization method, we can compute the moments of interest by simulation for a given distribution of monetary shocks. In fact, since certainty equivalence applies when focusing on linear approximations, the equations governing the stochastic model are exactly those derived above. We generate a random sample of home and foreign money shocks, where the shocks are white noise with zero mean and a standard deviation of unity. In addition, the shocks are permanent, so that $E_{t+1} M_{t+1} = \hat{M}_t$, $(E_{t+1} \hat{M}^*_t = \hat{M}^*_t)$. In the sample that we analyze, the correlation of the two money shocks is very small—about 0.01.

Simulation results are shown in Table 1. Since (for $s < 1$) monetary shocks

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17 A comprehensive quantitative examination of these issues along the lines of the International Real Business Cycle Literature would require a fully dynamic model, including endogenous physical capital accumulation and perhaps multiperiod price stickiness.
18Moments are constructed using 100 replications over a draw of 100 periods.
will generate permanent effects on consumption and employment levels, the appropriate procedure is to compute moments using first differences in the simulated data. The first row of the table reports, for varying values of $s$, the value of variance of the change in the exchange rate relative to the monetary fundamentals, i.e. the number

$$\frac{\sigma^2_c}{\sigma^2_m}$$

where $m = \hat{M}_t - \hat{M}_t^*$. The second row of the table reports the value of the same variance ratio, but for the real exchange rate. The third and fourth rows of the table report the values of cross country consumption correlations and cross country output correlations for different values of $s$.

In comparing the economy with full PPP ($s = 0$) and one with full PTM ($s = 1$), we see that the latter implies considerable greater volatility of the exchange rate, in response to either type of shock. Even for $s = 0.5$, exchange rate volatility is 30 percent greater than that of the economy without PTM, while full PTM raises exchange rate variability by more than 70 percent relative to the PPP based economy. Thus, the model suggests that the presence of pricing-to-market can generate a quantitatively significant increase in exchange rate volatility. Clearly, as shown in the second row of the table, increasing nominal exchange rate volatility, with PTM, implies increasing real exchange rate volatility.

The table also shows that the ranking of the cross country correlations of output and consumption are reversed as $s \to 1$. While for $s = 0$, outputs are almost perfectly negatively correlated, and consumption almost perfectly positively correlated, the output correlation rises towards one as $s$ rises to unity. At the same time, the consumption correlation falls towards zero.

6. International welfare effects of money shocks

What are the welfare implications of money shocks in this model? Because there is a distortion due to monopoly pricing, a surprise monetary expansion can raise welfare in a closed economy version of this model (as in Blanchard and Niyotaki, 1987 for instance). But in a global setting, does a domestic welfare gain come at the expense of the foreign country? Traditional reasoning about the incentive to engage in “competitive depreciation” through surprise monetary expansions might lead one to think so. But Obstfeld and Rogoff (1995) show that

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19With a high value of $\rho$ (the elasticity of substitution between goods), the expenditure switching effects on output remain very strong even with a high value of $s$. In our simulations, the cross country output correlation remains negative until $s \approx 0.9$. With lower values of $\rho$, the correlation becomes positive at much lower values of $s$. 

in a model with full PPP, this reasoning is not correct. A surprise monetary expansion in the home economy will raise both home and foreign welfare. Here we show that the presence of PTM reverses this finding, causing monetary shocks to generate negative international welfare effects.

Write date \( t \) utility as

\[
U_t = U_t^R + U_t^M
\]

where

\[
U_t^R = \log C_t + \eta \log (1 - h_t)
\]

and

\[
U_t^M = \frac{\gamma}{1 - \epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon}
\]

Similarly, foreign utility may be decomposed as \( U_t^{R*} \) and \( U_t^{M*} \).

Using the results of the previous section, it is easy to see that

\[
dU_t^R = \frac{\hat{C}_t^w}{\rho} + (1 - n) \hat{\sigma} \hat{\epsilon}_t - \frac{(1 - n)(\rho - 1)(1 - s) \sigma}{\frac{\sigma}{r} + 1} \frac{\sigma}{r} \hat{\epsilon}_t
\]

\[
dU_t^{R*} = \frac{\hat{C}_t^w}{\rho} - n s \hat{\epsilon}_t + \frac{n(\rho - 1)(1 - s) \sigma}{\frac{\sigma}{r} + 1} \hat{\epsilon}_t
\]

In the same manner, we can show that, from period \( t+1 \) onwards, the impact on utility is

\[
dU_{t+1}^R = \frac{\sigma(1 - n)(\rho - 1)(1 - s)}{1 + \sigma/r} \hat{\epsilon}_t
\]

\[
dU_{t+1}^{R*} = - \frac{\sigma n(\rho - 1)(1 - s)}{1 + \sigma/r} \hat{\epsilon}_t
\]

where \( i = 1, 2, \ldots, \) etc, since the impact of monetary policy on period \( t+1 \) utility persists for all future periods.

Then, using (39) and (41), as well as (40) and (42), we obtain the full impact of monetary shocks on home and foreign welfare.

\[
dV_t^R = \frac{\hat{C}_t^w}{\rho} + (1 - n) \hat{\sigma} \hat{\epsilon}_t
\]

\[^{20}\text{The second component of welfare; } U_t^{R*}, \text{ and } U_t^{M*} \text{ will also change for each country. As argued by Obstfeld and Rogoff (1995) however, this component is likely to be small so we ignore it.}\]
\[ dV^w_i = \frac{\hat{C}^w_i}{\rho} - n\hat{e}_i, \]

where \( V_i = U_i^R + (1/r)U^R_{i+1}, \) and analogously for the foreign country.

In the absence of pricing-to-market, home and foreign welfare are affected identically by home or foreign money shocks. Moreover, since \( \rho < \infty, \) a money shock raises welfare, as it increases employment and consumption, given that both are initially at an inefficiently low equilibrium level. Thus, money shocks have positive international welfare spillover effects.

With PTM however, the welfare impacts of monetary shocks are quite different. Substituting for the exchange rate solutions into the two previous equations, we get

\[ dV^R_i = \epsilon \left( \frac{n\hat{M}_i + (1-n)\hat{M}_i^*}{\rho} \right) \]

\[ + \frac{(1-n)\epsilon(\hat{M}_i - \hat{M}_i^*)(1 + \sigma/r)}{(1-s)(\epsilon(1 + \sigma/r) + \rho - 1) + s(1 + \sigma/r) \left( \frac{1 + 1/r}{1 + 1/er} \right)} \]

\[ dV^R_{i+1} = \epsilon \left( \frac{n\hat{M}_i + (1-n)\hat{M}_i^*}{\rho} \right) \]

\[ - \frac{n\epsilon(\hat{M}_i - \hat{M}_i^*)(1 + \sigma/r)}{(1-s)(\epsilon(1 + \sigma/r) + \rho - 1) + s(1 + \sigma/r) \left( \frac{1 + 1/r}{1 + 1/er} \right)} \]

A money shock in the home (foreign) country raises home (foreign) country utility, for any value of \( s. \) However, the direction of the international welfare spillover effect is now ambiguous. A home (foreign) money shock will reduce foreign (home) utility if

\[ (1-s)\left(\epsilon + \frac{\rho - 1}{1 + \sigma/r}\right) < \left(\rho - \frac{(1 + 1/r)}{(1 + 1/er)}\right)s \]

When \( s = 1, \) this is satisfied for all reasonable values of \( \rho, \epsilon \) and \( r. \) In particular, it is always satisfied for \( \epsilon = 1. \)

From this it follows that the overall cross country welfare effects of monetary shocks will generally be negative in the presence of a high degree of PTM. Monetary policy becomes a “beggar-thy-neighbor” instrument in an environment of PTM. The intuition behind this is quite easy to see. In the presence of PTM, monetary surprises reduce the foreign terms of trade. Foreign residents get less benefit of the world output expansion in consumption, but increase their labour supply to meet the expansion in demand from the other country. This makes them worse off.
Therefore, the more important is PTM, the greater the likelihood that monetary policy will be governed by “competitive depreciation” incentives. By orchestrating an unanticipated exchange rate depreciation, a country may improve its terms of trade in an environment of PTM, and raise its consumption and welfare, at the expense of its trading partners. Although foreign output will increase, foreign consumption will increase by less, the greater is the home country terms of trade improvement.

Of course this cannot be done systematically, as both the private sector and foreign authorities would understand the incentives involved and rationally predict the rate of depreciation. Nevertheless, the fact that the strategic implications of international monetary policy are so radically different in this environment highlights the importance of PTM in the understanding of international macroeconomic fluctuations.

7. Conclusions

This paper has shown that the presence of national market segmentation by monopolistic competitive firms, in conjunction with prices that are sticky in the currency of sale, has important implications for exchange rate volatility, international macroeconomic transmission and welfare. In an environment in which PPP holds, the expenditure switching effects of exchange rate changes naturally dampen the overall movement of the exchange rate. This expenditure switching is absent in an environment with PTM. As a result, exchange rate volatility can be far higher with PTM. It is also shown that the presence of PTM restricts consumption risk sharing between countries. As a result, in the world economy with PTM, cross-country consumption correlations tend to be low, and cross-country output correlations are very high. Finally, we show that in an international environment with PTM, monetary policy is a beggar-thy-neighbor instrument. A surprise exchange rate depreciation will raise a country’s terms of trade and raise domestic welfare at the expense of foreign welfare.

The results, in combination with the empirical evidence on the deviations from the law-of-one-price in international data, suggest that goods market segmentation and local-currency pricing may be an important phenomenon for understanding international quantity and price fluctuations.

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Appendix A

It is first shown how Eqs. (10) and (11) are derived. For period \( t+1 \) (the period after the shock), the equilibrium conditions may be written as

\[
\frac{M_{t+1}}{P_{t+1}} = \left( \frac{\gamma C_{t+1}}{1 - d_{t+1}} \right)^{\frac{1}{\epsilon}}
\]  
(A1)

\[
\frac{M^*_t}{P^*_t} = \left( \frac{\gamma C^*_t}{1 - d_{t+1} e_{t+1}} \right)^{\frac{1}{\epsilon}}
\]  
(A2)

\[
P_{t+1} C_{t+1} + P_{t+1} G_{t+1} + d_{t+1} F_{t+1} = p_{t+1} y_{t+1} + F_t
\]  
(A3)

\[
P^*_{t+1} C^*_{t+1} + P^*_{t+1} G^*_{t+1} + (d_{t+1} e_{t+1}) F^*_{t+1} = p^*_{t+1} y^*_{t+1} + \left( \frac{F^*_{t+1}}{e_{t+1}} \right)
\]  
(A4)

\[
y_{t+1} = \left( \frac{P^*_{t+1}}{P_{t+1}} \right)^{-\rho} (n(C_{t+1} + G_{t+1}) + (1 - n)(C^*_{t+1} + G^*_{t+1}))
\]  
(A5)

\[
y^*_{t+1} = \left( \frac{P^*_{t+1}}{P_{t+1}} \right)^{-\rho} (n(C_{t+1} + G_{t+1}) + (1 - n)(C^*_{t+1} + G^*_{t+1}))
\]  
(A6)

\[
d_{t+1} e_{t+2} = \beta C_{t+1}^* P_{t+1}
\]  
(A7)

\[
d_{t+1} e_{t+2} C_{t+1}^* = \beta P^*_{t+1} C^*_{t+1}
\]  
(A8)

\[
\frac{\eta}{1 - h_{t+1}} = \frac{\rho - 1}{\rho} A_{t+1} P_{t+1}
\]  
(A9)

\[
\frac{\eta}{1 - h^*_{t+1}} = \frac{\rho - 1}{\rho} A^*_{t+1} P^*_{t+1}
\]  
(A10)

Since, in period \( t+1 \), all prices are fully anticipated, PTM is irrelevant, and there is full PPP. Thus, (A5) and (A6) represent market clearing conditions for all commodities.

By taking a linear approximation of (A1) and (A2), and using the fact that, from period \( t+1 \) onwards, \( d_t \) and the exchange rate are constant, we may derive Eq. (10). To derive Eq. (11) take a linear approximation of (A5) and (A6), as well as (A9) and (A10). Substitute these into the approximations of (A3) and (A4), using the fact that the initial government spending is zero. Note also that, because the
steady state response to a shock is attained in period $t+1$, it must be that $dF_{t+1} = dF_t$.

Appendix B

One of the characteristics of the preferences used in the text is that only one parameter, $\epsilon$, governs the magnitude of both the consumption elasticity and the interest elasticity of money demand. Moreover, with a reasonable magnitude of the consumption elasticity of demand, the interest elasticity is unrealistically large.

Take the more general preferences given by

$$U = \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\omega}}{1-\omega} + \frac{\gamma}{1-\epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} + \eta \log(1-h_t) \right)$$

These preferences allow for a separation of the interest elasticity of money demand from the consumption elasticity. It is easy to solve the model under this specification. The new money demand schedule is given by

$$\frac{M_t}{P_t} = \left( \frac{\gamma}{1-\epsilon} \left( \frac{C_t^\omega}{(1-d_j)} \right) \right)^{1/\epsilon} \tag{B1}$$

The first order conditions for the household consumption and labour supply become

$$\frac{\eta}{1-h_j} = \frac{W_t}{P_t C_t^\omega}$$

$$d_t P_{t+1} C_{t+1}^\omega = \beta P_t C_t^\omega$$

Using these conditions, the money demand schedule (B1), and performing the same operations as in the text, the solution for the exchange rate response under this specification may be derived as

$$\hat{\epsilon}_t = \frac{\left( \frac{\epsilon}{\omega} \right) (\hat{M}_t - \hat{M}_t^*) \left( 1 + \frac{\sigma}{r} \right) + \frac{dG_t - dG_t^*}{C^\omega} + \frac{1}{r} \frac{dG_t + dG_t^*}{\hat{C}^\omega}}{\Delta} \tag{B2}$$

where

$$\Delta = (1-s)(\rho-1) + s \left( 1 + \frac{\sigma}{r}\omega \right) + \frac{(1-s) + \frac{1}{e_r} \left( \frac{\epsilon - s}{\epsilon} \right) \left( 1 + \frac{\sigma}{r} \right)}{\omega} \left( 1 + \frac{1}{e_r} \right)$$

Table B1 computes the variance of exchange rate changes relative to fundamentals in the same way as before, except using the specification underlying (B2). All parameters except $\epsilon$ are as above. We now set $\epsilon$ so as to give an interest elasticity
of money demand equal to 1. The variable $\omega$ may then be set so as to achieve a consumption elasticity of money demand equal to 1, as before.

Table B1 illustrates the variability of the exchange rate relative to nominal fundamentals in the same way as Table 1 of the text is computed. The implications of PTM for exchange rate volatility in this setup are quite dramatic. In the PPP case, the variance ratio is quite similar to before. But as $s$ rises, the volatility of the exchange rate rises very sharply. For $s=1$, the variance ratio is 3.15. In this extended model, PTM leads to a very large magnification of exchange rate variability, relative to fundamentals.

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