Preference Orderings

Resnik

xPy ≡ the agent prefers x to y
yPx ≡ the agent prefers y to x
xIy ≡ the agent is indifferent between x and y

Strict preference: xPy just in case the agent prefers x to y and not vice versa.

Alternative

Weak preference: $x \succ y$ just in case the agent either prefers x to y or is indifferent between them.

Def. (Indifference) xIy ≡ $x \succ y$ and $y \succ x$.
Def. (Strict Preference) xPy ≡ $x \succ y$ and not xIy.

Ordering Conditions

(O1) (Reflexivity) $x \succeq x$

- the agent prefers or is indifferent between *x* and *x*.

(O2) (Transitivity) If $x \succeq y$ and $y \succeq z$, then $x \succeq z$.

- if the agent prefers *x* to *y* and *y* to *z*, then prefers *x* to *z*

(O3) (Connectedness) For any outcomes *x* and *y*, $x \succeq y$ or $y \succeq x$

- for any two outcomes, one is (weakly) preferred

Corollaries:

1. All of Resnik's ordering conditions

2. Results about *Indifference*

   a) *x*I*x* follows from (O1). So indifference is *reflexive*.

   b) *x*I*y* implies *y*I*x*, by definition. So indifference is *symmetric*.

   c) If *x*I*y* and *y*Iz, then $x \succeq z$ and $z \succeq x$, so *x*Iz. So indifference is *transitive*.

   - Indifference as an *equivalence relation*
   - Outcomes fall into *indifference classes*

Justification

- (O1) is unproblematic.

- (O2) is much discussed.

  Empirically false. Sorites examples: imperceptible differences can add up.

  Justifiable as an idealization?

  Money pump argument.

- (O3) is the most unreasonable constraint.

  Example: rescuing Ames or Burns from a fire

  Argument from *revealed preferences*: actual choices demonstrate preferences.

  Objection: choice may be made using a mechanism unrelated to preferences

  Real motivation for (O3) is theoretical: to allow construction of utility functions

Utility functions (numerical preference rankings)

Proposition: If a preference ordering satisfies (O1) - (O3), then we can assign to each outcome x a number $u(x)$, called the *utility* of x, such that:

1) $u(x) > u(y)$ iff $x\mathrm{P}y$;
2) $u(x) = u(y)$ iff $x\mathrm{I}y$.

u is called a *utility function* or *utility scale*.

Ordinal transformations

An *ordinal transformation* $t(u)$ is a function such that for all utility values u and v,

$t(u) \geq t(v)$ iff $u \geq v$.

Positive linear transformations

$t(u) = a\,u + b$, where $a > 0$

Utility functions specified up to a positive linear transformation are called *interval scales*.

Criteria for evaluating decision principles

1. Invariance under ordinal transformations
2. Invariance under expansion of options
3. Ability to take advantage of opportunities
4. Intuitive counterexamples
5. Probabilistic pre-suppositions
6. Arbitrariness

1. Maximin Rule

a) Simple Version

- Find the minimum value for each act.
- Choose the act whose minimum value is maximal.

b) Lexical Version

- In case of a tie, maximize the next-to-minimum value (etc.)

Rationale: Conservatism — avoids worst outcome.

Objections:

1) Lost opportunities
2) Probabilistic pre-suppositions
3) Intuitive counter-examples

2. Minimax Regret

- **Regret value** for each act-state pair: MAX value for state – value for the pair
- For each act, find maximum regret value on its row
- Choose the act which minimizes the maximum regret value

- Can make it a lexical rule in case of ties

Rationale: Make the decision that will minimize lost opportunity.

Objections:

1) not invariant under ordinal transformations; pre-supposes an *interval scale*

2) Not invariant under act expansion

3) Intuitive counterexamples

3. The 'best average' rule

- For each act, find MAX and MIN on its row
- Compute AVG = (MAX + MIN) / 2
- Choose the act which maximizes AVG

Rationale: Avoid acts that might be catastrophic and acts that miss out on great opportunities.

Objections:

1) Pre-supposes interval scale
2) Same counter-example as for Minimax Regret

4. Principle of Insufficient Reason

- Treat each state as equally probable
- Maximize expected utility on this basis.

(Shortcut: sum utilities for the row, and choose the maximal row.)

Rationale: With no good reason to assign any probabilities, assign all equal probability.

Objections:

1) Pre-supposes interval scale.
2) Arbitrariness of assumption of equi-probability.
3) Possibility of catastrophe
4) Incoherence of the equi-probability assumption.