CHAPTER XII

THE CONSUMPTION FUNCTION

1. INTRODUCTION

The literature on the consumption function aims to provide a theory that explains some of the time series and cross section correlations between consumption and income, and sometimes also correlations among consumption, wealth, and interest rates. The subject has served as an important laboratory for building dynamic theories that are sufficiently simple and tractable to guide interpretation of the data. Thus, important efforts in applied dynamics of Milton Friedman, John F. Muth, Robert E. Lucas, Jr. and Robert E. Hall were inspired by the subject of the consumption function. These efforts have had ramifications for applied work throughout dynamic macroeconomics.

Initial work on the subject was inspired by paradoxes that emerged from applying the early Keynesian consumption function of the form $C_t = a + bY_t$. From the cross sections came estimates of the marginal propensity to consume b that were much less than estimates from the time series, which seemed to require reconciling. In aggregate time series, consumption is a much smoother series over the business cycle than is income. To model this pattern statistically, consumption was posited to be a distributed lag of income, making consumption a smoothed version of income. Economic theories were sought to interpret this distributed lag. This led to Friedman's work on permanent income, which was inspired by the idea of using Irving Fisher's theory of intertemporal consumption allocation to account for the dynamics in the data. Friedman did not make completely tight or explicit the link between Fisher's theory and the regressions that he studied, but subsequent work has. John F. Muth soon built upon Friedman's work and discovered specifications of the stochastic process for income that assured optimality for Friedman's assumed adaptive expectations, or geometric lag distribution, for permanent income. Implicit in Muth's argument was Lucas's critique of econometric policy evaluation. In 1973 Lucas used the consumption function of Friedman and Muth as an example to illustrate his critique. By using stochastic optimization theory explicitly, Robert

Hall was able to characterize additional interesting and easily testable implications of the permanent income theory. In effect, Hall noted that the household's Euler equation restricts the projection of consumption on lagged information. Hansen and Sargent did work that obtains additional characterizations involving projections in alternative directions to the one used by Hall.

This chapter studies a version of the permanent income theory of consumption. The theory is developed by studying the intertemporal optimum problem faced by a single consumer who is confronted with an exogenous stochastic process for labor income and the opportunity to borrow or lend a limited amount at a fixed interest rate.[1] Using the tools for prediction and dynamic optimization that are described in Chapters XI and IX, we can describe precisely the restrictions that this theory places on time series data. We use these tools to compute the projections of consumption on various combinations of assets and income that are implied by the theory. In particular, we compute the projection of consumption on three subspaces. We compute the projection of consumption on current and past labor income alone, the projection of consumption on current and past values of "total income" which includes labor income plus interest on assets, and the projection of consumption on current and past labor income and current assets. These projections are the theoretical counterparts of various sample regressions that have been computed in the empirical literature on consumption.

2. THE CONSUMER'S CHOICE PROBLEM

A representative consumer is assumed to choose a strategy for consumption that maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1 \tag{1}$$

where c_t is consumption at t, E_t is the mathematical expectation operator conditioned on information known to the consumer at t, and u is a one period utility function satisfying $u' > 0$, $u'' < 0$.[2] The objective (1) is maximized with respect to $\{c_t, A_{t+1}\}_{t=0}^{\infty}$ subject to the sequence of budget constraints

$$A_{t+1} = R_t[A_t + y_t - c_t], \qquad t = 0, 1, \ldots \qquad A_0 \text{ given}, \tag{2}$$

where $R_t \geq 1$ is the gross rate of return on savings between periods t and $t+1$, A_t is assets (or indebtedness, if negative), and y_t is noncapital or labor income at t. We assume that y_t is a given stochastic process, outside of the control of the agent, with features to be specified below. The consumer is assumed to know the

[1] Multiple agent equilibrium versions of the permanent income model are described in Sargent (1986). See especially the chapter and exercises on the Bewley-Townsend model.

[2] The quadratic utility function used below, $u_0 + u_1 c_t - (u_2/2)c_t^2$, satisfies these restrictions only for c_t in the interval $0 \leq c_t < u_1/u_2$.

law of motion of y_t. At time t, the consumer's information is assumed to include at least $\{y_{t-s}, s \geq 0\}$ and A_t. In the interests of obtaining a version of the permanent income model, we assume that $R_t = R > 1$ for all t. We shall assume further that $\beta R^2 > 1$. We also assume that $\{y_t\}$ is a stochastic process of mean exponential order less than β^{-1}, so that $\lim_{j\to\infty} E_t y_{t+j}\beta^j = 0$ for all t.

In this model, some device is needed to rule out indefinitely large and growing borrowing on the part of the consumer. We shall impose the condition

$$E_0 A_t \geq M > -\infty \qquad \text{for all } t \tag{3}$$

where M is a finite but possibly very large negative number. Subject to the condition (3) on borrowing, we solve the difference equation (2) with $R_t = R$ to obtain

$$\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j E_t c_{t+j} = A_t + \sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j E_t y_{t+j}, \tag{4}$$

which states that the expected present value of consumption at t equals the expected present value at t of labor income plus initial nonlabor assets A_t.

Substituting the constraints (2) into the objective function (1), the problem is to choose a strategy for A_t to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(y_t + A_t - R^{-1}A_{t+1})$$

subject to the boundary conditions (3) and A_0 given. Proceeding as in Chapter IX and XIV, the Euler equations can be found to be[3]

$$E_t R\beta u'(c_{t+1}) = u'(c_t) \qquad t = 0, 1, \ldots$$

or

$$E_t u'(c_{t+1}) = (\beta R)^{-1} u'(c_t) \tag{5}$$

where $c_t = y_t + A_t - R^{-1}A_{t+1}$. Notice that if a condition like (3) were not imposed, (5) could be satisfied with $u'(c_{t+1}) = u'(c_t) = 0$, which is an instruction to consume an unbounded amount each period, and to borrow at the gross real rate R to support this plan. This would imply an unbounded volume of indebtedness that would violate (3).

[3] Using the method of Chapter IX, the Euler equation for a nonstochastic version of the problem is found to be $R\beta u'(c_{t+1}) = u'(c_t)$. The appropriate Euler equation for the stochastic version of the problem is found formally by replacing all functions of random variables in the Euler equation for the nonstochastic version of the problem with conditional expectations, conditioned on information available at t. (Technically, this involves an exchange of order of an integration and a differentiation operation.) Stochastic Euler equations and the relationship between stochastic and nonstochastic linear-quadratic control problems are described more thoroughly in Chapter XIV.

Equation (5) is Robert Hall's (1978) result that the marginal utility of consumption follows a univariate first order Markov process, and that no other variables Granger cause the marginal utility of consumption. In the special case that $(\beta R)^{-1} = 1$, (5) predicts that the marginal utility of consumption is a "martingale."[4] For specific forms of the utility function, Hall tested this implication by estimating vector autoregressions including the marginal utility of consumption and testing the zero restrictions associated with lack of Granger causality.

To obtain tighter restrictions, we consider the special case in which utility is quadratic,

$$u(c_t) = u_0 + u_1 c_t - \frac{u_2}{2} c_t^2 \tag{6}$$

with $u_0, u_1, u_2 > 0$, so that $u'(c_t) = u_1 - u_2 c_t$. We also assume that the satiation level of consumption, u_1/u_2, is large relative to the typical value of y_t. More precisely, we assume for all $t \geq 0$, with probability arbitrarily close to one we have the condition

$$E_t \sum_{j=0}^{\infty} R^{-j} y_{t+j} < u_1/u_2.$$

Such a condition prevents simply consuming at the satiation level u_1/u_2 for $t \geq 0$.

When (6) is assumed, (5) implies that

$$E_t c_{t+1} = \alpha + (\beta R)^{-1} c_t \tag{7}$$

where

$$\alpha = \frac{u_1(1 - (\beta R)^{-1})}{u_2}.$$

Recursions on equation (7) imply that

$$E_t c_{t+j} = \alpha\left[\frac{1-\gamma^j}{1-\gamma}\right] + \gamma^j c_t, \tag{8}$$

where $\gamma \equiv (\beta R)^{-1}$. Substituting (8) into (4), using $\beta R^2 > 1$ and $R > 1$, and solving for c_t gives

$$c_t = \left(\frac{-\alpha}{R-1}\right) + \left(1 - \frac{1}{\beta R^2}\right)\left[\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j E_t y_{t+j} + A_t\right]. \tag{9}$$

[4] A "martingale" is a stochastic process x_t that satisfies $E_t x_{t+1} = x_t$.

Equations (9) and (8) characterize the optimal consumption plan under the borrowing limit (3) and our assumption that $\{y_t\}$ is small relative to u_1/u_2.[5]

In the special case that is often considered, it is assumed that $\beta R = 1$, in which case (8) and (9) become the consumption plan

$$E_t c_{t+j} = c_t, \qquad \text{for all } j \geq 1 \tag{10}$$

$$c_t = \left(1 - \frac{1}{R}\right)\left[\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j E_t y_{t+j} + A_t\right]. \tag{11}$$

Equations (10) and (11) form a version of the "permanent income theory." The expression on the right side of (11) can be interpreted as the consumer's "permanent income:" it is the rate at which the consumer expects to be able to consume while leaving intact total wealth, including both the "nonhuman" part A_t and the human part $\sum_{j=0}^{\infty}\left(\frac{1}{R}\right)^j E_t y_{t+j}$. Since equation (11) is exact and involves no random disturbance, equations (10) and (11) immediately imply Hall's result that permanent income follows a "martingale."[6]

3. THE PROJECTION OF CONSUMPTION ON LABOR INCOME

Collecting results in the special case in which $\beta R = 1$, we have the structure

$$A_{t+1} = R[A_t + y_t - c_t] \tag{2}$$

$$E_t c_{t+j} = c_t \qquad \text{for } j \geq 1. \tag{10}$$

$$c_t = \left(1 - \frac{1}{R}\right)\left[\sum_{j=0}^{\infty} R^{-j} E_t y_{t+j} + A_t\right]. \tag{11}$$

[5] While the model described in the text is useful for expositing the permanent income theory of consumption, it has some undesirable limiting properties. The equilibrium law of motion for assets satisfies

$$A_{t+1} = \left(\frac{1}{\beta R}\right)A_t + \frac{\alpha R}{(R-1)} + R y_t - \left(R - \frac{1}{\beta R}\right)\sum_{j=0}^{\infty} R^{-j} E_t y_{t+j}.$$

This equation is derived by substituting (9) into the budget constraint (2). When $\beta R > 1$, the above equation implies that $E_t A_{t+j}$ converges as $j \to \infty$. Further, from equation (8) and the definition of α, when $\beta R > 1$, $\lim_{j\to\infty} E_t c_{t+j} = u_1/u_2$. Thus, it happens that, when $\beta R > 1$, the system can be expected to converge to a level of asset holdings such that consumption varies stochastically around the bliss level of consumption u_1/u_2. This is unsatisfactory because it implies that eventually the marginal utility of consumption fluctuates about zero. Despite such undesirable limiting behavior, the model behaves sensibly at low levels of consumption.

[6] See Hall's paper (1978) for implementation of econometric tests of this martingale characterization. Further work along this line is executed by Hansen and Singleton (1982) who provide an econometric method for imposing and testing restrictions implied by Euler equations in which the econometrician faces no unobserved components. An alternative econometric approach to implementing the restrictions on a vector imposed by an Euler equation is described by Hansen and Singleton (1983). The papers by Hansen and Singleton (1982, 1983) and the references in them describe the relationship between consumption and asset prices.