

# A Model of Evidence Production and Optimal Standard of Proof and Penalty in Criminal Trials\*

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## Abstract

A model of evidence production by the litigating parties is developed in a criminal context. The defendant can be of two types, innocent or guilty. The defendant knows her type, but neither the court nor the prosecutor possesses this information. The court convicts the defendant if and only if its posterior probability of the guilt of the defendant is greater than a certain threshold value, *the standard of proof*. The posterior depends on the evidence presented by the parties to the court. Evidence production is a stochastic process which is costly. This model of evidence production is then used to analyze optimal choice of standard of proof and penalties for criminal cases in the second part of the paper. As one would expect, it can be shown that the optimal standard of proof is increasing in the cost of convicting an innocent defendant and decreasing in the cost of acquitting a guilty defendant. More surprisingly, it is also shown that an increase in the penalty imposed on the defendant in the case of her conviction may increase the probabilities of *both* false conviction *and* false acquittal.

## 1 Introduction

This paper consists of two parts. In the first part, a model of costly evidence production by litigating parties is developed for criminal (and some civil) cases. This model is used in the second part to analyze the optimal choice of standard of proof and penalties.

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Much of the law and economics literature has focused on pre-trial settlement issues.<sup>1</sup> The actual trial process itself has received little attention. This cannot be justified by the fact that most cases settle prior to the trial stage, since what would happen in the court will affect the settlement negotiations. Parties' assessment of their likelihood of prevailing in the trial will serve as "threat points" in the pre-trial bargaining.

The probability of, say, the defendant prevailing in the trial is not completely determined by the merits of the case in question; it also depends on the litigation expenditures of parties in dispute.<sup>2</sup> The first part of this study can be seen as an attempt to endogenize expenditure decisions of the litigants in criminal cases.<sup>3,4</sup> Moreover, unlike most existing models of endogenous litigation expenditures, it does not start with exogenous, "reduced form" formulae relating the decision of the court to parties' expenditures, rather it develops a primitive model of evidence gathering by the parties involved in the trial and information processing by the court.

In the model, the defendant can be of two types, innocent or guilty. The defendant knows her type, but the court and the prosecutor do not. There are two types of evidence; one of them is favorable to the defendant and the other is favorable to the prosecution. The court convicts the defendant if and only if its posterior probability of guilt of the defendant is greater than a certain threshold, *the standard of proof*. The posterior depends on the evidence presented by the parties to the court. Evidence production is a costly stochastic process; for both parties, the probability of finding a piece of evidence favorable to the defendant (respectively, the prosecutor) is higher when the defendant is innocent (respectively, guilty). When worked out, these will result in "sensible" reduced form functions linking the court's probability of conviction to the standard of proof, expenditures of the litigating parties, and, of course, the culpability of the defendant: The probability of conviction is decreasing in the standard of proof and the defendant's expenditures, and increasing in the prosecutor's expenditures. Also, guilty defendants face a higher probability of conviction than the innocent defendants when they spend the same amount.

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<sup>1</sup>See, for example, Bebchuk (1984), P'ng (1983), Reinganum and Wilde (1986), Shavell (1982) and Spier (1992).

<sup>2</sup>There are previous studies that analyzed endogenous litigation expenditures by the parties in civil cases. See, for example, Braeutigam, Owen and Panzar (1984) and Katz (1988). Symmetric information between the plaintiff and the defendant about the liability issue is a central feature of these models. It is reasonable to think that in most criminal (and in some civil) cases, the defendant is better informed about her liability than the prosecutor (or the plaintiff).

<sup>3</sup>The analysis is applicable to some civil cases as well. See the discussion in the last section.

<sup>4</sup>See Rubinfeld and Sappington (1987) and Miceli (1990) for models of one-sided litigation expenditures in a criminal context. These papers are discussed further below.

There is some previous work on the production of evidence by litigating parties.<sup>5</sup> The model of Froeb and Kobayashi (1996) is the closest one to the model developed in the first part of this study. However, the focus and the goal of their study is very different than this one. They model civil cases for which there is comparative negligence tort standard and analyze the ability of the courts to evaluate selectively produced evidence, whereas in this paper, a model of evidence production is developed for criminal cases and used to analyze the optimal standard of proof and penalty. There are important differences in the models used as well. In their model, liability of the defendant is represented by a continuous variable,  $p \in [0, 1]$ . However, in most criminal (and some civil) cases  $p$  is a discrete variable which takes only two values; the defendant is either innocent or guilty (either exercised due care or did not). The court's decision is also modelled as a continuous variable (the court's posterior of  $p$ ). In criminal cases (and civil cases with contributory negligence standard) the court's decision is dichotomous: the defendant is either convicted or acquitted (found either liable or not). The discrete nature of the verdict is the *raison d'être* of the standard of proof and allows it to be incorporated in the analysis. Another important difference is that in their model parties have symmetric and perfect information about the liability of the defendant, they both know  $p$ .

In the second part of the paper, a restricted version of the model of evidence production developed in the first part is used to analyze the optimal choice of standard of proof and penalties for criminal cases. The basic analysis closely follows the seminal work of Rubinfeld and Sappington (1987) on this topic. The court is assumed to minimize the sum of the following: (i) the expected cost of *false conviction* (convicting an innocent defendant; also known as *type I error* in the literature), (ii) the expected cost of *false acquittal* (acquitting a guilty defendant; also known as *type II error*), and (iii) the total expected expenditures by the defendant and the prosecutor. There are two instruments available to the court: The standard of proof and the penalty which will be imposed on the defendant if she is convicted. Given the standard of proof and the penalty, expenditures of the prosecutor and the defendant are determined endogenously.

It is shown that, for a given level of standard of proof, a change in the penalty has ambiguous consequences in terms of likelihoods of committing legal errors: An increase in the penalty may result in more or less false convictions or false acquittals. Moreover, it is *possible* that a penalty increase will result in higher probabilities of *both* false conviction *and* false acquittal. For any given standard of proof, how will the optimal amount of penalty respond to changes in the costs of legal errors? (Note that finding the optimal penalty involves considering the expenditures of the litigating

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<sup>5</sup>See, e.g., Froeb and Kobayashi (1996), Hay and Spier (1997), Milgrom and Roberts (1986), and Sanchirico (1997a and b), and Shin (1994a and b).

parties as well as likelihoods of committing legal errors.) Again, the answer is indeterminate.

As one would expect, for any given amount of penalty, the probability of a false acquittal increases and the probability of a false conviction decreases when there is an increase in the standard of proof. Furthermore, the optimal standard of proof is increasing in the cost of a false conviction and decreasing in the cost of a false acquittal.

The court chooses the standard of proof *and* the penalty to minimize the sum of expected legal costs and expenditures of the litigating parties. After characterizing the optimal standard of proof and penalty, it is shown that, the optimal standard of proof increases as the cost of false conviction increases, and decreases as the cost of a false acquittal increases. This result is consistent with the fact that in criminal cases the prosecution has the burden of proof beyond a reasonable doubt, whereas in most civil cases only proof by a fair preponderance of evidence is required. The consistency follows from the common viewpoint that in criminal cases the cost of a false conviction is much higher than the cost of a false acquittal, whereas in most civil cases they are close to each other.<sup>6</sup> The direction of change in the optimal amount of penalty, however, is indeterminate as the costs of legal errors change.

Together these results *suggest* the importance and the effectiveness of the standard of proof, relative to the penalty imposed on the convicted defendants, as an instrument of efficient judicial decision making. Again, the distinction between criminal and civil cases provides support for this claim. For the purposes of this paper, criminal and civil cases differ mainly in the relative magnitudes of social costs of legal errors, as discussed above. When one considers the standard of proofs and penalties across different classes of cases, criminal ones, generally, require higher standard of proofs, whereas there is no clear ranking in terms of penalties: A very large penalty may be imposed on an individual, or a firm, in a civil trial with a fair preponderance of evidence, whereas to convict a defendant in a criminal trial, the prosecution has the burden of proof beyond a reasonable doubt even when the possible penalty is minor.

Many lawmakers and legal scholars are concerned about setting penalties optimally for given crimes. The results and discussions above demonstrate the necessity of being very careful in this course: A policy change that has positive *direct* effects may have unintended detrimental consequences because of its *indirect* effects via the induced change in equilibrium behavior of the parties involved.<sup>7</sup> Another point, while evident given the structure of the model, may be worth mention-

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<sup>6</sup>See, for example, Posner (1986, pp. 518-21).

<sup>7</sup>Lewis and Poitevin (1997), in their study of information disclosure in regulatory hearings, forcefully make the case that one has to consider the full equilibrium effects of policy changes.

ing. The optimal penalty cannot be set independently of the optimal standard of proof. These two instruments are closely intertwined through the strategic evidence production decisions of the litigating parties.

There are two closely related previous articles on the issue of the optimal choice of the standard of proof and the penalty: Rubinfeld and Sappington (1987) and Miceli (1990). The former considers litigation expenditures only by the defendant, and the latter only by the prosecutor. However, excluding one of the parties from the analysis is excluding the effects of the *interaction* between expenditure decisions of the prosecutor and the defendant. For example, in Rubinfeld and Sappington (1987) an increase in the penalty causes increases in both type of defendants' expenditures. When the prosecution is also taken into account, this is not always true: An increase in the penalty may increase the prosecutor's expenditures, and this diminishes the incentives of the defendant to expend resources when the return to the defendant's expenditures are decreasing in the prosecutor's expenditures. Another departure from the results of Rubinfeld and Sappington (1987) is the direction of change in the optimal penalty as costs of legal errors change, keeping the standard of proof constant. In their model the optimal penalty increases as cost of false conviction (respectively, false acquittal) increases (respectively, decreases). The reason is that an increase in the penalty raises expenditures of both types of defendants, thereby decreasing (respectively, increasing) the probability of a false conviction (respectively, false acquittal). In the more general model, an increase in the penalty may increase the prosecutor's expenditures rendering the effect on probabilities of legal errors ambiguous. Actually, as discussed above, an increase (respectively, decrease) in the cost of false conviction (respectively, false acquittal) may *decrease* the optimal penalty, for a given level of standard of proof.

The rest of the paper is organized as follows: In Section 2 the model of evidence production is developed. The optimal choice of the standard of proof and the penalty is studied in Section 3. Section 4 has some concluding remarks.

## 2 A Model of Evidence Production

Suppose that a crime was committed, and that there is a suspect for it. The suspect (defendant, hereafter) can be of two types, guilty ( $G$ ) or innocent ( $I$ ). The defendant knows whether she is guilty or not, but neither the court nor the prosecutor can observe her type. Plea bargaining is ignored, therefore the defendant has to be tried. The ex ante probability of the defendant being type  $G$  is  $\alpha$ . There are two types of evidence (signals),  $i$  and  $g$ . Searching for and providing evidence

requires effort (expenditure) and hence is costly. Each investigation by either of the parties will independently produce  $i$  with probability  $p$  and  $g$  with probability  $1 - p$  when the defendant is innocent, where  $p \in (\frac{1}{2}, 1)$ . When the defendant is guilty each investigation will produce  $i$  with probability  $1 - p$ , and  $g$  with probability  $p$ .<sup>8</sup> So, when the defendant is innocent any investigation is more likely to lead to  $i$  than to  $g$ , and conversely. Let  $q$  be the standard of proof used by the court: The defendant will be convicted if and only if the posterior probability of the defendant's guilt is greater than or equal to  $q$ . It is assumed that the court does not observe expenditures of the parties. All of these and the way the court's posterior is formed are common knowledge.

Let  $m$  be the number of  $g$  signals and  $n$  the number of  $i$  signals presented to the court. The court's posterior of the guilt of the defendant will depend on  $m$ ,  $n$ , and  $\alpha$ . Denote the posterior by  $\rho_\alpha(m, n)$ . The court will convict the defendant if and only if  $\rho_\alpha(m, n) \geq q$ .

The court is assumed to be *naive, yet Bayesian*.<sup>9</sup> The court is naive, in the sense that it ignores the fact that the evidence is produced by interested parties; it is Bayesian because, not surprisingly, it applies Bayes' Rule to update the probability of the defendant's guilt. The probability of obtaining  $m$   $g$  signals out of a total of  $m + n$  signals is  $\binom{m+n}{m} p^m (1 - p)^n$  when the defendant is guilty, and  $\binom{m+n}{m} (1 - p)^m p^n$  when she is innocent. Hence,

$$\rho_\alpha(m, n) = \frac{\alpha \binom{m+n}{m} p^m (1 - p)^n}{\alpha \binom{m+n}{m} p^m (1 - p)^n + (1 - \alpha) \binom{m+n}{m} (1 - p)^m p^n} = \frac{1}{1 + \frac{1 - \alpha}{\alpha} \left(\frac{1 - p}{p}\right)^{m - n}}.$$

The way the court is assumed to update its prior probability of the defendant's guilt is in the spirit of the rule that the court should not use circumstantial evidence. The verdict should be based on the evidence presented to the court. One implication of this, in terms of the model studied in this paper, is that the court will utilize all the signals presented to it irrespective of whether the prosecutor or the defendant presented them. Therefore, a certain amount of "naivete" on the part of the court is required. Another implication is that the court does not use the parties' expenditures to reach its decision even if these were observable.<sup>10</sup>

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<sup>8</sup>Equality of probabilities of getting  $g$  when the defendant is type  $G$  and getting  $i$  when she is type  $I$  is assumed only for expositional simplicity.

<sup>9</sup>The phrase is due to Froeb and Kobayashi (1996).

<sup>10</sup>Notice that the posterior of the guilt of the defendant depends on the prior, which may seem to be in contradiction with the requirement that the court should base its decision only on the evidence presented. However, there is a qualitative difference between the prior and, say, the money that the defendant spends. The prior represents how the court views the outside world; it is best thought as the proportion of the guilty defendants in the population of suspects who are brought to trial. The readers who are still uncomfortable with the existence of the prior in the formula of the posterior might want to take it to be  $\frac{1}{2}$ : nothing in what follows will be affected.

The defendant knows whether she is guilty or not. Under the “no-lying” (or, verifiable reports) assumption (so that parties cannot *lie*, but are allowed to withhold any information or evidence they may have) the “rationality” of the court implies the well-known “unraveling result” (e.g., Milgrom and Roberts (1986)): The court will simply ask the defendant whether she is guilty or not, and the guilty ones will “babble”. Therefore, the court is always able to *infer* the culpability of the defendant. However, this “unraveling result” should be taken with a grain of salt in judicial settings, since the arguments used to obtain it do not exactly match the characteristics of the trial process. For example, in the United States, the court is supposed to make no inferences when a defendant uses her right against self-incrimination, which is protected by the Fifth Amendment to the constitution. Similarly, in many systems derived from British Common Law, the defendant cannot be compelled to testify against herself. In this paper the “no-lying” assumption is kept: the prosecutor and the defendant can withhold evidence (i.e., signals) that are unfavorable, but cannot forge favorable evidence. However, the “fully rational” court is replaced by the “naive, yet Bayesian” court that bases its decision on the evidence presented by the litigating parties.

Notice that  $\rho_\alpha(m, n)$ , the posterior of the defendant’s guilt, is an increasing function of  $\alpha$ , the prior. Also,  $\rho_\alpha(m, n) \rightarrow 0$  as  $\alpha \rightarrow 0$ , and  $\rho_\alpha(m, n) \rightarrow 1$  as  $\alpha \rightarrow 1$ . From here onwards, the dependence of  $\rho_\alpha(m, n)$  on  $\alpha$  is suppressed in the notation, since it will not be utilized. Let  $\rho(m - n) \equiv \rho(m, n)$ , and notice that it is an increasing function of  $m - n$ ; the posterior of the guilt of the defendant is increasing in the number of  $g$  signals in excess of  $i$  signals.<sup>11</sup> Moreover  $\rho(0) = \alpha$ ; when the number of  $g$  and  $i$  signals are equal, the posterior is equal to the prior; also  $\rho(m - n) \rightarrow 1$  as  $m - n \rightarrow \infty$ , and  $\rho(m - n) \rightarrow 0$  as  $m - n \rightarrow -\infty$ .

Let  $e_d$  (respectively,  $e_p$ ) be the expenditure of the defendant (respectively, the prosecutor). The cost of acquiring a signal is normalized to 1, so  $e_d$  (respectively,  $e_p$ ) will also represent the number of signals acquired by the defendant (respectively, the prosecutor).<sup>12</sup> It is assumed that each party

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<sup>11</sup>The court is similar to Sanchirico’s (1997b) court in that it announces a decision rule as a function of evidence presented: If  $m - n$ , the number of  $g$  signals in excess of  $i$  signals, is greater than a certain number (i.e.,  $\rho(m - n) \geq q$ ), then the defendant is convicted, otherwise she is acquitted. In his model the court can choose any decision rule, whereas here the court is restricted to *monotonic* rules, which is quite natural: If the defendant is convicted when  $m - n$  is equal to a certain number, then she should be convicted when  $m - n$  is greater than that number as well.

<sup>12</sup>It is a shortcoming of the model that parties commit to the number of signals they will acquire. This is not a problem for the defendant since she does not learn anything from her signals. The prosecutor, however, may, and generally will, change her beliefs about the defendant’s culpability as she collects more evidence. See Celik (2001) for a model of evidence production in which the expert searches sequentially for evidence to present to the decisionmaker. This issue of commitment vs. sequential search is immaterial for the study of optimal standard of proof and penalty in Section 3, since both parties search for one piece of evidence in that model.

presents only evidence that is favorable to her cause, i.e., the defendant only presents  $i$  and the prosecutor  $g$ . This amounts to assuming that there is no full disclosure.<sup>13</sup> If the defendant is innocent and spends  $e_d$ , the probability that she will get  $n$  signals of type  $i$  is:

$$\Pr(\#i = n \mid e_d, I) = \binom{e_d}{n} p^n (1-p)^{e_d-n}.$$

When the defendant is innocent, the probability that the prosecutor gets  $m$  signals of type  $g$  if she spends  $e_p$  is:

$$\Pr(\#g = m \mid e_p, I) = \binom{e_p}{m} p^{e_p-m} (1-p)^m.$$

Similarly, when the defendant is guilty:

$$\Pr(\#i = n \mid e_d, G) = \binom{e_d}{n} p^{e_d-n} (1-p)^n,$$

$$\Pr(\#g = m \mid e_p, G) = \binom{e_p}{m} p^m (1-p)^{e_p-m}.$$

Let  $F^I(q, e_d, e_p)$  be the probability of the conviction of the defendant when she is innocent, spends  $e_d$ , the expenditure of the prosecutor is  $e_p$ , and the standard of proof is  $q$ . Let  $F^G(q, e_d, e_p)$  be the probability of conviction when the defendant is guilty.

$$F^J(q, e_d, e_p) \equiv \Pr(\rho(m-n) \geq q \mid J), \quad J = I, G.$$

Since  $\rho(m-n)$  is increasing in  $m-n$ ,  $\rho(m-n) \geq q \Leftrightarrow m-n \geq t$ , where  $\rho(t) = q$  (ignoring integer problems). Thus,

$$F^J(q, e_d, e_p) \equiv \Pr(m-n \geq t \mid J), \quad J = I, G.$$

One would like to write  $F^J(\cdot)$  as a function of  $e_d, e_p$ , and  $q$ , instead of  $m, n$ , and  $t$ , because the parties' decision variables are  $e_d, e_p$  and  $q$ , not  $m, n$  or  $t$ . If the prosecutor acquires  $m$   $g$  signals,

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<sup>13</sup>The behavior of the parties actually follow from their preferences discussed below; i.e., the defendant is assumed to minimize the sum of the expected penalty and the litigation expenditures and the prosecutor is assumed to maximize the expected punishment to the defendant net of the litigation expenditures. The laws governing discovery assures that the defendant has access to all of the prosecutor's evidence against her, but in practice this access can be rather limited; see, e.g., the discussion in Reinganum (1988) and the references therein. See, for example, Sobel (1989) for an economic analysis of discovery rules.

the probability that the defendant gets  $m - t$  or less  $i$  signals, given that she is actually innocent is:

$$\sum_{n=0}^{\min\{m-t, e_d\}} \binom{e_d}{n} p^n (1-p)^{e_d-n}.$$

Noticing that  $t = \rho^{-1}(q)$ ,

$$F^I(q, e_d, e_p) \equiv \Pr(m - n \geq t \mid I) = \sum_{m=\max\{0, \rho^{-1}(q)\}}^{e_p} \left[ \sum_{n=0}^{\min\{m-\rho^{-1}(q), e_d\}} \binom{e_d}{n} p^n (1-p)^{e_d-n} \right] \binom{e_p}{m} p^{e_p-m} (1-p)^m.$$

Similarly,

$$F^G(q, e_d, e_p) \equiv \Pr(m - n \geq t \mid G) = \sum_{m=\max\{0, \rho^{-1}(q)\}}^{e_p} \left[ \sum_{n=0}^{\min\{m-\rho^{-1}(q), e_d\}} \binom{e_d}{n} p^{e_d-n} (1-p)^n \right] \binom{e_p}{m} p^m (1-p)^{e_p-m}.$$

The following observations can be made about the probability of conviction:<sup>14</sup>

**Proposition 1** For  $J = I$  and  $G$ ,

- a)  $F^J(q, e_d, e_p)$  is decreasing in  $q$ .
- b)  $F^J(q, e_d, e_p)$  is decreasing in  $e_d$ , and increasing in  $e_p$ .
- c)  $F^I(q, e_d, e_p) \leq F^G(q, e_d, e_p)$ ,  $\forall q, e_d, e_p$ .

The proofs are straightforward, and hence omitted.

Thus, reduced form functions with intuitive properties have been derived: The probability of a conviction decreases (respectively, increases) as the defendant's (respectively, the prosecutor's) expenditures increases, *ceteris paribus*. Increases in the standard of proof decrease the probability of conviction, again *ceteris paribus*. Guilty defendants face a higher probability of conviction than the innocent defendants when they spend the same amount.

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<sup>14</sup>Throughout this paper, increasing (respectively, decreasing) is used for weakly increasing (respectively, decreasing).

## 2.1 Determination of litigation expenditures by the parties

Let  $N$  be the penalty (in terms of dollars) imposed on the defendant if she is found guilty. The defendant is assumed to be risk neutral and to minimize the sum of the expected penalty and the litigation expenditures. The objective function of the type  $J$  defendant is then,

$$u_d^J(e_d^J, e_p; \bar{q}, N) = -F^J(q, e_d^J, e_p)N - e_d^J.$$

Let  $e_d^J(e_p; q, N)$  be the optimal expenditure of the type  $J$  defendant when the prosecutor spends  $e_p$ , the standard of proof is  $q$ , and the penalty is  $N$ , i.e.,

$$e_d^J(e_p; q, N) \in \arg \max_e u_d^J(e, e_p; q, N), \quad J = I, G.$$

The following observations can be made about  $e_d^J(e_p; q, N)$ :

**Proposition 2** a)  $0 \leq e_d^J(e_p; q, N) \leq N$ .

b)  $e_d^J(e_p; q, N)$  is increasing in  $N$ .

c) If  $F_{23}^J(q, e_d, e_p) \leq (\geq) 0$  for all  $e_d$ , then  $e_d^J(e_p + 1; q, N) \geq (\leq) e_d^J(e_p; q, N)$ .<sup>15</sup>

d) If  $F_{12}^J(q, e_d, e_p) \leq (\geq) 0$  for all  $e_d$ , then  $e_d^J(e_p; q', N) \geq (\leq) e_d^J(e_p; q, N)$ , where  $q'$  is the (higher) standard of proof that requires one more unit of guilty signal than  $q$  for conviction.<sup>16</sup>

The proofs can be found in the Appendix.

The defendant will never spend more than the penalty, because the best payoff she can obtain by spending more than the amount of penalty is lower than the worst payoff she can obtain by not spending at all. Increasing the penalty increases the return (i.e., the reduction in the expected amount of penalty) from spending an extra dollar, but does not affect its cost. Hence, the defendant's expenditures are increasing in the amount of penalty.

What happens to the defendant's expenditure when the expenditure of the prosecutor increases? If the increase in the prosecutor's expenditure causes an increase in the marginal benefit of spending an extra dollar, i.e.,  $F_{23}^J(\cdot) \leq 0$ , then the defendant's expenditure increases as well. When the standard of proof is too high, or the expenditure of the prosecutor is too low, i.e.,  $\rho^{-1}(q) > e_p + 1$ ,

<sup>15</sup>  $F_{23}^J(q, e_d, e_p) = [F^J(q, e_d + 1, e_p + 1) - F^J(q, e_d, e_p + 1)] - [F^J(q, e_d + 1, e_p) - F^J(q, e_d, e_p)]$ . There is a qualification for this result and similar ones below: The same set (with two or more elements) of expenditure levels can be optimal for both  $(e_p; q, N)$  and  $(e_p + 1; q, N)$ . If one considers the minimum (or maximum) optimal expenditure, then the result holds.

<sup>16</sup>  $F_{12}^J(q, e_d, e_p) = [F^J(q', e_d + 1, e_p) - F^J(q', e_d, e_p)] - [F^J(q, e_d + 1, e_p) - F^J(q, e_d, e_p)]$ , where  $q'$  is such that  $\rho^{-1}(q') = \rho^{-1}(q) + 1$ .

so that the defendant is acquitted independent of the outcome of the prosecutor's search and her expenditure level, she will choose not to exert any effort, and  $F_{23}^J(q, e_d, e_p) = 0$ . Similarly,  $F_{23}^J(q, e_d, e_p) = 0$  when the standard of proof is too low, or the expenditure of the defendant is too low, i.e.,  $-\rho^{-1}(q) > e_d + 1$ . A sufficient, though not necessary, condition for  $F_{23}^I(q, e_d, e_p) < 0$  when the standard of proof is higher than the prior probability of guilt, i.e.,  $q > \alpha$ , is  $p > 1 - \frac{\rho^{-1}(q)}{e_p + 1}$ . Numerical calculations show that for any given standard of proof there is a threshold probability of getting the "right" signal in each state, i.e.,  $p^*$ , such that  $F_{23}^I(\cdot) \leq 0$  if  $p > p^*$ , and  $F_{23}^I(\cdot) \geq 0$  if  $p < p^*$ . Similarly, for any given  $p$ , there is a threshold standard of proof  $q^*$  such that  $F_{23}^I(\cdot) \leq 0$  if  $q > q^*$ , and  $F_{23}^I(\cdot) \geq 0$  if  $q < q^*$ . High standard of proof and high probability of getting the right signal causes the innocent defendant to spend more when the prosecutor increases her spending by increasing the chances of getting a favorable result for spending more money. For the guilty defendant  $1 - p$  replaces  $p$  in the conditions above: High  $q$  and low  $p$  (both favorable to the guilty defendant) will cause the guilty defendant to increase her expenditure when the prosecutor spends more.

If an increase in the standard of proof increases the marginal benefit of spending an extra dollar for the defendant, i.e.,  $F_{12}^J(\cdot) \leq 0$ , then the defendant's expenditure increases as a response. Suppose  $0 < \rho^{-1}(q) + 1 < e_p$  to avoid uninteresting cases. A sufficient condition for  $F_{12}^I(q, e_d, e_p) > 0$  is  $p \geq \frac{e_p - \rho^{-1}(q)}{e_d + 1}$  when  $e_p \leq e_d + \rho^{-1}(q)$ , and  $p \geq \frac{e_d}{e_d + 1}$  and  $p(\frac{e_p}{\rho^{-1}(q)}) \geq (1 - p)(\frac{e_p}{e_d + \rho^{-1}(q) + 1})$  when  $e_p > e_d + \rho^{-1}(q)$ . These conditions suggest and numerical calculations show that when  $p$  is high, the innocent defendant decreases her expenditure as the standard of proof increases. Fix the expenditure of the prosecutor. An increase in the standard of proof will cause a decrease in the probability of conviction. When  $p$  is high, this decrease is smaller for higher expenditures of the defendant. So, for high  $p$ , the innocent defendant will decrease her spending as the standard of proof increases. For the guilty type defendant, again, the role of  $p$  will be taken by  $1 - p$ .

The prosecutor is assumed to be risk-neutral and to maximize the expected punishment net of the litigation expenditures. Landes (1971) used this objective function in one of the first studies in the economic analysis of law, Miceli (1990) employed a similar one. Justifications for using this strong assumption, as summarized in Miceli (1990, p. 197), are:<sup>17</sup>

First, the adversarial system itself naturally places the prosecutor and defense attorney on opposing sides of an all-or-nothing contest. Consequently, each side has an incentive to present its most favorable case to the court. Second, the heavy caseload facing

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<sup>17</sup>See Grosman (1969) for a detailed study of prosecutors and support for these arguments.

most prosecutors necessitates that the majority of cases be settled out of court by plea bargains. Therefore, to encourage such pleas, and to insure that the prosecutor is in a strong bargaining position, it is important that she establish a record of consistent success at trial...Finally, the prosecutor's salary, prestige, and her chances for promotion or reelection are presumably linked to her record of convictions as a measure of her success.

The objective function of the prosecutor is therefore assumed to be,

$$u_p(e_p, e_d^I, e_d^G; q, N) = [\alpha F^G(q, e_d^G, e_p) + (1 - \alpha)F^I(q, e_d^I, e_p)]N - e_p.$$

Let  $e_p(e_d^I, e_d^G; q, N)$  be the optimal expenditure of the prosecutor when the type  $J$  defendant spends  $e_d^J$ , the standard of proof is  $q$ , and the penalty is  $N$ , i.e.,

$$e_p(e_d^I, e_d^G; q, N) \in \arg \max_e u_p(e, e_d^I, e_d^G; q, N).$$

The following observations can be made about  $e_p(e_d^I, e_d^G; q, N)$  :

**Proposition 3** a)  $0 \leq e_p(e_d^I, e_d^G; q, N) \leq N$ .

b)  $e_p(e_d^I, e_d^G; q, N)$  is increasing in  $N$ .

c) If  $F_{23}^I(q, e_d, e_p) \leq (\geq) 0$  for all  $e_p$ , then  $e_p(e_d^I + 1, e_d^G; q, N) \leq (\geq) e_p(e_d^I, e_d^G; q, N)$ . Similarly when  $I$  is replaced by  $G$ .

d) If  $\alpha F_{13}^G(q, e_d, e_p) + (1 - \alpha)F_{13}^I(q, e_d, e_p) \leq (\geq) 0$  for all  $e_p$ , then  $e_p(e_d^I, e_d^G; q', N) \leq (\geq) e_p(e_d^I, e_d^G; q, N)$ , where  $q'$  is the (higher) standard of proof that requires one more unit of guilty signal than  $q$  for conviction.<sup>18</sup>

The proofs and interpretations are analogous to those of Proposition 2, and hence omitted.

For any given standard of proof and penalty, the prosecutor and the defendant choose their litigation expenditures simultaneously.

**Definition 1** Given  $q$  and  $N$ ,  $(\bar{e}_d^I(q, N), \bar{e}_d^G(q, N), \bar{e}_p(q, N))$  is a (pure-strategy) Nash equilibrium if

$$\bar{e}_d^J(q, N) \in e_d^J(\bar{e}_p; q, N) \text{ and } \bar{e}_p(q, N) \in e_p(\bar{e}_d^I, \bar{e}_d^G; q, N).$$

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<sup>18</sup>  $F_{13}^J(q, e_d, e_p) = [F^J(q', e_d, e_p + 1) - F^J(q', e_d, e_p)] - [F^J(q, e_d, e_p + 1) - F^J(q, e_d, e_p)]$ , where  $q'$  is such that  $\rho^{-1}(q') = \rho^{-1}(q) + 1$ .

Establishing the existence of (a possibly mixed-strategy) equilibrium is trivial. A pure-strategy equilibrium need not exist, though. Extending the definition to include mixed strategies is a straightforward task, which is avoided here for notational simplicity.

### 3 The Optimal Standard of Proof and Penalty

In this section the court's optimal choice of standard of proof and penalty will be analyzed using the model of evidence production and transmission to the court by the litigating parties developed above. The analysis of the court's problem follows Rubinfeld and Sappington (1987) closely.

Let  $q$  be the *standard of proof* used by the court, i.e., the verdict would be "guilty" if the assessment of the guilt of the defendant by the court is more than  $q$ . If the litigation expenditures of the prosecutor and the defendant of type  $J$  are  $e_p$  and  $e_d^J$ , respectively, this has probability  $F^J(q, e_d^J, e_p)$ . Let  $N$  be the penalty (in terms of dollars) imposed on the defendant if she is found guilty. The procedural process should determine  $q$  and  $N$  optimally.

A court can commit two types of errors when it makes a decision: An innocent person can be convicted, or a guilty person can be acquitted. Respective probabilities of false conviction and false acquittal are:

$$\begin{aligned}\Pr(fc) &= \Pr(\text{innocent}) \Pr(\text{conviction} \mid \text{innocent}) = (1 - \alpha)F^I(q, e_d^I, e_p), \text{ and} \\ \Pr(fa) &= \Pr(\text{guilty}) \Pr(\text{acquittal} \mid \text{guilty}) = \alpha[1 - F^G(q, e_d^G, e_p)],\end{aligned}$$

where  $\alpha$  is the ex ante probability of the defendant being guilty.

Notice that, keeping  $e_d^J$  and  $e_p$  constant, an increase in the standard of proof decreases the probability of false conviction and increases the probability of false acquittal. Of course, a change in the standard of proof affects the incentives of the prosecutor and the defendant as well. A change in the penalty has no direct effect on the probabilities of committing legal errors, it only has indirect effects via the expenditure decisions of the parties. Let  $L^{fc}$  (respectively,  $L^{fa}$ ) be the cost of a false conviction (respectively, false acquittal) to the society. So, the expected legal cost of a decision to the society is

$$\Pr(fc)L^{fc} + \Pr(fa)L^{fa} = (1 - \alpha)F^I(q, e_d^I, e_p)L^{fc} + \alpha[1 - F^G(q, e_d^G, e_p)]L^{fa}.$$

Besides minimizing this cost, another objective imputed to the procedural system is to sustain the efficient use of resources devoted to it. (See, for example, Posner (1986, ch.21).) The objective

function then becomes:<sup>19</sup>

$$(1 - \alpha)F^I(q, e_d^I, e_p)L^{fc} + \alpha[1 - F^G(q, e_d^G, e_p)]L^{fa} + (1 - \alpha)e_d^I + \alpha e_d^G + e_p.$$

The cost of the procedural process itself is ignored; if this cost is constant, then nothing changes in the analysis.

The standard of proof ( $q$ ) and the penalty ( $N$ ) shall be chosen to minimize the above objective function.<sup>20</sup> The optimal choice of  $q$  and  $N$  depend on litigation expenditures of the defendant and the prosecutor, as well as on  $\alpha$ ,  $L^{fc}$ , and  $L^{fa}$ , but these expenditures are, in turn, functions of  $q$  and  $N$ . Given  $q$  and  $N$ , litigation expenditures of the parties are determined noncooperatively as explored in the previous section. Let  $(\bar{e}_d^I(q, N), \bar{e}_d^G(q, N), \bar{e}_p(q, N))$  be a Nash equilibrium. Let,

$$\begin{aligned} C(q, N) = & (1 - \alpha)F^I(q, \bar{e}_d^I(q, N), \bar{e}_p(q, N))L^{fc} + \alpha[1 - F^G(q, \bar{e}_d^G(q, N), \bar{e}_p(q, N))]L^{fa} \\ & + (1 - \alpha)\bar{e}_d^I(q, N) + \alpha\bar{e}_d^G(q, N) + \bar{e}_p(q, N). \end{aligned}$$

Note that the dependence of the total cost on exogenous variables ( $\alpha, L^{fc}, L^{fa}$ ) are suppressed in the notation.

The court's problem can be written as,

$$\min_{q, N} C(q, N).$$

Hereafter, a special case of the model in which each party can acquire at most one signal, is studied. Tractability is the most important reason for this restriction. Secondly, it is possible to generate some general insights with this restricted model which will hold in the unrestricted model. Thirdly, for any given level of the standard of proof and the penalty, each party will obtain at most one signal for extreme values of  $p$ : If the result of an investigation is (almost) perfectly informative about the innocence of the defendant ( $p$  close to 1), then neither party has any incentive to obtain more than one signal. A case where a single important piece of evidence, almost perfectly, reveals the culpability of the defendant is an example. Similarly, if the result is very uninformative ( $p$  close

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<sup>19</sup>The model abstracts from deterrence effects of the standard of proof and penalty. See Schrag and Scotchmer (1994) for a model of endogenous deterrence.

<sup>20</sup>Note that the model here abstracts from the cost imposed on the society by the crime. Since the procedural process is for a given crime, this is without loss of generality. Alternatively, one could assume that a crime with a given cost to society has been committed, and analyze the relation of  $N$  with this cost, as is done in Miceli (1990). This would be a straightforward extension, and is not done here in order to keep the exposition as simple as possible.

to  $\frac{1}{2}$ ), then neither party will exert any effort since the court's posterior will be very close to its prior irrespective of the evidence presented to the court.<sup>21</sup>

The ex ante probability of the defendant being guilty is assumed to be  $\frac{1}{2}$  for *expositional simplicity*. The posterior of guilt, then, becomes

$$\rho(m, n) = \frac{1}{1 + \left(\frac{1-p}{p}\right)^{m-n}}, \quad m, n \in \{0, 1\}.$$

In particular,  $\rho(0, 0) = \rho(1, 1) = \frac{1}{2}$ ,  $\rho(1, 0) = p$ , and  $\rho(0, 1) = 1 - p$ . The verdict is guilty if and only if  $\rho(\cdot) \geq q$ . Since  $\rho(1, 0) > \rho(0, 0) = \rho(1, 1) > \rho(0, 1)$ , the court (effectively) has four choices for standard of proof:

1)  $q \in [0, 1 - p]$  : The defendant is always convicted.

2)  $q \in (1 - p, \frac{1}{2}]$  : The defendant is acquitted if and only if she presents an  $i$  and the prosecutor does not present any evidence to the court.

3)  $q \in (\frac{1}{2}, p]$  : The defendant is convicted if and only if she does not present any evidence to the court and the prosecutor presents a  $g$ .

4)  $q \in (p, 1]$  : The defendant is always acquitted.

The intervals in each of these cases will be denoted by their upper end-points, hence  $q \in \{1 - p, \frac{1}{2}, p, 1\}$ . For each possible choice of the standard of proof, first the equilibrium expenditures by the prosecutor and the defendant (both types) will be found for any given amount of penalty; then the optimal choice of the penalty (as a function of costs of both types of legal errors) will be analyzed. Afterwards, the optimal choice of the standard of proof, again as a function of costs of both types of legal errors, will be studied.

The determination of expenditures by the parties, given the standard of proof and the penalty, was studied in the previous section. Here it will be sketched briefly to incorporate mixed strategies. Let  $s_p$  denote the mixed strategy of the prosecutor in which  $Pr(e_p = 1) = s_p$ , and  $s_d^J$  denote the mixed strategy of the type  $J$  defendant where  $Pr(e_d^J = 1) = s_d^J$ . Clearly,  $(s_d^I, s_d^G, s_p) \in [0, 1]^3$ . Let

$$\begin{aligned} F^J(q, s_d^J, s_p) &\equiv s_d^J s_p F^J(q, 1, 1) + s_d^J (1 - s_p) F^J(q, 1, 0) + \\ &\quad (1 - s_d^J) s_p F^J(q, 0, 1) + (1 - s_d^J) (1 - s_p) F^J(q, 0, 0). \end{aligned}$$

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<sup>21</sup>Formally,  $\forall (q, N) \exists p_1, p_2 \in (\frac{1}{2}, 1)$  s.t.  $\forall p \in (p_1, 1]$  and  $\forall p \in [\frac{1}{2}, p_2)$ ,  $\bar{e}_d^J(q, N), \bar{e}_p(q, N) \in \{0, 1\}$ . The reason is that as  $p \rightarrow 1$ ,  $F^J(q, e_d, e_p) \rightarrow 0 \forall e_d > 0$ , and  $F^G(q, e_d, e_p) \rightarrow 1 \forall e_p > 0$ ; and as  $p \rightarrow \frac{1}{2}$ ,  $F^J(q, e_d, e_p) \rightarrow 1$  if  $q < \alpha$ , and  $F^J(q, e_d, e_p) \rightarrow 0$  if  $q \geq \alpha$ , so that there is no need to obtain more than one signal. Notice that  $p_1$  and  $p_2$  may depend on  $q$  and  $N$ . Also the cost of acquiring a signal is held constant when taking these limits, as it is done throughout the paper.

The payoff function of the defendant of type  $J$  is,

$$u_d^J(s_d^J, s_p; q, N) = -F^J(q, s_d^J, s_p)N - s_d^J.$$

The payoff function of the prosecutor is,

$$u_p(s_p, s_d^I, s_d^G; q, N) = \frac{1}{2}[F^I(q, s_d^I, s_p) + F^G(q, s_d^G, s_p)]N - s_p.$$

For any  $q$  and  $N$ ,  $(\bar{s}_d^I, \bar{s}_d^G, \bar{s}_p)$  is a Nash equilibrium if

$$u_d^J(\bar{s}_d^J, \bar{s}_p; q, N) \geq u_d^J(s_d^J, \bar{s}_p; q, N), \forall s_d^J, \text{ and}$$

$$u_p(\bar{s}_p, \bar{s}_d^I, \bar{s}_d^G; q, N) \geq u_p(s_p, \bar{s}_d^I, \bar{s}_d^G; q, N), \forall s_p.$$

### 3.1 The Optimal Penalty for any Given Level of the Standard of Proof

Now the four possible choices for the standard of proof are analyzed.

#### 3.1.1 $q = 1 - p$

The standard of proof is so low that, the defendant is convicted regardless of the evidence presented to the court. Since the verdict is insensitive to expenditures by the parties,  $(s_d^I, s_d^G, s_p) = (0, 0, 0)$  is the unique equilibrium for any given amount of penalty ( $N$ ). Both types of the defendant are convicted with probability one, hence  $\Pr(fc) = \frac{1}{2}$  and  $\Pr(fa) = 0$ , where  $\Pr(fc)$  and  $\Pr(fa)$  are probabilities of false conviction and false acquittal, respectively. The total cost is

$$C(1 - p, N) = \Pr(fc)L^{fc} + \Pr(fa)L^{fa} + \frac{1}{2}(s_d^I + s_d^G) + s_p = \frac{1}{2}L^{fc}.$$

#### 3.1.2 $q = \frac{1}{2}$

In this case, the defendant is acquitted if and only if she presents evidence favorable to her case, and the prosecutor does not present anything to the court. It will be shown that the optimal penalty depends only on the cost of false conviction: If the cost of a false conviction is high enough ( $L^{fc} \geq \frac{1}{p}$ ), then any  $N \in (\frac{1}{p}, \frac{1}{1-p})$ , given which only the innocent defendant produces evidence, is optimal. If the cost of a false conviction is low ( $L^{fc} \leq \frac{1}{p}$ ), then any  $N \in [0, \frac{1}{p})$  is optimal; in this case nobody produces any evidence.

The equilibria will depend on the value of  $p$  and  $N$ . In particular, whether  $p > \frac{2}{3}$  or not, will change the equilibria. But in both cases optimal penalties are the same for all  $L^{fc}$  and  $L^{fa}$ , so attention will be restricted to the case where  $p > \frac{2}{3}$ , without loss of generality. For the following  $N$  there is a unique equilibrium; probabilities of false conviction and false acquittal, and the overall costs are also listed:<sup>22</sup>

$N$	$(\bar{s}_d^I, \bar{s}_d^G, \bar{s}_p)$	$\Pr(fc)$	$\Pr(fa)$	$C(\frac{1}{2}, N)$
$[0, \frac{1}{p})$	$(0, 0, 0)$	$\frac{1}{2}$	0	$\frac{1}{2}L^{fc}$
$(\frac{1}{p}, \frac{1}{1-p})$	$(1, 0, 0)$	$\frac{1}{2}(1-p)$	0	$\frac{1}{2}(1-p)L^{fc} + \frac{1}{2}$
$(\frac{1}{1-p}, \frac{1}{p(1-p)})$	$(1, 1, 0)$	$\frac{1}{2}(1-p)$	$\frac{1}{2}(1-p)$	$\frac{1}{2}(1-p)L^{fc} + \frac{1}{2}(1-p)L^{fa} + 1$
$(\frac{1}{p(1-p)}, \frac{2}{p(1-p)})$	$(1, a, b)$			
$(\frac{2}{p(1-p)}, \frac{1}{(1-p)^2})$	$(1, 0, 1)$	$\frac{1}{2}(1-p^2)$	0	$\frac{1}{2}(1-p^2)L^{fc} + \frac{3}{2}$
$(\frac{1}{(1-p)^2}, \infty)$	$(1, 1, 1)$	$\frac{1}{2}(1-p^2)$	$\frac{1}{2}(1-p)^2$	$\frac{1}{2}(1-p^2)L^{fc} + \frac{1}{2}(1-p)^2L^{fa} + 2$

where  $a = \frac{2-p(1-p)N}{p(1-p)N}$  and  $b = \frac{(1-p)N-1}{p(1-p)N}$ .

As the penalty increases the equilibrium expenditures by the prosecutor and the innocent defendant increase. The equilibrium expenditures by the guilty defendant may go either way. Increasing the penalty affects the expenditures of the parties both directly and indirectly. The direct effect is always positive, i.e., when the penalty is higher, ceteris paribus, both types of the defendant and the prosecutor spend more, since the stakes are higher (Prop. 2b and 3b). On the other hand, expenditures of both types of the defendant are decreasing in the expenditure of the prosecutor and the expenditure of the prosecutor is increasing in the expenditures of both types of the defendant.<sup>23</sup> Remember that the defendant is convicted unless she presents an innocency signal and the prosecutor does not present any evidence. The low standard of proof favors the prosecution; it does not pay for the defendant to spend more when the prosecutor increases her expenditures. The combination of these direct and indirect effects results in higher expenditures for the innocent defendant and the prosecutor when the penalty is increased, whereas the indirect effect may dominate the direct effect for the guilty defendant, so that she may spend less. The difference between the effects on the innocent and guilty defendant's expenditures is the result of the assumption that an investigation will yield favorable evidence to the defendant when she is innocent. In Rubinfeld

<sup>22</sup>For the mixed strategy equilibrium, the cost will be a convex combination of the costs for  $(1, 0, 0)$ ,  $(1, 1, 0)$ ,  $(1, 0, 1)$ , and  $(1, 1, 1)$ , each of which can already be induced as a unique equilibrium for some  $N$ . Hence the cost is not calculated for  $N \in (\frac{1}{p(1-p)}, \frac{2}{p(1-p)})$ . The same argument applies for  $N = \frac{1}{p}, \frac{1}{1-p}, \frac{1}{p(1-p)}, \frac{2}{p(1-p)}$ , and  $\frac{1}{(1-p)^2}$ .

<sup>23</sup>These follow from Prop. 2c and 3c and the fact that  $F_{23}^J(p, 0, 0) = -p(1-p) < 0$ ,  $J = I, G$ .

and Sappington (1987), an increase in the penalty results in increases in the expenditures of both type of defendants. As it is explained above, equilibrium considerations may change the picture.

Now notice that any  $N \in (\frac{1}{p}, \frac{1}{1-p})$  dominates, i.e., the cost is smaller for *any*  $L^{fc}, L^{fa} \geq 0$ , any  $N \in (\frac{1}{1-p}, \infty)$ . An increase in the amount of penalty increases *both*  $\Pr(fc)$  and  $\Pr(fa)$  in these cases. When  $N \in (\frac{1}{p}, \frac{1}{1-p})$ , in equilibrium, only the innocent defendant spends effort. Since the defendant is acquitted if and only if she presents a favorable signal and the prosecutor cannot present any signals, the probability of convicting an innocent defendant is the probability of her not obtaining an innocency signal, i.e.,  $1 - p$ ; moreover all guilty defendants are convicted. First, notice that total expenditures of the parties do not decrease as the penalty increases. If an increase in the penalty makes the guilty defendant spend effort, the probability of a false acquittal will be higher, since, with positive probability, the guilty defendant will acquire an innocency signal and the prosecutor will not obtain any guiltiness signals. If an increase in the penalty makes the prosecutor spend effort, the probability of a false conviction will be higher. Hence, any penalty in the interval  $(\frac{1}{p}, \frac{1}{1-p})$  dominates any penalty higher than  $\frac{1}{1-p}$ .

Two candidates for the optimal penalty remain. Let  $N = \frac{1}{p}$  represent  $N \in [0, \frac{1}{p})$ , since when  $N = \frac{1}{p}$  the unique perfect equilibrium is  $(0, 0, 0)$ . For the same reason, let  $N = \frac{1}{1-p}$  represent  $N \in (\frac{1}{p}, \frac{1}{1-p})$ . When  $N = \frac{1}{p}$ , in equilibrium, nobody spends effort and hence both types of defendants are convicted. Increasing the penalty to  $\frac{1}{1-p}$ , by causing the innocent defendant to spend, decreases the probability of a false conviction without increasing the probability of a false acquittal. So, the trade-off is between the social benefit arising from the decrease in the probability of a false conviction and the social loss from the expenditure of the defendant:

$$C(\frac{1}{2}, \frac{1}{p}) \leq C(\frac{1}{2}, \frac{1}{1-p}) \Leftrightarrow L^{fc} \leq \frac{1}{p}$$

$N = \frac{1}{p}$  is optimal when  $L^{fc} \leq \frac{1}{p}$ , and  $N = \frac{1}{1-p}$  is optimal when  $L^{fc} \geq \frac{1}{p}$ . If the cost of a false conviction is high enough, it is optimal to choose a high enough (but not too high) penalty such that the innocent defendant, and only the innocent defendant, is induced to produce evidence.

### 3.1.3 $q = p$

In this case, the defendant is convicted if and only if the prosecutor presents evidence favorable to her case and the defendant does not present any evidence. It is shown that if the cost of a false conviction is high relative to the cost of a false acquittal, then any  $N \in [0, 2]$  is optimal. When  $N \in [0, 2]$  nobody produces any evidence, and hence the defendant is acquitted. If the costs of legal errors are both, relatively, close to each other and high, then any  $N > \frac{1}{p(1-p)}$ , which induces

the prosecutor and both types of the defendant to produce evidence, is optimal. Finally, if the cost of a false acquittal is high relative to the cost of a false conviction, which is unlikely for criminal cases, then any  $N \in (2, \frac{1}{p(1-p)}]$ , which induces only the prosecutor to produce evidence, is optimal.

Considering only perfect equilibria, each penalty induces a unique equilibrium.<sup>24</sup> There are effectively three choices for the penalty. The probabilities of false conviction, false acquittal, and costs associated with these choices are:

$N$	$(\bar{s}_d^I, \bar{s}_d^G, \bar{s}_p)$	$\Pr(fc)$	$\Pr(fa)$	$C(p, N)$
$[0, 2]$	$(0, 0, 0)$	0	$\frac{1}{2}$	$\frac{1}{2}L^{fa}$
$(2, \frac{1}{p(1-p)}]$	$(0, 0, 1)$	$\frac{1}{2}(1-p)$	$\frac{1}{2}(1-p)$	$\frac{1}{2}(1-p)L^{fc} + \frac{1}{2}(1-p)L^{fa} + 1$
$(\frac{1}{p(1-p)}, \infty)$	$(1, 1, 1)$	$\frac{1}{2}(1-p)^2$	$\frac{1}{2}(1-p^2)$	$\frac{1}{2}(1-p)^2L^{fc} + \frac{1}{2}(1-p^2)L^{fa} + 2$

In this case the equilibrium expenditures by the prosecutor and both types of the defendant increase as the penalty increases. Expenditures of both types of the defendant are increasing in the expenditure of the prosecutor and the expenditure of the prosecutor is decreasing in the expenditure of both types of the defendant.<sup>25</sup> The combination of these effects with the direct effect of an increase in the penalty results in higher equilibrium expenditures by all the parties.

Notice that, as the penalty increases the probability of a false conviction first increases and then decreases. On the other hand, as the penalty increases the probability of a false acquittal first decreases and then increases. Unlike the case when  $q = \frac{1}{2}$ , there is no relation of domination between penalties. A decrease in the probability of one type of error is accompanied by an increase in the other type of error. Let  $N = 2$  represent  $[0, 2]$ ,  $\frac{1}{p(1-p)}$  represent  $(2, \frac{1}{p(1-p)}]$ , and  $\bar{N}$  represent  $(\frac{1}{p(1-p)}, \infty)$ .

Straightforward calculations reveal that,

- $N = 2$  is optimal if  $L^{fa} \leq \min\{\frac{1-p}{p}L^{fc} + \frac{2}{p}, (\frac{1-p}{p})^2L^{fc} + \frac{4}{p^2}\}$ ,
- $N = \frac{1}{p(1-p)}$  is optimal if  $L^{fa} \geq \max\{L^{fc} - \frac{2}{p(1-p)}, \frac{1-p}{p}L^{fc} + \frac{2}{p}\}$ , and
- $\bar{N}$  is optimal if  $(\frac{1-p}{p})^2L^{fc} + \frac{4}{p^2} \leq L^{fa} \leq L^{fc} - \frac{2}{p(1-p)}$ .

<sup>24</sup>For  $N \in [0, 2)$ ,  $(s_d^I, s_d^G, s_p) = (0, 0, 0)$  is the unique equilibrium. When  $N = 2$ ,  $(0, 0, \pi_1)$ , where  $\pi_1 \in [0, 1]$ , are all equilibria. But, since  $e_p = 0$  weakly dominates  $e_p = 1$  (when  $N = 2$ ), the unique perfect equilibrium is  $(0, 0, 0)$ . For  $N \in (2, \frac{1}{p(1-p)})$ , the unique equilibrium is  $(0, 0, 1)$ . When  $N = \frac{1}{p(1-p)}$ ,  $(\pi_2, \pi_3, 1)$ , where  $(\pi_2, \pi_3) \in [0, 1]^2$ , are all equilibria, but since  $e_d^J = 0$  weakly dominates  $e_d^J = 1$ , the unique perfect equilibrium is  $(0, 0, 1)$ . For  $N > \frac{1}{p(1-p)}$ , the unique equilibrium is  $(1, 1, 1)$ .

<sup>25</sup>Since  $F_{23}^J(p, 0, 0) = -p(1-p) < 0$ ,  $J = I, G$ .

(Insert Figure 1 about here)

When the cost of a false conviction is high relative to the cost of a false acquittal,  $N = 2$ , which induces the smallest possible probability of false conviction, is optimal. When the costs of both types of errors are both, relatively, close to each other and high,  $\bar{N}$  is optimal, balancing these costs. When the cost of a false acquittal is high relative to the cost of a false conviction, the optimal penalty is  $\frac{1}{p(1-p)}$ , inducing the lowest possible probability of false acquittal.

### 3.1.4 $q = 1$

The standard of proof is so high that, the defendant is acquitted regardless of the evidence presented to the court. Since the verdict is insensitive to the evidence presented,  $(0, 0, 0)$  is the unique equilibrium for any  $N$ . Both types of the defendant are acquitted with probability one, so  $\Pr(fc) = 0$  and  $\Pr(fa) = \frac{1}{2}$ ;  $C(1, N) = \frac{1}{2}L^{fa}$ .

## 3.2 The Optimal Standard of Proof

In this section the optimal standard of proof and penalty are derived for given costs of legal errors. These findings are discussed in the next section.

Notice that  $C(1, N) = C(p, 2)$  and  $C(1-p, N) = C(\frac{1}{2}, \frac{1}{p})$ ,  $\forall N$ . This means that one can get the same results of an extreme standard of proof with a more moderate standard of proof and a suitably chosen penalty. Also,

$$C(p, \frac{1}{p(1-p)}) > C(\frac{1}{2}, \frac{1}{1-p}) \quad \forall L^{fc}, L^{fa} \geq 0.$$

Hence, only  $(q, N) = (\frac{1}{2}, \frac{1}{p}), (\frac{1}{2}, \frac{1}{1-p}), (p, 2)$  and  $(p, \bar{N})$  need to be considered. Comparing these,

- $(\frac{1}{2}, \frac{1}{p})$  is optimal if  $L^{fc} \leq \min\{\frac{1}{p}, L^{fa}\}$ ,
- $(\frac{1}{2}, \frac{1}{1-p})$  is optimal if  $L^{fc} \geq \frac{1}{p}$  and  $L^{fa} \geq \max\{(1-p)L^{fc} + 1, \frac{p}{1+p}L^{fc} - \frac{3}{1-p^2}\}$ ,
- $(p, 2)$  is optimal if  $L^{fa} \leq \min\{L^{fc}, (1-p)L^{fc} + 1, (\frac{1-p}{p})^2L^{fc} + \frac{4}{p^2}\}$ , and
- $(p, \bar{N})$  is optimal if  $L^{fa} \leq \min\{L^{fc} - \frac{2}{p(1-p)}, \frac{p}{1+p}L^{fc} - \frac{3}{1-p^2}\}$  and  $L^{fa} \geq (\frac{1-p}{p})^2L^{fc} + \frac{4}{p^2}$ .

The optimal standard of proof and penalty for any given  $L^{fc}$  and  $L^{fa}$  are shown in Figure 2.

(Insert Figure 2 about here)

When  $p \leq \frac{1}{2}(\sqrt{5} - 1)$ ,  $(p, \bar{N})$  is not optimal for any  $L^{fc}$  and  $L^{fa}$  and this graph simplifies to:

(Insert Figure 3 about here)

It may be interesting to see what happens as the signals become very precise, i.e., as  $p \rightarrow 1$ .

(Insert Figure 4 about here)

Remember that the cost of an investigation was normalized to 1. As the signals become very precise, whenever costs of both legal errors exceed the cost of investigation, the optimal standard of proof is  $\frac{1}{2}$ , and the optimal penalty is any penalty greater than 1. When  $q = \frac{1}{2}$ , the defendant is acquitted if and only if she presents evidence favorable to her case and the prosecutor does not provide any evidence to the contrary. In this case one can choose the penalty high enough (but not too high), i.e.,  $\frac{1}{p} < N < \frac{1}{1-p}$ , so that in equilibrium only the innocent defendant searches for evidence. The guilty defendant is always convicted in this case since she will not be able to produce favorable evidence. Hence, the probability of a false acquittal is zero. The innocent defendant, however, will be acquitted if and only if her search produces favorable evidence, which has probability  $p$ . The probability of a false conviction is, therefore,  $1 - p$ . So, as the evidence becomes more reliable, i.e.,  $p \rightarrow 1$ , the probability of committing either type of legal error will approach to zero. For  $p = 1$ , this equilibrium is short of the first-best only due to the expenditure of the innocent defendant to signal her innocence.

### 3.3 Comparative Statics

#### 3.3.1 For a fixed level of the standard of proof

As the penalty increases the equilibrium expenditures by the prosecutor and the innocent defendant increase. The equilibrium expenditures by the guilty defendant may go either way. Increasing the penalty has both direct and indirect effects on the expenditures of the parties. When the penalty is higher, ceteris paribus, both types of the defendant and the prosecutor spend more, since the stakes are higher. The direction of the indirect effect is indeterminate: Expenditures of both types of the defendant are decreasing in the expenditure of the prosecutor when the standard of proof is low (i.e.,  $q = \frac{1}{2}$ ) and increasing when it is high (i.e.,  $q = p$ ). It follows that the prosecutor's expenditure is increasing (respectively, decreasing) in the expenditures of both types of the defendant when the standard of proof is low (respectively, high). The combination of these direct and indirect effects results in higher expenditures for the innocent defendant and the prosecutor when the penalty is

increased, whereas the indirect effect may dominate the direct effect for the guilty defendant, so that she may spend less.

As the penalty increases probabilities of committing legal errors can go either way, as was shown for the case  $q = p$ . Moreover, it is possible that an increase in the penalty results in both higher  $\Pr(fc)$  and  $\Pr(fa)$ . Take the case where  $q = \frac{1}{2}$ , i.e., the defendant is acquitted if and only if she presents an innocency signal and the prosecutor does not present any guiltiness signal. When  $N \in (\frac{1}{p}, \frac{1}{1-p})$ , only the innocent defendant will spend effort, so that  $\Pr(fc) = \frac{1}{2}(1-p)$  and  $\Pr(fa) = 0$ . If the penalty is sufficiently high (i.e.,  $N > \frac{1}{(1-p)^2}$ ), then, in the equilibrium, the prosecutor and both types of the defendant will spend effort. This, in turn, will result in higher probabilities of both types of error, since, with positive probability, the effort of the prosecutor will produce a guilty signal when the defendant is innocent and the effort of the guilty defendant will produce an innocence signal.

What happens to the optimal level of penalty as the costs of legal errors change for a given level of standard of proof? A glance at Figure 1 reveals that the effect is indeterminate.

These findings are in sharp contrast with both Rubinfeld and Sappington (1987) and Miceli (1990), showing that leaving one of the litigating parties out of the analysis may be misleading. More importantly, they demonstrate the necessity of being cautious when considering increases in penalties: It may increase the probabilities of *both* false conviction and false acquittal.

### 3.3.2 For a fixed amount of penalty

Equilibrium expenditures by both parties may go either way as the standard of proof increases.<sup>26</sup>

The probability of a false conviction is decreasing and the probability of a false acquittal is increasing in the standard of proof for any given level of the penalty.<sup>27</sup>

The optimal standard of proof is increasing (respectively, decreasing) in the cost of a false conviction (respectively, false acquittal).<sup>28</sup>

<sup>26</sup>Let  $N > \frac{1}{(1-p)^2}$ . When  $q = 1-p$ ,  $(\bar{s}_d^I, \bar{s}_d^G, \bar{s}_p) = (0, 0, 0)$ ; when  $q = \frac{1}{2}$  and  $q = p$ ,  $(1, 1, 1)$ ; when  $q = 1$ ,  $(0, 0, 0)$  is the unique equilibrium.

<sup>27</sup>It is possible that an increase (respectively, decrease) in the standard of proof may strictly decrease  $\Pr(fc)$  (respectively,  $\Pr(fa)$ ) while unaffected  $\Pr(fc)$  (respectively,  $\Pr(fa)$ ).

<sup>28</sup>There is a minor qualification. Two different levels of standard of proof may be optimal for *both*  $(L^{fc}, L^{fa})$  and  $(\bar{L}^{fc}, L^{fa})$ , where  $\bar{L}^{fc} > L^{fc}$ . If one considers the minimum (or the maximum) optimal standard of proof, then the result holds.

### 3.3.3 Effects of changes in the costs of legal errors on the optimal standard of proof and penalty

As the cost of a false conviction (respectively, false acquittal) increases the optimal standard of proof increases (respectively, decreases). This result is consistent with the fact that in criminal cases the prosecution has the burden of proof beyond a reasonable doubt, whereas in most civil cases only proof by a fair preponderance of evidence is required, since in criminal cases cost of a false conviction is much higher than cost of a false acquittal, whereas in most civil cases they are close to each other.

The direction of change in the optimal amount of penalty is indeterminate as the costs of legal errors change. See Figure 3.

These results *suggest* the importance and the effectiveness of the standard of proof, relative to the penalty imposed on the defendant who is convicted, as an instrument of judicial decision making. Again, the distinction between criminal and civil cases provides support for this claim. The main difference, for the purposes of this paper, between criminal and civil cases is that in criminal cases the social cost of a false conviction is much higher than the social cost of a false acquittal, whereas in most civil cases these costs are close to each other. When one looks at the standard of proofs and penalties across different classes of cases, criminal cases, generally, require higher standard of proofs, whereas there is no clear ranking in terms of penalties. It is possible to impose a very large penalty on an individual, or a firm, in a civil trial with a fair preponderance of evidence, whereas to convict a defendant in a criminal trial, the prosecution has the burden of proof beyond a reasonable doubt even when the penalty imposed in the case of conviction is minor.

Finally, despite being evident given the formal model, another point may be worth mentioning: The optimal penalty cannot be set independently of the optimal standard of proof. These two instruments are closely intertwined through the strategic evidence production decisions of the litigating parties.

## 4 Concluding Remarks

In this paper a model of evidence production by the litigating parties is developed. Even though the model was laid down in terms of a criminal case, it can be reinterpreted to incorporate some civil cases as well. With appropriate changes in the terminology it is a model of the following type of civil cases: Suppose that a “victim” sues a person (real or legal) for a tort. It is known that the plaintiff exercised due care. The defendant is either negligent (i.e., did not exercise due

care), and hence liable, or not. The defendant knows whether she is liable or not, but neither the plaintiff nor the court possesses this information. Suppose also that the amount of damage to the plaintiff is uncontested. Given these, the outcome of the litigation will be dichotomous under both comparative and contributory negligence tort standards: the defendant is found either liable or not, and in the former case she has to compensate the victim's damage.

The model of evidence production and the decision process of the court was then used to analyze optimal choice of standard of proof and penalty in criminal trials. There are other questions that seem suitable to study using this model. One is the comparison of Common Law and Civil Law litigation procedures (i.e., "adversarial" and "inquisitorial" systems). In the inquisitorial system, the judge is involved and active in the trial, whereas in the adversarial system she is (traditionally) more passive. Another difference is in the use of experts. These differences may be studied in a model similar to the one above.<sup>29</sup> Another question that may be tackled by the use of the model developed here is the possible consequences of having budget constrained defendants or prosecutors.<sup>30</sup>

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<sup>29</sup>See Zweigert and Kotz (1987) for a detailed study of comparative law. See Froeb and Kobayashi (1993) for a comparison of adversarial and court-appointed expert testimony.

<sup>30</sup>Roberts (1998) studies the budget constrained prosecutor's plea bargaining allocation problem.

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# Appendix

## Proof of Proposition 1:

a) Since  $F^J(q, e_d, e_p) \in [0, 1]$ , the defendant can guarantee a payoff of at least  $-N$  by not spending:

$$u_d^J(0, e_p; \bar{q}, N) = -F^J(q, 0, e_p)N \geq -N,$$

and can obtain at most  $-e_d^J(e_p; q, N)$  by spending this amount:

$$u_d^J(e_d^J(e_p; q, N), e_p; \bar{q}, N) \leq -e_d^J(e_p; q, N).$$

Hence,  $0 \leq e_d^J(e_p; q, N) \leq N$ .

b) Let  $e_i^J \in \arg \max_e u_d^J(e, e_p; q, N_i)$ ,  $J = I, G, i = 1, 2$ , where  $N_2 > N_1$ . By optimality of  $e_1^J$  and  $e_2^J$ :

$$-F^J(q, e_1^J, e_p)N_1 - e_1^J \geq -F^J(q, e_2^J, e_p)N_1 - e_2^J$$

$$-F^J(q, e_2^J, e_p)N_2 - e_2^J \geq -F^J(q, e_1^J, e_p)N_2 - e_1^J.$$

Arranging these inequalities:

$$[F^J(q, e_1^J, e_p) - F^J(q, e_2^J, e_p)]N_2 \geq [F^J(q, e_1^J, e_p) - F^J(q, e_2^J, e_p)]N_1.$$

Since  $N_2 > N_1$ ,  $F^J(q, e_1^J, e_p) \geq F^J(q, e_2^J, e_p)$ . Now, suppose that,  $e_1^J > e_2^J$ . Since  $F^J(q, e_d, e_p)$  is decreasing in  $e_d$ , it must be that  $F^J(q, e_1^J, e_p) = F^J(q, e_2^J, e_p)$ . But this means that  $e_2^J$  is a better response than  $e_1^J$  to  $N_1$ , a contradiction. Hence,  $e_2^J \geq e_1^J$ .

c) Let  $e_1^J \equiv \min e_d^J(e_p; q, N)$  and  $e_2^J \equiv \min e_d^J(e_p + 1; q, N)$ ,  $J = I, G$ . Optimality of  $e_1^J$  and  $e_2^J$  implies:

$$-F^J(q, e_1^J, e_p)N - e_1^J \geq -F^J(q, e_2^J, e_p)N - e_2^J$$

$$-F^J(q, e_2^J, e_p + 1)N - e_2^J \geq -F^J(q, e_1^J, e_p + 1)N - e_1^J.$$

Arranging,

$$F^J(q, e_1^J, e_p + 1) - F^J(q, e_1^J, e_p) \geq F^J(q, e_2^J, e_p + 1) - F^J(q, e_2^J, e_p),$$

and if  $e_1^J \neq e_2^J$ , then the inequality is strict.

Note that if  $F_{23}^J(q, e_d, e_p) \leq 0$ , for all  $e_d$ , then  $F^J(q, e_d^J, e_p + 1) - F^J(q, e_d^J, e_p)$  is decreasing in  $e_d^J$ . Hence,  $e_2^J \geq e_1^J$ . Similarly, if  $F_{23}^J(q, e_d, e_p) \geq 0$ , then  $e_2^J \leq e_1^J$ .

*d)* The proof is very similar to the one in *c)*, and therefore omitted.

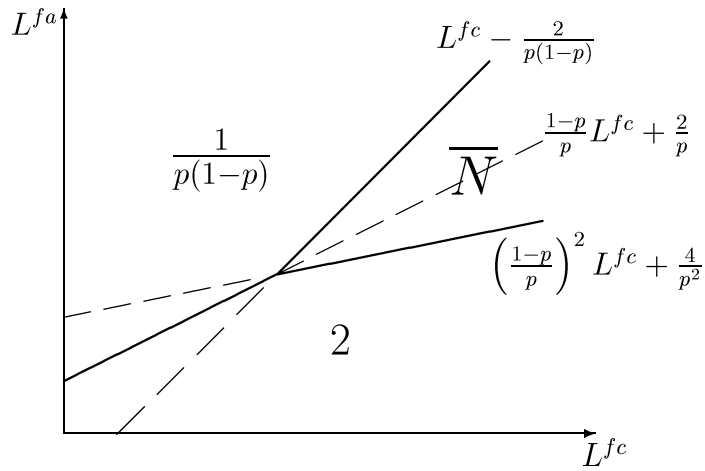


Figure 1  
(Optimal  $N$  when  $q = p$ )

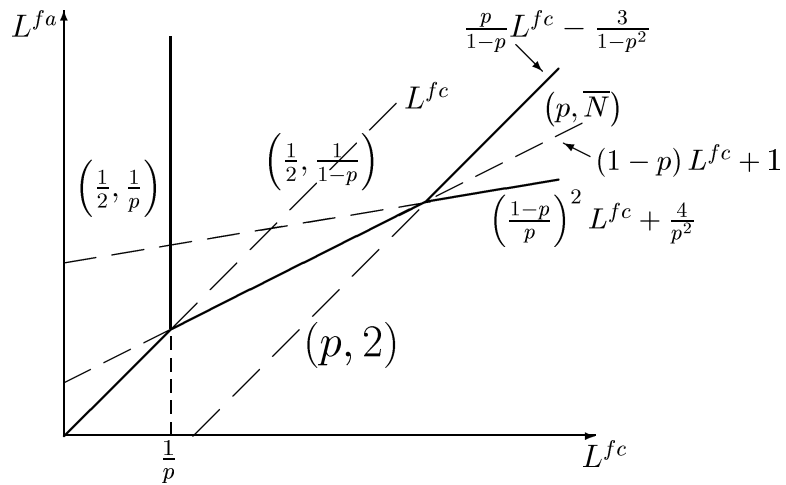


Figure 2  
(Optimal  $(q, N)$ )

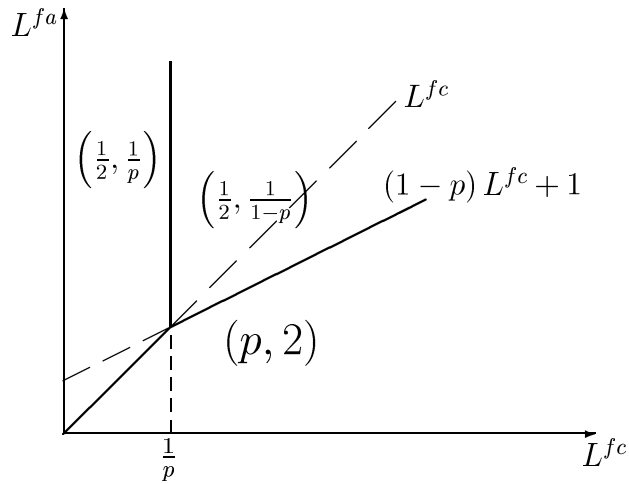


Figure 3

(Optimal  $(q, N)$ ;  $p \leq \frac{1}{2}(\sqrt{5} - 1)$ )

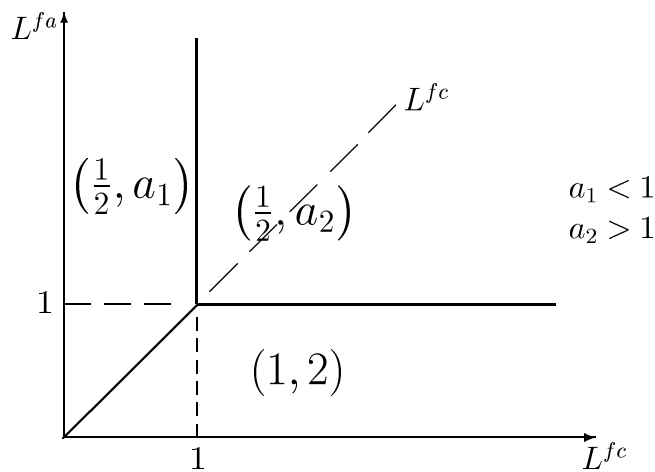


Figure 4

(Optimal  $(q, N)$  when  $p = 1$ )