

Sharing the Burden: Monetary and Fiscal Responses to a World Liquidity Trap *

David Cook[†] and Michael B Devereux[‡]

December 19, 2011 (first draft, June 2011)

Abstract

With integrated trade and financial markets, a sudden collapse in aggregate demand in a large country can cause ‘natural real interest rates’ to fall below zero in other countries, giving rise to a global ‘liquidity trap’. This paper explores the optimal policy response following such a shock, when governments cooperate on both fiscal and monetary policy. Adjusting to a large negative demand shock requires raising world aggregate demand, as well as redirecting demand towards the source (home) country. The key feature of demand shocks in a liquidity trap is that relative prices respond perversely. A negative demand shock concentrated in the home country causes an *appreciation* of the home terms of trade, exacerbating the slump in that country. At the zero bound, the home country cannot counter this shock. In this situation, it may be optimal for the foreign policy-maker to *raise* interest rates. Strikingly, the foreign country may choose to have a positive policy interest rate, even though its ‘natural real interest rate’ is below zero. An optimal policy involves a combination of relatively tight monetary policy in the foreign country combined with substantial fiscal expansion in the home country. Thus, in response to conditions generating a global liquidity trap, there is a critical mutual interaction between monetary and fiscal policy.

Keywords: Liquidity Trap, Monetary Policy, Fiscal Policy, International Spillovers

JEL: E2, E5, E6

* Devereux thanks SSHRC, the Bank of Canada, and the Royal Bank of Canada for financial support. The opinions in this paper are those of the authors alone and cannot be ascribed to the Bank of Canada. We thank seminar participants at the Bank of Japan, participants of the ECB-Bundesbank joint lunchtime workshop, the Bundesbank Spring Conference 2011, and the Paris School of Economics Conference October 2011, for comments. In addition, we thank Paul Beaudry, Nao Sudou, Ippei Fujiwara, Takahashi Ito, Giovanni Lombardo, Luca Dedola, Gernot Muller, Florin Bilbie and John Leahy for comments.

[†]Hong Kong University of Science and Technology, davcook@ust.hk

[‡]CEPR, NBER, University of British Columbia, devm@interchange.ubc.ca

1 Introduction

This paper is concerned with global policy responses to a world liquidity trap. For the last three years, many areas of the world economy have experienced depressed aggregate demand and policy interest rates close to zero. But the incidence has not been uniform across countries or regions. This suggests that policy responses may differ among countries. In normal times, with positive interest rates, an active monetary policy can be used to perfectly offset shocks to aggregate demand. In the standard New Keynesian model the ‘divine coincidence’ means that monetary policy can simultaneously deliver zero inflation and a zero output gap. In such a setting, monetary policy makers may act independently - there is no need for international policy cooperation across countries. But in face of large negative demand shocks, one or more countries may find that optimal policy is constrained by the zero lower bound on interest rates. This leads policy choices to become interdependent across countries. This paper explores how optimal policy should be designed in such an environment.

A key question is how the adjustment to a global recession should be allocated across countries that experience the downturn at different levels of severity. The specific focus of the paper is to identify an optimal policy response to a world liquidity trap in which two trading partners are well integrated through financial markets but less than perfectly integrated in goods markets. The international propagation of negative demand shocks depends critically upon three factors; a) the size of the shock, b) the degree of integration in financial markets, and c) the degree of integration in goods markets. For small shocks, monetary policy can be used to offset the shock in each country, and there is no need for either international cooperation, or the use of other policy tools, such as fiscal policy. But for very large shocks which push desired policy rates below zero, and when international financial markets are highly integrated (as we assume throughout the paper), the propagation of shocks depends critically on the degree of trade integration. With highly open trade linkages, all countries face similar macroeconomic response, regardless of the source of the shock. In particular, a liquidity trap in one country is mirrored in all other countries. This implies that policy responses should be uniform across countries, and the optimal policy response is to have interest rates as low as possible, and an equal fiscal expansion in all countries.

However, the benchmark of fully open trade does not closely approximate the current configuration of the world economy, where large, but relatively closed economies, such as Japan and the United States, are stuck in a liquidity trap. In these countries, exports make up substantially less than 20% of GDP. With home bias in consumption baskets, which acts

so as to reduce trade linkages between countries, both the propagation of demand shocks and the optimal response of policy to shocks takes on very different characteristics. Typically, a large negative demand shock in one country will push down the desired real interest rate in that country more than those of its trading partners. This is the scenario we examine in the paper.

The key feature of the environment with home bias in trade is that a shock that precipitates a liquidity trap generates a perverse response of the terms of trade. Under ‘normal’ monetary policy, a fall in demand in one country will reduce domestic real interest rates and lead to a compensating terms of trade deterioration, channelling more world demand towards the country directly affected by the shock, thus mitigating the effects of the shock. But when the interest rate is at the zero bound, this same shock generates a terms of trade *appreciation*, since it tends to raise domestic real interest rates by pushing down inflation expectations. Hence, the response of the terms of trade exacerbates the effect of the shock. Typically, in order to alleviate a terms of trade appreciation, a country could engage in expansionary monetary policy. But when interest rates are zero, the home country (which is the source of the shock) cannot do this. But instead, the foreign country can raise its interest rate. We find that, such an interest rate increase will in general form part of an optimal policy response. Strikingly, we find that the optimal policy is for the foreign country to tighten monetary policy, even when using the standard criterion from the closed economy logic, it should still be in a liquidity trap (where its ‘natural’ real interest rate is below zero). The foreign interest rate increase acts so as to weaken the appreciation of the home terms of trade caused by the original demand shock, limiting the degree of world expenditure switching away from the home economy. Overall, it is best for both countries to have higher interest rates in the trading partner, when the source country shock requires zero home interest rates.

Our results in fact show that the response of policy interest rates in a global liquidity trap are piecewise functions of the degree of trade-openness, as measure by the parameter of ‘home bias’ in preferences. When preferences are identical, trade is fully open, and a global liquidity trap is associated with zero policy rates in all countries. For a shock coming from the home country, home policy rates are always set equal to zero. As preferences display more home bias, both policy rates are still zero for some interval. But at a critical threshold level of home bias, foreign interest rates are raised, even when the foreign natural real interest rate is negative. As the degree of home bias rises, foreign policy rates rise more and more, and are always set above the foreign natural real interest rate.

The message is that the open economy dimension has very substantial implications for

both the occurrence of a liquidity trap, in the sense that it predicts that policy is not restricted by the zero lower bound even when traditional indicators (which look at the value of the ‘natural real interest rate’) say that it should be, and for the way in which policy is designed when the world economy ‘on average’ is in a liquidity trap. More generally, the model predicts that the ‘burden of adjustment’ to a global liquidity trap may be spread quite unequally across countries, and implies some apparently counterintuitive policy responses¹

²

We extend the model to allow for the choice of fiscal policy response as well as monetary response. Again, the results critically depend on the degree of trade integration. With fully open trade, both countries are in a liquidity trap, and the optimal policy is to have an identical fiscal expansion. But when trade is limited, we find that the optimal policy response is to have a large fiscal expansion in the home country, coupled with a small fiscal expansion, but a positive interest rate in the foreign country. The results indicate that monetary and fiscal policy should be used in a mutually supportive way in responding to a global liquidity trap shock. If monetary policy were set in a conventional way, so that policy rates were equal to natural real interest rates, except when the latter variables were below zero, then all the burden of adjustment would be on fiscal policy. In this case, in order to facilitate expenditure adjustment and expenditure switching, a policy response would require a large home fiscal expansion and foreign fiscal *contraction*. The benefit of adjusting foreign interest rates optimally is that it relieves (but does not eliminate) the need for large fiscal responses in each country.

The paper builds on a substantial recent literature on monetary and fiscal policy in a liquidity trap. In particular, with the experience of Japan in mind Krugman (1999), Eggertson and Woodford (2003, 2005), Jung et al. (2005), Svensson (2003), Auerbach and Obstfeld (2004) and many other writers explored how monetary and fiscal policy could be usefully employed even when the authorities have no further room to reduce short term nominal interest rates. Recently, a number of authors have revived this literature in light of the very similar problems now encountered by the economies of Western Europe and North America. Papers by Christiano et al (2009), Devereux (2010), Eggertson (2009), Taylor et al. (2008) have explored the possibility for using government spending expansions, tax

¹By burden of adjustment we refer to the agreement among countries to engage in macroeconomic response to the global recession. This was a central part of the G20 process in 2008 and 2009, and led to commitments to engage in fiscal stimulus. Separately, various swap agreements among central banks indicated a substantial degree of cooperative monetary stimulus.

²Although the paper is a theoretical analysis primarily, it is worth noting that in the aftermath of the recession of 2008-2009, other jurisdictions such as the eurozone raised policy rates while the Federal Reserve kept rates close to zero throughout the recession and beyond.

cuts, and monetary policy when the economy is in a liquidity trap. For the most part, these papers did not focus on the international dimension of liquidity traps. Some recent expectations are Fujiwara et al. (2009, 2010), Erceg et al. (2009) and Jeanne (2009). Jeanne (2009) examines a ‘global liquidity trap’ in a model of one-period ahead pricing similar to that of Krugman (2009). Erceg. et al (2009) use a fully specific two country DSGE model to examine the international transmission of shocks when one country is in a liquidity trap, but do not focus on optimal monetary policy or fiscal policy choices. Fujiwara et al. (2009) examine the optimal monetary problem with commitment in a multi country situation, but do not examine the determination of fiscal policy, or the transmission of demand shocks across countries. Fujiwara et al. (2010) look at the impact of the international effects of fiscal policy in a liquidity trap, examining the sign and size of domestic and international fiscal multipliers. Our paper may be seen as complementary to theirs in that we extend the analysis to incorporate trade frictions, but more importantly, investigate the determination of optimal policy³.

The rest of the paper is organized as follows. The next section develops the basic model. Section 3 examines the solution under sticky prices. Then in section 4 we analyze the impact of demand shocks at the zero lower bound. Section 5 examines the optimal policy making problem in a global cooperative agreement, including the possibility of using both monetary and fiscal policy. Section 7 illustrates a numerical analysis of optimal policy. Some conclusions are then offered.

2 A two country model

Assume that there are two countries in the world economy. In each country, households consume both private and government goods, and supply labor. Denote the countries as ‘home’ and ‘foreign’, with foreign variables denoted with an asterisk superscript. The population of each country is normalized to unity. Each country produces a range of differentiated goods. Complete asset markets allow full insurance of consumption risk across countries. Households also hold their own country’s nominal government bonds. Firms produce private goods, while governments produce government goods which are distributed uniformly across households. Firms production and supply is constrained by sticky prices. Governments have access to lump sum taxation.

³In addition, a previous paper (Cook and Devereux, 2011) examines the linkages of natural real interest rates, the determination of fiscal multipliers and optimal fiscal policy in a simpler version of the model of the present paper, but does not allow for the endogenous response of monetary policy.

2.1 Households

Utility of a representative infinitely lived home household evaluated from date 0 is:

$$U_t = E_0 \sum_{t=0}^{\infty} (\beta)^t (U(C_t, \xi_t) - V(N_t) + J(G_t)) \quad (1)$$

where U , V , and J represent the utility of the composite home consumption bundle C_t , disutility of labour supply N_t , and utility of the government supplied public good G_t , respectively. The variable ξ_t represents a shock to preferences or a ‘demand’ shock. We assume that $U_{12} > 0$.

Composite consumption is defined as

$$C_t = \Phi C_{Ht}^{v/2} C_{Ft}^{1-v/2}, \quad v \geq 1$$

where $\Phi = \left(\frac{v}{2}\right)^{\frac{v}{2}} \left(1 - \left(\frac{v}{2}\right)\right)^{\frac{v}{2}}$, C_H is the consumption of the home country composite good by the home household, and C_F is consumption of the foreign composite good. If $v > 1$ then there is a home preference bias for domestic goods. The case $v > 1$ is most realistic for thinking about policy in large open economies.

Consumption aggregates, C_H and C_F are composites, defined over a range of home and foreign differentiated goods, with elasticity of substitution θ between goods. Price indices for home and foreign consumption are:

$$P_H = \left[\int_0^1 P_H(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad P_F = \left[\int_0^1 P_F(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}},$$

while the aggregate (CPI) price index for the home country is $P = P_H^{v/2} P_F^{1-v/2}$ and for the foreign is $P_t^* = P_F^{*v/2} P_H^{*1-v/2}$

Demand for each differentiated good ($j = H, F$) is

$$\frac{C_j(i)}{C_j} = \left(\frac{P_j(i)}{P_j} \right)^{-\theta}$$

The law of one price holds for each good so $P_j(i) = S_t P_j^*(i)$, where S_t is the nominal exchange rate (home price of foreign currency).

Home government spending falls on the home composite good and foreign government spending on the foreign composite good. Thus, government spending is assumed to have full ‘home bias’. In addition, we assume that government spending demand for each variety of home goods has price elasticity θ , the same as that for private spending.

The household's implicit labor supply at nominal wage W_t is:

$$U_C(C_t, \xi_t)W_t = P_t V'(N_t). \quad (2)$$

Optimal risk sharing implies

$$U_C(C_t, \xi_t) = U_C(C_t^*, \xi_t^*) \frac{S_t P_t^*}{P_t} = U_C(C_t^*, \xi_t^*) T_t^{v-1}, \quad (3)$$

Nominal bonds pay interest, R_t . Then the consumption Euler equation is:

$$\frac{U_C(C_t, \xi_t)}{P_t} = \beta R_t E_t \frac{U_C(C_{t+1}, \xi_{t+1})}{P_{t+1}}. \quad (4)$$

Foreign household preferences and choices can be defined exactly symmetrically. The foreign representative household has weight $v/2$, $(1 - v/2)$ on the foreign (home) composite good in preferences.

2.2 Firms

Each firm i employs labor to produce a differentiated good.

$$Y_t(i) = N_t(i),$$

Profits are $\Pi_t(i) = P_{Ht}(i)Y_t(i) - W_t H_t(i)^{\frac{\theta-1}{\theta}}$ indicating a subsidy financed by lump-sum taxation to eliminate steady state first order inefficiencies. Each firm re-sets its price according to Calvo pricing with probability of adjusting prices equal to $1 - \kappa$. Firms that adjust their price set new price given by $\tilde{P}_{Ht}(i)$:

$$\tilde{P}_{Ht}(i) = \frac{E_t \sum_{j=0} m_{t+j} \kappa^j \frac{W_{t+j}}{A_{t+j}} Y_{t+j}(i)}{E_t \sum_{j=0} m_{t+j} \kappa^j Y_{t+j}(i)}. \quad (5)$$

where the stochastic discount factor is $m_{t+j} = \frac{P_t}{U_C(C_t, \xi_t)} \frac{U_C(C_{t+j}, \xi_{t+j})}{P_{t+j}}$. In the aggregate, the price index for the home good then follows the process given by:

$$P_{Ht} = [(1 - \kappa) \tilde{P}_{Ht}^{1-\theta} + \kappa P_{Ht-1}^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (6)$$

The behavior of foreign firms and the foreign good price index may be described analogously.

2.3 Market Clearing

Equilibrium in the market for good i as

$$Y_{Ht}(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\theta} \left[\frac{v}{2} \frac{P_t}{P_{Ht}} C_t + \left(1 - \frac{v}{2}\right) \frac{S_t P_t^*}{P_{Ht}} C_t^* + G_t \right],$$

where G_t represents total home government spending. Aggregate market clearing in the home good is:

$$Y_{Ht} = \frac{v}{2} \frac{P_t}{P_{Ht}} C_t + \left(1 - \frac{v}{2}\right) \frac{S_t P_t^*}{P_{Ht}} C_t^* + G_t. \quad (7)$$

Here $Y_{Ht} = V_t^{-1} \int_0^1 Y_{Ht}(i) di$ is aggregate home country output, where we have defined $V_t = \int_0^1 \left(\frac{P_{Ht}(i)}{P_{Ht}} \right)^{-\theta} di$. It follows that home country employment (employment for the representative home household) is given by $N_t = \int_0^1 N(i) di = Y_{Ht} V_t$.

An equilibrium in the world economy with positive nominal interest rates may be described by the equations (2), (4), (5), (6), and (7) for the home country, and the analogous equations for the foreign economy. Together with (3), and for given values of V_t and V_t^* , given monetary rules (to be discussed below) and given government spending policies, these equations determine an equilibrium sequence for the variables $C_t, C_t^*, W_t, W_t^*, S_t, P_{Ht}, P_{Ft}^*, \tilde{P}_{Ht}, \tilde{P}_{Ft}^*, R_t, R_t^*$, and N_t, N_t^* .

3 Demand Shocks and Natural Interest Rates

3.1 Defining Natural Interest Rates

Define $\sigma \equiv -\frac{U_{CC}\bar{C}}{U_C}$ as the inverse of the elasticity of intertemporal substitution in consumption, $\phi \equiv -\frac{V''\bar{H}}{V'}$ as the elasticity of the marginal disutility of hours worked, and $\sigma_g \equiv -\frac{J''\bar{G}}{J'}$ as the elasticity of marginal utility of public goods. In addition, we assume that $\sigma_g = \sigma > 1$. Finally, $\varepsilon_t = \frac{U_{C\xi}}{U_C} \ln(\xi_t)$ is the measure of a positive demand shock in the home country, with an equivalent definition for the foreign country. Define $c_y = \frac{C}{Y}$ is the steady state share of consumption in output.

We assume that any preference shock is unanticipated, and reverts back to zero with probability $1 - \mu$ in each period. Because there are no predetermined state variables in the model, this implies that all variables in the world economy will inherit the same persistence as the shock itself, in expectation. Thus, for any variable x_t , we may write $E_t(x_{t+1}) = \mu x_t$. After the shock expires, all variables will then revert to their zero initial equilibrium.

We first derive a measure of Wicksellian, or ‘natural’ real interest rates for each country, defined as the interest rates that would hold in a purely flexible price equilibrium of the world economy where there are no monopolistic distortions, and in addition where governments choose an optimal fiscal spending rule with access to lump-sum taxes. In this case, the government spending rate for the home economy will be determined by:

$$V'(N_t) = J'(G_t) \quad (8)$$

For any variable x_t , define the world average and world relative level, $x_t^W = \frac{x_t + x_t^*}{2}$ and $x_t^R = \frac{x_t - x_t^*}{2}$. In a competitive equilibrium with optimal government spending in both countries as in (8), the natural real interest rates of the home and foreign economy are defined as:⁴

$$\tilde{r}_t = \bar{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W + \frac{\phi c_y (v - 1)}{\Delta} \varepsilon_t^R \right) (1 - \mu) \quad (9)$$

$$\tilde{r}_t^* = \bar{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W - \frac{\phi c_y (v - 1)}{\Delta} \varepsilon_t^R \right) (1 - \mu) \quad (10)$$

where $\Delta \equiv \phi c_y D + \phi(1 - c_y) + \sigma$, $\sigma > D \equiv (\sigma v(2 - v) + (1 - v)^2) > 1$, and \bar{r} is the non-stochastic steady state interest rate. These are critical variables for our analysis, since they govern the degree to which monetary policy can be efficiently employed to stabilize the economy. In particular, our model has the characteristic that when (9) and (10) are both positive, then monetary policy can perfectly achieve the joint target of zero inflation and zero output gaps, since home and foreign policy rates can simply be set to equal (9) and (10), respectively. In addition, as seen below, there will then be no need to have fiscal gaps differ from zero.

Note that in the no home bias case, when $v = 1$, the natural interest rate for both economies should be the same, so that $\tilde{r}_t = \tilde{r}_t^* = \bar{r} + \left(\frac{\phi c_y}{\phi + \sigma} \varepsilon_t^W \right) (1 - \mu)$. The reason is that, with no home bias, demand shocks have no effect on the terms of trade. As a result, with financial market integration, PPI based real interest rates are equalized across countries.

For concreteness, we look at the case where the home country is the source of the shocks. In particular, we will assume that home consumers are subject to preference shocks which affect their propensity to save, whereas consumers in the foreign economy is not directly affected by these shocks. Of course foreign consumers will be indirectly affected by the shock, since integrated financial markets lead to linkages between interest rates. Thus, a saving shock with its source in the home economy, pushing the monetary authority into a liquidity trap,

⁴Note that this is defined as the value of $r_t - E_t \pi_{Ht+1}$ in a flexible price economy, or in other words, the PPI based real interest that would hold with flexible prices.

may have similar effects on the foreign economy, even though the foreign consumers are not directly affected by the shock.

Making this assumption, we have in this case, $\varepsilon_t^* = 0$ and $\varepsilon_t^W = \varepsilon_t^R = \frac{\varepsilon_t}{2}$ and we can write the natural real interest rates as:

$$\tilde{r}_t = \bar{r} + \left(\frac{\Delta + (\phi + \sigma)(v - 1)}{(\phi + \sigma)\Delta} \right) (1 - \mu)\phi c_y \frac{\varepsilon_t}{2} \quad (11)$$

$$\tilde{r}_t^* = \bar{r} + \left(\frac{\Delta - (\phi + \sigma)(v - 1)}{(\phi + \sigma)\Delta} \right) (1 - \mu)\phi c_y \frac{\varepsilon_t}{2} \quad (12)$$

We may rewrite the natural real interest rate expressions in shorthand as $\tilde{r}(\varepsilon_t, v)$ and $\tilde{r}^*(\varepsilon_t, v)$. If the home country shock is sufficiently negative, then it may drive natural real interest rates below zero. We define $\varepsilon_H(v)$ and $\varepsilon_F(v)$ respectively as the size of the shock such that $\tilde{r}(\varepsilon_H, v) = 0$, and $\tilde{r}^*(\varepsilon_F, v) = 0$. Clearly, for $v \geq 1$, $\varepsilon_H \geq \varepsilon_F$. Figure 1 illustrates the two functions $\tilde{r}(\varepsilon_t, v)$ and $\tilde{r}^*(\varepsilon_t, v)$. For $v = 1$, they coincide, while for $v = 2$, the foreign natural real interest rate is simply \bar{r} . As the countries move from being more open to more closed, the impact of the shock on the home country natural real interest rate rises, while the impact on the foreign natural real interest rate falls.

In the discussion below, we will mostly focus on a ‘large’ shock, such that $\varepsilon_t < \varepsilon_H(1)$. This means that whatever is v , the home country natural real interest rate is always below zero.

3.2 The World and Relative Economy

We derive a sticky price log-linear approximation of the model in terms of inflation and output gaps in a similar manner to Clarida et al. (2002) and Engel (2010). Let \hat{x}_t be the percentage deviation of a given variable x_t from the efficient zero flexible price equilibrium. Thus, \hat{x}_t is interpreted as a ‘gap’ variable. As defined before, $D \equiv \sigma v(2 - v) + (1 - v)^2 > 1$. In addition, let $s \equiv \frac{\sigma}{c_y}$, and $s > s_D \equiv \frac{s}{D} > 1$.

In order to explore the implications of the zero lower bound constraint, we begin with the standard forward looking inflation equations and open economy IS relationships, expressed in terms of world averages and world relatives. The world average equations are:

$$\pi_t^W = k(\phi + s)\hat{n}_t^W - ks \cdot \hat{c}g_t^W + \beta E_t \pi_{t+1}^W \quad (13)$$

$$sE_t(\hat{n}_{t+1}^W - \hat{n}_t^W) - sE_t(\hat{c}g_{t+1}^W - \hat{c}g_t^W) = E_t(r_t^W - \tilde{r}_t^W - \pi_{t+1}^W) \quad (14)$$

The world ‘relative’ variables are written as:

$$\pi_t^R = k(\phi + s_D)\widehat{n}_t^R - ks_D\widehat{g}_t^R + \beta E_t\pi_{t+1}^R \quad (15)$$

$$s_DE_t(\widehat{n}_{t+1}^R - \widehat{n}_t^R) - s_DE_t(\widehat{c}g_{t+1}^R - \widehat{c}g_t^R) = E_t(r_t^R - \widetilde{r}_t^R - \pi_{t+1}^R) \quad (16)$$

where $\widehat{c}g_t^W \equiv (1 - c_y)\widehat{g}_t^W$ and $\widehat{c}g_t^R \equiv (1 - c_y)\widehat{g}_t^R$. The coefficient k depends on the degree of price rigidity. Note that, approximated around the steady state, $\widehat{n}_t \approx \widehat{y}_t$, $\widehat{n}_t^* \approx \widehat{y}_t^*$, so the labor gap for each country will stand in for the output gap.

If the natural interest rates of both economies are always above zero, then the monetary and fiscal authorities can achieve perfect price and output stability by setting the nominal interest rate equal to the natural real interest rate and keeping the fiscal gaps, \widehat{g}_t^W and \widehat{g}_t^R equal to zero. However, if one or both countries have a natural real interest rate below zero then this cannot occur, because then the world and relative policy interest rates cannot be set to equal world and relative natural rates without at least one policy rate being below zero.

Note that both systems of equations (for the world average and the world relative economies) are in the canonical form of the New Keynesian closed economy equations. The only difference comes in the parameterization of the inverse elasticity of consumption: s , in the case of the average economy; and, s_D , in the case of the relative world economy. Note that $s_D < s$, so the world average level of demand is less sensitive to the average interest rate than the relative level of demand is sensitive to the relative interest rate. This reflects the expenditure switching effect of terms of trade changes. When worldwide interest rates are relatively low, then (for intertemporal substitution reasons) world demand will be relatively high. Analogously, when the relative interest rate is low, demand will be relatively high in the low interest rate country. But in addition, in order to satisfy interest rate parity, a relatively low real interest rate country must have an anticipated terms of trade appreciation. This implies a current terms of trade depreciation, leading world aggregate demand to move towards the low interest rate country through the expenditure switching channel.

This distinction is key to the understanding of the response to shocks that create liquidity traps. When a negative demand shock in one country drives down its relative interest rate, then the terms of trade will deteriorate, cushioning the full impact of the demand shock. But when the demand shock leads the home relative interest rate to rise, then the terms of trade appreciates, and the relative price effect is destabilizing. In the next section, we show that this is the response characteristic of a liquidity trap.

4 The International Effects of Negative Demand Shocks

If demand shocks are sufficiently small (i.e. so that $\varepsilon_t > \varepsilon_H(v)$), then policy rates can adjust to eliminate the effects of shocks, so that all gaps are zero⁵. In this case, a negative demand shock will lead to a decline in policy rates in both countries precisely equal to the fall in the natural rates $\tilde{r}(\varepsilon, v)$ and $\tilde{r}^*(\varepsilon, v)$. For $v > 1$, the home policy rate falls by more than the foreign policy rate. In this case, the home terms of trade deteriorates.

For comparison purposes it is useful to illustrate the impact of a small shock that satisfies $\varepsilon_t > \varepsilon_H(v)$, but where instead of adjusting policy rates to offset the shocks, we assume that monetary rules are more constrained. In particular, assume that monetary policy in each country follows a simple Taylor rule. This comparison is useful in that it provides a contrast to the effect of shocks when interest rates are constrained by the zero lower bound.

4.1 Demand Shocks under a Taylor rule

The movement of natural real interest rates is as in (9) and (10). But assume that, instead of offsetting the movement in natural real interest rates, policy interest rates are set such that:

$$r_t = \bar{r} + \gamma\pi_{Ht}, \quad r_t^* = \bar{r} + \gamma\pi_{Ft}, \quad \gamma > 1 \quad (17)$$

Interest rates adjust downwards when a shock reduces current inflation rates. Using (17) in the solutions for world and relative output gaps, gives us:

$$\begin{aligned} \Delta_1 \hat{n}_t^W &= -(1 - \beta\mu)(\bar{r} - \tilde{r}_t^W) \\ \Delta_1^D \hat{n}_t^R &= (1 - \beta\mu)\tilde{r}_t^R \end{aligned}$$

where $\Delta_1 \equiv s(1 - \beta\mu)(1 - \mu) + (\gamma - \mu)k(\phi + s) > 0$, and $\Delta_1^D = s_D(1 - \beta\mu)(1 - \mu) + (\gamma - \mu)k(\phi + s_D) > 0$, with $\Delta_1 > \Delta_1^D$.

A negative demand shock in the home country ensures that $\bar{r} - \tilde{r}_t^W > 0$ and $\tilde{r}_t^R < 0$. Thus, both \hat{n}_t^W and \hat{n}_t^R fall. The home and foreign output gaps are written respectively as $\hat{n}_t = \hat{n}_t^W + \hat{n}_t^R$, and $\hat{n}_t^* = \hat{n}_t^W - \hat{n}_t^R$. Thus:

$$\hat{n}_t = (1 - \beta\mu) \left[\frac{-(\bar{r} - \tilde{r}_t^W)}{\Delta_1} + \frac{\tilde{r}_t^R}{\Delta_1^D} \right]$$

⁵Note that in order to implement this as an equilibrium outcome, interest rates must follow a rule consistent with a unique equilibrium, as discussed in Galí, 2008.

$$\widehat{n}_t^* = (1 - \beta\mu) \left[\frac{-(\bar{r} - \widetilde{r}_t^W)}{\Delta_1} - \frac{\widetilde{r}_t^R}{\Delta_1^D} \right]$$

The home output gap falls. The response of the foreign output gap is ambiguous, however, and depends upon both the strength of the shock as well as the openness of total trade. When $v = 1$, $\widetilde{r}_t^R = 0$, and home and foreign output gaps fall by equal amounts. Note that the first term inside the square brackets in each equation is independent of v . Then as v rises above unity, \widetilde{r}_t^R falls, Δ_1^D rises, so that the foreign output gap responds by less, and the home output gap by more.

The negative demand shock always reduces home country inflation. Foreign inflation is defined as $\pi_t^W - \pi_t^R$, which may be written as:

$$\pi_t^* \propto k(\phi + s)\widehat{n}_t^* + k(s - s_D)\widehat{n}_t^R$$

A sufficient condition for foreign inflation to fall is that the foreign output gap falls. But even if the foreign output gap rises, foreign inflation may still fall as a result of the reduction in the home output gap reducing demand and marginal cost in the foreign economy.

Finally, we may compute the impact of the demand shock on the terms of trade. We may derive the home terms of trade response in the following way. From interest rate parity, it must be that (up to a first order), we have:

$$r_t - E_t\pi_{Ht+1} = r_t^* - E_t\pi_{Ft+1} + E_t(\widehat{\tau}_{t+1} - \widehat{\tau}_t) \quad (18)$$

Under the Taylor rule, the endogenous response of interest rates must be such that $r_t - r_t^* \leq$, so that the home interest rate falls at least as much as the foreign rate. Now, using the assumption on persistence of all variables, the fact that the steady state terms of trade is zero (in logs), and the Taylor rule, we may write the response of the current terms of trade as:

$$\tau_t = -2\frac{\gamma - \mu}{1 - \mu}\pi_t^R \quad (19)$$

Since π_t^R is negative when $v > 1$, the terms of trade must depreciate. Hence, when policy interest rates are above their zero lower bound, and policymakers follow a Taylor rule, a negative demand shock in one country is associated with a depreciation in that country's terms of trade, which cushions the impact of the shock on inflation and the output gap.

4.2 Demand Shocks in a liquidity trap

Now assume that the demand shock satisfies $\varepsilon < \varepsilon_H(1)$. Then either one or both countries are constrained by the zero lower bound on nominal interest rates. The effect of these shocks obviously depends on the policy response, both the current and anticipated future responses. As stated above, we focus only on discretionary policy, assuming that the current policy-maker cannot credibly make announcements over future monetary policy actions. The next section examines the *optimal* policy response to a demand shock. But first, we explore the consequences of following the conventional policy, described as

$$r_t = \max(0, \tilde{r}_t), \quad r_t^* = \max(0, \tilde{r}_t^*) \quad (20)$$

Under this conjectured policy, each country will set its policy rate to target the natural interest rate, if this is feasible. Otherwise, policy interest rates will be zero. This is a natural extension of the optimal discretionary monetary rule in the closed economy literature on the ‘zero bound’ (e.g. Eggertson and Woodford 2003, Jung et al. 2005)⁶.

The impact of the shock on home and foreign output gaps depends, for a given shock, on the actual value of v . We focus on two cases. In both cases, the home policy interest rate is zero, but the foreign policy rate is only zero for $v \leq v_F$. If $v > v_F$, then by rule (20), the foreign monetary authority will set $r_t^* = \tilde{r}_t^*$.

Case 1. For $v \leq v_F$, we have

$$\hat{n}_t = (1 - \beta\mu) \left[\frac{\tilde{r}_t^W}{\Delta_2} + \frac{\tilde{r}_t^R}{\Delta_2^D} \right]$$

$$\hat{n}_t^* = (1 - \beta\mu) \left[\frac{\tilde{r}_t^W}{\Delta_2} - \frac{\tilde{r}_t^R}{\Delta_2^D} \right]$$

where $\Delta_2 \equiv s(1 - \beta\mu)(1 - \mu) - \mu k(\phi + s) > 0$, and $\Delta_2^D = s_D(1 - \beta\mu)(1 - \mu) - \mu k(\phi + s_D) > 0$, with $\Delta_2 > \Delta_2^D$.⁷

In this case, the home output gap must fall, while the foreign output gap may rise or fall, depending on the size of v .

Case 2. For $v > v_F$, we have $\tilde{r}_t^W = \tilde{r}_t^R = \frac{\tilde{r}_t}{2}$. Then we get:

⁶In order to implement (20), the authorities would need to follow an interest rate feedback rule which guarantees uniqueness of equilibrium. See, e.g. Gali (2009)

⁷These terms must be positive in order that the equilibrium be determinate. This puts a limit on the degree of persistence of the demand shock.

$$\widehat{n}_t = (1 - \beta\mu) \frac{\widetilde{r}_t}{2} \left[\frac{1}{\Delta_2} + \frac{1}{\Delta_2^D} \right]$$

$$\widehat{n}_t^* = (1 - \beta\mu) \frac{\widetilde{r}_t}{2} \left[\frac{1}{\Delta_2} - \frac{1}{\Delta_2^D} \right]$$

Again, the home output gap must fall. But in this case, the foreign output gap will always *rise*, because, from the definitions above, we have $\Delta_2 > \Delta_2^D$.

It is straightforward to show that a negative demand shock causes the output gap in the home economy to fall by more when the economy is in a liquidity trap than under a Taylor rule. A fall in demand during a liquidity trap causes a persistent fall in inflation, which, given no adjustment in the nominal interest rate, causes a rise in the real interest rate, which causes a further fall in demand. So long as $\Delta_2 > 0$, this process converges when output falls by a sufficient amount.

In the open economy, however, there is a further effect at work. The fall in relative home country expected inflation leads to a rise in the home real interest rate, relative to the foreign real interest rate. In case 1 above, neither country's policy interest rate responds. By condition (18), this requires an anticipated terms of trade depreciation for the home country. Since the shock is temporary, an anticipated terms of trade depreciation can only be satisfied by an immediate terms of trade appreciation. Thus, the home country terms of trade must *appreciate*. The analogue of condition (19) under a liquidity trap in both countries is thus:

$$\tau_t = 2 \frac{\mu}{1 - \mu} \pi_t^R$$

Since in this case, $\pi_t^R < 0$, the home country terms of trade appreciates. Thus, in a liquidity trap, relative prices move in the 'wrong direction', leading to a further fall in demand for home goods, following the initial negative demand shock. This appreciation helps to explain why the cross country spillover impact of a negative demand shock may be positive⁸

In case 2, the appreciation in the terms of trade of the home country is diminished by

⁸While our model is obviously not designed to match the historical data, the incidence of real exchange rate appreciation in a liquidity trap is not inconsistent with the experience of Japan and the US in the wake of the financial crisis of October, 2008. The Bank of Japan and United States Federal Reserve Bank cut their policy rates so that, by December 2008, the uncollateralized call money rate and the Fed Funds rate were effectively at a lower bound. Simultaneously, both countries real exchange rates appreciated sharply, despite expansionary monetary policy. The CPI based Real Effective Exchange Rate for the US and Japan, appreciated by 15% and 30% respectively, between June and November 2008. Of course, other factors were at work, including the 'flight to safety' in global financial markets, and the reversal of the foreign exchange 'carry trade'. A full analysis of the impact of the episode on real exchange rates would require a careful decomposition of these separate effects.

the increase in the foreign interest rate. The terms of trade response is described as:

$$\tau_t = 2 \frac{\mu}{1 - \mu} \pi_t^R - \frac{\tilde{r}_t^*}{1 - \mu}$$

The first term is again negative, but the second term is positive. In general, this can go in either direction. But in the quantitative analysis below, we see that, even in the case where the foreign central bank adjusts the policy rate when $\tilde{r}_t^* > 0$, the home terms of trade still appreciates.

5 Optimal Policy in a Liquidity Trap

We now turn to the analysis of the optimal policy response to a liquidity trap shock. We explore optimal *cooperative* monetary and fiscal policy responses. While a complete analysis of the determination of fiscal and monetary policy in a global liquidity trap would also require an exploration of the strategic interaction between non-cooperative policy authorities, this raises difficult technical issues which we avoid here.⁹ Focusing on the cooperative problem is a desirable first approach, since it sets out a benchmark for choosing a policy so as to maximize world welfare in response to a negative demand shock that undermines the normal mechanism of monetary policy¹⁰.

In order to analyze optimal policy, we first need to define an objective function. A second order approximation to an equally weighted world social welfare can also be constructed in world averages and world differences¹¹.

⁹Clarida et al. (2002) describe a non-cooperative monetary policy equilibrium in a model with Calvo pricing and producer currency price setting, similar to our model, but without home bias in preferences. But Benigno and Benigno (2005) note that the environment of Clarida et al. is quite a special case, and in general, a Nash equilibrium in a monetary policy game between countries involves a convolution of the goal of stabilization policy (or closing ‘gaps’) and a goal of manipulating the terms of trade in a country’s favor. Our environment, where home bias in preferences is critical to the interaction between countries, precludes use of the special case of Clarida et al. (2002). Moreover, in a liquidity trap, a Nash equilibrium becomes even more complex due to the restricted strategy space of one or both countries. Finally, it is worth noting that in the case of home bias in preferences, it would not be possible to derive a simple analytical solution to the non-cooperative policy problem, since, as shown in Benigno and Benigno, it requires computing a second-order approximation to the necessary conditions of the model.

¹⁰The cooperative approach to fiscal policy in a global liquidity trap is not necessarily unrealistic. In the immediate aftermath of the financial crash of 2008, the G20 group agreed on a joint policy response to the crisis which assigned target levels of fiscal stimulus to each member country.

¹¹See Cook and Devereux (2011)

$$\begin{aligned}
V_t = E_t \sum_{s=t} & (\beta^{s-t}) - (\hat{n}_s^R)^2 \cdot \frac{A}{2} - (\hat{n}_s^W)^2 \cdot \frac{B}{2} - (\hat{c}g_s^R)^2 \cdot \frac{F}{2} - (\hat{c}g_s^W)^2 \cdot \frac{H}{2} - J(\hat{n}_s^R)(\hat{c}g_s^R) \\
& - L(\hat{n}_s^W)(\hat{c}g_s^W) - \frac{\theta}{4k}(\pi_s^W + \pi_s^R)^2 - \frac{\theta}{4k}(\pi_s^W - \pi_s^R)^2
\end{aligned} \tag{21}$$

where

$$\begin{aligned}
A & \equiv \left\{ \frac{(1 + \phi c_y)}{c_y^2} + \frac{(\sigma - D)}{D} \left(1 + \frac{(1 - c_y^2)}{c_y^2 D} \right) \right\} = \frac{(s_{DD} + \phi)}{c_y} \\
s_{DD} & \equiv \frac{(D - 1)(1 - c_y^2)}{c_y D} + \frac{(\sigma)}{D} \left(\frac{1 + c_y^2(D - 1)}{c_y D} \right) < s_D \\
B & \equiv \frac{(\sigma + \phi c_y)}{c_y^2} = \frac{(s + \phi)}{c_y}, \\
H & \equiv \frac{1}{(1 - c_y)} \frac{\sigma}{c_y^2} = \frac{1}{(1 - c_y)} \frac{s}{c_y} \quad L \equiv \frac{-\sigma}{c_y^2} = \frac{s}{c_y} \\
J & \equiv \left[-\frac{1}{c_y^2} - \frac{(\sigma - D)}{c_y^2 D^2} (1 + (v - 1)(D - 1)c_y^2) \right] \\
F & \equiv \frac{((1 - c_y) + c_y \sigma)}{(1 - c_y)c_y^2} + \frac{(\sigma - D)}{c_y^2 D^2} (1 + (v - 1)(D - 1)c_y^2)
\end{aligned}$$

Thus, the social welfare function faced by the policy maker depends upon output gaps, inflation rates, fiscal gaps, and the interaction between these variables.

5.1 Optimal Monetary Policy

First, we focus on monetary policy alone. Assume that all fiscal gaps are zero, and the only policy instrument available is the policy interest rate in each country. The key question is whether (20), the conventional interpretation of optimal monetary policy under the zero lower bound, is a correct description of optimal monetary policy. Then using the welfare approximation (21), the optimal cooperative policy problem under discretion is described by the Lagrangean:

$$\begin{aligned}
& \max_{\widehat{n}_t^R, \widehat{n}_t^W, \pi_t^W, \pi_t^R, r_t, r_t^*} L_t = V_t + \lambda_{1t} [\pi_t^W - k(\phi + s)\widehat{n}_t^W - \beta E_t \pi_{t+1}^W] \\
& \quad + \lambda_{2t} [\pi_t^R - k(\phi + s_D)\widehat{n}_t^R - \beta E_t \pi_{t+1}^R] \\
& \quad + \psi_{1t} \left[s E_t (\widehat{n}_{t+1}^W - \widehat{n}_t^W) - E_t \left(\frac{r_t + r_t^*}{2} - \widetilde{r}_t^W - \pi_{t+1}^W \right) \right] \\
& \quad + \psi_{2t} \left[s_D E_t (\widehat{n}_{t+1}^R - \widehat{n}_t^R) - E_t \left(\frac{r_t - r_t^*}{2} - \frac{\widetilde{r}_t^R}{2} - \pi_{t+1}^R \right) \right] \\
& \quad \quad \quad + \gamma_{1t} r_t + \gamma_{2t} r_t^*
\end{aligned}$$

The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative ‘IS’ equations. The final two constraints are the non-negativity constraint on the two policy interest rates. The policy optimum involves the choice of the output gaps, the inflation rates and interest rates to maximize this Lagrangean. The first order conditions are:

$$-A\widehat{n}_t^R = \lambda_2 k(\phi + s_D) + s_D \psi_2 \quad (22)$$

$$-B\widehat{n}_t^W = \lambda_1 k(\phi + s) + s \psi_1 \quad (23)$$

$$k\lambda_1 = \theta \pi_t^W \quad (24)$$

$$k\lambda_2 = \theta \pi_t^R \quad (25)$$

$$\psi_{2t} + \psi_{1t} = \gamma_{1t} \quad (26)$$

$$\psi_{1t} - \psi_{2t} = \gamma_{2t} \quad (27)$$

$$\gamma_{1t} \geq 0, \quad r_t \geq 0, \quad \gamma_{1t} r_t = 0 \quad (28)$$

$$\gamma_{2t} \geq 0, \quad r_t^* \geq 0, \quad \gamma_{2t} r_t^* = 0 \quad (29)$$

Together with the conditions (13)-(16), these equations determine the optimal policy solutions for the variables $n_t^R, n_t^W, \pi_t^R, \pi_t^W, r_t, r_t^*, \lambda_{1t}, \lambda_{2t}, \psi_{1t}, \psi_{2t}, \gamma_{1t}$, and γ_{2t} .

Conditions (26) and (27) together with the comparative slackness conditions (28) and (29), are the key equations of interest. They say that if the *sum* of the multipliers on the world and relative IS curves is positive, then the home country interest rate is at the zero bound (i.e. $\gamma_{1t} > 0$), while condition (27) says that if the *difference* between the multipliers is positive, then the foreign country is at the zero bound, so that $\gamma_{2t} > 0$.

In the Appendix, we show that, using (22)-(27) together with (13)-(16), we can derive a two equation relationship between the policy rates and the multipliers ψ_1 and ψ_2 . This is

given by:

$$\Omega_D \left(\frac{r_t - r_t^*}{2} - \tilde{r}_t^R \right) = \psi_2 \quad (30)$$

and using (53) and (55), and rearranging, we arrive at:

$$\Omega \left(\frac{r_t + r_t^*}{2} - \tilde{r}_t^W \right) = \psi_1 \quad (31)$$

where $\Omega_D \equiv \frac{\Psi_D}{\Delta_2^D s_D}$ and $\Omega \equiv \frac{\Psi}{\Delta_2^s}$. As shown in the Appendix, $\Omega_D \geq \Omega$, with strict inequality when $v < 2$.

From (30) and (31), we can now characterize the jointly optimal monetary policy in terms of the properties of the policy interest rates r and r^* . The key question is to see the conditions under which either the home, the foreign, or both non-negativity conditions on interest rates are binding; i.e. what determines when the zero lower bound is reached for each country? Note that since $\varepsilon < \varepsilon_H(v)$, it must be that $r = \tilde{r}(\varepsilon, v) < 0$, so clearly the unconstrained optimal policy is not a feasible solution for any value of v .

5.2 Characteristics of the optimal policy

The description of the optimal policy may be obtained from conditions (30) and (31), in conjunction with the characteristics of the natural real interest rates (9) and (10).

From (26), the home policy interest rate is zero whenever $\psi_1 + \psi_2 > 0$ and from (27) the foreign rate is zero when $\psi_1 - \psi_2 > 0$. In the case $v = 1$, $\tilde{r}_t^R = 0$, and $\tilde{r}_t^W < 0$. Setting $r_t = r_t^* = 0$ in (30) and (31), we find that $\psi_1 > 0$ and $\psi_2 = 0$, so that both constraints are binding. Thus $r_t = r_t^* = 0$ is a solution when $v = 1$.

In the more general case, we can establish the following proposition.

Proposition 1 *For $\varepsilon < \varepsilon_H(1)$, the optimal policy is characterized by the conditions; a) $r_t = 0$, b) there exists a critical value \bar{v} , such that (i) for $1 \leq v \leq \bar{v}$, $r_t^* = 0$, (ii), for $v > \bar{v}$, r_t^* satisfies:*

$$r_t^* = \tilde{r}^*(\varepsilon_t, v) - \frac{(\Omega_D - \Omega)}{\Omega_D + \Omega} \tilde{r}(\varepsilon_t, v) > 0.$$

In addition, $r_t^ > 0$, $r_t^* > \tilde{r}^*(\varepsilon_t, v)$, and $\bar{v} < v_F$.*

Proof. To prove the proposition, initially assume that a) holds, so that $r_t = 0$. Then from (30) and (31), we may write

$$\gamma_2 = \psi_1 - \psi_2 = \Omega \left(\frac{r_t^*}{2} - \tilde{r}_t^W(\varepsilon, v) \right) + \Omega_D \left(\frac{r_t^*}{2} + \tilde{r}_t^R(\varepsilon, v) \right) \quad (32)$$

Define the right hand side of (32) as $J(\varepsilon, v)$. If $J(\varepsilon, v) > 0$, then $r^* = 0$ must hold. Setting $r^* = 0$ in (32) gives

$$J(\varepsilon, v) = -\Omega \tilde{r}_t^W(\varepsilon, v) + \Omega_D \tilde{r}_t^R(\varepsilon, v) \quad (33)$$

By the definition of $\tilde{r}_t^W(\varepsilon, v)$ and $\tilde{r}_t^R(\varepsilon, v)$, it must be that $\tilde{r}_t^R(\varepsilon, 1) = 0$, so that $J(\varepsilon, 1) = -\Omega \tilde{r}_t^W(\varepsilon, 1) > 0$. Hence the condition $\gamma_2 > 0$ is satisfied at $v = 1$. Conversely, we have $\tilde{r}_t^W(\varepsilon, 2) = \frac{\tilde{r}_t(\varepsilon, 2) + \bar{r}}{2} > \frac{\tilde{r}_t(\varepsilon, 2) - \bar{r}}{2} = \tilde{r}_t^R(\varepsilon, 2)$, so that $J(\varepsilon, 2) = -\Omega \tilde{r}_t^W(\varepsilon, 2) + \Omega_D \tilde{r}_t^R(\varepsilon, 2) < 0$, since from the definitions above, $\Omega = \Omega_D$ when $v = 2$. Then $\gamma_2 > 0$ is not satisfied at $v = 2$. Hence, by continuity, there exists a value \bar{v} defined by the condition $J(\varepsilon, \bar{v}) = -\Omega \tilde{r}_t^W(\varepsilon, \bar{v}) + \Omega_D \tilde{r}_t^R(\varepsilon, \bar{v}) = 0$.¹²

Taking v such that $1 \leq v \leq \bar{v}$, and setting $r_t = r_t^* = 0$ in (30) and (31) implies that $J(\varepsilon, v) > 0$, which confirms the conjecture that both zero bound constraints are strictly binding, so both policy rates are zero. At $v = \bar{v}$, $J(\varepsilon, \bar{v}) = 0$, and the home constraint is strictly binding while the foreign constraint is just binding. For $\bar{v} < v \leq 2$, $J(\varepsilon, v) < 0$. Then the home country constraint is binding, but the foreign constraint is not binding. Then for $v \geq \bar{v}$, given that the foreign constraint is not binding, we set $\gamma_{2t} = 0$ in (27), which implies that $\psi_1 = \psi_2$. We use (with $r_t = 0$) in (30) and (31), to solve for the equilibrium foreign country interest rate as

$$r_t^* = \tilde{r}^*(\varepsilon, v) - \frac{(\Omega_D - \Omega)}{\Omega_D + \Omega} \tilde{r}(\varepsilon, v) > 0 \quad (34)$$

Note that for $v \geq \bar{v}$, this is strictly positive, since from the definition of $J(\varepsilon, v)$, we have $r_t^* = -\frac{2}{\Omega_D + \Omega} J(\varepsilon, v) > 0$, for $v \geq \bar{v}$. In addition, (34) implies that $r_t^* > \tilde{r}^*(\varepsilon, v)$ since for $\varepsilon < \varepsilon_H$, we have $\frac{(\Omega_D - \Omega)}{\Omega_D + \Omega} \tilde{r}(\varepsilon, v) < 0$.

Moreover, the critical value \bar{v} must satisfy $\bar{v} < v_F$. This is because, given the definition of the natural interest rates, it must be that $\tilde{r}_t^W(\varepsilon, v_F) = \frac{\tilde{r}_t(\varepsilon, v_F)}{2}$, and $\tilde{r}_t^R(\varepsilon, v_F) = \frac{\tilde{r}_t(\varepsilon, v_F)}{2}$. Hence $J(\varepsilon, v_F) = -(\Omega - \Omega_D) \frac{\tilde{r}_t(\varepsilon, v_F)}{2} < 0$, since $\Omega_D > \Omega$. Therefore, the foreign policy rate is strictly positive, for $v \geq \bar{v}$, even in the range $[\bar{v}, v_F]$, for which the foreign natural real interest rate is strictly *negative*.

This establishes part b) of the proposition. To show that part a) holds, assume that $r_t = 0$. Then for $v \leq \bar{v}$,

$$\gamma_1 = \psi_1 + \psi_2 = -\Omega \tilde{r}_t^W(\varepsilon, v) - \Omega_D \tilde{r}_t^R(\varepsilon, v) > 0$$

¹²We have not shown that \bar{v} is unique. However, in extensive simulation over different parameter settings, we did not find any instances of non-uniqueness.

so that $r_t = 0$ is confirmed. For $\bar{v} < v \leq 2$, using (34), we have

$$\begin{aligned}
\gamma_1 = \psi_1 + \psi_2 &= -\Omega \frac{(\tilde{r}_t(\varepsilon, v) + \tilde{r}_t^*(\varepsilon, v))}{2} - \Omega_D \frac{(\tilde{r}_t(\varepsilon, v) - \tilde{r}_t^*(\varepsilon, v))}{2} + (\Omega - \Omega_D) \frac{r_t^*}{2} \\
&= -(\Omega + \Omega_D) \frac{\tilde{r}_t(\varepsilon, v)}{2} - (\Omega - \Omega_D) \frac{\tilde{r}_t^*(\varepsilon, v)}{2} + (\Omega - \Omega_D) \left(\frac{\tilde{r}_t^*(\varepsilon, v)}{2} - \frac{(\Omega_D - \Omega)}{(\Omega_D + \Omega)} \tilde{r}(\varepsilon, v) \right) \\
&= -2 \frac{\Omega \Omega_D}{(\Omega_D + \Omega)} \tilde{r}_t(\varepsilon, v) > 0
\end{aligned}$$

where the second line equality follows from use of (34), and the third line follows by cancellation and rearrangement. Hence, $r_t = 0$ is satisfied for $\bar{v} < v \leq 2$. ■¹³

This proposition makes it clear that whether the two countries are in a liquidity trap is critically determined not by whether their respective natural real interest rates are negative, but by the strength of the shock and the size of the trade flows between the countries. Figure 2 illustrates the behavior of the foreign interest rate for various values of v . The figure is based on the calibration discussed in section 7 below. For a shock that would be large enough to drive the world natural real interest rate below zero in the fully open world economy (i.e. $\varepsilon < \varepsilon_H(1)$), the foreign country will also set the interest rate at the zero bound, if it is sufficiently open to trade with the home country ($v \leq \bar{v}$). But no matter how big is the shock, there is always a \bar{v} such that, for $v \geq \bar{v}$, the foreign country will keep its policy interest rate above zero, and also above its natural interest rate. Moreover, there is always an interval $[\bar{v}, v_F]$ for which the foreign policy rate is above zero, even though its natural interest rate below zero. An interesting feature of this figure is that for some range of v , the foreign country may set its interest rate above the non-stochastic steady state rate. Thus, the interest rate in an optimal monetary regime may be not only higher than the natural real interest rate, but also higher than the rate that would apply if the foreign economy were completely closed, and unaffected by the negative demand shock emanating from the home country.

The key intuition behind the optimal monetary policy rule comes from the benefits of tempering the home country terms of trade appreciation that occurs in a global liquidity trap. As we discussed above, when $v > 1$, the home terms of trade exacerbates the negative demand effects of the liquidity shock on the home country, drawing world demand away from home goods rather than cushioning the impact of the fall in demand. The only way in which the home monetary authority could limit this is to reduce its interest rate; but of course at the zero bound, it cannot engage in any further interest rate reduction. But the

¹³Note that although we use time subscripts in the definition of policy solutions, the policy responses are the same across all periods in which the demand shock is non-zero.

foreign country can limit the home terms of trade appreciation by increasing its own interest rate. For higher and higher values of v , this is of direct benefit to the foreign country, since in those circumstances, it is more likely that the movement in the terms of trade causes an expansion in the foreign output gap. We can write the home terms of trade as follows;

$$\tau_t = 2 \frac{\mu}{1 - \mu} \left(\frac{k(\phi + s_D)}{\Delta_2^D} (\tilde{r}_t^W + \frac{r_t^*}{2}) + \frac{r_t^*}{(1 - \mu)} \right)$$

For $1 \leq v < \bar{v}$, then $\tau_t < 0$, and the terms of trade appreciates. But as $\bar{v} \leq v \leq 2$, the appreciation is mitigated by a rise in the foreign policy rate.

The behavior of the foreign interest rate depends not just on the value of v , but also the size of the shock. There is a trade-off between the size of home bias and the size of the shock in the assessment of whether a liquidity trap in one country spills over into another country. Figure 3 illustrates this. The Figure illustrates a downward sloping locus of points in $v - \varepsilon$ space. Above and to the right of the locus, the foreign country sets a positive policy rate higher than the foreign natural real interest rate. Below and to the left of the locus, the foreign country is constrained by the zero lower bound. Note that the locus become steeper as v increases, because the foreign country is less and less sensitive to foreign demand shocks, the higher is v . Literally, as v approaches 2, the required negative home demand shock that would put the foreign country into a liquidity trap becomes infinitely large.

So far we have focused on large negative demand shocks, defined as $\varepsilon < \varepsilon_H(1)$. In this case, there is always a region $[1, \bar{v}]$ such that both countries are in a liquidity trap. Figure 4 illustrates an opposite case, where the negative demand shock is smaller. In this case, $\varepsilon > \varepsilon_H(1)$. Then, with fully open trade, *neither* country is in a liquidity trap. But if the shock is still sufficiently large so that $\varepsilon < \varepsilon_H(\hat{v})$, for some $\hat{v} \in [1, 2]$, then the home country is in a liquidity trap for values of $v \geq \hat{v}$. Then in this case, condition (34) applies as before. For $1 < v \leq \hat{v}$, both countries can close all gaps through adjusting policy interest rates. For $v > \hat{v}$, the home economy is constrained by the zero bound. Now the foreign economy raises its interest rate *above* its natural real interest rate. Interestingly, this is a case where more open trade reduces policy interdependence. With fully open trade, due to the spreading of demand shocks through the two countries each country can act independently to adjust its policy rate. But with more restricted trade, the home economy is more affected by the negative shock, and the inability to adjust interest rates in a downward direction leads to a reliance on tighter foreign monetary policy to adjust to the shock. Hence, paradoxically, policy interdependence is more important, the less interdependence in trade flows.

5.2.1 Consequences of the conventional policy

Some more insight into the motivation for the foreign country to raise interest rates by looking at the alternative, where the foreign country set $r_t^* = \tilde{r}^*(\varepsilon, v)$, when $v > v_F$.

Proposition 2 *If the foreign central bank closes the interest gap, $r_t^* = \tilde{r}_t^*$, there is a contraction and deflation in the home economy, $\hat{n}_t < 0$, $\pi_{Ht} < 0$ and an expansion and inflation in the foreign economy $\hat{n}_t^* > 0$, $\pi_{Ft} > 0$.*

Proof. See Appendix ■

The intuition for this proposition is that, when $r_t^* = \tilde{r}^*(\varepsilon, v)$, the only shock to system (13)-(16) is to the home natural real interest rate $\tilde{r}(\varepsilon, v)$, and the shock enters the same way in both the average and relative systems. It is then possible to show that \hat{n}^W and \hat{n}^R fall - both world average output relative home output falls. But, as discussed above, home bias in preferences leads the relative elasticities of response to a given real interest rate shock to exceed the average elasticity. Thus, we can show that $\hat{n}^R < \hat{n}^W$. This implies that $\hat{n} < 0$, and $\hat{n}^* > 0$. A similar argument applies to inflation, so we may show that $\hat{\pi}^R < \hat{\pi}^W$, and $\hat{\pi} < 0$ and $\hat{\pi}^* > 0$.

Thus, in the region $v > v_F$, where the foreign central bank actually has the capacity to close its interest rate gap, it would find such a policy excessively expansionary and inflationary. Thus, although the monetary stance outlined in Proposition 1 is part of a global cooperative agreement, this suggests that it is also in the interest of the foreign central bank to follow a tighter monetary rule than implied by the conventional rule (20). This tighter rule minimizes the terms of trade appreciation of the home country, and reduces the inflationary forces affecting the foreign economy. In fact, it can be further shown (see Appendix) that under the optimal rule, the foreign central bank will temper the appreciation to such an extent that it imports deflation - i.e. the inflation rate under the optimal policy rule is negative.

5.3 Optimal Monetary and Fiscal Policy

We now extend the analysis to encompass the joint determination of monetary and fiscal policy. If natural interest rates were always positive, then under an optimal policy, interest rates would be set equal to natural rates, and all fiscal gaps would be zero. But recognizing the zero lower bound gives a role for active fiscal policy, which involves having non-zero fiscal gaps.

Again, the cooperative optimal policy response to a liquidity trap involves maximizing (21) in each period, taking expectations of all future variables as given, subject to the inflation

equations for world averages and differences, given by (13) and (15), and subject to the non-negativity constraints on nominal interest rates in each country. Since from the results of the previous section we know that the non-negativity constraint on the home country policy rate will always bind for the duration of the shock, we only impose the non-negativity condition on the foreign interest rate.

Given this, we have the Lagrangean expression:

$$\begin{aligned}
\max_{\hat{n}_t^R, \hat{n}_t^W, \hat{c}g_t^R, \hat{c}g_t^W, \pi_t^W, \pi_t^R, r_t^*} L_t = & -(\hat{n}_t^R)^2 \cdot \frac{A}{2} - (\hat{n}_t^W)^2 \frac{B}{2} - (\hat{c}g_t^R)^2 \cdot \frac{F}{2} - (\hat{c}g_t^W)^2 \cdot \frac{H}{2} \\
& - J(\hat{n}_t^R)(\hat{c}g_t^R) - L(\hat{n}_t^W)(\hat{c}g_t^W) - \frac{\theta}{4k}(\pi_t^W + \pi_t^R)^2 - \frac{\theta}{4k}(\pi_t^W - \pi_t^R)^2 \\
& + \lambda_{1t} [\pi_t^W - k(\phi + s)\hat{n}_t^W + ks \cdot \hat{c}g_t^W - \beta E_t \pi_{t+1}^W] \\
& + \lambda_{2t} [\pi_t^R - k(\phi + s_D)\hat{n}_t^R + ks_D \hat{c}g_t^R - \beta E_t \pi_{t+1}^R] \\
& + \psi_{1t} \left[s E_t (\hat{n}_{t+1}^W - \hat{n}_t^W) - s E_t (\hat{c}g_{t+1}^W - \hat{c}g_t^W) - E_t \left(\frac{r_t^*}{2} - \tilde{r}_t^W - \pi_{t+1}^W \right) \right] \\
& + \psi_{2t} \left[s_D E_t (\hat{n}_{t+1}^R - \hat{n}_t^R) - s_D E_t (\hat{c}g_{t+1}^R - \hat{c}g_t^R) - E_t \left(-\frac{r_t^*}{2} - \frac{\tilde{r}_t^R}{2} - \pi_{t+1}^R \right) \right] \\
& + \gamma_t [r_t^*]
\end{aligned}$$

The problem is a straightforward generalization of the previous one. The policy optimum involves the choice of the output gaps, the government spending gaps, the inflation rates and the foreign interest rate to maximize this Lagrangean. The first order conditions of the maximization are:

$$-A\hat{n}_t^R - J(\hat{c}g_t^R) = \lambda_2 k(\phi + s_D) + s_D \psi_2 \quad (35)$$

$$-B\hat{n}_t^W - L(\hat{c}g_t^W) = \lambda_1 k(\phi + s) + s \psi_1 \quad (36)$$

$$F\hat{c}g_t^R + J(\hat{n}_t^R) = ks_D \lambda_2 + s_D \psi_2 \quad (37)$$

$$H\hat{c}g_t^W + L(\hat{n}_t^W) = ks \lambda_1 + s \psi_1 \quad (38)$$

$$k\lambda_1 = \theta \pi_t^W \quad (39)$$

$$k\lambda_2 = \theta \pi_t^R \quad (40)$$

$$\psi_{2t} - \psi_{1t} + \gamma_t = 0 \quad (41)$$

These equations, in conjunction with (13)-(16), give the conditions determining average and relative output gaps, inflation rates, fiscal gaps, Lagrange multipliers, and the value of

either γ or r_t^* . As in the previous subsection, we can reduce these equations into a condition which determines whether the foreign country's policy rate is positive or constrained by the zero bound. But now this is simultaneously determined with the size of the average and relative fiscal gaps. First, take (36), (38) and (39). Combine these with (13) and (14), to get the relationship between the world average fiscal gap and the interest rate gap as follows:

$$[(\Delta_2 HL + \Delta_3 BL) + \phi f(1 - \mu)s] \widehat{c}g_t^W = [f(\phi + s) + (1 - \beta\mu)BL] \left(\frac{r_t^*}{2} - \widetilde{r}_t^W \right) \quad (42)$$

where $\Delta_3 = \Delta_2 + k\phi > 0$, $HL \equiv H + L > 0$, $BL \equiv B + L > 0$ and $f \equiv \phi\theta k > 0$. Since $r_t^* \geq 0$, from this, it is clear that when the world average natural rate falls below zero, the world average fiscal gap must increase.

When at least one of the the policy rates is constrained by the zero lower bound, the world output gap is negative, and inflation is negative. Then fiscal spending, by creating anticipated inflation, can reduce real interest rates, stimulate private demand, and reduce the current world output gap.

We may use a similar procedure to compute the relationship between the relative fiscal gap and the relative interest rate gap. This gives us the condition:

$$[\Delta_2^D FJ + \Delta_3^D JA + f\phi s_D(1 - \mu)] \widehat{c}g_t^R = -[f(\phi + s_D) + (1 - \beta\mu)JA] \left(\frac{r_t^*}{2} + \widetilde{r}_t^R \right) \quad (43)$$

where $\Delta_3^D = \Delta_2^D + k\phi > 0$, $FJ \equiv F + J > 0$, and $JA \equiv J + A > 0$.

When $v = 1$, given that $\varepsilon < \varepsilon(1)$, it must be that both countries are constrained by the zero bound. In addition, it must be that $\widetilde{r}_t^R = 0$. Therefore, both countries must have identical and positive fiscal gaps.

For the case $v \geq 1$, since $r_t^* \geq 0$ and $\widetilde{r}_t^R \leq 0$, the expression (43) cannot immediately be signed. But it is shown below that $r_t^* - \widetilde{r}_t^* \leq \widetilde{r}_t$. Hence the *relative* fiscal gap is always non-negative. It follows then that the home country fiscal gap will always be positive.

Finally, we may use (41), (36), (35), (39), (40), in conjunction with (13) and (16) to compute a solution for γ as:

$$\gamma = \psi_1 - \psi_2 = \Gamma \left(\frac{r_t^*}{2} - \widetilde{r}_t^W \right) - \Gamma_D \left(\frac{r_t^*}{2} + \widetilde{r}_t^R \right) \quad (44)$$

where $\Gamma(v)$ and $\Gamma_D(v)$ satisfy the condition that $\Gamma_D(v) \geq \Gamma(v)$, with $\Gamma_D(2) = \Gamma(2)$ ¹⁴

¹⁴The expressions are defined as follows: $\Gamma_D \equiv \Omega_D + \frac{(J\Delta_2^D + A\Delta_3^D)(1 - \beta\mu) + (\phi + s_D)\theta k\phi s_D(1 - \mu)}{\Delta_3^D s_D} g^R$, $\Gamma \equiv \Omega + \frac{(L\Delta_2 + B\Delta_3)(1 - \beta\mu) + (\phi + s)\theta k\phi s(1 - \mu)}{\Delta_3 s} g^W$, $g^R = -\frac{[f(\phi + s_D) + (1 - \beta\mu)JA]}{[\Delta_2^D FJ + \Delta_3^D JA + f\phi s_D(1 - \mu)]}$, and $g^W = \frac{[f(\phi + s) + (1 - \beta\mu)BL]}{[\Delta_2 HL + \Delta_3 BL + \phi f(1 - \mu)s]}$.

Equation (44) satisfies the same properties as (32) in the previous subsection. When $v = 1$, then $r_t^* = 0$, $\tilde{r}_t^R = 0$, and so the right hand side of (44) is positive at $r_t^* = 0$, ensuring that the foreign zero bound constraint is binding. Alternatively, in the case $v = 2$, then the right hand side of (44) is negative at $r_t^* = 0$, so the zero bound constraint is non-binding. As before, there is a critical value for v , denoted \bar{v} , such that for $v \geq \bar{v}$, then $r_t^* > 0$. In this case, since $\gamma = 0$, we may derive the optimal value of r_t^* from (44) itself. In summary, we may then define the behavior of the foreign policy interest rate in the same way as before. Thus:

$$\begin{aligned} \text{For } 1 \leq v \leq \bar{v}, & \quad r_t^* = 0 \\ \text{For } \bar{v} < v < 2, & \quad r_t^* = \tilde{r}_t^*(\varepsilon, v) - \frac{(\Gamma_D - \Gamma)}{(\Gamma_D + \Gamma)} \tilde{r}_t(\varepsilon, v) \end{aligned} \quad (45)$$

With the condition that $\Gamma_D \geq \Gamma$, this ensures that $r_t^* - \tilde{r}_t^* \leq \tilde{r}_t$, as stated above. Thus, the characteristics of monetary policy are similar to those of the last section. The difference is that now monetary policy response on the part of the foreign country is augmented by positive fiscal gaps on the part of one or both governments. Note also that the stance of monetary policy will affect the optimal fiscal gaps chosen by each country. Only when there is substantial trade openness, so that $v \leq \bar{v}$, and $r_t^* = 0$, will monetary policy play no role in an optimal policy. More generally, there is an interaction between the optimal fiscal and monetary responses to a liquidity trap in one country. The way in which this takes place is explored in the following section.

6 Numerical Analysis of Optimal Policy

We now provide a numerical illustration of the jointly optimal cooperative monetary fiscal policy. To evaluate the economy quantitatively, we adopt the following parameter assumptions. Let $\beta = 0.99$, so each period is a quarter, and this translates to a value of the steady state interest rate $\bar{r} = 0.01$. The Frisch labor supply elasticity is set at $\phi = 1$. Price stickiness is $\kappa = 0.85$, so that $k = 0.027$, as in Christiano et al. (2009). Let the share of government in output be 20 percent, so that $c_y = 0.8$. We assume the inverse of the intertemporal demand elasticity σ , is equal to 2. The persistence of the demand shock is set at 0.8 ($\mu = 0.8$) implying an expected length of the slump to be 5 quarters. We set the elasticity of substitution between individual good varieties within a country, θ , equal to 5. Finally, we set the preference shock in the home country ε so that at $v = 1$ (the case without

any home bias), the natural real interest rate at the quarterly frequency would fall from 1 percent to -1.7 percent, with persistence μ .

Figure 5 illustrates the response of home and foreign output gaps, home and foreign government spending gaps, home and foreign inflation, the foreign country optimal policy rate, as well as the foreign natural real interest rate, and the home country terms of trade, for different values of v , when the optimal fiscal and monetary policy response is chosen. The Figure takes account of condition (45), so that, at each value of v , the non-negativity constraint on r_t^* is tested, and if it is not binding, the optimal foreign policy rate is chosen to satisfy (45). The first thing to note is that at $v = 1$, then clearly the zero bound is binding in both countries, and all variables respond in the same way in the two countries. The output gap falls by over 7 percent in both countries, and this is coupled with a fall in the rate of inflation by equal amounts. Since both countries are affected equally, and interest rates are zero, adjusting the fiscal gaps is the only possible policy response to the shock. The Figure shows a positive response of the fiscal gap in each country. Thus, fiscal policy should behave counter-cyclically, and equally so in each country for a world without home bias in preferences.

Now, as v rises above unity, we know that the impact of the shock on the foreign natural interest rate becomes muted, while the opposite occurs for the home natural interest rate. The negative response of the foreign output gap is then reduced, while that of the home output gap is increased. As v rises more and more, holding the foreign policy rate constant, the foreign output gap may actually increase. This is due to the sharp terms of trade appreciation of the home country, leading to an expenditure switching towards foreign output. A similar dynamic occurs in the response of the inflation rates in the two countries - home inflation becomes more and more negative as v rises, while the negative response of foreign inflation becomes less and less. The optimal response of fiscal policy gaps is illustrated in panel b of the Figure. As v rises, home fiscal policy becomes more aggressive, while the foreign fiscal policy becomes more muted.

Panel d illustrates the optimal response of the foreign country policy rate, alongside the foreign country natural real interest rate. Note that at $v = 1$, the foreign policy rate is stuck at zero, while the natural real interest rate is at -0.017 . As v rises, the response of the foreign natural interest rate becomes less and less, as is obvious from the formula (45). Eventually, as v rises to 2, the foreign country would be entirely unaffected by the shock, and the foreign natural interest rate would rise to 0.01, the steady state natural interest rate. But the key feature of panel d is that the foreign country will raise its policy rate above zero for values of $\tilde{r}_t^* < 0$. That is, the foreign country will choose positive interest rates

after point \bar{v} as part of an optimal cooperative policy package, even though, by the usual closed economy logic, it should be still in a liquidity trap, since its natural rate of interest is below zero. Equivalently, the foreign country will not follow a policy of offsetting the movement in the foreign natural interest rate to the greatest extent that it can, so long as the policy rate is above the zero bound. Rather, it chooses to raise policy rates, even though $\tilde{r}_t^* < 0$. Quantitatively panel d shows that for high values of v , the foreign policy make imply an interest rate *above the steady state natural rate of interest*. Thus, by any definition of the term, the optimal monetary stance for the foreign country, in face of the home liquidity trap, is to tighten its monetary policy.

Hence, an optimal cooperative policy response to a liquidity trap can be characterized by expansionary fiscal policy in all countries, but contractionary monetary policy in the least affected country. As v rises, the home economy is significantly more affected by the negative demand shock. An optimal policy response is to raise world demand, and to re-orient world demand towards the home country. Raising world demand is accomplished by expansionary fiscal policy, and particularly so in the country which is the source of the demand shock. But reorientation of demand towards the source country is achieved by tighter monetary policy in the least affected country. The rise of the foreign policy rate is associated with an depreciation of the home terms of trade, which generates an additional expenditure switching of demand towards the home country. Since the impact of the home country shock on foreign output is positive in any case, when v is sufficiently greater than unity, the rise in the foreign policy rate has the additional benefit that it helps to minimize the response of the foreign output gap to the home country shock. The Figure shows that the tightening of the policy rate in the foreign country as v rises reduces the degree to which the home terms of trade appreciates in response to the initial savings shock.

We note that, when an optimal foreign monetary policy is used, the foreign country has a very small fiscal gap. While it is optimal for the foreign country to follow an expansionary fiscal policy, quantitatively, the size of the fiscal expansion is much less than that of the home country.

Figure 6 provides further illustration of the key interaction between monetary and fiscal policy in responding to the liquidity trap in the home country. The Figure contrasts the optimal policy for fiscal and monetary policy to that where fiscal policy is set optimally, but monetary policy is set according to the conventional rule (20). Thus, the foreign country sets the policy rate equal to zero when the natural real interest rate is negative, and equal to the natural real interest rate when it is above zero. The Figure shows that the response of fiscal policy under this alternative (non-optimal) monetary rule is substantially different when

$v > 1$. The key feature of this policy is that it is excessively expansionary for the foreign economy, relative to the optimal rule. As v rises more and more, the foreign economy experiences a boom, which is countered by a *contractionary* fiscal policy. At the same time, the outcome of expansionary monetary and contractionary fiscal policy in the foreign country leads to an excessive contraction in the home economy, which then requires a much greater fiscal expansion than would take place under the optimal policy. This comparison makes clear that the *optimal* foreign monetary policy adjustment in effect reduces the extent to which the home country has to engage in expansionary fiscal policy in response to the liquidity trap. It does so precisely by tempering the sharp terms of trade appreciation of the home economy. Note from panel e that under the non-optimal monetary rule (20), the terms of trade appreciates much more for the home economy than it would under the optimal policy. In addition, under this non-optimal rule, the foreign economy experiences inflation, while the deflation in the home economy is substantially greater than it would be under the optimal policy.

7 Conclusions

The experience of major recessions in many of the world's largest economic regions, together with low or zero interest rates, has reduced confidence in the ability of monetary policy to respond to economic shocks, and suggests that only fiscal policy can be used as a countercyclical device. This paper shows that in a world economy where countries are affected in different ways by 'liquidity trap' shocks, monetary and fiscal policy may be used in mutually supportive ways, and in some cases the standard prescriptions for monetary policy response to a liquidity trap may fail to apply. A relatively tight monetary policy in the least hit country facilitates an efficient redirection of world spending, and reduces the extent to which fiscal expansion must be used to raise world expenditure. The key useful feature of monetary policy in our model is that it tempers the perverse response of real exchange rates to shocks that occurs in a liquidity trap. The underlying message of the paper is that in a liquidity trap, the exchange rate response may exacerbate rather than ameliorate the impact of negative demand shocks.

Appendix

Derivation of Conditions (30) and (31)

Combining (24) and (25) with (22) and (23), we obtain the relationship between world and relative output gaps, inflation rates, and the multipliers ψ_{2t} and ψ_{1t} . Since the underlying demand shock is either a constant (negative) number, or zero, the solution for all variables during the period of the shock will be time invariant. Hence we can drop the time notation. Thus:

$$-A\widehat{n}^R = \theta\pi^R(\phi + s_D) + s_D\psi_2 \quad (46)$$

$$-B\widehat{n}^W = \theta\pi^W(\phi + s) + s\psi_1 \quad (47)$$

Then, solving the conditions (13)-(16) using the fact that the shock to the natural real interest rates will revert to zero with probability $1 - \mu$ per period, we have:

$$\pi^R(1 - \beta\mu) = k(\phi + s_D)\widehat{n}^R \quad (48)$$

$$s_D(\mu - 1)\widehat{n}^R = \frac{r - r^*}{2} - \widetilde{r}^R - \mu\pi^R \quad (49)$$

$$\pi^W(1 - \beta\mu) = k(\phi + s)\widehat{n}^W \quad (50)$$

$$s(\mu - 1)\widehat{n}^W = \frac{r + r^*}{2} - \widetilde{r}^W - \mu\pi^W \quad (51)$$

From (48) and (49), we can derive the partial solution for the relative output gap as:

$$\Delta_2^D \widehat{n}^R = -\left(\frac{r - r^*}{2} - \widetilde{r}^R\right)(1 - \beta\mu) \quad (52)$$

Likewise, the world output gap that solves (50) and (51) is

$$\Delta_2 \widehat{n}^W = -\left(\frac{r + r^*}{2} - \widetilde{r}^W\right)(1 - \beta\mu) \quad (53)$$

Now using (48) in (22) we get:

$$- [A(1 - \beta\mu) + \theta k(\phi + s_D)^2] \widehat{n}_t^R \equiv -\Psi_D \widehat{n}_t^R = s_D\psi_2(1 - \beta\mu) \quad (54)$$

and (50) in (23) we get

$$- [B(1 - \beta\mu) + \theta k(\phi + s)^2] \widehat{n}_t^W \equiv -\Psi \widehat{n}_t^W = s\psi_1(1 - \beta\mu) \quad (55)$$

Using (52) and (54), and rearranging, we arrive at (30) and (31), where $\Omega_D \equiv \frac{\Psi_D}{\Delta_2^D s_D}$ and $\Omega \equiv \frac{\Psi}{\Delta_2 s}$. By the properties already defined above, it must be that $\Omega_D \geq \Omega$, with strict inequality when $v < 2$.

Proof of Proposition 2

Proof. Note that $r_t = 0$. $\frac{r_t^*}{2} = \frac{\tilde{r}_t^*}{2}$, so $r_t^W - \tilde{r}_t^W = r_t^R - \tilde{r}_t^R = \frac{\tilde{r}_t}{2} < 0$. Write (52) and (53)

$$\Delta_2 \hat{n}_t^W = \Delta_2^D \hat{n}_t^R = (1 - \beta\mu) \frac{\tilde{r}_t}{2} < 0 \quad (56)$$

Since $\Delta_2 > \Delta_2^D > 0$, $\hat{n}_t^R < \hat{n}_t^W < 0$. Since the average level of output drops by less than the relative level, $\hat{n}_t < 0$ and $\hat{n}_t^* > 0$. Add both sides of (50) and (48)

$$(1 - \beta\mu)\pi_t = k(\phi + s)\hat{n}_t^W + k(\phi + s_D)\hat{n}_t^R < 0 \quad (57)$$

Since both $\hat{n}_t^R, \hat{n}_t^W < 0$, $\pi_t < 0$. Subtract (48) from (50) Subtract the world Phillips curve

$$(1 - \beta\mu)\pi_t^* = k(\phi + s)\hat{n}_t^W - k(\phi + s_D)\hat{n}_t^R$$

Multiply both sides of the equation by $\Delta_2^D \Delta_2$. We can write $\Delta_2^D \Delta_2 \hat{n}_t^W = \Delta_2^D (1 - \beta\mu) \frac{r_t}{2}$ and $\Delta_2^D \Delta_2 \hat{n}_t^R = \Delta_2 (1 - \beta\mu) \frac{r_t}{2}$

$$\begin{aligned} \Delta_2^D \Delta_2 (1 - \beta\mu)\pi_t^* &= k(\phi + s)\Delta_2^D \Delta_2 \hat{n}_t^W - k(\phi + s_D)\Delta_2^D \Delta_2 \hat{n}_t^R \\ &= [\phi(\Delta_2^D - \Delta_2) + (s\Delta_2^D - s_D\Delta_2)] k(1 - \beta\mu) \frac{\tilde{r}_t}{2} \end{aligned}$$

Note $(s\Delta_2^D - s_D\Delta_2) = (s_D - s)\mu\phi k$, so

$$\Delta_2^D \Delta_2 (1 - \beta\mu)\pi_t^* = (1 - \beta\mu)k\phi[(\Delta_2^D - \Delta_2) + (s_D - s)\mu k] \frac{\tilde{r}_t}{2} > 0$$

Since $(\Delta_2^D - \Delta_2) < 0$ and $(s_D - s) < 0$ ■

Characteristics of the optimal Monetary Rule

Proposition 3 *If the foreign central bank follows an optimal $r_t^* > 0$, then the home output gap will be negative and both home and foreign inflation will be negative.*

Proof. If $\gamma_2 = 0$

$$\psi_1 = \psi_2 \equiv \psi$$

Multiply both sides of (54) by D to equalize the right side with the right side of (55)

$$D\Psi_D\widehat{n}_t^R = \Psi\widehat{n}_t^W \quad (58)$$

Add (52) and (53) and premultiply by $D\Psi_D$ to get

$$D\Psi_D\Delta_2\widehat{n}_t^W + \Delta_2^D D\Psi_D\widehat{n}_t^R = (1 - \beta\mu)D\Psi_D\widetilde{r}_t$$

Insert the optimal monetary condition

$$[D\Psi_D + \Delta_2^D\Psi]\widehat{n}_t^W = (1 - \beta\mu)D\Psi_D\widetilde{r}_t$$

So $\widehat{n}_t^W < 0$. From (58) we have $\widehat{n}_t^R < 0$ so $\widehat{n}_t < 0$. From (50) and (48) we have $\pi_t^W < 0$ and $\pi_t^R < 0$, so $\pi_t < 0$. Insert (50) and (48) into (58):

$$[(1 - \beta\mu)B + \theta k(\phi + s)^2] \frac{(1 - \beta\mu)}{k(\phi + s)} \pi_t^W = D [(1 - \beta\mu)A + \theta k(\phi + s_D)^2] \frac{(1 - \beta\mu)}{k(\phi + s_D)} \pi_t^R$$

Cancel $\frac{(1-\beta\mu)}{k}$ and $B = \frac{(\phi+s)}{c_y}$ so the leftand side is

$$\frac{\left[\frac{(1-\beta\mu)}{c_y}(\phi + s) + \theta k(\phi + s)^2\right]}{(\phi + s)} \pi_t^W = \left[\frac{(1 - \beta\mu)}{c_y} + \theta k(\phi + s)\right] \pi_t^W$$

Multiply both sides by $(\phi + s_D)$

$$\left[\frac{(1 - \beta\mu)}{c_y}(\phi + s_D) + \theta k(\phi + s)(\phi + s_D)\right] \pi_t^W = D \left[\frac{(1 - \beta\mu)}{c_y}(\phi + s_{DD}) + \theta k(\phi + s_D)^2\right] \pi_t^R$$

Define

$$\begin{aligned} \Pi_1 &\equiv \left[\frac{(1 - \beta\mu)}{c_y}(\phi + s_D) + \theta k(\phi + s)(\phi + s_D)\right] \\ \Pi_2 &\equiv D \left[\frac{(1 - \beta\mu)}{c_y}(\phi + s_D) + \theta k(\phi + s_D)^2\right] \end{aligned}$$

so

$$\Pi_1\pi_t^W = \Pi_2\pi_t^R$$

Calculate

$$\begin{aligned}
\Pi_2 - \Pi_1 &= \\
\frac{(1 - \beta\mu)}{c_y}(D - 1)\phi + \frac{(1 - \beta\mu)}{c_y}(Ds_{DD} - s_D) + \theta k(\phi + s_D) [D(\phi + s_D) - (\phi + s)] &= \\
\frac{(1 - \beta\mu)}{c_y}(D - 1)\phi + \frac{(1 - \beta\mu)}{c_y}(Ds_{DD} - s_D) + \theta k(\phi + s_D) \left[(D - 1)\phi + (Ds_{D_0} - s) \right] &= \\
\frac{(1 - \beta\mu)}{c_y}(D - 1)\phi + \frac{(1 - \beta\mu)}{c_y}(Ds_{DD} - s_D) + \theta k(\phi + s_D) [(D - 1)\phi] &> 0
\end{aligned}$$

Hence, the fall in inflation of the home country is less than the world average decline. Therefore inflation must also fall in the foreign country. ■

References

- [1] Auerbach, Alan, and Maurice Obstfeld (2006) “The Role for Open Market Operations in a Liquidity Trap”, American Economic Review.
- [2] Beetsma Roel and Henrik Jensen, (2005), “Monetary and Fiscal Policy Interactions in a Micro-Founded Model of a Monetary Union”, Journal of International Economics, 67, 320-352.
- [3] Benhabib, Jess, Stephanie Schmitt Grohe, and Martin Uribe, (2002) “Avoiding Liquidity Traps”, Journal of Political Economy.
- [4] Benigno, Gianluca and Pierpaolo Benigno (2006), “Designing Targeting Rules for International Monetary Policy Cooperation”, Journal of Monetary Economics, 53, 473-506.
- [5] Blanchard, Olivier and Roberto Perotti, (2002), “An Empirical Characterization of the Dynamic Effects of changes in Government Spending and Taxes on Output”, Quarterly Journal of Economics, 117, 1329-1368.
- [6] Bodenstein, Martin, Christopher J. Erceg, and Luca Guerrieri (2009) “The Effect of Foreign Shocks When Interest Rates are at Zero” IFdp 983, Board of Governors of the Federal Reserve System.
- [7] Christiano, Larry, Martin Eichenbaum, and Sergio Rebelo, (2009) “When is the Government Spending Multiplier Large?”, Northwestern University.
- [8] Cook, David, and Michael B. Devereux, (2011) “Optimal Fiscal Policy in a World Liquidity Trap”, European Economic Review, forthcoming.

- [9] Cogan, John, Tobias Cwik, John Taylor, and Volker Wieland, (2009) “New Keynesian Versus Old Keynesian Government Spending Multipliers”, mimeo
- [10] Davig, Troy and Eric M. Leeper (2009) “Monetary Fiscal Interactions and Fiscal Stimulus”, NBER d.p. 15133.
- [11] Devereux, Michael B. (2010) “Fiscal Deficits, Debt, and Monetary Policy in a Liquidity Trap”, Central Bank of Chile, Working Paper 581.
- [12] Engel, Charles (2010), “Currency Misalignments and Optimal Monetary Policy: A Reinvestigation”, mimeo
- [13] Eggerston, Gauti, (2010) “What Fiscal Policy is Effective at Zero Interest Rates?”, NBER Macroeconomics Annual, forthcoming.
- [14] Eggertson Gauti and Michael Woodford, (2003) “The Zero Interest Bound and Optimal Monetary Policy”, Brookings Papers on Economic Activity.
- [15] Eggertsson, Gauti and Michael Woodford, (2005) “Optimal Monetary and Fiscal Policy in a Liquidity Trap”, ISOM Annual.
- [16] Faia, Ester and Tommaso Monacelli, (2008) “Optimal Monetary Policy in a Small Open Economy with Home Bias,” *Journal of Money, Credit and Banking*, 40, 721-750.
- [17] Fujiwara, Ippei, Nao Sudo, and Yuki Teranishi, (2009) “The Zero Lower Bound and Monetary Policy in a Global Economy: A Simple Analytical Investigation”, *International Journal of Central Banking*.
- [18] Fujiwara, Ippei, Tomoyuki Nakajimaz, Nao Sudo, and Yuki Teranishi, (2010) “Global Liquidity Trap”, mimeo, Bank of Japan
- [19] Fujiwara, Ippei, and Kozu Ueda (2010) “The Fiscal Multiplier and Spillover in a Global Liquidity Trap”, IMES d.p. 2010-E-3. Bank of Japan.
- [20] Jeanne, Olivier (2009) “The Global Liquidity Trap”, mimeo
- [21] Krugman, Paul, (1998), “It’s baaack: Japan’s Slump and the Return of the Liquidity Trap”, *Brookings Papers on Economic Activity*, 2, 137-87.
- [22] Jung, Taehun, Yuki Terinishi, and Tsutomu Watanabe, (2005) “The Zero Bound on Nominal Interest Rates and Optimal Monetary Policy”, *Journal of Money Credit and Banking*.

- [23] Monacelli, Tomasso and Roberto Perotti, (2008), “Fiscal Policy, Wealth Effects and Markups”, NBER d.p. 14584.
- [24] Perotti, Roberto, (2007) “In Search of the Transmission Mechanism of Fiscal Policy”, NBER 13143.
- [25] Rotemberg, J. and M. Woodford (1998) “An Optimization Based Econometric Framework for the Evaluation of Monetary Policy”, NBER Technical Working Paper 233.
- [26] Svensson, Lars E. (2003) “Escaping from a Liquidity Trap and Deflation: The Foolproof Way and Others” *Journal of Economic Perspectives* 17, 145-166.
- [27] Woodford, Michael (2003) *Interest and Prices*, MIT Press
- [28] Woodford, Michael (2010) “Simple Analytics of the Government Expenditure Multiplier”, mimeo, Columbia University.

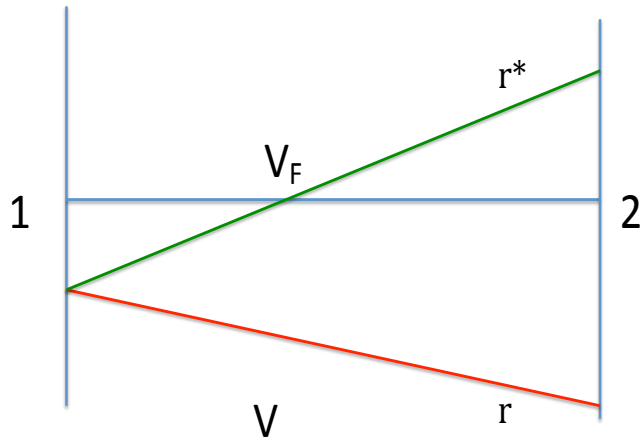
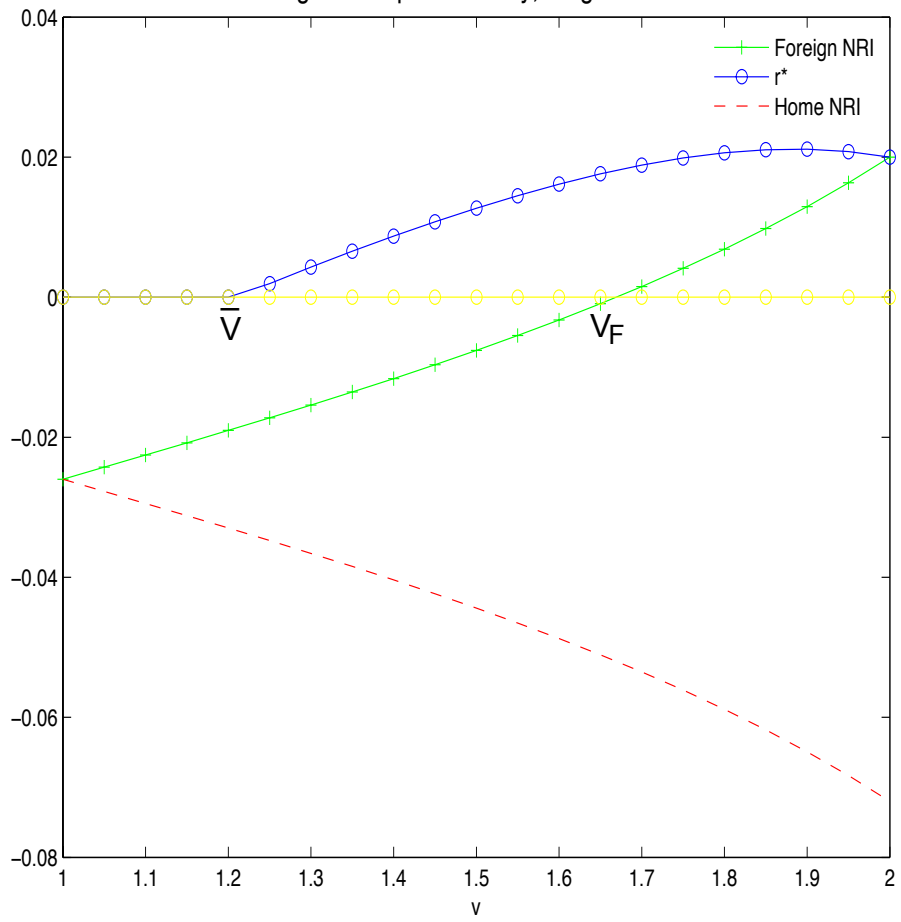


Figure 1

Figure 2: Optimal Policy, Large Shock



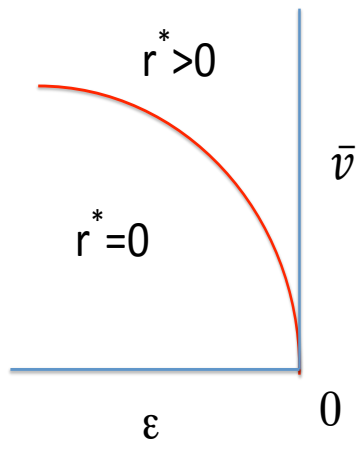


Figure 3

Figure 4: Optimal Policy, Small Shock

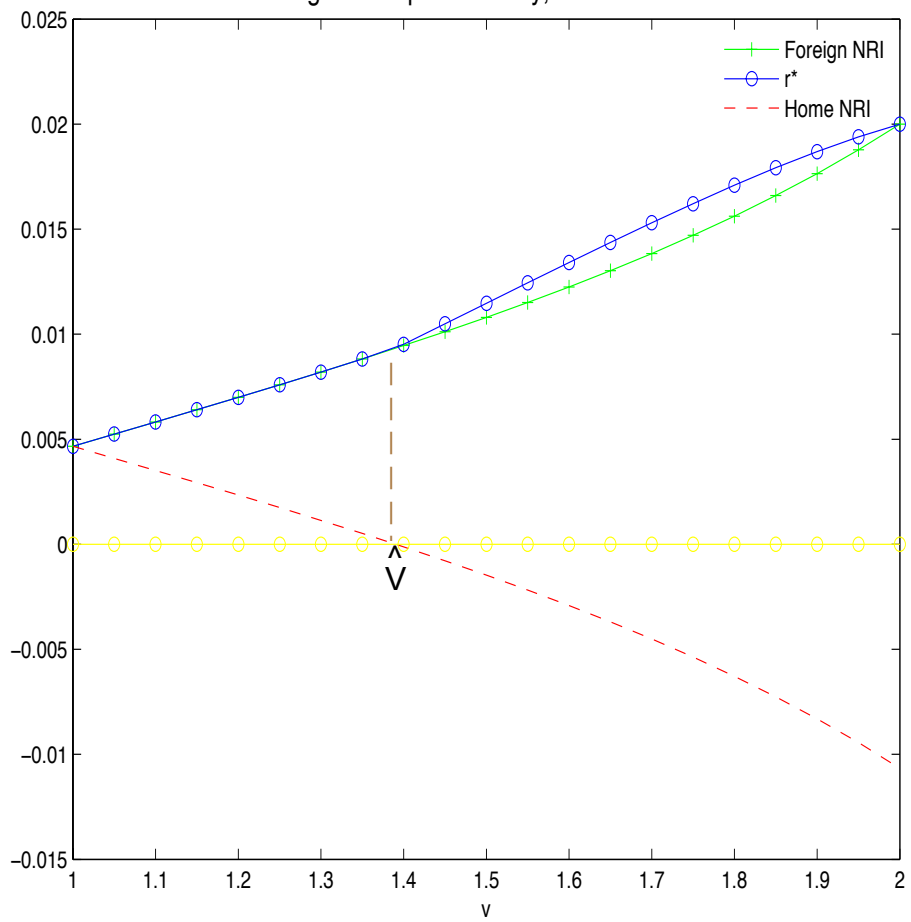


Figure 5: Optimal Policy

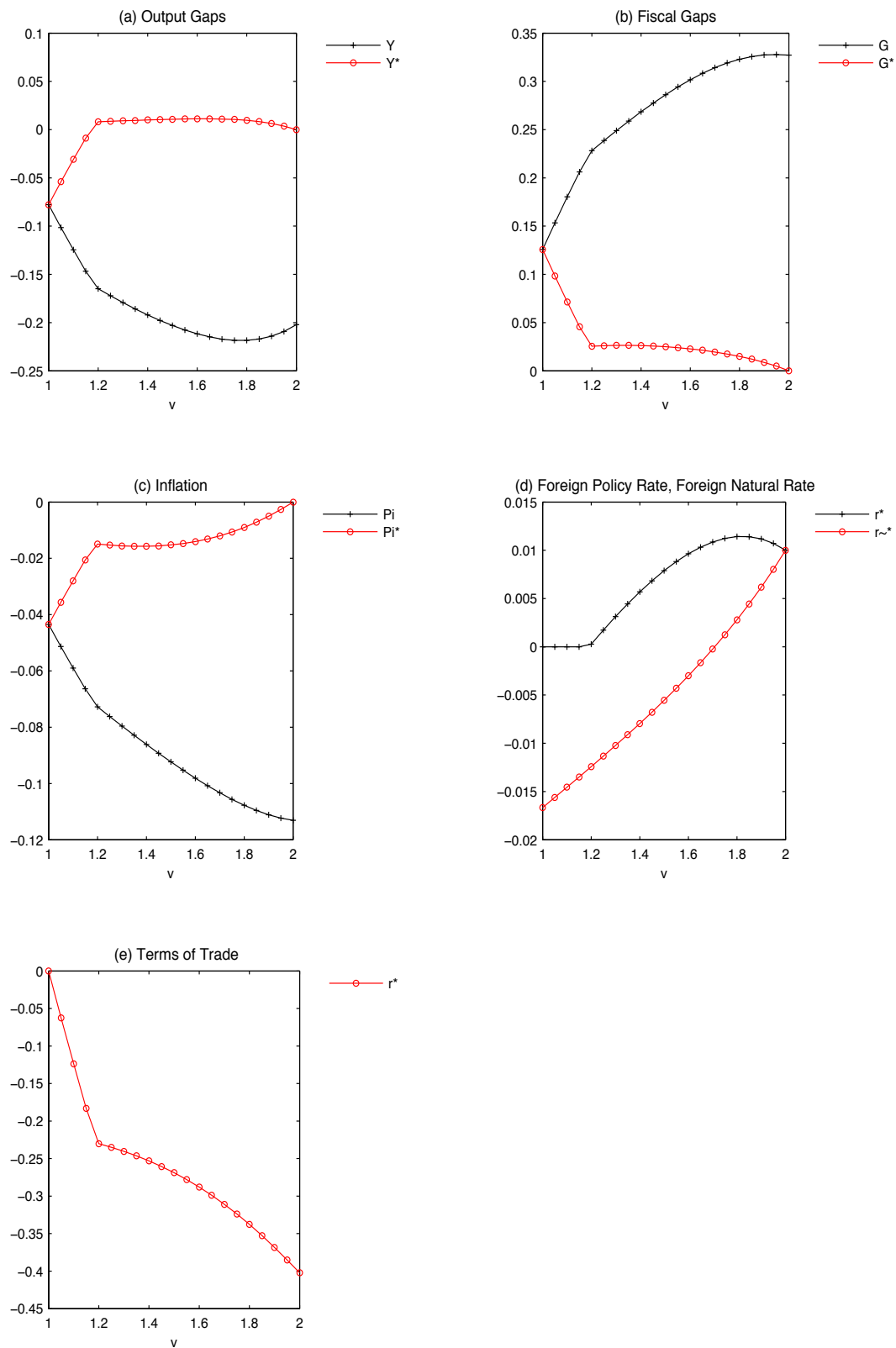


Figure 6: Optimal and Constrained Policy

