I. **Labour Supply**

*B. Intertemporal/Life-cycle Behaviour*

*Plan*

1. **Recent Trends: Age-LFP Profiles**

2. **Life-Cycle/Intertemporal Labour Supply Model**

3. **Estimating the life-cycle model**
1) Recent Trends: Age-LFP Profiles and Cohort Effects

Men

A. Changes over Time

B. Changes over the Life-Cycle

Women

A. Changes over Time

B. Changes over the Life-Cycle

Figure 3: Employment rate by age (1977-2007)

A. Male 1977

B. Male 2007

C. Female 1977

D. Female 2007

Source: Blundell, Bozio, and Laroque (2011)
Figure 2: Total Hours by Age (1977-2007)

A. Male 1977

B. Male 2007

C. Female 1977

D. Female 2007
Labour supply is clearly part of a life-time decision-making process: individuals attend school early in life, accumulate wealth while in the labour force and make retirement decisions later in life.

- As shown by men’s life-cycle LFP profiles

- Women’s life-cycle LFP may be complicated by labour force withdrawal when they have children (generally in their mid-20s to mid-30s)
  - Because fertility decisions have changed across generations, women’s life-cycle or age-profile LFP profiles are complicated by the presence of cohort effects, which may it difficult to disentangle to the “pure effect” of age from a birth cohort effect (e.g. the fact that women born between 1940-45 waited for their children to leave school before entering the labour market, but not women born between 1960-65)
  - These two effects cannot be disentangled using a single cross-section of data.

- Women’s LFP has increased dramatically over the twentieth century

- Wealth and/or savings from labour earnings are used to sustain individuals during periods when they are out of the labour market.

- Life-cycle/Intertemporal Labour Supply (ILS) models seek to understand variations in labour supply as individuals aged as well as year-to-year variations.
• There are three main "dimensions" of ILS:
  1) Wages have a hump-shaped pattern over the lifecycle. Do people respond to this? What determines the shape of the life-cycle profile?

  2) A person may have temporarily higher wages in some period. How does he/she respond? What is the source of idiosyncratic (i.e. person-specific) changes in year-to-year labour supply?

  3) The "macro" dimension. Hours of work vary over the business cycle. Can we interpret this as an endogenous response to wages?

    ➢ What is the effect of a transitory change in wage by comparison with a permanent modification to the life-cycle wage profile?

• The intertemporal model of labour supply makes the point that there are three types of wage shocks: permanent, transitory, and anticipated, and the labour supply response (elasticity) will depend on what type of shock occurs.
There are 3 types of substitutions elasticities relevant for predicting the response of hours of work to changes in the wage rate:

- The intertemporal elasticity determines the labour-supply response to wage changes resulting from life-cycle wage growth.
- The uncompensated and compensated elasticities determine the hours of work response to shift in wage profiles.

An important assumption of the intertemporal model of labour supply is **time separability**, this assumption rules out “habit-formation” or “habit persistence”, which has been shown to exist empirically.

What else does it rule out?
2. Intertemporal Labour Supply (ITLS) Model

• A consumer makes his or her choices of consumption and leisure (or hours of work) over a life-cycle that goes from an initial time period 0 to a ending time period T.

• Standing at time $t$, the full life-cycle model is characterized by a utility function of the form

$$U_t = U(C_t, H_t, X_t; C_{t+1}, H_{t+1}, X_{t+1}; ... C_T, H_T, X_T),$$

where $C_t$ is consumption, $H_t$ is labour and $X_t$ are individual characteristics.

• Most often the life-cycle utility is assumed separable over time, we thus consider a utility function of the form

$$\sum \beta^t U(C_t, H_t, X_t),$$

where $\beta = 1/(1+\rho)$ is the subjective discount rate.

• In models with uncertainty, preferences are considered additive over states of the world and time, so that uncertainty is handled by taking the expectation of (1).

• In this world, individuals have the opportunity to save and therefore earn an interest rate $r_t$ between period $t$ and $t-1$. 
• The second element of the model is the wealth constraint. Letting the change in the value of assets be

\[
A_1 = A_0 (1 + r_1) + w_1 H_1 - C_1 \\
A_2 = A_1 (1 + r_2) + w_2 H_2 - C_2 \\
\ldots \\
A_{t+1} = A_t (1 + r_{t+1}) + w_{t+1} H_{t+1} - C_{t+1}
\]

(2)

where the price of consumption is normalized to 1 so that \( w_t \) is the real wage rate, \( r_t \) is the real interest rate in period \( t \) and \( A_t \) is the real value of assets, or equivalently

\[
A_0 + \sum_{t=0}^{T} \frac{1}{\prod_{s=0}^{t-1} (1 + r_{t-s})} [w_t H_t - C_t] = 0,
\]

(2')

so that there are no bequests.

• But borrowing and lending is permitted, and so the budget constraint essentially summarizes the time path of wealth (or assets).

• What would if consumers were not allowed to have a negative asset position \( A_t \geq 0, \forall t = 1, \ldots, T \) ? For example, hold students loans borrowed against future human capital, rather than collateral. (Hint: static labour supply model, see Deaton (Econometrica Sept 1991))
• There are many ways to solve this problem, but given the time separability, the simplest one is simply to set up a Lagrangian

\[ L(C_t, H_t, \lambda_t) = \sum_{t=0}^{T} \beta^t U(C_t, H_t, X_t) - \sum_{t=0}^{T} \lambda_t [A_t - A_{t-1}(1+r_t) - w_t H_t + C_t] \]

• The **FOC conditions** for an interior solution to the maximization of (1) subject to (2) are

\[ \beta^t U_C(C_t, H_t, X_t) - \lambda_t = 0 \quad t = 0, \ldots, T \quad (3a) \]

\[ \beta^t U_H(C_t, H_t, X_t) - w_t \lambda_t = 0 \quad t = 0, \ldots, T \quad (3b) \]

and the Euler equation

\[ \lambda_t - \beta(1+r_t)\lambda_{t+1} = 0 \quad (3c) \]

where \( \lambda_t \) is the **marginal utility of wealth** in period \( t \).

• Equation (3c) determines the rule for the allocation of wealth across periods. It says that the consumer chooses savings so that the marginal utility of wealth \( \lambda_t \) in period \( t \) **equals** the discounted value of the marginal utility of wealth in period \( t+1 \), where the marginal rate of discount is \( \beta(1+r_t) \):

\[ \frac{\lambda_t}{\lambda_{t+1}} = \beta(1+r_t) = \frac{(1+r_t)}{(1+\rho)} \quad (3c') \]
Equation (3c) defines the optimal strategy and the lifetime problem decomposes into two levels. At the first level, an individual allocates his wealth over his life such that his marginal utility of wealth evolves as he ages according to equation (3c).

At the second level, conditional upon wealth allocated to a given period, the within period allocation problem is addressed. The first two conditions (3a) and (3b) are the same as in the static model and require that the

\[ \frac{U_c}{U_h} = MRS_{HC} = w_t \]

in every period as a consequence of time separability.

➢ Time-separability transforms the intertemporal allocation problem into a two-stage budgeting problem

The FOC conditions imply consumption and hours-of-work supply functions of the form

\[ C^*_t = C(\lambda_t, w_t, X_t) \]
\[ H^*_t = H(\lambda_t, w_t, X_t) \geq 0 \]

These are commonly referred to as Frisch demand functions or constant-marginal-utility-of-wealth labour supply functions.
• Variables such as future wealth, wages, or personal characteristics affect labour supply and consumption only by changing the value of \( \lambda_t \), which serves the role of sufficient statistic capturing all information from other periods that is relevant.

• Whereas Marshallian functions held income constant, Hicksian functions held utility constant, Frisch functions hold marginal utility of wealth constant.

• As shown in MaCurdy (1981) and Browning et al. (1985), the Frisch elasticities are the largest of the corresponding three wage elasticities, when an interior solution is choosen
\[
\frac{\partial H^M}{\partial w} < \frac{\partial H^H}{\partial w} < \frac{\partial H^F}{\partial w}.
\]

• The Euler equation implies a time path for \( \lambda \) of the form
\[
\ln \lambda_t = b_t + \ln \lambda_{t-1},
\]
where \( b_t = -\ln[\beta(1 + r_t)] \).

• Repeated substitution yields
\[
\ln \lambda_t = \sum_{j=1}^{t} b_j + \ln \lambda_0
\] (4)
• Hence the $\lambda$ term in the Frisch supply functions can be captured as an **individual fixed effect** $\lambda_0$ plus a **function of age** which is **common** across consumers.

• In the world of perfect certainty, the path of $\lambda$ through time is determined solely by the known path of interest rates and discount factor.

• Then apart from taste changes and a geometric trend, the life-cycle **profile of labour supply** is completely determined by the profile of **wages**.

• A priori, $\lambda_0$, which is obtained by substituting $C_i^*$ and $H_i^*$ into the wealth constraint (2’), is affected by all future values of wages received by the consumer over his/her lifetime.

• Thus a **transitory change** in a single $w_t$ is unlikely to have a large effect on $\lambda_0$, but the effects of a **permanent change** in the wage profile is likely to have an effect on $\lambda_0$. 
In the life-cycle model wage changes may have different effects on labour supply.

1. Evolutionary change in wages along the expected wage profile.
   - no wealth effect (as $\Delta w$ is fully expected)
   - only SE: work more in times when $w$ is high

2. Change in one particular wage (without affecting the whole wage profile).
   - only small wealth effect
   - SE: work more in high $w$ period

3. Changes in the complete wage profile.
   - large wealth effect: could reduce hours
   - SE: work more with higher $w$
• With concavity in preferences, as shown in Heckman (1974, 1976), the FOC conditions imply:
  
  \[ \frac{\partial H_t}{\partial w_t} \bigg|_{\lambda_0=\text{constant}} \geq 0, \text{ effect of a movement along anticipated wage profile, also called evolutionary wage changes} \]
  
  \[ \frac{\partial H_t}{\partial \lambda_0} \geq 0, \text{ effect of a change in the marginal utility of wealth} \]
  
  \[ \frac{\partial \lambda_t}{\partial A_0} \leq 0, \text{ marginal utility of wealth is declining} \]
  
  \[ \frac{\partial \lambda_t}{\partial w_t} \leq 0, \text{ wages generate wealth effects} \]
  
  ➢ The last two propositions say that the marginal utility of wealth (based on initial assets) diminishes with wages and initial assets.

  • The underlying concept of dynamic equilibrium is key to understand the model: it yields a set of equilibrium time profiles for labour supply and consumption determined for a given value of initial wealth \( A_0 \) and for a given profile of wages, \( w_t \) for all periods \( t \).
Note that \textit{anticipated effects} are just movements along a given labour supply profile, and should be analyzed as an issue of dynamic equilibrium, not comparative dynamics.

It is natural to think that simple \textit{efficiency} considerations would dictate that the individual will work the most during periods when the wage (the opportunity cost of leisure) is highest.

What would you do if you were told that your wage was raised to $50,000 per hour for just one day?

But suppose your wage was raised to $50,000 per hour, effective for the rest of your life. Comparing your remaining total lifetime labor supply under that scenario to what you are currently planning, would it be less or more?

So, what happens if \( w_i \uparrow \)

- If this increase was anticipated, the profile of work and consumption is not affected.
- But if this increase is unanticipated then we will observe a wealth effect and a substitution effect.
- The former is for sure going to reduce labour supply, the second will increase labour supply in the surprise period, and depending on the cross-substitution effect will increase or reduce leisure at times different from that $t$.

- Temporary transfer has a small effect on labour supply in all periods. But in reality, temporary transfers seem to have large effects on labor supply [e.g., severance payments, Card-Chetty-Weber (2008)]

- The intuition for these results has to do with the fact that a one-period wage increase generates little or no wealth effects when income is pooled across all periods of the lifetime. In contrast to the static, or liquidity-constrained case where you might decide to take it easy when wages rise, you should always “make hay when the sun shines” if you have access to capital markets.
Adding uncertainty

- Under uncertainty, FOC (3a) and (3b) remain the same, whereas \( \lambda_t \) becomes a random variable that follows the stochastic process described by

\[
\lambda_t = E_t \left[ \lambda_{t+1} \beta' (1 + r_{t+1}) \right]
\]

- Assuming rational expectations \( \lambda_{t+1} = E_t \lambda_t + \mu_{t+1} \), where \( \mu_{t+1} \) is an innovation to the marginal utility of wealth, and deterministic interest rate, the expression can be reduced to

\[
\ln \lambda_{t+1} - \ln \lambda_t \approx \rho - r_{t+1} + \frac{(1 + r_{t+1}) \mu_{t+1}}{(1 + \rho) \lambda_t}
\]

- The implication of the presence of uncertainty is that we need a measure of anticipated wage change between \( t-1 \) and \( t \)
3. Estimating the life-cycle model

a. Structural Approach

- Earlier attempts at evaluating the elasticity of labour supply in Frisch’s sense used a structural approach by positing a specific functional form for the utility.

- For example, MaCurdy (1981), used the addilog utility function, which is additive in log consumption and hours:

\[
U_i (C_{it}, H_{it}; X_{it}, \varepsilon_{it}) = b_{it} (C_{it})^{\gamma_i} - \tilde{\gamma}^{-1} a_{it} (H_{it})^{\tilde{\gamma}},
\]

where \( \tilde{\gamma} = \gamma^{-1} + 1, \quad \gamma > 0, \quad 0 < \gamma_{1} < 1, \quad a_{it} = \exp[\gamma^{-1}(-\beta X_{it} - \varepsilon_{it})], \)

- The Frisch hours of work equation for individual \( i \) at age \( t \) is then log-linear in hours and wages:

\[
\ln H_{it} = \gamma \ln \lambda_{0i} + \eta \ln W_{it} + \beta X_{it} + \gamma (\ln \theta) t + \varepsilon_{it}, \quad (4)
\]

where \( \theta = (1 + \rho I (1 + r)). \)

- Here, the parameter \( \eta \) is the intertemporal substitution elasticity: it measures the proportional change in hours of work induced by a proportional increase in wage rates
as a **worker age**. It is necessarily positive and greater than the Hicksian elasticity if leisure is a normal good.

- The parameter $\gamma$ represents the elasticity of hours with respect to the marginal utility of wealth, it must also be positive if leisure is a normal good.

- MaCurdy (1981) used ten annual observations on 513 white, continuously married, men from the PSID who were aged 25-46 in 1967 and observed in each of the ten years between 1967 and 1976. He finds estimates of $\eta$ ranging from 0.14 to 0.35 with standard errors going from 0.07 to 0.016.

**b. Basic empirical specification**

- A typical empirical specification for estimating the intertemporal elasticity of substitution begins with the common log-linear specification,

  $$\ln H_{it} = a_{it} + \eta \ln W_{it} + bX_{it} + \delta \ln \lambda_{it},$$

- In a stochastic environment, the consumer maximizes her/his expected utility and equation (3c) becomes

  $$\lambda_t - \beta (1 + r_t) E[\lambda_{t+1}] = 0$$

  - Letting $\phi_{it} = \ln \lambda_{it} - E_{t-1} \ln \lambda_{it}$ is the one-period-ahead forecast error in the logarithm of the marginal utility of wealth and $\xi_{it} = -E_{t-1} [\exp (\phi_{it})].$
• The first-differenced equation becomes
\[ \Delta \ln H_{it} = \Delta a_{it} + \eta \Delta \ln W_{it} + b \Delta X_{it} - \delta \cdot (r_{t-1} - \rho) + \delta \phi_{it} + \delta \xi_{it}, \]

• Thus, the change in labour supply consists of
  - a component due to changes in tastes (\( \Delta a_{it} \)) and characteristics (\( \Delta X_{it} \))
  - a component due to variation in wages (here not sure what type?),
  - a component due to the difference between the real interest rate and the rate of time preference,
  - a component due to any updating in the logarithm of the marginal utility of income. Since there is no closed form solution for the latter, it is often treated as a nuisance parameter.

• How well does the lifecycle model explain the lifecycle of hours?
  - Well, there is much growth in both hours and wages at the beginning of the lifecycle (between ages 20-30), but at midlife (ages 30-50) the quasi-absence of growth in hours does not match the continued growth in wages and towards retirement (ages 50-60) hours fall more than wages.
• Consider the following table (from Card, 1991) based on six cohorts of male:

<table>
<thead>
<tr>
<th>group</th>
<th>wage change</th>
<th>hours change</th>
<th>employment rate change</th>
</tr>
</thead>
<tbody>
<tr>
<td>aged 20–30</td>
<td>+40–45%</td>
<td>+45%</td>
<td>+10 % points</td>
</tr>
<tr>
<td>aged 30–40</td>
<td>+10–15%</td>
<td>0%</td>
<td>-5 % points</td>
</tr>
<tr>
<td>aged 50–60</td>
<td>-5%</td>
<td>-5–10%</td>
<td>-20 % points</td>
</tr>
<tr>
<td>0–8 years school (30–50)</td>
<td>+40%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>12 years school (30–50)</td>
<td>+20%</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>&gt;= 16 years school (30–50)</td>
<td>+10%</td>
<td>0%</td>
<td></td>
</tr>
</tbody>
</table>

• The table shows that the age profile of wage change and the profile of hours change are hardly parallel, in contrast to what the lifecycle labor supply model would predict.

• Also, looking across education groups, over the life-cycle the wage growth of college graduates in much higher than that of high school graduates but the hours profile of different education groups are very similar. A fairly elaborate set of taste parameters would be needed to explain these data.

• An important problem in evaluating the validity of the lifecycle model is that very little is know about the evolution of the marginal utility of income or the size of the wealth elasticity $\delta$. 
Figure 4
Lifecycle Profiles of Wages
Men Age 18-66 in CPS, 1977-89

Source: Card (1991)
Figure 5
Lifecycle Profiles of Hours
Men Age 18-66 in CPS, 1977-89

Source: Card (1991)
• Pistaferri (2003) improves on this point by using data on expectations of future wages collected in the 1989-93 panel section of the Bank of Italy SHIW to isolate anticipated and unanticipated wage changes and to construct a measure of wage risk.
  o He estimates an intertemporal substitution elasticity of around 0.7.

• Another problem is that individual hours are influenced directly by employer-specific demand conditions. Much recent research has focused on evaluating the intertemporal elasticity of labour supply in case where workers are free to choose their hours of work.
Additional readings:
