Phil 321
Assignment 3

Due: November 30

1. Solve the following zero-sum games, using pure or mixed strategies.

a)

7	5	3
4	1	2

b)

7	8	7
4	6	12
10	8	9

c)

7	5	6
4	8	5

d)

-2	1
3	0

2. Consider the following game. Each of two players has a pile of stones. On alternate turns, each player must place either one or two stones in a circle, which is initially empty. Assume that player A goes first, and B second.

a) Suppose that the player who places the fifth stone wins the game. Draw the game tree, and use it to determine who will win.

b) Can you determine who will win for any number of stones? (There is no need to draw the game tree!)

3. Do the following non-zero-sum games have solutions? Explain.

a)

(1, 5)	(-1, 2)	(3, 1)
(2, 2)	(1, 3)	(4, 2)
(5, 1)	(-1, 2)	(4, 1)

b)

(-1, -1)	(3, 1)
(1, 3)	(2, 2)

4. Suppose that ABC Builders and XYZ Builders are bidding on a contract for an office expansion. The estimated costs of the work are $200,000 for ABC and $220,000 for XYZ. Each company wants to make some profit; assume that the possible bids are $225,000 and $260,000. The lowest bidder will get the contract; in case of a tie, each company has a 50% chance of winning. What should each company's strategy be?

5. Does the "Co-ordination Theorem" on p. 131 of Resnik hold good for 3-person zero-sum games? That is, if two strategy profiles (A, B, C) and (A′, B′, C′) represent equilibria, must the payoffs to each player be the same in each of these equilibria?

6. Consider the following version of the Prisoner's Dilemma, where the numbers reflect positive payoffs:

		B: Co-operate	B: Defect
A	Co-operate	(6, 6)	(2, 8)
	Defect	(8, 2)	(4, 4)

Suppose that the population includes four different groups, each initially comprising 25% of the total population, with the following strategies:

Group A (your group)
Group B (always cheat)
Group C (always co-operate)
Group D (tit-for-tat, but defect on last round)

However, everybody looks alike: you can't tell which group anybody belongs to.

Each year, every individual meets an individual from one of the groups (including possibly his/her own) and they play 3 games. Assume that individuals from the four groups are encountered in the proportions their groups currently represent in the population – for instance, in the first year, you are equally likely to encounter a member of any group. At the end of each year, the proportions of the four groups are adjusted to reflect the total payoff earned by each group relative to all other groups.

More formally, let Pr(A) be the probability of encountering a member of group A, and so on for B, C and D. Initially, Pr(A) = 0.25 and the same is true of B, C and D. Let v(A, B) be the payoff to a member of A from three games with a member of B, and so on. At the end of one year, we will have

$$v(A) = [0.25\, v(A, A) + 0.25\, v(A, B) + 0.25\, v(A, C) + 0.25\, v(A, D)]$$

as the expected total payoff for group A, and a similar expression for groups B, C and D. Then the new probability (or proportion) of A becomes:

$$Pr_1(A) = v(A) / [v(A) + v(B) + v(C) + v(D)]$$

Your objective is to find a strategy for group A that will boost it to 30% population share within two years. This may or may not be possible!

a) Define your strategy.

b) Draw up a 4 by 4 matrix that indicates the outcome when two individuals meet and play 3 games over the year.

c) Calculate the total payoff earned by each group in the first year, and the adjusted populations.

d) Repeat for the second year.

(If you can't find a strategy that boosts group A to 30%, just select the best strategy you can find. If you think it's impossible to reach 30%, explain why.)