1. Problem Set 3

1c)

	q_1	q_2	$1 - (q_1 + q_2)$
p	7	5	6
1-p	4	8	5

Leads to something like:

$EU(Row) = 3pq_1 - 6pq_2 + 2p - q_1 + 3q_2 + 4$

Write in two ways:

$= p(3q_1 - 6q_2 + 2) + (3q_2 - q_1 + 4)$

$= q_1(3p - 1) + q_2(-6p + 3) + 2p + 4$

Row wants to maximize this; Col wants to minimize it.

Step 1: Set co-efficient of p to 0

$= p(3q_1 - 6q_2 + 2) + \ldots$

So $3q_1 - 6q_2 + 2 = 0$

(*) $q_1 = 2q_2 - 2/3$.

If this is not met, Row will want to set $p = 0$ or $p = 1$.

If $p = 0$, Col will set $q_1 = 1$, and then Row wants to switch to $p = 1$.

If $p = 1$, Col will set $q_2 = 1$, and then Row wants to switch to $p = 0$.

So (*) needed for an equilibrium.

Step 2: Given (*), find q_1 and q_2 that yield minimum value for EU(Row).

Here, want to minimize $3q_2 - q_1 + 4$. Just consider endpoints (0, 1/3) and (5/6, 1). Clearly (0, 1/3).

Step 3: Finally, the second way of writing tells us $p = ½$. For if $p < 1/3$, want to make $q_1 > 0$; if $p < ½$, set $q_2 = 1$; etc.

Skyrms, chapter 1: follow-up

1) Gender equality.

1. Suppose proportion of males in the population is p and $p < 0.5$.
2. Same three strategies:

		Expected # of grandchildren
A	2M	4
B	2F	4[p/1-p]
C	1M, 1F	2 + 2(p/1-p) = 2/(1-p)

3. Heredity: offspring use same strategies. And # of offspring determined by payoffs.

Initial conditions:

Let the initial proportions using strategies A, B and C be 1/3 each.
Total population size N.

Show: Proportion of males gets steadily closer to 1.

Calculation

At the end of one round:

using A = 4N/3
using B = 4N(p/1-p) / 3
using C = (2N/1-p) / 3

So proportion using A = (4N /3) / [4N/3 + 4N(p/1-p) / 3 + (2N/1-p) / 3]
= 2 / [2 + 2p/1-p + 1/1-p]
= 2/3 · (1-p)

The proportion using B is 2/3 p, while the proportion using C is 1/3.

The *grandchildren* that are male: all the A's and half the C's.
The *grandchildren* that are female: all the B's and half the C's.

So: proportion of males is now 2/3 · (1-p) + 1/6.
Can show that this number is between ½ and 5/6, so it has risen above 0.5.
But more importantly,

2/3 (1-p) + 1/6 - ½ = 1/3 -2/3 p = 1/3 (1 - 2p) = 2/3 (0.5 - p),

so that the distance to ½ is now 2/3 of what it was.

2) Male excess.

Claim: A higher male mortality prior to maturity leads to a higher expected number of grandchildren for those who produce more males, until the birth rate/mortality rate combine for roughly equal numbers at maturity.

Three factors:

1. Mortality rate
2. Gender ratio at maturity
3. Discount rate/parental cost for producing males

Result:

1 and 2 cancel out, exactly as in the polygamy argument, yielding no net tendency to produce other than 50/50 birth rate.

Only if we add the assumption that the average cost for raising a male is discounted according to the mortality rate do we get the desired result of an equilibrium with elevated male birth rate.

Justification for discount: mortality spread out over the period of childcare in a normal way.

[**Note:** If instead of this assumption, males all died on the eve of maturity, there would be no discount rate and the birth rate would remain 50/50.]