Cheating in Rank-Order Tournaments*

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Abstract

Tournaments are a commonly used mechanism for the allocation of resources. Examples include promotion tournaments, sporting events, patent races, and the classroom environment (especially when there exists a prescribed grade distribution). Lazear and Rosen (1981) demonstrate that tournaments in the workplace can actually dominate a wage scheme whereby the workers are paid their marginal product in terms of efficiency. In a tournament, however, competitors have incentive to engage in undesirable activities, or “cheating”, in order to gain an advantage. Examples of such activities include the taking of steroids, plagiarism, and “creative accounting”. This paper considers the problem of deterrence of such activities and finds that there exist special considerations that are not present in a traditional model of crime deterrence. For example, an agent’s decision to cheat depends on the whether others are cheating or not, and so there may exist multiple equilibria. In addition, if the effort that competitors in the tournament exert is valuable, then complementarities between effort and cheating means that a low (but non-zero) expected sanction, and especially a low probability of detection, can be optimal. Finally, in situations where it is important that the tournament determine a winner, and that the winner not have cheated, this paper demonstrates that awarding the prize to the (ex-post) highest ranked contestant can lead to less effort being expended by every contestant.

Key Words: Crime, Enforcement, Cheating, Tournament Theory, Law and Economics;
JEL: K42; J41

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1 Introduction

Tournaments are a commonly used mechanism for the allocation of resources. Examples include promotion tournaments, sporting events, patent races, and the classroom environment (especially when there exists a prescribed grade distribution). Indeed, rank-order tournaments have many desirable features as a mechanism, particularly in the work place (see, for example Lazear and Rosen (1981), Carmichael (1983), Waldman (1990), Prendergast (1993), Bernhardt (1995), Bognanno (2001), Zabojnik and Bernhardt (2001), and Choi and Gulati (2004)). In a tournament, however, competitors have incentive to engage in undesirable activities, or “cheating”, in order to gain an advantage. Examples of such activities include the taking of steroids, plagiarism, “creative accounting” and sabotage. This paper considers the problem of deterrence of such activities and finds that there exist special considerations that are not present in a traditional model of crime deterrence. For example, an agent’s decision to cheat depends on the whether others are cheating or not, and so there may exist multiple equilibria. In addition, if the effort that competitors in the tournament exert is valuable, then complementarities between effort and cheating means that a low (but non-zero) expected sanction, and especially a low probability of detection, can be optimal. Finally, in situations where it is important that the tournament determine a winner, and that the winner not have cheated, this paper demonstrates that awarding the prize to the (ex-post) highest ranked contestant can lead to less effort being expended by every contestant.

The fact that tournaments can provide incentive for agents to engage in undesirable activities has already been noted in the economic literature. In particular, it has been noted that agents have incentive to try to sabotage each other. Lazear (1989) noted that the ability of agents to sabotage each other reduces the attractiveness of a tournament. Similarly, Konrad (2000) shows that sabotage in rent-seeking contests can be an attractive option for the contestants, but will be welfare reducing. However, since sabotage helps not only the saboteur, but all other agents not directly affected, sabotage is a kind of public good. Thus as the number of contestant increases, the free rider problem becomes more severe, and the amount of sabotage decreases. Chen (2003) considers equilibrium decisions to sabotage and demonstrates that more able
contestants are the target of more sabotage\(^1\), leading to the result that the highest ability contestant is not necessarily most likely to win the tournament. Chen then considers modifications to the tournament design, such as pay equality, promotion based on seniority, or group compensation, which can reduce the incentive to invest in sabotage. This paper considers the deterrence of sabotage, or cheating, in the traditional law and economics sense. That is, we consider a tournament in which the principal is able to monitor\(^2\) the contestants to try to detect cheating behaviors. If any contestants are caught cheating, then the principal can impose some penalty. This paper notes that the tournament environment provides some novel issues for deterrence.

In the classic model of crime (see, for example, Becker (1968) and Ehrlich (1996)), an individual’s decision to commit crime depends only on the benefit derived from the crime, the probability of being caught, and the penalty. If the agent is risk neutral, then the probability of being caught and the penalty can be reduced to simply the expected penalty. In general, it does not depend on the decisions of others to commit crime\(^3\). When agents compete in a tournament, however, one agent’s decision to cheat affects the expected payoff of the others. Most notably, the benefit to cheating may increase when others cheat. This can lead to multiple equilibria. It should be noted, however, that the extent to which the benefit derived from cheating increases depends on the punishment scheme. If winners are stripped of the prize when caught cheating, and the prize then handed to the second place contestant (provided they did not get caught cheating), then the benefit to cheating is significantly reduced, provided the probability with which the other cheaters are caught is above some minimal level. Stripping winners of their prize and awarding it to the next best contestant can thus be a more effective deterrent than levying a large fine (in particular, larger than the value of the prize) on cheaters, but not stripping the winner of the prize.

It should further be noted that an important condition for the optimality of tour-

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\(^1\)Experimental results by Harbring and al. (2004) provide support for this.
\(^2\)Kim, Qin and Yu (2002) talk about monitoring, but in a very different environment where third party monitoring is used to deter organizer from accepting bribe for the contestant.
\(^3\)There are exceptions to this. For example, there may exist congestion effects in deterrence, social norms about the stigma associated with crime, or coordination issues surrounding occupational choice (see Sha (1991) or Murphy, Schleifer and Vishny (1993) for further discussion on those topics).
naments in Lazear and Rosen (1981) is homogeneity among the contestants. If one of the contestants has a lower marginal productivity, (or higher cost of effort), it can seriously handicap his or her chance to “win” the tournament (or to rank well), resulting in lower incentive to provide effort. However, in small tournaments, the reduction of effort by one contestant also reduces the incentive for the more able contestants to provide effort. In other words, heterogeneity can lead to under-provision of effort by all agents. Lazear and Rosen propose to organize tournaments with multiple distinct groups, so that agents in each tournament are similar. However, this may not always be possible, especially when the number of possible contestants is small. Singh and Wittman (2001) propose that the tournament by designed such that the winner is not simply the one with the greatest output. In particular, they propose that agents receive “lottery tickets” in proportion to their output, and that low ability contestant get more tickets per unit of output. This has the effect of levelling the field, leading to greater incentive to exert effort. However, this solution requires that the principal know the ability of each contestant. In both papers, it is assumed that agents comply with the rules of the tournament.

This paper considers a tournament with heterogeneous agents that choose not only to provide productive effort, but can also engage in cheating activities. Such activities are unproductive in the sense that they do not increase the real output (they may in fact reduce real output, as in the case of sabotage), but they are beneficial to the agent because they increase the chance of winning the tournament. In this case, cheating (especially on behalf of lower ability contestants) can have the effect of making the tournament more competitive (equalize the probabilities of winning). As noted above, this leads to greater incentive to provide effort on behalf of every agent, potentially resulting in a higher output compared to an environment without cheating. As a result, the organizer may not want to deter cheating completely. In particular, the principal may wish to choose the penalty and probability of detection such that low ability agents will cheat. It should be noted that it is not necessary for the tournament organizer to observe the individual ability, as in Singh and Wittman (2001).
2 The Model

We consider the following model of a rank-order tournament, based on Lazear and Rosen (1981). Two risk neutral agents compete in a given tournament, where the winner receives a payoff of $A$. The loser’s payoff is normalized to zero. It should be noted that all results generalize to an $N$-player tournament. If both agents comply with the rules of the tournament (do not cheat), then agent 1 wins the tournament with probability $p$. Agents, however, have the ability to engage in illegal activities in order to improve their chances of winning. If agent 1 cheats and 2 does not, then the probability that 1 wins the tournament is $p' > p$. If 2 cheats and 1 does not, the probability that 1 wins is $p'' < p$. For simplicity, it is assumed that if both cheat then the effects cancel out and the probability with which 1 wins is $p$.

Cheating is not a productive activity. As such, the tournament organizer, or principal, may wish to discourage it. The principal can therefore choose to monitor the participants to try to detect any cheating. Denote by $\pi$ the probability that the principal catches an agent who cheats. The principal can also set the penalty for cheating. We consider two possible sanction schemes. The first scheme dictates that if the winner of the tournament is caught, then they are stripped of the award. The second penalty scheme also strips the winner of the prize (if caught cheating), but then awards the prize to the other contestant (provided that they did not cheat). If both agents are caught cheating, the prize is not awarded. This latter scheme shall be referred to as “re-awarding”.

The timing is as follows. First, the tournament organizer announces the prize for the winner, the probability of detection, and the penalty scheme. Agents decide whether to cheat or not simultaneously. A winner is determined and the organizer audits the players for cheating. If any cheaters that are detected, the penalty is imposed. The prize is then awarded.

2.1 No Re-Award of Prize

We begin by considering the case in which the winner of the tournament is stripped of the prize if they are discovered to have cheated, but the prize is not re-awarded.
For a given award and probability of detection, $A$ and $\pi$ respectively, the decision to cheat or not is a $2 \times 2$ game. This $2 \times 2$ game is given by

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<th>Don’t Cheat</th>
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<td>$(1 - \pi) p' A$</td>
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<tr>
<td>Don’t</td>
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<td>$p A$</td>
</tr>
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It should first be noted that the benefits to cheating are not independent of the decision of the other player. For example, when agent 2 is not cheating, the benefits from cheating for agent 1 are

$$[(1 - \pi) p' - p] A$$

which may be either positive or negative, depending on $p, p'$ and $\pi$. When agent 2 is cheating, the benefits to agent 1 from cheating are

$$[(1 - \pi) p - p''] A$$

which again may be either negative or positive. As mentioned above, these benefits are typically not the same. Specifically, the benefits to cheating when 2 does not cheat are equal to the benefits when 2 does cheat only when $(1 - \pi) p' - p = (1 - \pi) p - p''$, which can be rewritten as $\pi = \frac{p' + p'' - 2p}{p' - p}$. Note that it is possible for the right hand side of this last equality to be negative, in which case there is no $\pi \in [0, 1]$ such that the benefits to player 1 from cheating are the same when 2 cheats as when 2 does not.

The fact that the benefits to cheating depend on the choice of the other player does not rule out dominant strategies, however. When $\pi = 0$ so that cheaters are never caught, each player has a dominant strategy to cheat. Further, when $\pi = 1$ so that cheaters are always caught, each player has a dominant strategy not to cheat. For some intermediate values, however, the decision to cheat depends on the decision of the other player. In fact, as is demonstrated in the following example, there may exist values of $\pi$ such that there are multiple equilibria to the game.
Example 1: Let $p = \frac{1}{2}$, $p' = \frac{3}{4}$ and $p'' = \frac{1}{4}$. In this case, there exists an equilibrium in which neither agent cheats when $\pi \geq \frac{1}{3}$ and an equilibrium in which both cheat when $\pi \leq \frac{1}{2}$. Thus when $\pi \in \left[\frac{1}{3}, \frac{1}{2}\right]$, both equilibria exist.

The following proposition gives the equilibria to this game as a function of $\pi$.

Proposition 1: Let $\bar{\pi} = \min \left\{ \frac{p - p''}{p}, \frac{p' - p}{1 - p} \right\}$ and let $\bar{\pi} = \max \left\{ \frac{p' - p}{p'}, \frac{p - p''}{1 - p''} \right\}$. There exists an equilibrium in which both agents cheat $\forall \pi \leq \bar{\pi}$. There exists an equilibrium in which neither agent cheats $\forall \pi \geq \bar{\pi}$. A sufficient condition for $\bar{\pi} > \bar{\pi}$ is that the game is symmetric. That is, $p = \frac{1}{2}$ and $p' = 1 - p''$.

It should be noted that if it is important to the tournament organizer to deter cheating, then she must choose $\pi > \bar{\pi}$ in order to eliminate cheating in all possible equilibria.

2.2 Re-Award of Prize

We now consider the case in which the winner of the tournament is stripped of the prize if they are discovered to have cheated and the prize is then given to the other contestant, provided they were not found to have cheated as well. Note that, for a given probability of detection, the probability with which an agent wins the tournament is weakly greater for each cheating profile when the prize is re-awarded than when it is not. In particular, the probability that an agent wins is higher under re-awarding when the other agent cheats. If neither agents cheat, then the probability that 1 wins is as before, $p$. If agent 1 is the only one that cheats, then the probability with which he/she wins is also as before, $(1 - \pi) p'$. However, agent 2 now wins the prize when 1 is found to have cheated. Thus agent 2’s probability of winning is $1 - p' + \pi p' = 1 - (1 - \pi) p'$, which is greater than when the prize is not re-awarded. If agent 2 is the sole cheater, agent 1 wins with probability $p'' + (1 - p'') \pi = \pi + (1 - \pi) p''$, while agent 2 wins with the same probability as when the prize is not re-awarded, $(1 - \pi) (1 - p'')$. Finally, if both agents cheat, then 1 wins with probability $(1 - \pi) [\pi + (1 - \pi)p]$ and 2 wins with probability $(1 - \pi) [1 - (1 - \pi) p]$. Both of these probabilities are greater than when the prize is not re-awarded. The $2 \times 2$ game
is given by

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When the probability of being caught when cheating is zero, each agent has a dominant strategy to cheat, as before. When the probability of being caught is one, agents have a dominant strategy not to cheat, also as before. Also as before, the benefits from cheating are not independent of the other contestant’s decision. In this case, however, when the other agent cheats, the benefits to cheating are reduced, as noted above. An interesting implication is that multiple equilibria is no longer possible.

**Lemma 1:** When prizes are re-awarded, multiple equilibria do not generically occur.

**Proposition 2:** The lowest level of monitoring required to obtain neither agent cheating as a unique equilibrium is lower when the prize is re-awarded than when it is not.

3 Conclusion

To come...
4 Appendix

Proof to Proposition 1:

First, let us consider when both agents will cheat in equilibrium. Given that agent 2 is cheating, agent 1 will choose to cheat when

\[(1 - \pi) p A \geq p^{''} A \]
\[\pi \leq \frac{p - p^{''}}{p} \]

Given that agent 1 is cheating, agent 2 will choose to cheat when

\[(1 - \pi) (1 - p) A \geq (1 - p') A \]
\[\pi \leq \frac{p' - p}{1 - p} \]

Thus there exists an equilibrium in which both agents cheat when \(\pi \leq \min \left\{ \frac{p - p^{''}}{p}, \frac{p' - p}{1 - p} \right\} = \bar{\pi} \).

Now suppose that agent 2 does not cheat. In this case, agent 1 will not cheat when

\[p A \geq (1 - \pi) p' A \]
\[\pi \geq \frac{p' - p}{p'} \]

When agent 1 is cheating, agent 2 will not cheat when

\[(1 - p) A \geq (1 - \pi) (1 - p'') A \]
\[\pi \geq \frac{p - p''}{1 - p''} \]

Thus there exists an equilibrium in which both agents cheat when \(\pi \geq \max \left\{ \frac{p' - p}{p'}, \frac{p - p''}{1 - p''} \right\} = \bar{\pi} \).

When \(\bar{\pi} \geq \bar{\pi} \), then both equilibria exist for all \(\pi \in [\bar{\pi}, \bar{\pi}] \). Let us consider the case in which \(p = \frac{1}{2} \) and \(p' = 1 - p'' \), so that the game is symmetric. In this case, \(\bar{\pi} = \max \left\{ \frac{p' - \frac{1}{2}}{p'}, \frac{p - \frac{1}{2}}{p'} \right\} = \frac{2p' - 1}{2p'} > 0 \), and \(\bar{\pi} = \min \left\{ \frac{p' - \frac{1}{2}}{2}, \frac{p - \frac{1}{2}}{2} \right\} = 2p' - 1 > 0 \). Since \(p' > p = \frac{1}{2} \), we have that \(2p' > 1 \) and so \(\bar{\pi} > \bar{\pi} \). ■

Proof to Lemma 1:
We first consider the conditions for both agents to cheat in equilibrium. Given that agent 2 is cheating, agent 1 will want to cheat if
\[
(1 - \pi) \left[ \pi + (1 - \pi) p \right] A \geq \left[ \pi + (1 - \pi) p' \right] A \\
\pi \left[ \pi + (1 - \pi) p \right] \leq (1 - \pi) (p - p'')
\]

Given that agent 1 is cheating, 2 will not cheat if
\[
(1 - \pi) \left[ 1 - (1 - \pi) p \right] A \geq \left[ 1 - (1 - \pi) p' \right] A \\
\pi \left[ 1 - (1 - \pi) p \right] \leq (1 - \pi) (p' - p)
\]

Thus there will be an equilibrium in which both agents cheat when these two conditions are satisfied.

Now let us consider 1’s decision when 2 is not cheating. In this case, 1 will not cheat when
\[
pA \geq (1 - \pi) p'A \\
p\pi \geq (1 - \pi) (p' - p)
\]

Given that 1 is not cheating, 2 will not cheat when
\[
(1 - p) A \geq (1 - \pi) (1 - p'') A \\
(1 - p) \pi \geq (1 - \pi) (p - p'')
\]

In order for multiple equilibria to exist, there must be a \( \pi \) such that all 4 conditions are satisfied. First, assume that \( \pi > 0 \), since each agent has a strictly dominant strategy to cheat in when \( \pi = 0 \). Then, note that it must be that
\[
\pi \left[ \pi + (1 - \pi) p \right] \leq (1 - \pi) (p - p'') \leq (1 - p) \pi,
\]
which requires
\[
\pi \geq 1 - 2p \\
\pi \leq \frac{1 - 2p}{1 - p}
\]

Since \( \pi > 0 \), it must be that \( p < \frac{1}{2} \).

It must also be that
\[
\pi \left[ 1 - (1 - \pi) p \right] \leq (1 - \pi) (p' - p) \leq p\pi,
\]
which requires (assuming \( \pi > 0 \))
\[
\pi \left[ 1 - (1 - \pi) p \right] \leq p \\
\pi \leq \frac{2p - 1}{p}
\]
This requires $p > \frac{1}{2}$, which is a contradiction.

**Proof to Proposition 2:**

From Lemma 1, we have that when neither agents cheat in equilibrium, it is unique. This occurs when $\pi \geq \max \left\{ \frac{\nu - p}{\nu'}, \frac{\nu - p''}{1 - p'} \right\}$. Recall from Proposition 1 that this is the same condition for there to exist an equilibrium in which neither agent to cheat when the prize is not re-awarded. However, since there exists the possibility of multiple equilibria when the prize is not re-awarded, the tournament organizer must choose $\pi \geq \max \bar{\pi}, \widetilde{\pi}$ in order to ensure that neither agent will cheat. When the prize is re-awarded, however, it is always sufficient for the organizer to choose $\pi \geq \max \left\{ \frac{\nu - p}{\nu'}, \frac{\nu - p''}{1 - p'} \right\} = \bar{\pi}$. ■
5 Bibliography


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