The Hierarchy of Fregean Senses

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Abstract

The question whether Frege's theory of indirect reference enforces an infinite hierarchy of senses has been hotly debated in the secondary literature. Perhaps the most influential treatment of the issue is that of Burge (1979), who offers an argument for the hierarchy from rather minimal Fregean assumptions. I argue that this argument, endorsed by many, does not itself enforce an infinite hierarchy of senses. I conclude that whether or not the theory of indirect reference can avail itself of only finitely many senses is pending further theoretical development.

Consider the occurrence of 'Opus 132 is a masterpiece' in

(1) Bela believes Opus 132 is a masterpiece

and compare it with its occurrence in

(2) Igor believes Bela believes Opus 132 is a masterpiece.

Fregean doctrine tells us that in (1), 'Opus 132 is a masterpiece' refers to the ordinary sense of 'Opus 132 is a masterpiece', a mode of presentation of the truth-value of 'Opus 132 is a masterpiece' as it occurs unembedded, the thought that Opus 132 is a masterpiece. And the doctrine is often taken to suggest that in (2), 'Opus 132 is a masterpiece' refers to the sense of 'Opus 132 is a masterpiece' in (1), a mode of presentation of a mode of presentation of the truth-value of 'Opus 132 is a masterpiece' as it occurs unembedded, a mode of presentation of the thought that Opus 132 is a masterpiece. Question: Might the referent of 'Opus 132 is a masterpiece' in (2) be the same as the referent of 'Opus 132 is a masterpiece' in (1), namely, the thought that Opus 132 is a masterpiece?

There is an influential argument due to Burge (1979) that is meant to show that it can't be. For any expression α, let '${}^{\mathrm{s}}\alpha^{\mathrm{s}}$' refer to the sense of α and assume for *reductio* that ${}^{\mathrm{ss}}\alpha^{\mathrm{ss}}={}^{\mathrm{s}}\alpha^{\mathrm{s}}$. (We treat single quotes as corners and assume the sense-of relation to be a one-many relation to expressions.) Call this assumption SC for "Sense Collapse".

Also assume a Sense Functionality principle SF that for any expressions $\alpha, \beta_1, ..., \beta_n$ of suitable type, ${}^{\mathrm{s}}\alpha(\beta_1, ..., \beta_n)^{\mathrm{s}}={}^{\mathrm{s}}\alpha^{\mathrm{s}}({}^{\mathrm{s}}\beta_1{}^{\mathrm{s}},...{}^{\mathrm{s}}\beta_n{}^{\mathrm{s}})$. Finally, assume the extensionality principle that for any expressions α, α', if $\alpha=\alpha'$ then $...\alpha... \leftrightarrow ...\alpha'...$.

Now, the Fregean analysis of (1) is:

(3) Believes(Bela, ${}^{\mathrm{s}}$Opus 132 is a masterpiece${}^{\mathrm{s}}$).

(2), on the other hand, is analyzed as

(4) Believes(Igor, ${}^{\mathrm{s}}$Bela believes Opus 132 is a masterpiece${}^{\mathrm{s}}$),

from which by synonymy and extensionality we get

(5) Believes(Igor, ${}^{\mathrm{s}}$Believes(Bela, ${}^{\mathrm{s}}$Opus 132 is a masterpiece${}^{\mathrm{s}}$)${}^{\mathrm{s}}$),

from which by SF and extensionality we get

(6) Believes(Igor, ${}^{\mathrm{s}}$Believes${}^{\mathrm{s}}$(${}^{\mathrm{s}}$Bela${}^{\mathrm{s}}$, ${}^{\mathrm{ss}}$Opus 132 is a masterpiece${}^{\mathrm{ss}}$)),

from which by SC and extensionality we get

(7) Believes(Igor, ${}^{\mathrm{s}}$Believes${}^{\mathrm{s}}$(${}^{\mathrm{s}}$Bela${}^{\mathrm{s}}$, ${}^{\mathrm{s}}$Opus 132 is a masterpiece${}^{\mathrm{s}}$)).

But if (5) is correct, then (7) isn't. For call what ${}^{\mathrm{s}}$Believes${}^{\mathrm{s}}$ refers to '**Believes**' (a dyadic first-level concept), call what ${}^{\mathrm{s}}$Bela${}^{\mathrm{s}}$ refers to '**Bela**' (an individual), and call what ${}^{\mathrm{s}}$Opus 132 is a masterpiece${}^{\mathrm{s}}$ refers to '**truth-value**' (a truth-value). Let '**Believes**' express ${}^{\mathrm{s}}$Believes${}^{\mathrm{s}}$, '**Bela**' express ${}^{\mathrm{s}}$Bela${}^{\mathrm{s}}$, and '**truth-value**' express ${}^{\mathrm{s}}$Opus 132 is a masterpiece${}^{\mathrm{s}}$. Then,

(8) **Believes**(**Bela**, **truth-value**)

expresses ${}^{\mathrm{s}}$Believes${}^{\mathrm{s}}$(${}^{\mathrm{s}}$Bela${}^{\mathrm{s}}$, ${}^{\mathrm{s}}$Opus 132 is a masterpiece${}^{\mathrm{s}}$). The latter sense is the second relatum for Igor's belief according to (7). But then, for any δ for which 'δ=**truth-value**' is true we get

(9) **Believes**(**Bela**, δ)

by extensionality. Such a consequence does not follow from (3), which expresses ${}^{\mathrm{s}}$Believes(Bela, ${}^{\mathrm{s}}$Opus 132 is a masterpiece${}^{\mathrm{s}}$)${}^{\mathrm{s}}$, which is the second relatum of Igor's belief according to (5). So (7) and (5) are distinct. And so, assuming the correctness of (5), SC should be given up. SC says that for any α, ${}^{\mathrm{ss}}\alpha^{\mathrm{ss}}={}^{\mathrm{s}}\alpha^{\mathrm{s}}$. The forgoing shows that for some α (i.e. 'Opus 132 is a masterpiece'), ${}^{\mathrm{ss}}\alpha^{\mathrm{ss}}\neq{}^{\mathrm{s}}\alpha^{\mathrm{s}}$. And so, SC is false.

Let us assume that Burge's argument supports the generalization that for any α, ${}^{\mathrm{ss}}\alpha^{\mathrm{ss}}\neq{}^{\mathrm{s}}\alpha^{\mathrm{s}}$. Question: Does this argument force upon us a hierarchy of senses? Burge (1979) concludes his presentation of the argument as follows (passage adjusted to the terminology and numbering of the reconstruction above and supplemented accordingly):

> The argument shows that on these assumptions 'Opus 132 is a masterpiece' in [(2)] cannot be represented by a term ['$^{\mathrm{s}}$Opus 132 is a masterpiece$^{\mathrm{s}}$'] denoting the [thought] that Opus 132 is a masterpiece. It is prima facie plausible to assume with Frege that the expression as it occurs in [(2)] should be represented by a term ['$^{\mathrm{ss}}$Opus 132 is a masterpiece$^{\mathrm{ss}}$'] denoting the sense of the expression ['Opus 132 is a masterpiece'] that represents ['Opus 132 is a masterpiece'] as it occurs in unembedded belief contexts [e.g. (1)]. Given this assumption, the argument can be replicated to show that the sentential expression ['Opus 132 is a masterpiece'] as it occurs in doubly embedded oblique contexts must be represented by yet another term [i.e. a term other than '$^{\mathrm{ss}}$Opus 132 is a masterpiece$^{\mathrm{ss}}$'] – and so on. (272)

It is often assumed that Burge's argument establishes a hierarchy of senses. Indeed, Burge writes: "In summary, the argument for a hierarchy... seems very powerful" (274). The assumption makes an appearance in a recent discussion by Salmon (2005: 1100 n. 31). And in an even more recent discussion Kripke writes: "I agree with Burge... that the hierarchy is an actual consequence of Frege's theory" (2008: 184 n. 9). This, I will now argue, is a mistake.

Call an expression without s-quote marks an expression of level 0. Independently of Burge's argument it seems plausible on Fregean grounds to suppose that for any expression α of level 0, $\alpha \neq {}^{\mathrm{s}}\alpha^{\mathrm{s}}$. (Think here of the example of Mont Blanc in the famous exchange between Frege and Russell.) Similar considerations could extend to $\alpha \neq {}^{\mathrm{s}(\times n)}\alpha^{\mathrm{s}(\times n)}$ for any $n \geq 1$, where 's$(\times n)$' designates n occurrences of 's'. And we assume that Burge's argument can be given for any $n \geq 1$ to the effect that ${}^{\mathrm{s}(\times n)}\alpha^{\mathrm{s}(\times n)} \neq {}^{\mathrm{s}(\times n+1)}\alpha^{\mathrm{s}(\times n+1)}$. But such an argument would still not launch a hierarchy per α.

For a hierarchy per α we would need to show that for any $n \geq 1$, the following conjunction is true:

(10) $\bigwedge_{1 \leq i \leq n} {}^{\mathrm{s}(\times i)}\alpha^{\mathrm{s}(\times i)} \neq {}^{\mathrm{s}(\times n+1)}\alpha^{\mathrm{s}(\times n+1)}$.

Now consider the following instance of (10) for $n = 2$:

(11) ${}^{\mathrm{s}}\alpha^{\mathrm{s}} \neq {}^{\mathrm{sss}}\alpha^{\mathrm{sss}} \wedge {}^{\mathrm{ss}}\alpha^{\mathrm{ss}} \neq {}^{\mathrm{sss}}\alpha^{\mathrm{sss}}$.

Burge's argument establishes, we suppose:

(12) ${}^{\mathrm{s}}\alpha^{\mathrm{s}} \neq {}^{\mathrm{ss}}\alpha^{\mathrm{ss}} \wedge {}^{\mathrm{ss}}\alpha^{\mathrm{ss}} \neq {}^{\mathrm{sss}}\alpha^{\mathrm{sss}}$.

Mont Blac-type considerations establish:

(13) $\alpha \neq {}^{\mathrm{s}}\alpha^{\mathrm{s}} \wedge \alpha \neq {}^{\mathrm{ss}}\alpha^{\mathrm{ss}} \wedge \alpha \neq {}^{\mathrm{sss}}\alpha^{\mathrm{sss}}$.

But we can see that the theory $\{(12), (13), \neg(11)\}$ is consistent by constructing a simple model $m = \langle\{o, o', o''\}, \sigma\rangle$, where $\{o, o', o''\}$ is the domain and σ is the interpretation function:

$$\begin{aligned}
&\sigma(\alpha) = o\\
&\sigma({}^{\mathrm{s}}\alpha^{\mathrm{s}}) = o'\\
&\sigma({}^{\mathrm{ss}}\alpha^{\mathrm{ss}}) = o''\\
&\sigma({}^{\mathrm{sss}}\alpha^{\mathrm{sss}}) = o'.
\end{aligned}$$

It is easily verified that $m \models \{(12), (13), \neg(11)\}$.

So now consider the following model $m^* = \langle\{o, o', o''\}, \sigma^*\rangle$ interpreting every expression of the form ${}^{\mathrm{s}\ldots\mathrm{s}}\alpha^{\mathrm{s}\ldots\mathrm{s}}$, where $n \geq 1$:

$$\begin{aligned}
&\sigma^*(\alpha) = o\\
&\sigma^*({}^{\mathrm{s}(\times 2n-1)}\alpha^{\mathrm{s}(\times 2n-1)}) = o'\\
&\sigma^*({}^{\mathrm{s}(\times 2n)}\alpha^{\mathrm{s}(\times 2n)}) = o''.
\end{aligned}$$

Let the Burge set B be $\{{}^{\mathrm{s}(\times n)}\alpha^{\mathrm{s}(\times n)} \neq {}^{\mathrm{s}(\times n+1)}\alpha^{\mathrm{s}(\times n+1)} \mid n \geq 1\}$ and let the Mont Blanc set M be $\{\alpha \neq {}^{\mathrm{s}(\times n)}\alpha^{\mathrm{s}(\times n)} \mid n \geq 1\}$. Clearly for any $n \geq 2$, $m^* \models \neg(10)$. And for any $n \geq 1$, $m^* \models {}^{\mathrm{s}(\times n)}\alpha^{\mathrm{s}(\times n)} \neq {}^{\mathrm{s}(\times n+1)}\alpha^{\mathrm{s}(\times n+1)}$ and $m^* \models \alpha \neq {}^{\mathrm{s}(\times n)}\alpha^{\mathrm{s}(\times n)}$. So $m^* \models \{B, M\}$. I conclude that Burge's argument does not enforce a hierarchy of senses per α after all.

In light of the foregoing, consider the following triply embedded occurrence of 'Opus 132 is a masterpiece':

(14) Zoltan believes Igor believes Bela believes Opus 132 is a masterpiece.

Fregean doctrine analyzes it as

(15) Believes(Zoltan, ${}^{\mathrm{s}}$Igor believes Bela believes Opus 132 is a masterpiece${}^{\mathrm{s}}$),

which by synonymy and extensionality gets us

(16) Believes(Zoltan, ${}^{\mathrm{s}}$Believes(Igor, ${}^{\mathrm{s}}$Believes(Bela, ${}^{\mathrm{s}}$Opus 132 is a masterpiece${}^{\mathrm{s}}$)${}^{\mathrm{s}}$)${}^{\mathrm{s}}$),

which by repeated applications of SF and extensionality gets us

(17) Believes(Zoltan, ${}^{\mathrm{s}}$Believes${}^{\mathrm{s}}$(${}^{\mathrm{s}}$Igor${}^{\mathrm{s}}$, ${}^{\mathrm{ss}}$Believes${}^{\mathrm{ss}}$(${}^{\mathrm{ss}}$Bela${}^{\mathrm{ss}}$, ${}^{\mathrm{sss}}$Opus 132 is a masterpiece${}^{\mathrm{sss}}$))).

Now, according to interpretation σ^* above, for any α, $\sigma^*({}^{\mathrm{sss}}\alpha^{\mathrm{sss}}) = \sigma^*({}^{\mathrm{s}}\alpha^{\mathrm{s}})$. Let us consider such an interpretation for 'Opus 132 is a masterpiece' and assume, accordingly, that

(18) ${}^{\text{sss}}$Opus 132 is a masterpiecesss${}^{\text{sss}}$ = ${}^{\text{s}}$Opus 132 is a masterpieces${}^{\text{s}}$.

From (17), (18), and extensionality, we get

(19) Believes(Zoltan, ${}^{\text{s}}$Believes${}^{\text{s}}$(${}^{\text{s}}$Igor${}^{\text{s}}$, ${}^{\text{ss}}$Believes${}^{\text{ss}}$(${}^{\text{ss}}$Bela${}^{\text{ss}}$, ${}^{\text{s}}$Opus 132 is a masterpiece${}^{\text{s}}$))).

Can we say here, as we did in the case of Burge's original argument, that if (16) is correct, then (19) isn't? Call what ${}^{\text{ss}}$Believes${}^{\text{ss}}$ refers to '**<u>Believes</u>**' (a mode of presentation of the relational concept **Believes**), call what ${}^{\text{ss}}$Bela${}^{\text{ss}}$ refers to '**<u>Bela</u>**' (a mode of presentation of the individual **Bela**), and call what ${}^{\text{s}}$Opus 132 is a masterpiece${}^{\text{s}}$ refers to '**truth-value**' as before. Let '**<u>Believes</u>**' express ${}^{\text{ss}}$believes${}^{\text{ss}}$, '**<u>Bela</u>**' express ${}^{\text{ss}}$Bela${}^{\text{ss}}$, and '**truth-value**' express ${}^{\text{s}}$Opus 132 is a masterpiece${}^{\text{s}}$ as before. Then

(20) **<u>Believes</u>**(**<u>Bela</u>**, **truth-value**)

expresses ${}^{\text{ss}}$Believes${}^{\text{ss}}$(${}^{\text{ss}}$Bela${}^{\text{ss}}$, ${}^{\text{s}}$Opus 132 is a masterpiece${}^{\text{s}}$)), which is the second relatum for Igor's belief according to Zoltan's belief according to analysis (19). Now, it does follow from (20) by extensionality that

(21) **<u>Believes</u>**(**<u>Bela</u>**, δ)

for any δ that agrees in truth-value with 'Opus 132 is a masterpiece'. But notice that (21) does not express the thought that Bela believes δ. With (21) we are in pre-theoretically unfamiliar territory. What (21) expresses is a complex consisting of a mode of presentation of a mode of presentation of a relational concept, a mode of presentation of a mode of presentation of an individual, and a thought that agrees in truth-value with the thought that Opus 132 is a masterpiece. To simply insist that such a consequence does not follow from whatever would express ${}^{\text{ss}}$Believes${}^{\text{ss}}$(${}^{\text{ss}}$Bela${}^{\text{ss}}$, ${}^{\text{sss}}$Opus 132 is a masterpiece${}^{\text{sss}}$)), the second relatum for Igor's belief according to Zoltan's belief according to (17), and so to conclude that the latter complex is distinct from the second relatum of Igor's belief according to Zoltan's belief according to (19), is to beg the issue at hand.

There might well be other theoretical reasons to resist (18) and more generally reasons to resist the collapse of third-level senses to first-level senses, fourth-level senses to second-level senses, fifth-level senses to first-level senses, sixth-level senses to second-level senses, and so on. For example, under the present scheme the usual Fregean assumption whereby for each $n > 1$, ${}^{\text{s}(\times n)}\alpha^{\text{s}(\times n)}$ refers to ${}^{\text{s}(\times n-1)}\alpha^{\text{s}(\times n-1)}$, entails that for any even number k, ${}^{\text{s}(\times k)}\alpha^{\text{s}(\times k)}$ refers to *and is referred by* ${}^{\text{s}(\times k-1)}\alpha^{\text{s}(\times k-1)}$, whereas for any odd number m, ${}^{\text{s}(\times m)}\alpha^{\text{s}(\times m)}$ will refer to *both* ${}^{\text{s}(\times m+1)}\alpha^{\text{s}(\times m+1)}$ *and* a truth-value (however this is to be modeled, ultimately). The present point isn't that we in fact need only two levels of sense. That is a question pending further theoretical development. The present point is that the familiar argument for the hierarchy fails to establish an infinite hierarchy of senses.

Whether Frege's theory of indirect reference is committed to an infinite hierarchy of senses is a question that is usually handled under the tacit assumption that the theory's pronouncements are to be taken in a realistic spirit. This is certainly Frege's original approach to the topic. What the assumption amounts to, at the very least, is the idea that the theory of indirect reference is to reveal the very meaning of attitude reports (as they are "in the wild", so to speak). By extension, the theory is also assumed to reveal the nature of the reported cognitive facts themselves, the facts of belief for example. But such large methodological assumptions are not compulsory. When thinking of the theory in a more instrumentalist spirit we can ask what explanatory work the theory can be expected to achieve in the first place and how much of the pre-theoretical situation surrounding the meaning of attitude reports should be captured. Under such instrumentalism, it is not obvious that finitely many levels of sense should not suffice for the theory's explanatory success. Perhaps, for example, we need as many levels of sense per α as there are humanly parsable embeddings of α and no more. The foregoing discussion aimed to illustrate that there is no principled barrier to such finitism.[1]

References

Burge, T. (1979) 'Frege and the Hierarchy', *Synthese* 40: 265-81.

Kripke, S. (2008) 'Frege's Theory of Sense and Reference: Some Exegetical Notes', *Theoria* 74: 181-218.

Salmon, N. (2005) 'On Designating', *Mind* 114: 1069-1133.

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Simchen, O. (unpublished) 'Realism and Instrumentalism in Metaphysical Explanation' (http://faculty.arts.ubc.ca/osimchen/drafts/RIME.pdf).

[1] For further discussion of such choice points between realist and instrumentalist attitude towards theoretical representations in various areas of philosophical pursuit, see Simchen (2019) and (unpublished).