Monetary policy rules and exchange rate flexibility in a simple dynamic general equilibrium model

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Abstract

This paper provides a complete analytical characterization of the positive and normative effects of alternative exchange rate regimes in a simple two-country sticky-price dynamic general equilibrium model with multiple shocks. A central question addressed is whether fixing the exchange rate prevents macroeconomic adjustment in relative prices from occurring. We find that in general, allowing the exchange rate to float does not facilitate relative price adjustment, in face of country-specific shocks. In a comparison of monetary policy rules which allow for differential degrees of exchange rate targeting, it is found that a rule in which both countries engage in a cooperative exchange rate peg welfare-dominates a rule which allows the exchange rate to float endogenously (as well as a one-sided peg). When optimal monetary rules can target both employment and exchange rates however, a cooperative exchange rate peg leads to lower welfare. Therefore, whether fixing the exchange rate involves a welfare cost depends critically upon the way in which monetary rules are designed. Quantitatively, we find that the welfare differences across different monetary rules (exchange rate regimes) are very small.

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1. Introduction

This paper explores the effects of alternative monetary rules that allow for varying degrees of exchange rate flexibility in a stochastic general equilibrium model, where prices are sticky, and there are a variety of different macroeconomic shocks. There has been a lively recent debate on the macroeconomic implications of alternative exchange rate regimes. The traditional literature, based on Friedman (1953) and Mundell (1961), argues for the importance of exchange rate flexibility in dealing with country specific disturbances. The cost of sacrificing the exchange rate as a macroeconomic adjustment device was seen as one of the key drawbacks of the European single currency (e.g. Feldstein, 1997). On the other hand, De Grauwe (1994) has set out the benefits of exchange rate stability within a single market, and recent evidence by Rose (2000) provides empirical support for the trade benefits of a single currency.

The aim of this paper is to map out the trade-offs involved in stabilizing the nominal exchange rate, both from a positive and normative perspective, within a model that makes use of recent developments in open economy macroeconomics due to Obstfeld and Rogoff (1995, 2000), Bachetta and Van Wincoop (2000), Corsetti and Pesenti (2001), and others. Specifically, we construct a two-country dynamic general equilibrium model in which prices are set in advance, in a stochastic environment, by profit-maximizing firms. The economy is subject to shocks to money demand and aggregate productivity. This framework can be used to address a variety of traditional questions in the comparison of exchange rate regimes. For instance, under what circumstances is exchange rate flexibility useful in offsetting country-specific macroeconomic disturbances? How does the manner in which the exchange rate is pegged matter—i.e. whether it is done cooperatively between monetary authorities or unilaterally by one country? Quantitatively, how important are the welfare differences between alternative exchange rate regimes? Finally, what are the characteristics of welfare maximizing monetary policies, and how much exchange rate flexibility is implied by these policies?

The model also allows us to focus on some non-traditional aspects of alternative exchange rate regimes. In particular, because the analysis allows for an exact analytical solution in a dynamic, stochastic environment, alternative monetary policies have implications for employment, the capital stock, and long run GDP.

An obvious but important point in the comparison of alternative exchange rate regimes is that the monetary policy rule through which exchange rates are targeted may be equally as important as the degree of exchange rate volatility itself. We focus on simple monetary rules where the policy variable can be targeted on some easily observable macroeconomic aggregate rather than directly on the underlying stochastic disturbances themselves. We first compare the implications of three different monetary rules that differ in the degree to which they target the exchange rate. The first rule simply involves a constant money growth rate, the second rule represents a unilateral exchange rate peg, while the third is a bilateral or cooperative peg. In each

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1 The addition of a government spending shock is straightforward—see the discussion below.
case the effects of these rules are compared to the allocation of an economy with fully flexible prices.

Traditional theory suggests that in the face of country-specific disturbances, it is better to have a floating exchange rate so as to allow relative prices to adjust more easily. Our model, perhaps surprisingly, indicates that a floating exchange rate, in the absence of specific activist monetary policies, does not facilitate relative price adjustment. The reason is that without activist monetary policies, there is no guarantee that the exchange rate will move in the right direction to achieve the correct response to country specific shocks. In the light of this result, we find that there is no presumption that a monetary rule which allows for exchange rate adjustment (the constant money growth rule) will produce higher welfare than a pegged exchange rate regime. In fact, we find that, in the absence of activist monetary policies, the cooperative peg will welfare-dominate both a one-sided peg as well as a constant-money growth floating exchange rate regime.

In all sticky price environments, we find that long run expected employment and GDP is lower than in the flexible price environment. Also, steady state employment and GDP are lower under a float than a cooperative peg.

We may also apply the model towards a quantitative analysis of the differences between regimes. Here we find quite sharp results—both in terms of volatilities, in expected values, and in welfare terms, the difference between alternative monetary rules is extremely small. When the model is calibrated based on a consensual set of parameter values, the allocations and welfare levels across rules are very similar. While a cooperative peg dominates a floating regime, agents in our model would sacrifice only a tiny fraction of consumption to move from the latter regime to the former. To a first approximation agents are indifferent between all types of exchange rate regimes. While the standard case for floating exchange rates is that they generate desired relative price adjustment, a second argument is that they allow monetary policy a free hand. If monetary policy is designed optimally, then it cannot but do better than if it were constrained by the requirements of an exchange rate peg. To explore this argument, we extend the model to allow for a set of monetary policies that are more activist. The optimal monetary rules in this model are shown to be particularly simple—monetary authorities in each country should target employment rates. When each monetary authority prevents employment from moving away from the 'natural rate', this sustains the first best allocation.\footnote{As in Rotemberg and Woodford (1997), we assume that optimal monetary rules are supplemented by production subsidies to eliminate the distortion from monopoly pricing.} Because such rules involve country specific monetary adjustment, by definition they require a system of exchange rate flexibility. But we also derive a constrained optimal monetary rule, that is consistent with a fixed exchange rate in which both countries pursue a cooperative peg. This rule requires each monetary authority to target an average index of global employment, as well as targeting the nominal exchange rate.\footnote{Benigno (2000) analyzes the optimal monetary policy within a currency area where different regions face specific shocks.
attain the desired response of real GDP in each country, it cannot attain the optimal employment response, in general.

The rest of the paper is structured as follows. Section 2 presents the basic model with money and technology shocks. Section 3 develops the results under flexible prices. Section 4 analyzes the case of sticky prices, and compares alternative exchange rate rules. Section 5 analyzes welfare across the different regimes. Section 6 derives the results under welfare maximizing monetary rules. Some conclusions follow.

2. A two-country model

The model has two countries, denoted “home” and “foreign”. Within each country, there exist consumers, firms and a government. Government issues fiat money.

We assume that there is continuum of goods varieties in the world economy of measure 1, and that the relative size of the home and foreign economy’s share of these goods is \( n \) and \( 1 - n \) respectively. We choose units so that the population of the home and foreign country is also \( n \) and \( 1 - n \), respectively.

Let the state of the world at time \( t \) be defined as \( z_t \). In each period \( t \), there is a finite set of possible states of the world. Let \( z_t \) denote the history of realized states between time 0 and \( t \), i.e., \( z_t = \{z_0, z_1, \ldots, z_t\} \). The probability of any history, \( z_t \), is denoted by \( \pi(z_t) \).

We may just describe the details of the model for the home country economy. The conditions for the foreign country are analogously defined in all cases, except where stated.

2.1. Consumers

Assume that preferences are identical across countries. In the home country, consumers have preferences given by

\[
EU = \sum_{i=0}^{\infty} \sum_{z'} \beta^i \pi(z') U \left( c(z'), \frac{m(z')}{P(z')}, 1 - h(z'), z' \right),
\]

where

\[
c(z') = \frac{c_h(z')^n c_f(z')^{1-n}}{n^n (1 - n)^{1-n}}, \quad c_h(z') = \left( n^{-1/\beta} \int_0^n c(i, z')^{1-1/\beta} \, di \right)^{1/\beta(1-1)},
\]

and

\[
c_f(z') = \left( (1 - n)^{-1/\beta} \int_n^1 c(i, z')^{1-1/\beta} \, di \right)^{1/\beta(1-1)}.
\]

In addition, we assume the specific functional form given by

\[
U \left( c(z'), \frac{m(z')}{P(z')}, 1 - h(z'), z' \right) = \ln c(z') + \chi(z') \ln \left( \frac{m(z')}{P(z')} \right) + \eta \ln(1 - h(z')).
\]

The consumer derives utility from a composite consumption good \( c(z') \), real home country money balances \( \frac{m(z')}{P(z')} \), and leisure, where \( h(z') \) represents hours worked. The
composite consumption good is broken up into two sub-composites, representing home and foreign goods consumption, with a unit elasticity of substitution between the two. Within each country-specific sub-composite, there is an elasticity of substitution of \( \lambda \) between any two consumption goods. The variable \( \chi(z') \) represents a random preference shock to the marginal utility of real balances. It plays the role of a velocity shock in the model.

The price index is defined as

\[
P(z') = P_h(z')^n (S(z')P_f(z'))^{1-n},
\]

where \( S(z') \) is the exchange rate, and the sub-price indices are defined as

\[
P_h(z') = \left[ \frac{1}{n} \int_0^n p_h(i,z')^{1-i} \, di \right]^{1/(1-\lambda)},
\]

\[
P_f(z') = \left[ \frac{1}{1-n} \int_n^1 p_f(i,z')^{1-i} \, di \right]^{1/(1-\lambda)}.
\]

The consumer price index in the home economy depends on the composite price of home goods \( P_h(z') \), and the exchange rate times the composite price of foreign goods, where \( P_f(z') \) is the foreign goods price, expressed in foreign currency. Note that the foreign consumer price index will be analogously defined as \( P^n(z') = \left( \frac{P_h(z')}{S(z')} \right)^n P_f(z')^{1-n} \), and therefore purchasing power parity must hold at all times in this economy.

The representative consumer in the home country receives income from wages and the return on physical capital holdings, profits from the ownership of domestic firms, income from international bond holdings and existing money balances, and receives transfers and/or pays taxes to the domestic government. Households then consume, accumulate capital and money balances, and purchase new assets.

Therefore the home consumer’s budget constraint is written as

\[
P(z')c(z') + m(z') + q(z')B(z') + P(z')v(z') = W(z')h(z') + R(z')k(z'\text{-}1) + \Pi(z') + m(z'\text{-}1) + B(z'\text{-}1) + TR(z'),
\]

where

\[
k(z') = v(z').
\]

The home consumer purchases home-currency denominated nominal bonds at price \( q(z') \). \( v(z') \) represents a composite investment good, which requires the same basket of goods as the consumer goods, and which forms next period’s capital holdings, given by \( k(z') \). The consumer also receives net transfers \( TR(z') \) from the government, and nominal domestic currency profits \( \Pi(z') \). \( R(z') \) denotes the nominal rental return on a unit of capital.

The consumer’s optimal consumption, money holdings, investment, and labor supply may be described by the following familiar conditions:

\[
q(z')c(z')^{-1} = \beta \sum_{z'+1} \pi(z'+1) \frac{P(z')}{P(z'+1)} c(z'+1)^{-1},
\]
\[
\frac{m(z')}{P(z')} = \chi(z')c(z') \left(1 + \frac{i(z')}{i(z')}\right),
\]
(6)

\[
\frac{\eta}{1 - h(z')} = \frac{W(z')}{P(z')c(z')},
\]
(7)

\[
c(z')^{-1} = \beta \sum_{z'=1} \pi(z'+1)c(z'+1)^{-1} \frac{R(z'+1)}{P(z'+1)}.
\]
(8)

Eq. (5) describes the choice of inter-temporal consumption smoothing, while (6) gives the implied demand for money of the consumer. The term \(i(z')\) represents the nominal interest rate, where \(\frac{1}{1+i(z')} = q(z')\). Eq. (7) describes the labor supply choice, while (8) results from the optimal choice of the investment good.

In a symmetric equilibrium, all households in the same country will have the same consumption, money holdings, investment, and labor supply. To reflect this, in what follows we will denote these variables by capital letters. In addition, due to unit substitution between home and foreign goods, the equilibrium current account is zero, and we have full consumption insurance, so that
\[
C(z') = C^*(z')
\]
(9)

2.2. Government

Governments in each country print money and levy taxes. We also assume that the government offers employment and capital subsidies to firms. These subsidies are introduced (as in Rotemberg and Woodford, 1997; Gali and Monacelli, 2002) so as to eliminate the distortion arising from the monopoly pricing by differentiated goods firms.

The home country government budget constraint is then
\[
M(z') - M(z'^{-1}) = TR(z') + W(z')H(z')s(z') + R(z')K(z')s(z'),
\]
(10)

where \(s(z')\) represents the government subsidy. We initially set this to zero.

2.3. Firms

Firms in each country hire capital and labor to produce output. For each home and foreign good, there is a separate, price-setting firm. The number of firms is sufficiently large that each firm ignores the impact of its pricing decision on the aggregate price index for that variety. A home firm of variety \(i\), has production function given by
\[
y(i,z') = \theta(z')k(i,z')^\alpha h(i,z')^{1-\alpha},
\]
where \(k(i,z')\) is capital usage and \(h(i,z')\) is labor usage. \(\theta(z')\) is a country-specific technology shock.

All firms will choose factor bundles to minimize costs. Thus, we must have
\[
W(z')(1 - s(z')) = MC(z')(1 - \alpha) \frac{y(i,j,z')}{h(i,j,z')},
\]
(11)
\[
R(z')(1 - s(z')) = MC(z')\frac{y(i,j,z')}{k(i,j,z')},
\]
where \(MC(z')\) is nominal marginal cost, which must be equal for all firms within the home economy.

### 2.4. Pricing

We assume that firms must set nominal prices one period in advance. \(^4\) Prices are set to maximize profits, where profits are evaluated using the marginal utility of money of the firm owners. The optimal price set by all firms in the home country will be identical, and equal to

\[
P_h(z'^{-1}) = \frac{\lambda}{\lambda - 1} \frac{\sum \pi(z')\Omega(z')MC(z')}{\sum \pi(z')\Omega(z')},
\]

where

\[
\Omega(z') = \left( \frac{C(z') + nV(z') + (1 - n)V^*(z')}{{C(z')}} \right).
\]

The equilibrium price will depend on the distribution of marginal costs, investment, and consumption. The foreign firm sets its price in an analogous fashion.

### 2.5. Market clearing

Within a country, all firms have the same capital labor ratio. Therefore we may aggregate across firms and sectors to define the aggregate output in the home economy as

\[
Y(z') = \theta(z')K(z'^{-1})^sH(z')^{1-s}.
\]

Output must equal aggregate demand. Total demand comes from the demand for consumption and investment of home and foreign consumers. Thus

\[
\theta(z')K(z'^{-1})^sH(z')^{1-s} = n \frac{P(z')}{P_h(z'^{-1})} [C(z') + nV(z') + (1 - n)V^*(z')].
\]

Similarly, for the foreign country,

\[
\theta'(z')K(z'^{-1})^sH(z')^{1-s} = (1 - n) \frac{P'(z')}{P_l(z'^{-1})} [C(z') + nV(z') + (1 - n)V^*(z')].
\]

### 2.6. Equilibrium

We may characterize the equilibrium of the two-country economy by collecting the equations above. The equilibrium is the sequence \(C(z'), C(z')^*, H(z'), H(z')^*\),

\(^4\) It would be possible to introduce more persistent price rigidity, along the lines of Calvo (1983) or Taylor (1979). But the results would be qualitatively similar, and we would no longer obtain a complete analytical solution to the model.
\( q(z'), q^*(z'), K(z'), K^*(z'), P_t(z'), P_t(z'), S(z'), MC(z'), M C^*(z'), W(z'), W^*(z'), R(z'), R^*(z') \) that gives a solution of the Eqs. (5)–(8), (11)–(13), and their counterparts for the foreign economy, as well as (9), (14) and (15). The model is sufficiently simple that we can solve it in closed form. However, it is necessary to give an explicit description of the structure of the shock processes. We take

\[
 z_t = \{ \chi_t, \chi_t^*, \theta_t, \theta_t^* \}.
\]

Velocity shocks are governed by the process

\[
 \chi_t = \exp(\zeta_t), \quad \zeta_t = \zeta_{t-1} + u_t,
\]

(16)

where \( u_t \) is a mean zero, i.i.d. process with variance \( \sigma_u^2 \).

We assume that the technology shocks follow the process

\[
 \theta_t = \exp(v_t), \quad v_t = q v_{t-1} + \epsilon_t,
\]

(17)

where \( \epsilon_t \) is a mean zero i.i.d. shock to technology with variance \( \sigma_e^2 \), and \( 0 < \rho < 1 \).

To keep the model as symmetric as possible, we assume that the foreign country velocity and technology shocks are taken from an identical distribution. Finally, we assume that the production subsidy is set at the constant rate \( s = 1/\lambda \). This will exactly offset the distortionary impact of monopoly pricing.

### 2.7. Monetary policies

The aim of the paper is to investigate the effects of alternative monetary policy rules. The baseline monetary rule is one where each government follows a constant growth rate of money. Thus, for the home government,

\[
 M_t = M_{t-1}(1 + \mu).
\]

(18)

In the baseline case, the foreign government uses an identical rule.

In the presence of nominal rigidities, a monetary rule that responds to ex-post disturbances may raise welfare. While in principle it may be possible for the authority to respond directly to the realizations of stochastic disturbances, it is more realistic to assume that monetary policy must be governed by simple rules, where the monetary rule must respond to an observable market price or quantity. This is the approach that is taken throughout the analysis.

With a constant money growth rule, the exchange rate will respond endogenously to domestic and foreign disturbances. An alternative money rule is where one or both governments target the exchange rate. But the impact of exchange rate targeting rules will depend on whether this is done unilaterally or bilaterally. A unilateral, or one-sided, exchange rate targeting rule involves one government (say home) adjusting its monetary rule to achieve an exchange rate target. This is described by the rule

\[
 M_t = \left( \frac{S_t}{\bar{S}} \right)^{-\sigma} M_{t-1}(1 + \mu),
\]

(19)

where \( \bar{S} \) is the target exchange rate. As \( \sigma \to \infty \), this rule approaches a one-sided peg.
But many exchange rate targeting rules involve bilateral or multilateral intervention (see Giavazzi and Giovannini (1989) for an account of EMS intervention rules, for instance). A cooperative exchange rate targeting rule may be described by the equations

\[ M_t = \left( \frac{S_t}{S \sigma(1-n)} \right) M_{t-1} (1 + \mu), \]  

\[ M_t^* = \left( \frac{S_t}{S \sigma n} \right) M_{t-1}^* (1 + \mu). \]

Under this rule, both governments target the same level of the exchange rate. The home government contracts the home money supply when the exchange rate depreciates, and the foreign government expands the foreign money supply in the same instance. The intervention elasticity is inversely proportional to country size. The smaller country does relatively more intervention, while the larger country more closely follows an independent money growth rule.  

In Section 5, we also allow for money rules that target employment. The rationale for these rules will be discussed below.

3. Solution of the model under flexible prices

We may solve the model in a series of steps. It is argued below that an optimal monetary policy will attempt to replicate the economy under fully flexible prices. Therefore, we first describe the equilibrium that would obtain if all prices could adjust ex-post to money and technology shocks.

With flexible prices, full monetary neutrality obtains, and the evolution of consumption and investment is independent of monetary policies. Then \( P_h(z_t) = \frac{z_t}{z_t-MC(z_t)} \) must hold; that is, price must equal ex-post marginal cost, adjusted for the monopolistic competitive markup. Given this, and the pricing equation, we may combine the factor pricing condition (12), the optimal investment condition (8), and the two market clearing conditions (14) and (15) to obtain solutions for investment, the terms of trade, and consumption. Because capital is constructed from the output of both countries, and because expected future income is pooled as described above, the current cost of capital and the expected return to investment is identical in both countries, and so investment rates are equal. Thus, we have \( K_t = K_t^* \).

The economy under flexible prices is then characterized by

\[ K_{t+1} = \beta \theta_i \theta_i^{(1-n)} K_t^* H_t^{(1-s)}. \]  

5 These rules are similar to those defined by Bachetta and Van Wincoo (2000).

6 In what follows, we omit the state contingent notation, since that configuration of shocks has now been explicitly defined.
\[
\frac{P_{ht}}{S_tP_t} = \frac{\theta_t^s}{\theta_t},
\]

\[
C_t = \frac{(1 - \beta x)}{\beta x} K_{t+1},
\]

\[
H_t = H_t^* = \frac{(1 - x)}{((1 - x) + \eta(1 - \beta x))}.
\]

This solution is intuitive. Investment is a constant fraction of real GDP. The relative price of home goods is inversely proportional to the relative size of home country total factor productivity. Since consumption is a constant fraction of real GDP, movements in the real wage are reflected proportionally in consumption. Wealth and substitution effects of wage increases on employment cancel out. Thus, equilibrium employment is actually constant.

4. Equilibrium with sticky prices

To compute the equilibrium under sticky prices, one initial result that is very useful is that under all monetary policy rules, the equilibrium nominal interest rate is constant. This result holds whether prices are sticky or not, and follow from the assumption of a random walk process (in logs) for the velocity shocks, combined with the form of the money demand function (6). Under the constant money growth rate rule, for instance, the equilibrium the common nominal interest rate is determined as

\[
\frac{1}{(1 + i)} = \frac{\beta}{1 + \mu} E_t \exp(u_t),
\]

which is constant.

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7 Real GDP per capita in the home country is \( P_{ht} \). With unit elasticity of substitution in preferences, this is equal to that of the foreign country; i.e., \( \frac{P_{ht}}{P_{ft}} = \theta_t^s \). This result also holds with sticky prices.

8 Intuitively, we may note that the nominal interest rate satisfies

\[
\frac{1}{(1 + i)} = \beta E_t \frac{P_t C_t}{P_{t+1} C_{t+1}}.
\]

An unanticipated permanent home country money shock will raise current consumption, increasing the price level (through exchange rate depreciation) by less than in proportion to the shock. In the next period, prices will rise by more and consumption by less (in the economy without capital, prices in the next period would rise by the full amount of the money shock, and consumption in the next period would be unchanged), but overall, the proportional response of nominal consumption will be equal to the proportional rise in the money supply. Alternatively, a technology shock has no impact on current consumption or the price level, but will lead to a rise in next periods consumption and a fall in next periods price level which in net leaves \( P_{t+1} C_{t+1} \) unchanged.
4.1. The exchange rate

Using the previous result, it follows from the money market clearing condition (5) (and its foreign country counterpart), the risk sharing condition (9), and the PPP condition that

\[
S_t = \frac{M_t}{M_t'} \frac{\chi_t}{\chi_t'}.
\]  

(26)

The nominal exchange rate depends only on relative money supplies, and relative velocity shocks.

4.2. Consumption and investment

Using (26) and the money market clearing conditions, it follows in addition that

\[
C_t = \left( \frac{M_t}{\chi_t} \right)^n \left( \frac{M_t'}{\chi_t'} \right)^{(1-n)} \frac{1}{P_{ht} P_{ft}^{1-n}}.
\]  

(27)

For preset prices, consumption within any period depends only on velocity shocks.

Since capital is subject to full depreciation, the trade-off between consumption and investment is actually the same as (24) above. That is, investment is proportional to consumption in each country

\[
K_{t+1} = \frac{\beta \chi}{1 - \beta \chi} C_t.
\]  

(28)

4.3. Employment

Given (27) and (28), investment rates are equalized across countries, and therefore so must be real GDP. Note from (27) that, for given prices \(P_{ht}\) and \(P_{ft}\), consumption, investment, and real GDP are independent of technology shocks. Domestic output, measured in terms of the local good (i.e. in terms of the GDP deflator) depends only on domestic velocity shocks, and is independent of both technology shocks and foreign velocity shocks. Moreover, since capital is fixed within the period, movements in output are driven by movements in employment. We may derive the solution for home employment by combining (27) and (28), and the market clearing equation (14), to obtain

\[
H_t = \left( \frac{P_t C_t}{P_{ht} \beta_t K_t'} \right)^{\frac{1}{\gamma - 2}} = \left( \frac{(1 - \beta \mu)}{(1 - \beta \chi \lambda) P_{ht} \beta_t K_t' \chi_t} \right)^{\frac{1}{\gamma - 2}}.
\]  

(29)

4.4. Price determination

The solution above takes as given the pre-set prices \(P_{ht}, P_{ft}\). But these prices must be determined optimally ex ante, using condition (13). Recognizing that
consumption and investment are in proportion to one another allows this condition to be significantly simplified. We get

\[ P'_{t-1} = \psi E_{t-1} \left( \frac{H(z^t)}{1 - H(z^t)} P'_{t-1} \right). \]

Eq. (29) implicitly describes the determination of expected employment. If \( \sigma_i^2 = \sigma_r^2 = 0 \), then \( \Theta = (\frac{H}{M})^{-(1 - \alpha)} \) would hold (since then the term \( (\frac{M_t}{M_{t-1}(1 + \mu)}) \) would be equal to unity for each money rule described above). But in general, the value of \( \Theta \) will depend on the distribution of \( \exp(-u + \varepsilon_i) \). Since \( E_{t-1}(H_t/(1 - H_t)) \) is a convex function of \( H_t \), an increase in the variance of \( H_t \) must reduce \( E_{t-1}(H_t/(1 - H_t)) \), in order to keep \( E_{t-1}(H_t/(1 - H_t)) \) constant. The model therefore implies that there is a negative relationship between the variance of employment, and the mean employment level.

4.5. Dynamics

Now using the solution for prices, we may describe the full solution path for all variables. Using (29), the equation for investment, (28), and consumption, (27), in the market clearing equation (14), we obtain the path of real GDP \( \tilde{y}_t \) (common across countries) as governed by

\[
y_t = \left( \frac{\beta (1 - \alpha)^2}{\Theta} \right) \left( \frac{M_t}{M_{t-1}(1 + \mu)} \right)^n \left( \frac{M_t^*}{M_{t-1}^*(1 + \mu)} \right)^{(1 - \eta)} \times \exp(-nu_t - (1 - n)\eta_t' \theta^\rho \theta^{(1 - \eta)^\rho}) \tilde{y}_{t-1}.
\]

Consumption and investment may then be obtained immediately, since they are proportional to real GDP. Eq. (30) says that GDP is directly affected by velocity shocks, but affected by technology shocks only with a one period delay. Within any period, a technological improvement has no impact on GDP. But prices are adjusted after one period. A persistent technology shock, for instance in the home country, will lead to a fall in home country prices in the next period, which allows for an increase in GDP for both countries. The rise in foreign GDP is achieved through a terms of trade deterioration for the home economy.

Note that GDP is also affected by unexpected movements in money to the extent that the money stock responds endogenously to economic shocks, through the exchange rate targeting rules (19) or (20), (21). This is explored in the next section.

Since (30) depends upon \( \Theta \), it follows that the mean level of GDP is also affected by the volatility of velocity and technology (and also by the exchange rate policy—see below). The unconditional mean level of ln GDP is given by:

\[
\ln(y) = \frac{1}{(1 - \alpha)}(\alpha \ln(\beta (1 - \alpha)^2) - \ln(\Theta)).
\]

This is also negatively related to the volatility of velocity and technology shocks. Thus, relative to the flexible price economy, the mean of log output is lower with sticky prices, under all possible monetary rules.
4.6. Alternative exchange rate rules

We now explore the influence of alternative monetary and exchange rate targeting rules on the economy with pre-set prices. The results may be summarized by focusing on the implications of the rules for real GDP and employment.

In rule (18) the money growth rate is constant and equal in each country. In the presence of country specific velocity and technology shocks, this amounts to a regime of freely floating exchange rates. The solution for GDP in this case is

\[ y_t = \left( \frac{\beta x^2}{\Theta} \right) \exp(-nu_t - (1 - n)u'_t) \theta_{t-1}^{\rho(1-n)\rho} y_{t-1}. \]

GDP is determined by a country-weighted average of home and foreign velocity shocks. Employment is then determined by

\[ H_t = \left( \frac{\exp(- (u_t + \varepsilon_t))}{\Theta} \right)^{1/\alpha}, \]

where \( \Theta \) is determined by Eq. (29), with \( \frac{M_t}{M_{t-1}(1+\mu)} = 1. \)

Now we turn to rule (19). In the limit when \( r \to 1 \), this represents a one-sided peg. Using the solution for the exchange rate (26) and the monetary rule, we can establish that as \( r \to 1 \), the home country money supply approaches

\[ M_t = M_t^* \left( \frac{Z_t}{Z_t^*} \right)^{\delta}. \]

Then, following the rule (18) used by the foreign monetary authority, we have

\[ \frac{M_t}{M_{t-1}(1+\mu)} = \exp(u_t - u'_t). \]

Substituting in (30), we arrive at the solution for GDP in the case of a one-sided peg:

\[ y_t = \left( \frac{\beta x^2}{\Theta} \right) \exp(-u'_t) \theta_{t-1}^{\rho(1-n)\rho} y_{t-1}. \]

GDP is affected only by the foreign velocity shock, since the home country intervention rule eliminates entirely the impact of the home velocity shock. GDP volatility is greater in the one-sided peg than under the free-floating exchange rate regime. Employment under the one-sided peg is then given by

\[ H_t = \left( \frac{\exp(- (u'_t + \varepsilon_t))}{\Theta} \right)^{1/\alpha}, \]

where again \( \Theta \) is determined by Eq. (29) with \( \frac{M_t}{M_{t-1}M_t} = \exp(u_t - u'_t). \) If the variance of velocity shocks is the same across countries (as we assume), then this solution indicates that the volatility of employment is identical in the floating exchange and

---

9 The intermediate case of positive but finite \( r \) falls between the two extremes of floating and a one-sided peg, and can easily be analyzed.
the one-sided peg. Employment is determined by a combination of the domestic money market shock (which includes the velocity shock and the monetary response) and the domestic technology shock. A one-sided peg just replaces the home country money market shock with that of the foreign country, and so leaves employment variability unchanged.

Rules (20) and (21) represents the case of a multilateral or cooperative peg, for the limit as $\sigma \to \infty$. In this case, both countries follow an intervention rule to target the exchange rate. Given the exchange rate solution (27), the resulting monetary rules followed by the home and foreign country are given by

$$M_t = M_{t-1}^nM_{t-1}^{n(1-n)} \left( \frac{\zeta_t}{\zeta'_t} \right)^{(1-n)} \overline{S}^{(1-n)},$$

$$M_t^* = M_{t-1}^nM_{t-1}^{n(1-n)} \left( \frac{\zeta'_t}{\zeta'_t} \right)^n \overline{S}^{-n}.$$  

From this, it follows that

$$\frac{M_t}{M_{t-1}(1+\mu)} = \exp((u_t - u'_t)(1-n))$$  and  $$\frac{M_t^*}{M_{t-1}^*(1+\mu)} = \exp((u_t^* - u_t)n),$$

so that the solution for GDP is

$$y_t = \left( \frac{\beta\alpha}{\theta} \right)^2 \exp(-nu_t - (1-n)u'_t)\theta^{\theta^n\theta^{(1-n)}}y_{t-1}^\theta.$$

This is identical to the case of a pure float! The explanation for this is easy to see. Under a pure float, GDP is affected by a country-weighted average of home and foreign velocity shocks. With a cooperative exchange rate peg of the form (20) and (21), the combination of velocity shocks and monetary response generates an identical shock from the money market of each country, and this shock is the same country-weighted average of home and foreign velocity shocks. Thus, the average world money market shock is equal to this country-weighted average also.

The solution for employment under the one-sided peg is

$$H_t = \left( \frac{\exp(-(nu_t - (1-n)u'_t + \epsilon_t))}{\theta} \right)^{1/\alpha}.$$  

Given that home and foreign velocity shocks are independent, this implies that employment variability is less under the cooperative peg than either a pure floating exchange rate or a one-sided peg. Under a cooperative peg the domestic money market disturbance is a combination of home and foreign velocity shocks, and has lower variance than the money market shock under a pure float or a one-sided peg. This leads to a lower variance of employment.

Table 1 documents the results from a comparison of the three exchange rate targeting rules for the variance of GDP and employment.

As we have already discussed, the variance of real GDP is identical under floating and a cooperative peg, while the variance of employment is the same under floating
Table 1
Volatility of real GDP and employment

<table>
<thead>
<tr>
<th></th>
<th>Real GDP</th>
<th>Employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Float</td>
<td>[ \frac{(1 - 2n(1 - n))\sigma_u^2}{1 - \alpha^2} + \left(1 + \frac{\alpha^2}{1 - \alpha^2} + \frac{\rho^2}{1 - \rho^2}\right) \frac{\sigma_i^2}{(1 - \alpha \rho)^2} ]</td>
<td>( \frac{(\sigma_i^2 + \sigma_u^2)}{(1 - \alpha)^2} )</td>
</tr>
<tr>
<td>One-sided peg</td>
<td>[ \frac{\sigma_u^2}{1 - \alpha^2} + \left(1 + \frac{\alpha^2}{1 - \alpha^2} + \frac{\rho^2}{1 - \rho^2}\right) \frac{\sigma_i^2}{(1 - \alpha \rho)^2} ]</td>
<td>( \frac{(\sigma_i^2 + \sigma_u^2)}{(1 - \alpha)^2} )</td>
</tr>
<tr>
<td>Cooperative peg</td>
<td>[ \frac{(1 - 2n(1 - n))\sigma_u^2}{1 - \alpha^2} + \left(1 + \frac{\alpha^2}{1 - \alpha^2} + \frac{\rho^2}{1 - \rho^2}\right) \frac{\sigma_i^2}{(1 - \alpha \rho)^2} ]</td>
<td>( \frac{((1 - 2n(1 - n))\sigma_u^2 + \sigma_i^2)}{(1 - \alpha)^2} )</td>
</tr>
</tbody>
</table>
and a one sided peg. Since the variance of employment is lowest under the cooperative peg, it follows from the analysis above that the mean employment level is highest under this system. Thus, the expected value of (the log of) real GDP is highest under a cooperative peg.

Note also from Table 1 that the component of the volatility of real GDP and employment that is attributable to technology shocks is identical across all regimes. This points to a basic property of this model. Since the exchange rate (see Eq. (26)) does not depend on technology shocks, the response of the economy to these shocks does not depend at all on the exchange rate regime. In fact, real GDP responds to a technology shock only with a one period lag. Thus, this model has a sharp implication for the debate on whether exchange rate targeting involves an efficiency cost by preventing the automatic adjustment of the real exchange rate to country specific technology shocks. We find that, as far as the response to country specific technology shocks are concerned, when the alternative to exchange rate targeting is a passive monetary rule (i.e. (18)), there is no efficiency cost involved in targeting, either using a one-sided peg or a cooperative peg.

5. Welfare and exchange rate regimes

We may conduct a welfare comparison of the alternative monetary rules. Welfare may be defined as the expected utility of the representative individual (in either country). Define the value function for an individual as a function of initial capital and the technology shocks from one period ago. In addition, following the approach of Obstfeld and Rogoff (2000) and Corsetti and Pesenti (2001), we abstract from the utility of real balances in the evaluation of welfare (this makes no essential difference for the results); Given the structure of the model, it is not surprising that we can solve for the exact form of the value function. It is given by

$$V(K, \theta_{t-1}, \theta'_{t-1}) = A + B \ln K + D_1 \ln(\theta_{t-1}) + D_2 \ln(\theta'_{t-1}),$$

where $A, B, D_1$ and $D_2$ are constants, given by

$$A = \Omega_0 + \frac{1}{(1 - \beta)} \left( \eta E_{t-1} \ln(1 - H_t) - \frac{1}{1 - \beta^2} \ln \Theta \right),$$

$$B = \frac{\alpha}{1 - \beta^2}, \quad D_1 = \frac{\rho n}{(1 - \beta \rho)(1 - \beta^2)}, \quad D_2 = \frac{\rho(1 - n)}{(1 - \beta \rho)(1 - \beta^2)}.$$
Given the structure of the economy, the monetary rule affects only the constant term \( A \) in the value function. Recall by (29), \( H_t \) depends upon both shocks and monetary rules.

Using (31) we can draw two conclusions. First, in welfare terms, a one-sided peg and a floating exchange rate are identical. This follows because the one-sided peg leaves the term \( \Theta \) unchanged, and because it leaves the variance of \( H_t \) unchanged, relative to the floating exchange rate regime. Second, the cooperative peg delivers higher welfare than under a floating exchange rate. This follows because (a) a cooperative peg reduces the volatility of employment, thereby increasing welfare directly through the second term in \( A \), and (b) a cooperative peg reduces \( \Theta \), thereby raising employment above its initial, inefficiently low level under floating exchange rates.

We can use the value function (31) to assess the welfare importance of alternative monetary rules. As discussed, the fact of sticky prices has both level and volatility effects. Likewise, the exchange rate rules also differ with respect to level and volatility. From the employment equation, we derive a quantitative approximation to the level effects of alternative regimes. The right-hand side of Eq. (31) may be approximated in the following way:

\[
E \left( \frac{H}{1-H} \right) \approx \frac{EH}{1-EH} + \frac{EH^2}{(1-EH)^2} \text{Var} \left( \ln H \right).
\] (32)

A rise in the variance of employment must reduce expected employment in the proportion \( \frac{EH}{1-EH} \), in order to hold the left-hand side constant. Using this, and the results of Table 1, we compute the mean levels of employment and GDP for the flexible price economy, and the three different monetary rules. The results are reported in Table 2.

Table 2
Moments from the model

<table>
<thead>
<tr>
<th></th>
<th>Mean output</th>
<th>Mean employment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible price model</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>Floating exchange rates</td>
<td>0.999</td>
<td>0.2994</td>
</tr>
<tr>
<td>One sided peg</td>
<td>0.9993</td>
<td>0.2984</td>
</tr>
<tr>
<td>Cooperative peg</td>
<td>0.9996</td>
<td>0.2997</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Output variance</th>
<th>Employment variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible price model</td>
<td>0.075</td>
<td>0</td>
</tr>
<tr>
<td>Floating exchange rates</td>
<td>0.15</td>
<td>0.3</td>
</tr>
<tr>
<td>One sided peg</td>
<td>0.25</td>
<td>0.3</td>
</tr>
<tr>
<td>Cooperative peg</td>
<td>0.15</td>
<td>0.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Welfare cost</th>
<th>Consumption cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexible price-floating</td>
<td>0.065</td>
<td></td>
</tr>
<tr>
<td>Flexible price-cooperative peg</td>
<td>0.034</td>
<td></td>
</tr>
</tbody>
</table>

Parameters: \( \alpha = 0.36, \beta = 0.96, \rho = 0.9, \lambda = 11, \sigma_u^2 = \sigma_e^2 = 0.0025 \).
In comparing across exchange rate rules, we see that the welfare implications are extremely small.

6. Optimal monetary policy rules

In this section we follow the analysis of Gali and Monacelli (2002) and Obstfeld and Rogoff (2000, 2002) by identifying the optimal monetary policy rules. In Obstfeld and Rogoff (2000) an optimal monetary policy rule will always replicate the flexible price equilibrium. In the present model, it is not automatically implied that an optimal monetary policy would replicate the flexible price economy. However, since we follow the procedure of Gali and Monacelli (2002) by assuming a production subsidy which eliminates the distortion from monopoly pricing, the optimal monetary policy should attempt to replicate the flexible price equilibrium. 12

Given that the optimal monetary policy should achieve the flexible price equilibrium, an unrestricted optimal monetary policy is easily identified in this economy. If the money supply in each country follows the process

\[ M_t = M_{t-1}(1 + \mu) \exp(u_t + \varepsilon_t), \tag{33} \]

then it is straightforward to see from Eqs. (29) and (30), that (a) real GDP will respond as in the flexible price equilibrium, and (b) employment will be constant and equal to the flexible price equilibrium. Thus, the monetary authorities should respond positively to both velocity and technology shocks.

But the monetary policy described in (33) requires that the monetary authority directly respond to the structural disturbances hitting the economy. As discussed above, this is an unrealistic feature for monetary policy. But (33) may in fact be implemented by a simpler monetary rule which directly targets domestic employment. Take the rule given by

\[ M_t = M_{t-1}(1 + \mu) \left( \frac{H_t}{H} \right)^{-\sigma_H}. \tag{34} \]

In this rule, the authorities follow a contractionary monetary policy whenever employment moves above its flexible price equilibrium level. Equivalently the rule acts to offset any deviations of employment from its ‘natural rate’.

Now substitute for employment from (29) into (34), and rearrange, to get

\[ M_t = M_{t-1}(1 + \mu)(\exp(u + \varepsilon))^{\frac{\sigma_H}{1 + \sigma_H}}(H \Theta^\frac{1}{\sigma_H})^{1 - \frac{1}{\sigma_H}}. \]

Now, letting \( \sigma_H \to \infty \), we see that this monetary rule replicates the optimal rule (33), which keeps employment exactly at its constant, flexible price level. 13

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12 Also note that because we are ignoring the utility of real balances in welfare, the optimal monetary policy will not attempt to achieve the Friedman rule.

13 Note that in the rule given by (34), as \( \sigma_H \to \infty \), the term \( (H \Theta^\frac{1}{\sigma_H})^{1 - \frac{1}{\sigma_H}} \) approaches unity, given the definition of \( \Theta \) from Eq. (29).
But it also leads to an increase in average employment, since mean employment is negatively related to employment volatility in this economy.

A critical aspect of (34) however is that it must operate through exchange rate movements. When both monetary authorities follow this rule, the exchange rate is

\[ S_t = \frac{M_0 \exp(\varepsilon_t)}{M_0 \exp(\varepsilon_t^*).} \]

Unless technology shocks are perfectly correlated across countries, the adjustment to technology shocks under the optimal monetary policy rule will require exchange rate adjustment. Intuitively, a technology shock in the home economy requires that the relative price of the home good should fall. A compensating monetary expansion in the home economy leads to an exchange rate depreciation, which achieves the required terms of trade deterioration. The optimal rule therefore entails country-specific monetary accommodation of technology and velocity disturbances. But under a fixed exchange rate regime, all monetary policy movements have to be exactly coordinated in the home and foreign economy. There can be no country specific monetary policy movements. Thus, loss of exchange rate flexibility has real consequences in an economy where optimal monetary policy can be employed.

Is there a limited monetary policy consistent with a fixed exchange rate that improves upon the environment without activist monetary policy? The optimal policy requires that each country’s monetary policy expand to raise aggregate demand in face of technology shocks, and offset velocity shocks. A compromise policy, consistent with fixed exchange rates, is that the world money supply expands in response to a technology shock in either country. Take the rule, followed by each country, given by

\[ M_t = M_{t-1}(1 + \mu) \exp(u_t) \exp(n_t) \exp((1 - n)\varepsilon_t). \] (35)

Substituting this into (30), we see that this policy ensures that real GDP responds to a technology shock in the same way as in the flexible price economy, and is independent of velocity shocks. Thus, even in the presence of country specific disturbances, a constrained optimal monetary policy can achieve the first-best optimum for the variance of GDP, and still remain consistent with exchange rate stability. But from (29), it is clear that the policy given by (35) does not stabilize employment. Employment (in the home economy) is now given by

\[ H_t = \left( \frac{(1 - \beta\mu)M_0 \exp((1 - n)\varepsilon_t - \varepsilon_t)}{(1 - \beta\varepsilon)} \frac{P_{t-1}M_0}{P_{t-1}M_0} \right)^{1/\pi}. \] (36)

While the monetary rule can ensure that real GDP in each country responds efficiently to technology shocks, the equilibrium response of employment requires terms of trade adjustment, which cannot occur under a fixed exchange rate. For instance, a technology shock in the home economy requires that real GDP increase in both the
home and foreign countries. This much can be achieved with a world monetary expansion. But in the floating exchange rate economy (with the optimal monetary policy), the increase in real GDP at home is achieved by a rise in production in proportion to the technology shock, and a fall in the terms of trade, while in the foreign economy it is achieved by no change in production, and an improvement in the terms of trade.

With a fixed exchange rate, the terms of trade cannot change. Therefore, GDP can only increase if production must increase in both economies. This means that production rises by too little in the home economy (as employment partially falls in response to a home technology shock, in place of the terms of trade deterioration that would take place under the optimal rule (34)), and too much in the foreign economy (as foreign employment increases in response to the technology shock, in place of the terms of trade improvement that would take place under (34)).

How much will the rules given by (34) and (35) affect welfare? The answer to this is actually implicit in the previous analysis. The unconstrained rule given by (34) achieves the first best outcome without price rigidities, but in welfare terms this has only a very minor benefit. Moreover, the welfare differences between the optimal and constrained optimal rule will be extremely small.

7. Conclusions

This paper has examined the trade-off between fixed and floating exchange rates in a sticky-price dynamic, stochastic general equilibrium model with capital accumulation. The results suggest that the trade-off is quite at variance with much of the discussion in the policy literature. In our model, allowing the exchange rate to float does not help at all in the response to country-specific supply or demand policy shocks. In fact, fixed exchange rates may do better, by increasing employment and long run GDP, as well as welfare. But if the benchmark comparison is one where monetary policy can be ‘activist’, where monetary rules can target employment, then giving up exchange rate flexibility will have real costs, both in terms of macroeconomic volatility and average long run GDP. Overall, however, when quantified in this model, the welfare differences between alternative exchange rate regimes is very small.

We have focused only on velocity shocks and productivity shocks. But it can be shown that the presence of government spending shocks has no affect on the welfare comparison between fixed and floating carried out in Section 5. Since the exchange rate will not in general respond directly to governments spending shocks, a passive floating exchange rate regime does not improve welfare relative to a peg, and moreover, for the same reasons as before, a cooperative peg still welfare dominates.

There are a number of caveats. Special functional forms are used in order to facilitate an analytical solution, and the model has only one period price setting. But in fact the result that productivity shocks do not directly affect the exchange rate extends to more general preference specifications (see Devereux and Engel,
2003). Also, it is worthwhile to note that the functional forms used are very close to those employed in the quantitative International Real Business Cycle literature, so it is not clear how limiting the present model is. A more general analysis would presumably need to allow for a more persistent degree of price rigidity, along the lines of Calvo (1983) or Taylor (1979). Extending our model to allow for this would make it necessary to employ numerical solution methods. This is left for future research.

Nonetheless, our analysis may throw some light on the debate about the costs and benefits of exchange rate flexibility. In this vein, we might interpret the results as providing a cautionary note about the adjustment properties of floating exchange rates.

Acknowledgements

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References