

Government Spending and Welfare with Returns to Specialization*

Michael B. Devereux

University of British Columbia, Vancouver, BC V6T 1Z1, Canada

Allen C. Head and Beverly J. Lapham

Queen's University, Kingston, ONT K7L 3N6, Canada

Abstract

We explore a novel channel through which government spending can stimulate consumption and welfare through its effects on aggregate productivity, without directly affecting either utility or production possibilities. In the presence of monopolistic competition and increasing returns to specialization, it is shown that government spending can partly alleviate the inefficiencies of monopolistic competition. This is because government spending generates an endogenous increase in total factor productivity by increasing the variety of intermediate goods. If the degree of increasing returns to variety is large enough, a rise in such wasteful government spending may increase consumption levels enough to increase welfare.

Keywords: Increasing returns to scale; government spending

JEL classification: E60

I. Introduction

It is well known that in the standard neoclassical growth model, increases in government spending tend to increase economic activity by stimulating labor supply; see, for example, Aiyagari, Christiano, and Eichenbaum (1992). Since increased employment can only occur if there is a fall in the real wage, aggregate consumption must fall, and thus welfare is reduced (unless government spending is a perfect substitute for private consumption). We demonstrate that in the presence of a market inefficiency due to monopolistic competition and increasing returns, government spending may be welfare improving, regardless of whether the spending is put to any useful purpose. Moreover, government spending stimulates productivity and welfare purely through its effect on overall economic activity, without directly affecting utility or production possibilities. We illustrate this in a simple, analytically tractable, version of the neoclassical growth model.

*We thank two anonymous referees for very insightful comments. We also gratefully acknowledge financial support from the Social Science and Humanities Research Council of Canada.

We analyze an economy in which monopolistically competitive firms supply differentiated intermediate inputs to a competitive final goods sector. In the presence of increasing returns to the variety of intermediate goods (returns to specialization), equilibrium variety is inefficiently low. As a result, employment, output, and consumption are all below their efficient levels. While there exists a first-best policy of subsidizing capital and labor income, we show also that government spending alone can be Pareto improving.

In our model, government spending not only stimulates employment but also increases the variety of intermediate products, which raises total factor productivity. If this effect is strong enough, the expansion in output may actually raise consumption. If the increase in consumption is sufficiently large, welfare may be increased, even if the government spending itself is entirely wasteful. We derive an optimal share of “wasteful” government spending in output for our economy. If returns to specialization are sufficient to generate endogenous growth, then this share is always strictly positive. It may also be positive for lesser degrees of returns to specialization. Our results do, however, require government spending to be financed by lump-sum taxation. Wasteful government spending financed by a flat income tax, a tax on labor income alone, or a tax on capital income alone would always reduce welfare.

In earlier work we used a similar model to study the effects of productivity and government spending shocks; see Devereux, Head, and Lapham (1996a, 1996b). This paper differs from our earlier studies in that it focuses on welfare in the steady state. Our findings highlight the differences between the *normative* effects of government spending in competitive environments and in environments with imperfect competition and increasing returns.

The remainder of the paper is organized as follows. In Sections II and III we introduce the economy and examine the social optimum. The effects of wasteful government spending on welfare are evaluated in Section IV, where we also derive conditions under which the optimal level of spending is positive. Section V concludes.

II. The Economy

Consider an economy in which there is a single final good that can be consumed or invested. This final good is produced using a range of differentiated intermediate inputs. Let m_{it} be the quantity of input i used in production of the final good at time t . The final good production technology is given by

$$Y_t = \left[\int_0^{N_t} m_{it}^\rho di \right]^{1/\rho} \quad \rho \in (0, 1), \quad (1)$$

where N_t represents the measure of intermediate inputs in production in the economy at time t . This production function exhibits a form of increasing returns to specialization that has been used in a variety of settings, for example by Romer (1987) in growth theory. If all intermediates are hired in the same quantity, m_t , output is given by $Y_t = N_t^{1/\rho} m_t > N_t m_t$. Thus, there are constant returns to the quantity employed of a fixed variety of intermediate goods, but increasing returns to an expansion of variety, holding fixed the quantity employed of each intermediate. The degree of returns to specialization is equal to $1/\rho$, where the parameter ρ governs the elasticity of substitution between any two intermediate inputs in final goods production.

Each intermediate input is supplied by a single monopolistically competitive firm. Intermediate goods producers use capital and labor to produce their product and sell to final goods producers at the profit-maximizing price. The production technology for intermediate good i is

$$m_{it} = k_{it}^\alpha h_{it}^{1-\alpha} - \phi, \quad (2)$$

where k_{it} and h_{it} denote capital and labor used by intermediate producer i , and ϕ is a fixed cost paid in units of intermediate good output. Physical capital available in period t is produced using the time $t - 1$ consumption good and depreciates fully at the end of period t .

There is a unit measure of identical households and we consider a representative household. Period preferences over individual consumption, c_t , and leisure, $1 - h_t$, are given by

$$U(c_t, 1 - h_t) = \ln(c_t) + \eta \ln(1 - h_t). \quad (3)$$

Here, the fixed endowment of time has been normalized to 1, h_t is hours worked, and $\eta > 0$ is a constant preference parameter. Lifetime utility of a representative household is then given by

$$E_t \sum_{t=0}^{\infty} \beta^t U(c_t, 1 - h_t), \quad (4)$$

where $\beta \in (0, 1)$ is the discount factor.

The government consumes a constant share, $\theta \geq 0$, of final good output in period t . Government consumption gives no direct utility to households, nor

does it have any direct effect on production. We restrict our attention to balanced-budget government fiscal plans. The government's budget constraint can be written

$$\theta Y_t = T_t. \quad (5)$$

Here T_t is a lump-sum tax at time t .

We consider a symmetric equilibrium with monopolistic competition. In equilibrium, intermediate goods producers hire labor and capital from the households in competitive factor markets and produce under conditions of monopolistic competition with free entry. Final goods producers behave competitively, purchasing inputs from the intermediate goods sector and selling output to households. Households choose consumption, investment, and hours worked to maximize lifetime utility, taking prices, aggregate quantities, the share of government consumption in output and the government's financing plan as given. Finally, the government's choice of consumption and taxes must satisfy its budget constraint.

We begin with the problems faced by firms in both the final and intermediate goods sectors. Because both types of firms face static optimization problems, we suppress time subscripts where possible. Final goods producers face a constant returns to scale technology and maximize profits taking the prices of intermediate inputs as given. The first-order condition for optimal choice of intermediate j by a final goods producer is given by

$$\left[\int_0^N m_i^\rho di \right]^{(1-\rho)/\rho} m_j^{\rho-1} = p_j \quad (6)$$

where p_j is the price of intermediate good j and the final good is the numeraire. To the producer of intermediate good j , equation (6) implies a demand curve for her product with constant elasticity $1/(1-\rho)$.

The producer of intermediate good j chooses k_j and h_j to maximize profits, Π_j :

$$\Pi_j = p_j m_j - w h_j - r k_j, \quad (7)$$

where w and r are the market wage and rental rate of capital, respectively, and p_j is given by equation (6). The following first-order conditions for optimal choice of capital and labor obtain:

$$\frac{p_j \alpha \rho (m_j + \phi)}{k_j} = r \quad (8)$$

$$\frac{p_j(1 - \alpha)\rho(m_j + \phi)}{h_j} = w. \tag{9}$$

Free entry into the intermediate goods industry guarantees zero profits for each producer, i.e., $p_j m_j = w h_j + r k_j \forall j$. We focus on a symmetric outcome where $p_j = p$, $m_j = m$, $k_j = K/N$, and $h_j = H/N \forall j$. From (6), we have

$$p = N^{(1-\rho)/\rho}. \tag{10}$$

Combining (8), (9), and the zero profit condition, we have w , r , and N as time-invariant functions of K and H :

$$w(K, H) = \Omega(1 - \alpha)K^{\alpha/\rho} H^{(1-\alpha-\rho)/\rho} \tag{11}$$

$$r(K, H) = \Omega\alpha K^{(\alpha-\rho)/\rho} H^{(1-\alpha)/\rho} \tag{12}$$

$$N(K, H) = \frac{(1 - \rho)}{\phi} K^\alpha H^{1-\alpha}, \tag{13}$$

where

$$\Omega \equiv \rho \left(\frac{1 - \rho}{\phi} \right)^{(1-\rho)/\rho}.$$

At this point it is also useful to note that using (1)–(2) and (13), output of the final good may be written as the following time-invariant function:

$$Y(K, H) = N^{(1-\rho)/\rho} \rho(K^\alpha H^{1-\alpha}) = \Omega(K^\alpha H^{1-\alpha})^{1/\rho}. \tag{14}$$

For $\rho < 1$, this equation illustrates that factors (indirectly) exhibit increasing returns in production of the final good. The first equality establishes that this arises from the presence of increasing returns in the variety of differentiated goods used in production of the final good. Hence, any change in the economy (such as an increase in government spending) which leads to entry in the intermediate goods sector will generate an endogenous increase in productivity of the primary factors in production of the final good.

The household’s problem can be written as follows:

$$\max_{\{c_t, h_t, k'_t\}_{t=0}^\infty} \sum_{t=0}^\infty \beta^t \{ \ln c_t + \eta \ln(1 - h_t) \}$$

subject to:

$$c_t + k'_t = w_t h_t + r_t k_t - T_t,$$

where k' denotes the consumer's next-period capital stock. In solving this problem, the consumer takes factor prices, taxes, and government spending as given. Solving the consumer's problem and substituting in the factor pricing functions (11) and (12), and the expression for aggregate output, (14), we can derive the following equilibrium aggregates as functions of the share of government consumption (and implicitly the current aggregate capital stock):

$$H(\theta) = \frac{(1 - \alpha)}{(1 - \alpha) + \eta[1 - \theta - \alpha\beta]}, \quad (15)$$

$$K'(\theta) = \alpha\beta Y(\theta) \quad (16)$$

$$C(\theta) = [1 - \theta - \alpha\beta]Y(\theta) \quad (17)$$

$$Y(\theta) = \Omega(K_t^\alpha H(\theta)^{1-\alpha})^{1/\rho}. \quad (18)$$

Given an initial capital stock, K_0 , and a government expenditure share, these equations characterize the equilibrium path of aggregate hours, capital, consumption, and output in this economy.

We restrict our attention to cases in which $\rho \geq \alpha$ as the economy will experience an accelerating growth rate when $\rho < \alpha$. If $\rho > \alpha$ and the share of government spending in output is constant at θ , the economy converges to a steady state characterized by employment given by (15), a capital stock given by,

$$\bar{K} = [\alpha\beta\Omega H(\theta)^{(1-\alpha)/\rho}]^{\rho/(\rho-\alpha)}, \quad (19)$$

and consumption given by (17) evaluated at the steady-state level of capital. If $\rho = \alpha$ the economy has a balanced growth path in which capital, output, and consumption all grow at the same rate given by

$$\gamma = \frac{K'}{K} = \alpha\beta\Omega H(\theta)^{(1-\alpha)/\rho}. \quad (20)$$

III. The Social Optimum

Owing to the imperfectly competitive behavior of intermediate goods producers and the presence of increasing returns to specialization, the equi-

brrium of the competitive economy is inefficient. We now compare this equilibrium to the outcome that a social planner would choose for the same economy. For comparative purposes, in this section we focus on the case with no government spending ($\theta = 0$) and reintroduce such spending in Section IV.

Restricting our attention to symmetric social optima, the social planner's problem can be written as follows:

$$\max_{\{C_t, H_t, K'_t, N_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \{ \ln C_t + \eta \ln(1 - H_t) \}$$

subject to:

$$C_t + K'_t = N_t^{(1-\rho)/\rho} (K_t^\alpha H_t^{1-\alpha} - \phi N_t), \tag{21}$$

where the RHS of equation (21) is derived by imposing symmetry and combining equations (1) and (2). It is straightforward to verify that the optimal choices in the absence of government spending as functions of the aggregate capital stock are given by,

$$H^s = \frac{1 - \alpha}{1 - \alpha + \eta(\rho - \alpha\beta)} \tag{22}$$

$$K'^s = \frac{\alpha\beta}{\rho} Y_t^s \tag{23}$$

$$C^s = \frac{(\rho - \alpha\beta)}{\rho} Y_t^s \tag{24}$$

$$N^s = \frac{(1 - \rho)}{\phi} K_t^\alpha H^{s1-\alpha}. \tag{25}$$

where $Y_t^s = \Omega(K_t^\alpha H^{s1-\alpha})^{1/\rho}$.

Comparing (13) and (25) it is clear that the social planner's rule for choosing the number of varieties of intermediate goods is the same as that followed in the market. But the planner takes returns to specialization into account, and in so doing devotes a larger share of final goods output to investment (compare (16) to (23)), a smaller share to consumption, and a greater share of the time endowment to production (compare (15) to (22)).

When $\rho > \alpha$, the optimal steady state is characterized by employment given by (22), capital stock given by

$$\bar{K}^s = \left[\frac{\alpha\beta\Omega}{\rho} H^{s[(1-\alpha)/\rho]} \right]^{\rho/(\rho-\alpha)} \quad (26)$$

and consumption given by (24) evaluated at \bar{K}^s and H^s . In this case, it is clear that employment, the capital stock, and consumption are all lower in the monopolistically competitive steady state than in the optimal steady state. If $\rho = \alpha$, the optimal growth rate given by

$$\gamma^s = \frac{K'^s}{K^s} = [\beta\Omega] H^{s[(1-\alpha)/\rho]} \quad (27)$$

is higher than the equilibrium growth rate given by (20).

The inefficiency and resulting divergence between the monopolistically competitive equilibrium and the social planner's solution is due to imperfect competition. In particular, factor payments do not reflect their true marginal social value. If the planner had the ability to choose optimal labor and capital taxes, then the social planning outcome could be achieved with a wage and rental subsidy. It is easy to show that a gross subsidy of $1/\rho$ paid on wage and capital income in the market equilibrium would achieve the planner's optimum.

If, however, optimal factor subsidies are unavailable or infeasible, other policies can be used to wholly or partially alleviate the market inefficiencies. In particular, in the next section, we show that a policy of wasteful government spending might actually be desirable in this environment.

IV. Welfare Effects of Government Spending

In an economy with perfect competition and constant returns to scale, wasteful government spending is necessarily welfare reducing. Government spending stimulates labor supply, increases the capital stock and output, but reduces consumption. Since consumption and leisure both fall, welfare is inevitably reduced. In an economy with increasing returns to scale such as this one, however, there is a possibility that wasteful government spending which leads to increases in production may actually increase welfare. We first describe the effects of government spending on our economy, and then consider the welfare implications.

Since employment (given by (15)) is increasing in the steady-state share of government expenditures, θ , the steady-state capital stock is increasing in θ . This is similar to an economy with perfect competition. A difference arises, however, in regard to the effect on the steady-state level of consumption, given by

$$\bar{C} = (1 - \theta - \alpha\beta)\Omega(\bar{K}^\alpha H(\theta)^{1-\alpha})^{1/\rho} \quad \rho > \alpha. \tag{28}$$

Using (15) and (18) it can be shown that

$$\frac{d \ln \bar{C}}{d \ln \theta} = \left[\frac{1 - \alpha}{\rho - \alpha} - 1 - \frac{1 - \alpha}{\eta(1 - \theta - \beta\alpha)} \right] \frac{d \ln H}{d \ln \theta}. \tag{29}$$

We may think of a perfectly competitive, constant returns economy as the limit of the economy under consideration here as $\rho \rightarrow 1$. In this case, since $d \ln H/d \ln \theta > 0$, $d \ln C/d \ln \theta < 0$. If, however, $\rho < 1$, consumption may be increasing in θ . In particular, if initially $\theta = 0$, a small increase in θ will increase long-run consumption if

$$\alpha < \rho < \hat{\rho} \equiv \frac{\eta(1 - \beta\alpha) + \alpha(1 - \alpha)}{\eta(1 - \beta\alpha) + (1 - \alpha)}. \tag{30}$$

Moreover, in the balanced growth case with $\rho = \alpha$, the growth rate of consumption given by equation (20) is always increasing in θ .

How does an increase in government spending raise both consumption and labor supply at the same time? From our knowledge of consumer preferences (i.e., (3)), it is clear that both consumption and labor supply can increase only if the wage also increases; see, for example, Barro and King (1984). With constant returns to specialization, expansion of government spending cannot increase the long-run real wage. But with $\rho < 1$, an increase in government spending generates an endogenous rise in productivity which increases the real wage, allowing for both consumption and labor supply to rise.

Some further intuition can be developed. Let us focus on the stationary case ($\rho > \alpha$). Consider the equations governing the determination of wages and employment in a long-run equilibrium. The household's first-order condition for optimal choices of consumption and leisure is:

$$w = \frac{\eta C}{1 - H}. \tag{31}$$

This condition may be thought of as determining a "labor supply" function. Fixing C , there is a positive relationship between the wage and aggregate labor supplied.

There is an analogous "labor demand" function that may be obtained by combining (11) and (13):

$$w(K, H; N) = \rho(1 - \alpha)N^{(1-\rho)/\rho} K^\alpha H^{-\alpha}. \tag{32}$$

For a given long-run N , the capital stock is

$$K = (\beta\alpha\rho)^{1/(1-\alpha)} N^{[1-\rho]/[\rho(1-\alpha)]} H.$$

Thus, in the long run, we may rewrite (32) as the relationship between w and H , for fixed N , as:

$$w(H; N) = \rho(1 - \alpha)(\beta\alpha\rho)^{\rho/(\rho-\alpha)} N^{[1-\rho]/[\rho(1-\alpha)]}. \quad (33)$$

If $\rho = 1$, then the long-run labor demand is horizontal, since, taking into account the endogenous response of the capital stock, the marginal productivity of labor is independent of H . In this case an increase in government spending has no effect on long-run labor demand. The real wage is constant. A rise in government spending will reduce consumption, shift the labor supply curve out to the right, and increase employment.

But when $\rho < 1$, the rise in government spending has a direct effect on the long-run productivity of labor, as N is dependent on K and H through equation (13) (and K depends on H through (19)). Then it is clear that the long-run labor demand curve, taking into account the endogenous response of N , is upward sloping in w , H space. If $\rho > \hat{\rho}$, then the long-run labor demand curve is less steep than the labor supply curve given by (31). In that case, a rise in government spending must have the same qualitative effects as before. Consumption falls, labor supply increases, and while the real wage rises, it is not enough to counter the negative relationship between consumption and employment implied by (31).

But if $\rho < \hat{\rho}$, the long-run labor demand curve is steeper than the labor supply curve. In this case, the endogenous productivity effect of government spending leads to a rise in the real wage which is sufficient to allow for an increase in both employment and consumption simultaneously. The increase in consumption shifts *back* the labor supply curve, which leads to a movement up the long-run labor demand curve, so that consumption, employment, and the real wage all increase.

We now consider the possibility that government spending will increase welfare. In particular, we ask whether it is possible that the optimal level of purely wasteful government spending in this economy is greater than zero.

Consider the problem of a government that wants to maximize social welfare by choosing the government spending share. We assume that the government can perfectly commit to a constant share θ at time 0, so that the optimal behavior of consumers contingent on the sequence of government consumption is given by (15)–(18). The fiscal authority takes these as given and chooses θ to solve:

$$\max_{\theta} \sum_{t=0}^{\infty} \beta^t \{ \ln(C(K_t, \theta)) + \eta \ln(1 - H(\theta)) \}$$

subject to:

$$0 \leq \theta \leq 1 \quad \text{and} \quad (15)\text{--}(18).$$

Let θ^* denote the optimal share of government spending in output. The following proposition summarizes conditions under which θ^* is strictly positive.

Proposition 1. *Given $\beta \in (0, 1)$ and $\eta > 0$, there is a unique capital share, $\bar{\alpha}$, such that for all $\alpha \leq \bar{\alpha}$, there exists a $\bar{\rho} \geq \alpha$, such that for $\alpha \leq \rho < \bar{\rho}$, the optimal share of government spending is positive and is given by:*

$$\theta^* = 1 - \beta\alpha - \frac{(1 - \alpha)(1 + \eta)(\rho - \beta\alpha)}{\eta[1 - \rho - \alpha(1 - \beta)]}. \tag{34}$$

Proof: See the Appendix.

Proposition 1 establishes that there is a unique, strictly positive, optimal share of wasteful government spending in output when the capital share, α , is sufficiently low and the degree of returns to specialization, $1/\rho$, is sufficiently high. Why is it that wasteful government spending can raise welfare? While the Appendix provides a rigorous demonstration, we give an intuitive account here. It is shown in the Appendix that the value to the government of choosing θ when the current capital stock is K_0 is given by

$$V(K_0, \theta) = \frac{1}{(1 - \beta)} \left[\ln(1 - \beta\alpha - \theta) + \eta \ln(1 - H(\theta)) + \frac{(1 - \alpha)}{(\rho - \beta\alpha)} \ln H(\theta) \right] + \Lambda + \frac{\alpha}{(\rho - \beta\alpha)} \ln K_0, \tag{35}$$

where Λ is a constant function of parameters and independent of θ .

A rise in the government spending share affects this in three ways, corresponding to the three separate terms inside the square brackets. The first expression indicates that a rise in θ directly reduces the share of consumption in GDP, and therefore reduces welfare. The second expression indicates that a rise in θ raises labor supply, reducing leisure, and therefore must also reduce welfare. The last expression captures the fact that a rise in government spending increases private consumption indirectly, by raising

employment, and by raising the capital stock. This last expression will be larger, the smaller is ρ . For $\rho = 1$ (constant returns to specialization), the first and second negative effects will offset the third positive effect, and government spending must reduce welfare. But as shown in the Appendix, with sufficiently low ρ (conditional on $\rho > \alpha$), it is possible that the indirect effects on consumption through employment and the capital stock dominate the other two negative effects. Clearly, however, this requires positive returns to specialization.

While wasteful government spending can improve welfare relative to the market equilibrium, it does not replicate the social optimum. Substituting θ^* from (34) into (15) we see that employment under the optimal spending policy is

$$H^* = \frac{1 - \alpha - \rho + \beta\alpha}{1 - \alpha + \eta(\rho - \beta\alpha)}, \quad (36)$$

which is less than the efficient level of employment given by (22). The optimal government spending policy is inferior to the first-best subsidization policy not only because output is wasted through government spending, but also because it increases investment only indirectly through its effect on employment, rather than through a direct subsidy to capital income.

Finally, as an illustration of Proposition 1, Figure 1 illustrates the relationship between θ and welfare for a specific set of parameter values.¹ For this example, the “optimal” share of government spending is approximately 9 percent of GDP.

V. Conclusion

With constant returns, wasteful government spending necessarily decreases private consumption and welfare. We have shown, however, that in the presence of sufficiently strong increasing returns, wasteful government spending may increase both consumption and welfare.

To illustrate the linkage between government spending and productivity with increasing returns to specialization, we have maintained a very simple model specification. Our welfare results may not necessarily extend to other cases. For instance, if government spending is financed with a distortionary income tax, then it can be shown that a rise in government spending will always reduce welfare. It would be interesting to explore the role of wasteful government spending in a richer environment (i.e., with more general utility and realistic depreciation), and to compare the results to those on optimal

¹The parameter values are $\beta = 0.96$, $\alpha = 0.45$, $\eta = 3$, $\rho = 0.65$, $\phi = 0.1$.

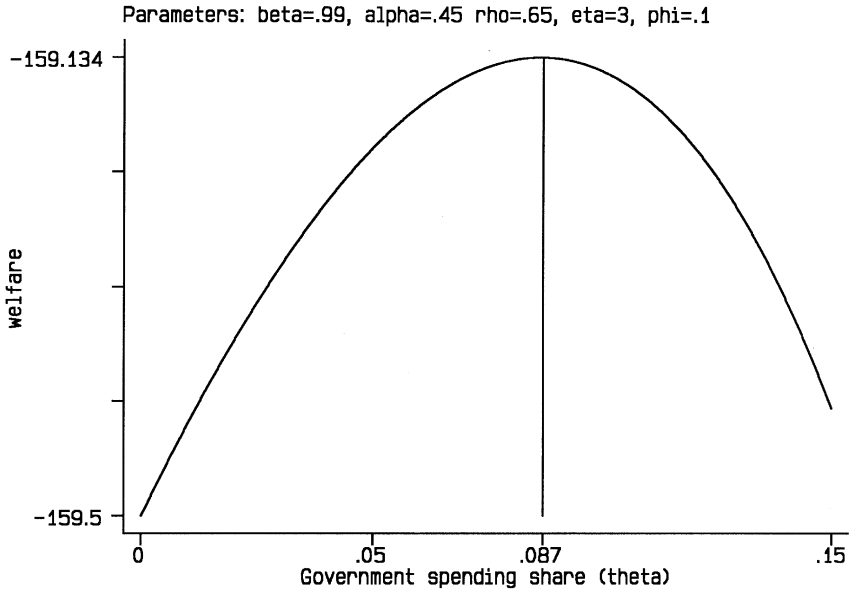


Fig. 1.

policies in the neoclassical model with perfect competition and constant returns; see Chari, Christiano, and Kehoe (1994). This would, however, be a substantial undertaking which we leave to further research.

Appendix

Proof of Proposition 1

Given the government's objective function, the return to the government for choosing θ when the current capital stock equals K_t must satisfy the following:

$$V(K_t, \theta) = \ln C_t(K_t, \theta) + \eta \ln(1 - H(\theta)) + \beta V(K_{t+1}, \theta), \quad (A1)$$

where $C_t(K_t, \theta)$ is given by (17), $H(\theta)$ is given by (15), and K_{t+1} is given by (16).

Given the form of utility in (3), it is an obvious conjecture that this function is log linear. Thus, we conjecture that

$$V(K_t, \theta) = A(\theta) + B \ln K_t, \quad (A2)$$

where $A(\theta)$ is an (as yet undetermined) time-invariant function of the government spending share and B is a constant undetermined coefficient.

By derivation, it is straightforward to establish that this conjecture is correct, and that

$$A(\theta) = \frac{1}{(1-\beta)} \left[\ln(1-\beta\alpha-\theta) + \frac{(1-\alpha)}{(\rho-\beta\alpha)} \ln H(\theta) + \eta \ln(1-H(\theta)) \right] + \Lambda \quad (\text{A3})$$

$$B = \frac{\alpha}{(\rho-\beta\alpha)}, \quad (\text{A4})$$

where Λ is a constant function of parameters, independent of θ .

Given these results we can establish that

$$\frac{\partial V}{\partial \theta} = \frac{1}{1-\beta} \left[\frac{\eta H(\theta)}{\rho-\alpha\beta} - \frac{\eta^2 H(\theta)^2}{(1-\alpha)(1-H(\theta))} - \frac{1}{1-\theta-\alpha\beta} \right]. \quad (\text{A5})$$

Now welfare will be increased by a small positive level of government spending relative to a situation with no government spending if (A5) is positive when evaluated at $\theta = 0$. Fixing β , η , and α , define $\Phi(\rho)$ as follows:

$$\Phi(\rho) \equiv [1-\alpha+\eta(1-\alpha\beta)][\alpha\beta-\rho] + \eta(1-\alpha)(1-\rho). \quad (\text{A6})$$

It can be easily shown that the sign of $\Phi(\rho)$ is equal to the sign of (A5) evaluated at $\theta = 0$, and that the function $\Phi(\cdot)$ is continuous and monotonically decreasing in ρ with $\Phi(1) < 0 < \Phi(0)$. Thus there exists a $\bar{\rho}$ such that $\Phi(\bar{\rho}) = 0$ with $\Phi(\rho) > 0$ for all $\rho < \bar{\rho}$ and $\Phi(\rho) \leq 0$ for all $\rho \geq \bar{\rho}$. This implies that for $\rho < \bar{\rho}$, a small increment of government spending improves welfare over the case where $\theta = 0$. It remains to be shown, however, that $\bar{\rho} > \alpha$, a requirement since the argument is only valid for $\rho > \alpha$. We now show that there is a unique $\bar{\alpha}$ such that for all $\alpha < (=)\bar{\alpha}$, $\bar{\rho}(\alpha) > (=)\alpha$. Setting (A6) equal to zero, we can solve for $\bar{\rho}$ as a function of α :

$$\bar{\rho}(\alpha) = \frac{\eta[\alpha\beta(1-\alpha\beta) + 1 - \alpha] + \alpha\beta(1-\alpha)}{\eta[(1-\alpha\beta) + (1-\alpha)] + (1-\alpha)}. \quad (\text{A7})$$

It can be shown through algebraic manipulation of (A7) that the condition $\alpha < \bar{\rho}$ is equivalent to

$$\Psi(\alpha) = \eta(1-\alpha)^2 - \alpha(\beta-1)[\eta(1-\alpha\beta) + (1-\alpha)] > 0. \quad (\text{A8})$$

Note that the function $\Psi(\alpha)$ is clearly continuous on $[0, 1]$ with $\Psi(1) < 0 < \Psi(0)$, and quadratic. Thus there exists an $\bar{\alpha} \in (0, 1)$ such that $\Psi(\bar{\alpha}) = 0$. Furthermore, since $\Psi(\cdot)$ is quadratic, it may change sign only once on the interval $[0, 1]$ and $\bar{\alpha} > 0$ is unique. Thus, for all $\alpha < \bar{\alpha}$, $\bar{\rho}(\alpha) > \alpha$.

These results imply that for all α and ρ such that $\alpha < \bar{\alpha}$ and $\alpha < \rho < \bar{\rho}$, a small increment of wasteful government spending improves welfare over the case where $\theta = 0$. In this case, solving the government's first-order condition given by equation (A5) set equal to zero gives a positive level of optimal government spending. Furthermore, as this equation is linear in θ , it will have a unique solution. It can be easily verified that this solution is given by equation (34).

References

- Aiyagari, S. R., Christiano, L. J. and Eichenbaum, M. (1992), The Output, Employment, and Interest Rate Effects of Government Consumption, *Journal of Monetary Economics* 30 (1), 73–86.
- Barro, R. J. and King, R. G. (1984), Time-separable Preferences and Intertemporal-substitution Models of Business Cycles, *Quarterly Journal of Economics* 99 (4), 817–839.
- Chari, V. V., Christiano, L. J. and Kehoe, P. J. (1994), Optimal Fiscal Policy in a Business Cycle Model, *Journal of Political Economy* 102 (4), 617–652.
- Devereux, M. B., Head, A. C. and Lapham, B. J. (1996a), Aggregate Fluctuations with Increasing Returns to Specialization and Scale, *Journal of Economic Dynamics and Control* 20 (4), 627–656.
- Devereux, M. B., Head, A. C. and Lapham, B. J. (1996b), Monopolistic Competition, Increasing Returns, and the Effects of Government Spending, *Journal of Money, Credit and Banking* 28 (2), 233–254.
- Romer, P. M. (1987), Growth Based on Increasing Returns Due to Specialization, *American Economic Review* 77 (2), 56–62.

First version submitted July 1998;
final version received July 1999.