I. Expressions commonly translated by relations

1. Transitive verbs.
   \[ A_{xy} \equiv x \text{ admires } y; \ C_{xy} \equiv x \text{ causes } y \]

2. Possessive relationships. (**See below.)
   \[ C_{xy} \equiv x \text{ is a cousin of } y; \ H_{xy} \equiv x \text{ has } y; \ P_{xy} \equiv x \text{ is a parent of } y \]

3. Prepositional relationships.
   \[ W_{xy} \equiv x \text{ is with } y; \ F_{xy} \equiv x \text{ is in front of } y \]

4. Comparative relationships.
   \[ S_{xy} \equiv x \text{ is smaller than } y; \ B_{xyz} \equiv x \text{ is between } y \text{ and } z \]

5. Combinations.
   \[ B_{xyz} \equiv x \text{ bought } y \text{ from } z; \ I_{xyz} \equiv x \text{ is introduced to } y \text{ by } z; \ W_{xyz} \equiv x \text{ wants } y \text{ at time } z \]

II. Symbolization

1. Singular sentences
   Put constants in for the variables.
   a) Follow the order in the dictionary.
      Winnipeg is between Vancouver and Toronto.
      \[ B_{xyz} \equiv x \text{ is between } y \text{ and } z \]
   b) Reflexive pronouns (myself, herself, etc.): use the constant twice.
      Betty likes herself. \[ L_{xy} \equiv x \text{ likes } y \]
      Eggbert gave himself a black eye. \[ G_{xyz} \equiv x \text{ gave } y \text{ to } z \]
   c) Passive voice.
      Dictionary can be either in active voice or passive voice.
      Susan and Alice are liked by Bill.
      \[ L_{xy} \equiv x \text{ likes } y \]
      OR \[ L_{xy} \equiv x \text{ is liked by } y \]
   d) Temporal expressions.
      Often these involve three-place relations, with the third slot for a time.
      I want to see my brother now.
      \[ W_{xyz} \equiv x \text{ wants to see } y \text{ at time } z \]
      The Minotaur was killed by Theseus in 1000 B.C.
      \[ K_{tmn} \text{ OR } K_{mtn} \]
2. Quantifier sentences

a) 2-place predicates.

Lxy ≡ x likes y

With one constant

- Everyone likes Chretien. (x)Lxc Chretien likes everyone. (x)Lcx
- Someone likes Chretien. (∃x)Lxc Chretien likes someone. (∃x)Lcx
- Not everyone likes Chretien. ~(x)Lxc Chretien doesn’t like everyone. ~(x)Lcx
- Nobody likes Chretien. ~∃x)Lxc Chretien likes nobody. ~∃x)Lcx

Exercise: Do all these in the passive voice (e.g., Chretien is liked by everyone.)

With no constants

- Everyone likes everyone. (x)(y)Lxy OR (y)(x)Lxy
- Somebody likes someone. (∃x)(∃y)Lxy OR (∃y)(∃x)Lxy

Replacement Rule: Two adjacent quantifiers of the same type can be interchanged.

- Everyone likes someone. (x)(∃y)Lxy; COMPARE (∃y)(x)Lxy
- Someone likes everyone. (∃x)(y)Lxy; COMPARE (y)(∃x)Lxy
- Everyone is liked by someone. (x)(∃y)Lyx; COMPARE (∃y)(x)Lyx
- Someone is liked by everyone. (∃x)(y)Lyx; COMPARE (y)(∃x)Lyx

Warning: Two adjacent quantifiers of different types may not be interchanged.

Examples:

1. Everyone has a mother.

2. Nobody is the mother of everybody.

b) n-place predicates, where n ≥ 2.

Method: Do these in stages.

1) Identify overall form and write down the initial quantifier, usually (x) or (∃x).
2) Write down in ‘pseudo-English’ what is true of x, and then repeat steps 1-2 for this sub-formula.

Examples:

1. For everybody, there is somebody smarter.
   (x)(there is somebody smarter than x)
   (x)(∃y)(y is smarter than x)
   (x)(∃y)Syx where Sxy ≡ x is smarter than y

2. Nothing is bought by everyone from someone.
   ~(∃x)(x is bought by everyone from someone)
   ~(∃x)(y)(y buys x from someone)
   ~(∃x)(y)(∃z)Byxz where Bxyz ≡ x buys y from z

c) QN Rules

Apply them one quantifier at a time.

- ~(x)(∃y)Lxy Not everybody loves somebody.
- (∃x)~(∃y)Lxy Somebody loves nobody. (For some x, there is no y such that x loves y.)
- (∃x)(y)~Lxy Somebody loves nobody. (For some x, for all y, x does not love y.)
Symbolizing Complex Categorical Sentences

Method:

1. Provide a dictionary if needed.
2. Figure out the overall form of the sentence (and write the skeleton sentence).
3. Identify subject class and predicate class.
4. Symbolize subject and predicate in turn, using the ideas of chapter 13 (compound subjects, modified subjects, complex predicates). You may need to repeat steps (2)-(4) for parts of the subject and predicate.

**Example:** Nobody on the yacht likes to swim.

\[\neg(\exists x)\left( (P \cdot O \cdot L) \right)\]

**a)** Handling possessive expressions.

These include explicit possessives (“John’s parents”, “anybody who has money”) as well as implicit possessives such as with (“anybody with money”) and possessive relationships (doctor-patient, parent-child, teacher-student, etc.). There are two methods you can use here:

i) A two-place relation.

\[P \equiv x \text{ is } y\text{'s parent}; \quad A \equiv x \text{ is angry} \]

\[(x)\left( (Pxj \supset Ax) \right)\]

ii) One-place relation plus \(H\)

\[P \equiv x \text{ is a parent}; \quad A \equiv x \text{ is angry}; \quad Hxy \equiv x \text{ has } y\]

\[(x)\left( (Px \cdot Hjx) \supset Ax \right)\]

Advice: use the first method if it will work. The second method will always work, but is more awkward.

**Ex. 1:** Anybody with a mortgage has stomach trouble. **careful with “any”**

\[\overline{(x)\left( (Px \cdot M \cdot H \cdot S) \right)}\]

\[\overline{(x)\left( (Px \cdot (\exists y)(My \cdot Hxy)) \supset (\exists z)(Sz \cdot Hxz) \right)}\]

**Ex. 2:** Not every student who has a parent who is a judge will win a prize.

\[\neg(x)\left( (S \cdot (x \text{ has a parent who is a judge}) \supset (x \text{ will win a prize}) \right)\]

\[\neg(x)\left( (Sx \cdot (\exists y)(Pyx \cdot Jy)) \supset (\exists z)(Pz \cdot Wxz) \right)\]

**Ex. 3:** No doctors treat all of their patients who have no ailments.

\[\neg(\exists x)\left( (D \cdot x \text{ treats all of } x\text{'s patients who have no ailments}) \right)\]

\[\neg(\exists x)\left( (D \cdot x \cdot (Pyx \cdot y \text{ has no ailments}) \supset Ty) \right)\]

\[\neg(\exists x)\left( (D \cdot x \cdot (Pyx \cdot (\exists z)(Az \cdot Hyz)) \supset Tx) \right)\]