I. **Labour Supply**

1. *Neo-classical Labour Supply*

1. Basic Trends and Stylized Facts

2. Static Model
   a. *Decision of whether to work or not*: Extensive Margin
   b. *Decision of how many hours to work*: Intensive margin

3. Comparative Statics

4. Estimation of Labour Supply Functions and Elasticities
1.1 Basic Trends and Stylized Facts

Statistics Canada, November 2016

- **Population Aged 15 and Older**: 29.7m (10 provinces)
- **Labor Force** (working or actively seeking work): 19.5m (LFPR=65.6%)
- **Not in Labor Force** (students, retired persons, household workers, etc.): 10.2m
- **Employed** (working, at work or not): 18.2m (EPR=61.2%)
- **Unemployed** (Not employed, but looking for work): 1.33m (UR=6.8%)

Labour Force Concepts:

- Labour Force = Employed + Unemployed
  - LF = E + U
  - Size of LF does not tell us about “intensity” of work

- Labour Force Participation Rate
  - LFPR = LF/P
  - P = civilian adult population 16 years or older not in institutions
1.1 Basic Trends and Stylized Facts

Statistics Canada, November 2016

**Labour Force Concepts:**

- Employment: Population Ratio (percent of population that is employed)
  - EPR = E/P
  - Employed at work and not at work (e.g. maternity or sick leave) sometimes distinguished

- Unemployment Rate
  - UR = U/LF

**Labour Force Concepts:**

- E
  - Employed (working, at work or not)
    - 18.2m
    - (EPR=61.2%)

- U
  - Unemployed (Not employed, but looking for work)
    - 1.33m
    - (UR=6.8%)

**P**
- Population Aged 15 and Older
  - 29.7m
  - (10 provinces)

**LF**
- Labor Force (working or actively seeking work)
  - 19.5m
  - (LFPR=65.6%)

**NLF**
- Not in Labor Force (students, retired persons, household workers, etc.)
  - 10.2m
Labour force participants include employed (at work or on-leave) and unemployed individuals.
Figure 3: Trends in Female and Male Labor Force Participation Rates, 1947-2014
(age 16 and over)


Source: Blau and Kahn (2016)
U.S. Female LFP over the Life-Cycle

Women's Labour Force Participation by Synthetic Birth Cohort

Source: Fortin (2016), Canadian Public Use Labour Force Surveys, ages 25 to 64
Women's Labour Force Participation by Synthetic Birth Cohort

Source: Fortin (2016), Canadian Public Use Labour Force Surveys, ages 25 to 64
1.2. Static Model

- In neo-classical theory, the individuals’ decisions of whether or not participate in the labour market and of how many hours to work each week (and weeks per year) are modeled in static framework of consumption-leisure choice.

- From a policy point of view, this model has been very important to evaluate the potentially negative effects on labour supply of tax and transfer programs.

- From a labour econometrics viewpoint, the analysis will provide us with a classic example of correction for selection biases.
• The estimation of “the” elasticity of labour supply (%Δh/%Δw) has long been an important quest for labour econometricians
  - differences across studies in labour supply estimates may come not only from differences in sampling or data differences but also in the underlying modeling assumptions.

• The more modern approaches have emphasized clearly sources of identification coming from natural and quasi-natural experiments, as well as field experiments.

  - An identification strategy describes the manner in which a researcher uses observational data to approximate a real experiment, i.e. a randomized trial.
• The standard static, **within-period** labour supply model is an application of the consumer’s utility maximization problem over consumption and leisure.

• Assume that each individual has a quasi-concave utility function:

\[ U(C, L; X) \]  \hspace{1cm} (1)

where \( C, L, \) and \( X \) are within-period consumption, leisure hours and individual attributes.

• Then utility is assumed to be maximized subject to the budget constraint

\[ p \cdot C + w \cdot L = Y + w \cdot T \]  \hspace{1cm} (2)

where \( w \) is the hourly wage rate, \( Y \) is the non-labour income, and \( T = H + L \) is the total time available, where \( H \) is the number of hours of work.

- \( M = Y + w \cdot T \) is sometimes called **full-income**.
- \( H(L), C \) are **endogenous**,
- \( T, Y, w \) and the preference-shifters \( X \) are **exogenous** in this model.
• The consumer may choose his/her hours of work $H(L)$ by selecting across employers offering different packages of hours of work and wages.

• FOC:

• In the case of an **interior solution**, the individual choose to participate in the labour market $L^* < T$, the first-order conditions equates the marginal rate of substitution ($MRS_{CL}$) to the real wage rate

$$\frac{U_L(C,L;X)}{U_C(C,L;X)}|_{L^*} = \frac{w}{p}$$

(3)

• It is important to distinguish the characteristics of the interior solution for hours of work, $H > 0, (L < T)$ from the corner solution, $H = 0 (L = T)$. 
• In the case of the corner solution, \( L^* = T \),

\[
\frac{w}{p} \leq \frac{w_R}{p} = \left. \frac{U_L(C;L,X)}{U_C(C;L,X)} \right|_{L=T} \tag{4}
\]

where the reservation wage, \( w_R \), is equal to the negative of \( MRS_{HL} \) of working hours for commodities at \( H = 0(L = T) \).

• Solving the FOC (3) or (4) yield the Marshallian demand functions for goods and leisure

\[
C^* = C(w,Y;X) \quad \text{and} \quad L^* = L(w,Y;X)
\]

or equivalently the labour supply function \( H^* = H(w,Y;X) \) \( \tag{5} \)
1.3 Comparative Statics

- The **comparative statics** of the impact of changes in income, \( \frac{\partial H}{\partial Y} \), and wage rate, \( \frac{\partial H}{\partial w} \), of the labour supply functions \( H^* = H(w, Y; X) \) are best illustrated in a diagram of consumption-leisure choice.
• The general effects are the following.

• **An increase in non-labour income**: An increase in non-labour income will shift the budget line outwards without changing the slope of the line: this is a pure income effect. The effect on the optimal amount of leisure consumed or hours worked can then be summarized as:
  - L will rise and H will fall if leisure is a normal good
  - L will fall and H will rise if leisure is an inferior good.

  There are very strong reasons to believe that leisure is a normal good, e.g. those who win the lottery (a large increase in non-labour income) are more likely to work less afterwards than before and certainly not the other way round. Hence, it is likely that an increase in non-labour income will reduce hours of work.

• **An increase in the real hourly wage**: An increase in the real hourly wage will pivot the budget line about the point where L=T making at the line steeper: here there are two effects:
  - *An income effect*. Individuals are better-off than before so there is a positive income effect that, because leisure is a normal good, makes individuals work fewer hours than before.
- *A substitution effect.* An hour of work now buys more consumption than previously so that there is an incentive to increase consumption and reduce leisure. Hours of work will rise as a result.

- Hence, the impact of a change in the wage on hours of work is theoretically **ambiguous**. They may rise or fall.

- There is one exception to this: for non-participants there is no income effect as they have no labour income so nobody can be induced to reduce hours of work to zero as a result of an increase in the wage.
How can we quantify these effects?

- Recall that the Hicksian labour supply function is the solution to the expenditure minimization problem
  \[
  E(w, p, \bar{U}) = \min(p \cdot C - wH) \quad \text{subject to } U(C, H) \geq \bar{U}
  \]

  and correspond to the following uncompensated labour supply function

  \[
  H(w, p, \bar{Y}) = H^C(w, p, \bar{U}) \quad \text{where } \bar{Y} = E(w, p, \bar{U})
  \]

- Differentiating with respect to \( w \) and applying the chain-rule
  \[
  \frac{\partial H}{\partial w} + \frac{\partial H}{\partial Y} \frac{\partial E}{\partial w} = \frac{\partial H^C}{\partial w} \bigg|_{\bar{U}}
  \]
• With the application of Sheppard Lemma and because $H$ is a factor (reverses the sign), we get the Slutsky equation

$$\frac{\partial H}{\partial w} = \frac{\partial H^C}{\partial w} \left|_{\bar{U}} \right. + H \cdot \frac{\partial H}{\partial Y}$$

substitution income

where the overall effect of a wage change is decomposed into a substitution effect plus an income effect.

• Multiplying the entire equation (6) by $\frac{w}{H}$ and the last term (income effect) by $\frac{Y}{Y}$

$$\frac{\partial H}{\partial w} \cdot \frac{w}{H} = \frac{\partial H^C}{\partial w} \left|_{\bar{U}} \right. \cdot \frac{w}{H} + \frac{w \cdot H}{Y} \cdot \frac{\partial H}{\partial Y} \cdot \frac{Y}{H}$$

or in terms of elasticities

$$\varepsilon_{Hw} = \varepsilon_{Hw}^C + s \cdot \eta_{HY}$$

(7)
• Thus, there are three “sufficient statistics” of labour supply
  o the uncompensated wage elasticity: the % change in labour supply resulting from 1% change in
    the wage rate; sign is theoretically ambiguous as the positive substitution effect can sometimes be
    dominated by the negative income effect
    \[
    \varepsilon_{Hw} = \frac{\partial H}{\partial w} \cdot \frac{w}{H} > 0 (< 0) ?
    \]
  o the compensated wage elasticity: the % change in labour supply resulting from 1% change in the
    wage rate, after compensation for the wage change; sign is positive as it reflects a pure substitution
    effect
    \[
    \varepsilon_{Hw}^C = \frac{\partial H^C}{\partial w} \cdot \frac{w}{H} > 0
    \]
  o the income elasticity: the % change in labour supply resulting from 1% change in non-labor
    income; sign is expected to be negative
    \[
    \eta_{HY} = \frac{\partial H}{\partial Y} \cdot \frac{Y}{H} < 0
    \]
The simple consumption-leisure model can be extended (altered) to analyze labour supply under various conditions:

- introducing the fixed (money) cost of working or time cost (commuting) of working
- moonlighting (2nd job) and overtime pay
- should a firm offer flexible hours (part-time) or hire only full-time workers
- family labour supply (actually more than a simple extension)
1.4 Estimating Labour Supply Functions and Elasticities

• We can proceed by assuming that the individuals have a direct utility function of the form:

\[ U(C, L) = C^\alpha L^\beta, \]

\[ \frac{\beta}{\alpha} \frac{L}{C} = \frac{w}{p}. \]

The FOC will become \[ \frac{\beta}{\alpha} \frac{L}{C} = \frac{w}{p}. \] Combining that equation with the budget constraint and using the fact that \( H^* = 1 - L^* \), we obtain

\[ H^* = 1 - \gamma - \gamma (Y / w) \]

\[ C^* = (1 - \gamma) [(w + Y) / p], \] where \( \gamma \equiv \beta / (\alpha + \beta) \).

• For example, see Abbott and Ashenfelter (1976) for the results of the estimation of a Stone-Geary utility function.
• See Stern (1986) for the functional forms that can be linked to a utility function.
• Because of the identification problems above, many studies focus directly on the wage elasticity of the Marshallian supply function and on the associated utility-constant Hicksian wage elasticity.

• Suppose that we have individual data on hours of work $H_i$, the wage rate $w_i$, and on non-labor income $Y_i$, we could estimate a simple OLS regression

$$H_i = \beta_0 + \beta_1 \frac{w_i}{p_i} + \beta_2 \frac{Y_i}{p_i} + \varepsilon_i$$

(8)

• Then the estimated effects, (setting $p_i=1$ as numeraire)
  - $\hat{\beta}_1$ will be the overall (uncompensated) effect, $\frac{\partial H}{\partial w}$
  - $\hat{\beta}_2 \overline{H}$ will be the income effect, $-H \cdot \frac{\partial H}{\partial Y}$, (evaluated at mean hours)
  - $\hat{\beta}_1 - \hat{\beta}_2 \overline{H}$ will be the substitution effect, $\frac{\partial H}{\partial w} - H \cdot \frac{\partial H}{\partial Y}$, (evaluated at mean hours)
  - $\hat{\beta}_2 \overline{\frac{Y}{H}}$ will be the income elasticity of labour supply, $\frac{\partial H}{\partial Y} \cdot \frac{Y}{H}$, (evaluated at mean hours and income)
• We may control for individual attributes

\[
H_i = \beta_0 + \beta_1 \frac{w_i}{p_i} + \beta_2 \frac{Y_i}{p_i} + \beta_3 X_i + \varepsilon_i
\]  

(9)

and we usually assume that the distribution of the \( \varepsilon_i \) would be a normal distribution.

• There have been many studies estimating labour supply and income elasticities of labour supply, and there have been many meta-analysis of these studies (e.g. Hansson and Stuart (1985) Killingsworth (1983), Killingsworth and Heckman (1986), Pencavel (1986), and Evers, de Mooij and van Vuuren (2008).

• Evers, de Mooij and van Vuuren (2008) conclude that an uncompensated elasticity of 0.5 for women and 0.1 for men is a good reflection of what the literature reveals, although for the US it may be negative for men, due to the income effect.
  o For male workers, small wage effects
  o For female workers, much larger elasticities with larger variations across studies and declining over time as women have become more attached to the labour market
• Bargain, Orsini and Peichl (2012) perform an extensive cross-country study of 17 European countries plus the US,
  - Argue for genuine differences across countries not necessarily linked to differences in tax code
    - the extensive (participation) margin dominates the intensive (hours) margin
    - for singles, this leads to larger labor supply responses in low-income groups
    - income elasticities are extremely small everywhere.
- the results for cross-wage elasticities in couples are opposed between regions, consistent with complementarity in spouses’ leisure in the US versus substitution in spouses’ household production in Europe.

Basic readings: