

External Appendix to “Overcoming Participation Constraints”;  
Detailed Calculations For the Example in Section 6.4.

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**Abstract**

This document provides step by step calculations and explanations that are omitted from the presentation of the example in Section 6.4 of the main paper.

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# 1 Introduction

To make this document self contained we are duplicating several expressions from the main document. The reader may also note that we are not adding the extra conditional lump sum payment  $\frac{\delta K}{4n}$  when we introduce the veto game. Clearly, adding this term would only increase the probability of a veto, so the conclusion that the probability of a veto converges to 1 as  $K$  goes to infinity extends immediately to the mechanism under consideration in the paper.

## 2 Primitives

The reader may note that there are many sets of primitives that generate the same maximized surplus function. The primitives below are picked mainly to make the calculations convenient. The reader may also take note that once the maximized surplus function has been derived, there is no need to return to the primitives, since interim expected payoffs from participation in the mechanisms depend only on the maximized surplus function. However, it is still important to specify some underlying primitives: not every surplus function  $s_k : \Theta_k \rightarrow R$  and reservation payoff  $r_k : \Theta_k \rightarrow R^n$  are consistent with surplus maximization, so we need to demonstrate existence of primitives generating the surplus function and reservation payoff function in the example.

### 2.1 Costs and Utility Functions

Let  $n = 2$  and assume that, for each  $k$  and  $i = 1, 2$ , the single issue  $k$  type space is given by  $\Theta^i = \{l, m, h\}$ . Assume that the set of alternative resolutions of issue  $k$  are  $D_k = \{d_k^0, d_k^l, d_k^m, d_k^h\}$ . In terms of interpretation it is useful to think of  $d_k^0$  as the “status quo” outcome, whereas  $d_k^l, d_k^m$  and  $d_k^h$  are surplus maximizing alternatives for type profiles  $ll, mm$  and  $hh$  respectively.

The cost function is given by

$$C_k(d_k) = \begin{cases} 0 & \text{if } d_k = d_k^0 \\ \frac{(k+1)^2}{k} & \text{if } d_k = d_k^l \\ \frac{k(k+1)^2}{2} & \text{if } d_k = d_k^m \\ 2(k+1)^3 & \text{if } d_k = d_k^h \end{cases},$$

where  $\varepsilon > 0$ . Further restrictions on  $\varepsilon$  will be derived below. Assume that the valuation functions for the three types are given by;

$$v_k^i(d_k, l) = \begin{cases} 0 & \text{if } d_k = d_k^0 \\ \frac{1}{2} \frac{(k+1)^2}{k} & \text{if } d_k = d_k^l \\ 0 & \text{if } d_k = d_k^m \\ \frac{(k+1)^2}{4} \left[ k - \frac{1}{k} \right] & \text{if } d_k = d_k^h \end{cases}$$

for  $\theta_k^i = l$ ,

$$v_k^i(d_k, m) = \begin{cases} 0 & \text{if } d_k = d_k^0 \\ \frac{3}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) & \text{if } d_k = d_k^l \\ \frac{k(k+1)^2}{2} - 2\varepsilon k(k+1)(k+2) & \text{if } d_k = d_k^m \\ \frac{k(k+1)^2}{4} - 3(k+1)^3 & \text{if } d_k = d_k^h \end{cases}$$

for  $\theta_k^i = m$ , and

$$v_k^i(d_k, h) = \begin{cases} 0 & \text{if } d_k = d_k^0 \\ -\varepsilon(k+1) & \text{if } d_k = d_k^l \\ -\frac{k(k+1)^2}{2} & \text{if } d_k = d_k^m \\ 3(k+1)^3 - \frac{k(k+1)^2}{4} & \text{if } d_k = d_k^h \end{cases}$$

for  $\theta_k^i = h$ .

## 2.2 Reservation Payoffs

Since  $C_k(d_k^0) = 0$  we take  $d_k^0$  as the status quo outcome. Hence,  $r_k^i(\theta_k^i) = v_k^i(d_k^0, \theta_k^i) = 0$  for all  $\theta_k^i$ .

## 3 The Surplus Maximizing Rule

The primitives in the example have been constructed to make sure that: i)  $d_k^l$  is optimal given type profiles  $lm$  and  $ml$  (which generates surplus of order  $k$ ), and; ii)  $d_k^m$  is optimal given type profile  $mm$  (surplus of order  $k^3$ ), and; iii) that  $d_k^h$  is optimal for profiles  $lh, hl$  and  $hh$  (surplus of order  $k^3$ , and, which is of some importance, that the surplus from  $hh$  is substantially larger than that from  $mm$ ). As will be seen below, this will make it possible for us to construct sequences where there is a significant effect on interim expected payoffs when types that are realized with an arbitrarily small probability opt out from the mechanism. Below, we provide the details of the derivation of the maximized surplus function.

### 3.1 Type Profile $ll$

It is immediate that  $v_k^1(d_k^0, l) + v_k^2(d_k^0, l) - C_k(d_k^0) = 0$ . If instead  $d_k = d_k^l$  we have that

$$v_k^1(d_k^l, l) + v_k^2(d_k^l, l) - C_k(d_k^l) = 2 \left[ \frac{1}{2} \frac{(k+1)^2}{k} \right] - \frac{(k+1)^2}{k} = 0,$$

whereas, if  $d_k = d_k^m$ ,

$$v_k^1(d_k^m, l) + v_k^2(d_k^m, l) - C_k(d_k^m) = -\frac{k(k+1)^2}{2} < 0$$

and, finally, if  $d_k = d_k^h$

$$v_k^1(d_k^h, l) + v_k^2(d_k^h, l) - C_k(d_k^h) = 2 \frac{(k+1)^2}{4} \left[ k - \frac{1}{k} \right] - 2(k+1)^3 = (k+1)^2 \left[ \frac{k}{2} - \frac{1}{2k} - k - 2 \right] < 0$$

We conclude that an efficient decision is  $x_k^*(ll) = d_k^l$  (or  $d_k^0$ ), which generates a maximized surplus of  $s_k(ll) = 0$ .

### 3.2 Type Profiles $lm$ and $ml$

Again it follows trivially that  $v_k^1(d_k^0, l) + v_k^2(d_k^0, m) - C_k(d_k^0) = 0$ , whereas the surplus generated by  $d_k = d_k^l$  is

$$v_k^1(d_k^l, l) + v_k^2(d_k^l, m) - C_k(d_k^l) = \frac{1}{2} \frac{(k+1)^2}{k} + \frac{3}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) - \frac{(k+1)^2}{k} = \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1).$$

If  $d_k = d_k^m$  is chosen

$$\begin{aligned} v_k^1(d_k^m, l) + v_k^2(d_k^m, m) - C_k(d_k^m) &= 0 + \frac{k(k+1)^2}{2} - 2\varepsilon k(k+1)(k+2) - \frac{k(k+1)^2}{2} \\ &= -2\varepsilon k(k+1)(k+2) < 0, \end{aligned}$$

and, for  $d_k = d_k^h$

$$\begin{aligned} v_k^1(d_k^h, l) + v_k^2(d_k^h, m) - C_k(d_k^h) &= \frac{(k+1)^2}{4} \left[ k - \frac{1}{k} \right] + \frac{k(k+1)^2}{4} - 3(k+1)^3 - 2(k+1)^3 \\ &= (k+1)^2 \left[ \frac{k}{2} - \frac{1}{4k} - 5k - 5 \right] < 0 \end{aligned}$$

Using symmetry, we conclude that the efficient decision is  $x_k^*(lm) = x_k^*(ml) = d_k^l$  and that the associated maximized surplus is

$$s_k(lm) = s_k(ml) = \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1)$$

### 3.3 Type Profile $mm$

Trivially,  $v_k^1(d_k^0, m) + v_k^2(d_k^0, m) - C_k(d_k^0) = 0$ . If instead  $d_k = d_k^l$  we have that

$$v_k^1(d_k^l, m) + v_k^2(d_k^l, m) - C_k(d_k^l) = 2 \left[ \frac{3}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) \right] - \frac{(k+1)^2}{k} = \frac{1}{2} \frac{(k+1)^2}{k} + 2\varepsilon(k+1) > 0$$

and if  $d_k = d_k^m$

$$\begin{aligned} v_k^1(d_k^m, m) + v_k^2(d_k^m, m) - C_k(d_k^m) &= 2 \left[ \frac{k(k+1)^2}{2} - 2\varepsilon k(k+1)(k+2) \right] - \frac{k(k+1)^2}{2} \\ &= \frac{k(k+1)^2}{2} - 4\varepsilon k(k+1)(k+2), \end{aligned}$$

while if  $d_k = d_k^h$  we have that

$$v_k^1(d_k^h, m) + v_k^2(d_k^h, m) - C_k(d_k^h) = 2 \left[ \frac{k(k+1)^2}{4} - 3(k+1)^3 \right] - 2(k+1)^3 = (k+1)^2 \left[ \frac{k}{2} - 8k - 8 \right] < 0$$

We conclude that  $d_k^l$  and  $d_k^m$  are the only remaining candidates that could maximize the social surplus. Define

$$\Delta(k) = \frac{k(k+1)^2}{2} - 4\varepsilon k(k+1)(k+2) - \frac{1}{2} \frac{(k+1)^2}{k} - 2\varepsilon(k+1).$$

If  $\Delta(k)$  is strictly positive  $d_k^m$  is the surplus maximizing decision, whereas if  $\Delta(k)$  is strictly negative, then  $d_k^l$  is the unique maximizer.

**Claim 1** *Suppose that  $\varepsilon \leq \frac{1}{24}$ , then  $\Delta(\cdot)$  is strictly increasing in  $k$  on the interval  $[1, \infty)$*

**Proof.** Rearrange to get

$$\begin{aligned} \Delta(k) &= \frac{k(k+1)^2}{2} - 4\varepsilon k(k+1)(k+2) - \frac{1}{2} \frac{(k+1)^2}{k} - 2\varepsilon(k+1) \\ &= \frac{1}{2}(k+1)^2 \left( k - \frac{1}{k} \right) - 4\varepsilon k(k+1)(k+2) - 2\varepsilon(k+1). \end{aligned}$$

Differentiation yields,

$$\Delta'(k) = (k+1) \left( k - \frac{1}{k} \right) + \frac{1}{2}(k+1)^2 \left( 1 + \frac{1}{k^2} \right) - 4\varepsilon [(k+1)(k+2) + k(k+2) + k(k+1)] - 2\varepsilon.$$

Simplify the bracketed expression to get

$$\begin{aligned} (k+1)(k+2) + k(k+2) + k(k+1) &= \underbrace{(k+1)^2 + (k+1)}_{=(k+1)(k+2)} + \underbrace{k(k+1) + k}_{=k(k+2)} + k(k+1) \\ &= (k+1)^2 + (k+1) + k(k+1) + (k+1) - 1 + k(k+1) = (k+1)^2 + 2(k+1) + 2k(k+1) - 1 \\ &= 3(k+1)^2 - 1. \end{aligned}$$

Hence

$$\begin{aligned} \Delta'(k) &= (k+1) \left( k - \frac{1}{k} \right) + \frac{1}{2}(k+1)^2 \left( 1 + \frac{1}{k^2} \right) - 4\varepsilon [3(k+1)^2 - 1] - 2\varepsilon \\ &= (k+1) \left( k - \frac{1}{k} \right) + \frac{1}{2}(k+1)^2 \left( 1 + \frac{1}{k^2} - 24\varepsilon \right) + 2\varepsilon. \end{aligned}$$

The claim follows since all terms are strictly positive given that  $k \geq 1$  and  $\varepsilon \leq \frac{1}{24}$ . ■

Evaluating we have that

$$\Delta(1) = 2 - 24\varepsilon - 2 - 4\varepsilon = -28\varepsilon < 0$$

for any  $\varepsilon > 0$ . We conclude that (somewhat unfortunately since it is an additional complication for the example)  $d_1^l$  is the surplus maximizing decision for  $k = 1$ . However

$$\Delta(2) = 9 - 96\varepsilon - \frac{9}{4} - 6\varepsilon$$

Clearly, for  $\varepsilon$  small enough (the exact bound is  $\varepsilon < \frac{27}{408}$ , where it may be noted that  $\frac{1}{24} = \frac{27}{648} < \frac{27}{408}$ ) we have that  $\Delta(2)$  is strictly positive. Combining with the fact that  $\Delta(k)$  is monotonically increasing in  $k$  under the condition that  $\varepsilon \leq \frac{1}{24}$  we conclude;

**Claim 2** Suppose that  $\varepsilon \leq \frac{1}{24}$ . Then  $x_1^*(mm) = d_1^l$  and  $x_k^*(mm) = d_k^m$  for every  $k \geq 2$ . The associated social surplus is

$$\begin{aligned} s_1(mm) &= \frac{1}{2} \frac{(k+1)^2}{k} + 2\varepsilon(k+1) \\ s_k(mm) &= \frac{k(k+1)^2}{2} - 4\varepsilon k(k+1)(k+2) \text{ for } k \geq 2 \end{aligned}$$

### 3.4 Type Profiles $lh$ and $hl$

As for any other type profile  $v_k^1(d_k^0, l) + v_k^2(d_k^0, h) - C_k(d_k^0) = 0$ . For the non-trivial alternatives we have that

$$v_k^1(d_k^l, l) + v_k^2(d_k^l, h) - C_k(d_k^l) = \frac{1}{2} \frac{(k+1)^2}{k} - \varepsilon(k+1) - \frac{(k+1)^2}{k} = -\frac{1}{2} \frac{(k+1)^2}{k} - \varepsilon(k+1) < 0,$$

and

$$v_k^1(d_k^m, l) + v_k^2(d_k^m, h) - C_k(d_k^m) = 0 - \frac{k(k+1)^2}{2} - \frac{k(k+1)^2}{2} = -k(k+1)^2 < 0,$$

and

$$\begin{aligned} v_k^1(d_k^h, l) + v_k^2(d_k^h, h) - C_k(d_k^m) &= \frac{(k+1)^2}{4} \left[ k - \frac{1}{k} \right] + 3(k+1)^3 - \frac{k(k+1)^2}{4} - 2(k+1)^3 \\ &= \frac{(k+1)^2}{4k} + (k+1)^3 = (k+1)^2 \left[ \frac{4k(k+1) - 1}{4k} \right] \end{aligned}$$

Hence, again using symmetry,  $x_k^*(lh) = x_k^*(hl) = d_k^h$  and the resulting maximized surplus is

$$s_k(lh) = s_k(hl) = (k+1)^2 \left[ \frac{4k(k+1) - 1}{4k} \right]$$

### 3.5 Type Profiles $mh$ and $hm$

Obviously,  $v_k^1(d_k^0, m) + v_k^2(d_k^0, h) - C_k(d_k^0) = 0$ . If instead  $d_k = d_k^l$

$$v_k^1(d_k^l, m) + v_k^2(d_k^l, h) - C_k(d_k^l) = \frac{3}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) - \varepsilon(k+1) - \frac{(k+1)^2}{k} = -\frac{1}{4} \frac{(k+1)^2}{k} < 0,$$

and if  $d_k = d_k^m$ ,

$$\begin{aligned} v_k^1(d_k^m, m) + v_k^2(d_k^m, h) - C_k(d_k^m) &= \frac{k(k+1)^2}{2} - 2\varepsilon k(k+1)(k+2) - \frac{k(k+1)^2}{2} - \frac{k(k+1)^2}{2} \\ &= -2\varepsilon k(k+1)(k+2) - \frac{k(k+1)^2}{2} < 0, \end{aligned}$$

whereas if  $d_k = d_k^h$ , we get that

$$\begin{aligned} v_k^1(d_k^h, m) + v_k^2(d_k^h, h) - C_k(d_k^h) &= \frac{k(k+1)^2}{4} - 3(k+1)^3 + 3(k+1)^3 - \frac{k(k+1)^2}{4} - 2(k+1)^3 \\ &= -2(k+1)^2 < 0 \end{aligned}$$

Hence,  $x_k^*(mh) = x_k^*(hm) = d_k^0$ , and the maximized surplus is  $s_k(mh) = s_k(hm) = 0$

### 3.6 Type Profile $hh$

Again,  $v_k^1(d_k^0, h) + v_k^2(d_k^0, h) - C_k(d_k^0) = 0$ , and

$$v_k^1(d_k^l, h) + v_k^2(d_k^l, h) - C_k(d_k^l) = -2\varepsilon(k+1) - \frac{(k+1)^2}{k} < 0$$

and

$$v_k^1(d_k^m, h) + v_k^2(d_k^m, h) - C_k(d_k^m) = -2 \left[ \frac{k(k+1)^2}{2} \right] - \frac{k(k+1)^2}{2} = -\frac{3k(k+1)^2}{2} < 0$$

and

$$v_k^1(d_k^h, h) + v_k^2(d_k^h, h) - C_k(d_k^h) = 2 \left[ 3(k+1)^3 - \frac{k(k+1)^2}{4} \right] - 2(k+1)^3 = 4(k+1)^3 - \frac{k(k+1)^2}{2} > 0$$

Hence,  $x_k^*(hh) = d_k^h$  and

$$s_k(hh) = 4(k+1)^3 - \frac{(k+1)^2}{2k} = \left[ 4(k+1) - \frac{k}{2} \right] (k+1)^2$$

### 3.7 Summary: An Optimal Decision Rule and the Maximized Surplus

Combining all the cases above and ignoring  $k = 1$  we have that an optimal social decision rule is<sup>12</sup>

$$x_k^*(\theta_k) = \begin{cases} d_k^0 & \text{if } \theta_k \in \{mh, hm\} \\ d_k^l & \text{if } \theta_k \in \{ll, lm, ml\} \\ d_k^m & \text{if } \theta_k = mm \\ d_k^h & \text{if } \theta_k \in \{lh, hl, hh\} \end{cases}, \quad (1)$$

and the maximized surplus (given  $k \geq 2$ ) is

$$s_k(\theta_k) = \begin{cases} 0 & \text{if } \theta_k \in \{ll, mh, hm\} \\ \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) & \theta_k \in \{lm, ml\} \\ \frac{k(k+1)^2}{2} - 4\varepsilon k(k+1)(k+2) & \text{if } \theta_k = mm \\ (k+1)^2 \left[ \frac{4k(k+1)-1}{4k} \right] & \text{if } \theta_k \in \{lh, hl\} \\ \left[ 4(k+1) - \frac{k}{2} \right] (k+1)^2 & \text{if } \theta_k = hh \end{cases}. \quad (2)$$

### 3.8 Interim Expected Payoffs from Participation in the Groves Mechanism

For  $k = 2, 3, \dots$  we assume that the probability distribution over  $\Theta_k^i$  be given as follow

$$(\Pr[\theta^i = l], \Pr[\theta^i = m], \Pr[\theta^i = h]) = \left( \frac{k(k+2)}{(k+1)^2}, \frac{1}{2(k+1)^2}, \frac{1}{2(k+1)^2} \right). \quad (3)$$

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<sup>1</sup>There is multiplicity for type profile  $ll$ :  $d_k^0$  and  $d_k^l$  are both optimal. We could easily get rid of this without affecting the maximized social surplus by adjusting the costs and preferences slightly. This would add some extra terms for the calculations, and, ultimately, only the maximized surplus is relevant, so we have opted to go with the simpler primitives.

<sup>2</sup>Hence, we will start our sequence at  $k = 2$ . Obviously, we could replace every  $k$  with  $k + 1$  and start the sequence at  $k = 1$ , but the formulas get somewhat less transparent, which is why we stick with the current formulation.

With probability distribution (3), the interim expected value of the maximized issue  $k$  surplus is

$$\begin{aligned}
\mathbb{E}[s_k(\theta_k) | l] &= \Pr[\theta^i = l] s_k(ll) + \Pr[\theta^i = m] s_k(lm) + \Pr[\theta^i = h] s_k(lh) \\
&= \frac{1}{2(k+1)^2} \left[ \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) \right] + \frac{1}{2(k+1)^2} \left[ (k+1)^2 \left[ \frac{4k(k+1)-1}{4k} \right] \right] \\
&= \frac{1}{2(k+1)^2} \left[ \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) + (k+1)^2 \left[ \frac{4k(k+1)-1}{4k} \right] \right] \\
&= \frac{1}{2} \left[ \frac{1}{4k} + \frac{\varepsilon}{(k+1)} + \frac{4k(k+1)-1}{4k} \right] \\
&= \frac{1}{2} \left[ \frac{1}{4k} + \frac{\varepsilon}{(k+1)} + (k+1) - \frac{1}{4k} \right] = \frac{1}{2} \left[ \frac{\varepsilon}{(k+1)} + (k+1) \right] = \frac{k+1}{2} + \frac{\varepsilon}{2(k+1)}
\end{aligned} \tag{4}$$

for  $\theta_i = l$ . For  $\theta_i = m$  we have that

$$\begin{aligned}
\mathbb{E}[s_k(\theta_k) | m] &= \Pr[\theta^i = l] s_k(ml) + \Pr[\theta^i = m] s_k(mm) + \Pr[\theta^i = h] s_k(mh) \\
&= \frac{k(k+2)}{(k+1)^2} \left[ \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) \right] + \frac{1}{2(k+1)^2} \left[ \frac{k(k+1)^2}{2} - 4\varepsilon k(k+1)(k+2) \right] \\
&= \frac{k+2}{4} + \frac{\varepsilon k(k+2)}{(k+1)} + \frac{k}{4} - \frac{2\varepsilon k(k+2)}{k+1} = \frac{k+1}{2} - \frac{\varepsilon k(k+2)}{(k+1)} \\
&= \frac{k+1}{2} - \frac{\varepsilon[k(k+1)+k]}{(k+1)} = \frac{k+1}{2} - \varepsilon k - \frac{\varepsilon k}{k+1}
\end{aligned} \tag{5}$$

Finally, for  $\theta_i = h$ , we have that

$$\begin{aligned}
\mathbb{E}[s_k(\theta_k) | h] &= \Pr[\theta^i = l] s_k(hl) + \Pr[\theta^i = m] s_k(hm) + \Pr[\theta^i = h] s_k(hh) \\
&= \frac{k(k+2)}{(k+1)^2} \left[ (k+1)^2 \left[ \frac{4k(k+1)-1}{4k} \right] \right] + \frac{1}{2(k+1)^2} \left[ 4(k+1) - \frac{k}{2} \right] (k+1)^2 \\
&= k(k+2) \left[ \frac{4k(k+1)-1}{4k} \right] + \left[ 2(k+1) - \frac{k}{4} \right] \\
&= k(k+1)(k+2) + 2(k+1) - \frac{k+2}{4} - \frac{k}{4} = k(k+1)(k+2) + 2(k+1) - \frac{k+1}{2} \\
&= (k+1) [k^2 + 2k + 1 + 1] - \frac{k+1}{2} = (k+1)^3 + (k+1) - \frac{k+1}{2} = (k+1)^3 + \frac{(k+1)}{2}
\end{aligned} \tag{6}$$

The ex ante expected surplus is thus

$$\begin{aligned}
\mathbb{E}[s_k(\theta_k)] &= \frac{k(k+2)}{(k+1)^2} \left[ \frac{k+1}{2} + \frac{\varepsilon}{2(k+1)} \right] + \frac{1}{2(k+1)^2} \left[ \frac{k+1}{2} - \frac{\varepsilon k(k+2)}{(k+1)} \right] \\
&\quad + \frac{1}{2(k+1)^2} \left[ (k+1)^3 + \frac{(k+1)}{2} \right] \\
&= \frac{k(k+2)}{2(k+1)} + \varepsilon \left[ \frac{k(k+2)}{2(k+1)^3} \right] + \frac{1}{4(k+1)} - \varepsilon \left[ \frac{\varepsilon k(k+2)}{2(k+1)^3} \right] + \frac{k+1}{2} + \frac{1}{4(k+1)} \\
&= \frac{k(k+2)}{2(k+1)} + \frac{1}{2(k+1)} + \frac{k+1}{2} = \frac{k^2 + 2k + 1}{2(k+1)} + \frac{k+1}{2} = \frac{(k+1)^2}{2(k+1)} + \frac{k+1}{2} = k+1
\end{aligned} \tag{7}$$

### 3.9 Summary of the Expected Surplus Calculations

We have shown that

$$\mathbb{E}[s_k(\theta_k)|l] = \frac{k+1}{2} + \frac{\varepsilon}{2(k+1)} \quad (8)$$

$$\mathbb{E}[s_k(\theta_k)|m] = \frac{k+1}{2} - \varepsilon k - \frac{\varepsilon k}{(k+1)} \quad (9)$$

$$\mathbb{E}[s_k(\theta_k)|h] = (k+1)^3 + \frac{(k+1)}{2} \quad (10)$$

$$\mathbb{E}[s_k(\theta_k)] = k+1 \quad (11)$$

## 4 The Probability of a Participation Constraint is Violated Converges to zero as $K \rightarrow \infty$ .

Since  $R_K^i(\theta^i) = 0$  for all  $\theta^i$ , the interim expected payoff from participation in the Groves mechanism under consideration simplifies to

$$U_K^i(\theta^i) = \mathbb{E}_{-i}[S_K(\theta)] - \frac{1}{2}\mathbb{E}[S_K(\theta)] = \sum_{k=2}^{K^*} \mathbb{E}_{-i}[s_k(\theta_k)] - \frac{1}{2} \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)] \quad (12)$$

Consider a type on the form  $(m, \dots, m, l, \dots, l)$ . Specifically, assume that  $\theta_k^i = m$  for  $k = 2, \dots, K^*$  and  $\theta_k^i = l$  for  $k = K^* + 1, \dots, K$  and denote this type by  $(\mathbf{m}_{K^*}, \mathbf{l}_{K^*-K})$ . Substituting (8), (9) and (11) into (12) we have that type  $(\mathbf{m}_{K^*}, \mathbf{l}_{K^*-K})$  earns an interim expected payoff of

$$\begin{aligned} U_K^i(\mathbf{m}_{K^*}, \mathbf{l}_{K^*-K}) &= \sum_{k=2}^{K^*} \mathbb{E}[s_k(\theta_k)|m] + \sum_{k=K^*+1}^K \mathbb{E}[s_k(\theta_k)|l] - \frac{1}{2} \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)] \\ &= \sum_{k=2}^{K^*} \left[ \frac{k+1}{2} - \varepsilon k - \frac{\varepsilon k}{(k+1)} \right] + \sum_{k=K^*+1}^K \left[ \frac{k+1}{2} + \frac{\varepsilon}{2(k+1)} \right] - \frac{1}{2} \sum_{k=2}^K (k+1) \\ &= \varepsilon \left[ \sum_{k=2}^{K^*} \left[ -k - \frac{k}{k+1} \right] + \frac{1}{2} \sum_{k=K^*+1}^K \left[ \frac{1}{k+1} \right] \right] \end{aligned} \quad (13)$$

Define

$$H(K^*, K) = \sum_{k=2}^{K^*} \left[ -k - \frac{k}{k+1} \right] + \frac{1}{2} \sum_{k=K^*+1}^K \frac{1}{k+1} \quad (14)$$

We note that;

1.  $H(K^*, K)$  is strictly decreasing in  $K^*$
2.  $H(1, K) = \frac{\varepsilon}{2} \sum_{k=2}^K \frac{1}{(k+1)} > 0$
3.  $H(K, K) = -\varepsilon \sum_{k=2}^K \left[ k + \frac{k}{(k+1)} \right] < 0$

These three properties imply that for every  $K \geq 2$  there exists a unique integer  $K^*(K) \in \{1, \dots, K\}$  such that

$$\begin{aligned} H(K^*(K), K) &> 0 \\ H(K^*(K) + 1, K) &< 0 \end{aligned} \quad (15)$$

Moreover,  $K^*(K)$  is monotonically increasing and goes (slowly) to infinity as  $K$  goes to infinity. To see this, we first observe that, for  $K^*$  fixed, the positive term in (14) is divergent. That is,

$$\begin{aligned} \sum_{k=K^*+1}^K \frac{1}{k+1} &= \sum_{k=K^*+1}^K \left[ \int_{k+1}^{k+2} \frac{1}{k+1} dz \right] > \sum_{k=K^*+1}^K \left[ \int_{k+1}^{k+2} \frac{1}{z} dz \right] \\ &= \int_{K^*+2}^{K+2} \frac{1}{z} dz = \ln(K+2) - \ln(K^*+2). \end{aligned} \quad (16)$$

For contradiction, assume that there exists some  $\bar{K}$  such that  $K^*(K) < \bar{K} - 1$  for all  $K$ . Then, for any  $K$  we have that

$$\begin{aligned} H(K^*(K) + 1, K) &> H(\bar{K}, K) = - \sum_{k=2}^{\bar{K}} \left[ k + \frac{k}{k+1} \right] + \frac{1}{2} \sum_{k=\bar{K}+1}^K \left[ \frac{1}{k+1} \right] \\ / \text{ using (16) } / &> \sum_{k=2}^{\bar{K}} \left[ -k - \frac{k}{k+1} \right] + \frac{\ln(K+2) - \ln(\bar{K}+2)}{2}. \end{aligned} \quad (17)$$

Since  $\ln(K+2) \rightarrow \infty$  as  $K \rightarrow \infty$  and the other two terms are finite we conclude that  $H(K^*(K) + 1, K) > 0$  for  $K$  sufficiently large, which contradicts the definition of  $K^*(K)$ .

Next, consider a type on form  $(\theta_2^i, \dots, \theta_{K^*(K)}^i, l, \dots, l)$ , where the signals for problems  $2, \dots, K^*(K)$  are arbitrary, and  $\theta_k^i = l$  for  $k = K^*(K) + 1, \dots, K$ . We denote such a type  $(\theta_{K^*(K)}^i, \mathbf{1}_{K^*-K(K)})$ . Since  $\mathbb{E}[s_k(\theta_k) | m] \leq \mathbb{E}[s_k(\theta_k) | \theta_k^i]$  for all  $\theta_k^i \in \{l, m, h\}$  and every  $k$  it follows that

$$\begin{aligned} U_K^i(\theta_{K^*(K)}^i, \mathbf{1}_{K^*-K(K)}) &= \sum_{k=2}^{K^*(K)} \mathbb{E}[s_k(\theta_k) | \theta_k^i] - \sum_{k=K^*(K)+1}^K \mathbb{E}[s_k(\theta_k) | l] - \frac{1}{2} \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)] \\ &\geq \sum_{k=2}^{K^*(K)} \mathbb{E}[s_k(\theta_k) | m] - \sum_{k=K^*(K)+1}^K \mathbb{E}[s_k(\theta_k) | l] - \frac{1}{2} \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)] \\ &= \varepsilon H(K^*(K), K) > 0. \end{aligned} \quad (18)$$

We conclude that the participation constraint holds for any such type. Since,

$$\Pr \left[ (\theta_{K^*(K)+1}^i, \dots, \theta_K^i) = (l, \dots, l) \right] = \prod_{k=K^*(K)+1}^K \frac{k(k+2)}{(k+1)^2} = \frac{(K^*(K)+1)(K+2)}{(K^*(K)+2)(K+1)} \rightarrow 1 \quad (19)$$

as  $K \rightarrow \infty$  it follows that the probability that  $\theta^i$  is on the form  $(\theta_{K^*(K)}^i, \mathbf{1}_{K^*-K(K)})$  tends to unity as  $K$  goes out of bounds. We conclude;

**Claim 3** *The probability that all participation constraints hold converges to 1 as  $K$  tends to infinity.*

Hence, the Groves mechanism is almost incentive feasible in this example.

## 5 Unraveling when the Veto Game is Introduced

Note that for type  $\mathbf{l} = (l, \dots, l)$

$$U_K^i(\mathbf{l}) = \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)|l] - \frac{1}{2} \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)] = \sum_{k=2}^K \left[ \frac{k+1}{2} + \frac{\varepsilon}{2(k+1)} - \frac{k+1}{2} \right] = \frac{\varepsilon}{2} \sum_{k=2}^K \frac{1}{(k+1)} \quad (20)$$

where

$$\sum_{k=2}^K \frac{1}{k+1} = \sum_{k=2}^K \left[ \int_k^{k+1} \frac{1}{k+1} dz \right] < \sum_{k=2}^K \left[ \int_k^{k+1} \frac{1}{z} dz \right] = \int_2^{K+1} \frac{1}{z} dz = \ln \left( \frac{K+1}{2} \right). \quad (21)$$

Hence,

$$U_K^i(\mathbf{l}) < \frac{\varepsilon}{2} \ln \left( \frac{K+1}{2} \right). \quad (22)$$

However, consider type  $\theta^i$  where all coordinates are  $l$  except for  $\theta_{k^*}^i = m$ . Denote this type  $\theta^i = \mathbf{l}|\theta_{k^*}^i = m$  and note that

$$\begin{aligned} U(\mathbf{l}|\theta_{k^*}^i = m) &= \sum_{k \neq k^*}^K \mathbb{E}[s_k(\theta_k)|l] + \mathbb{E}[s_{k^*}(\theta_{k^*})|m] - \frac{1}{2} \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)] \\ &= \underbrace{\sum_{k=2}^K \mathbb{E}[s_k(\theta_k)|l] - \frac{1}{2} \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)]}_{=U^i(\mathbf{l})} + \mathbb{E}[s_{k^*}(\theta_{k^*})|m] - \mathbb{E}[s_{k^*}(\theta_{k^*})|l] \\ /using (22)/ &\leq \frac{\varepsilon}{2} \ln \left( \frac{K+1}{2} \right) + \frac{k^*+1}{2} - \varepsilon k^* - \frac{\varepsilon k^*}{(k^*+1)} - \left[ \frac{k^*+1}{2} + \frac{\varepsilon}{2(k^*+1)} \right] \\ &= \varepsilon \left[ \frac{1}{2} \ln \left( \frac{K+1}{2} \right) - k^* - \frac{k^*}{(k^*+1)} - \frac{1}{2(k^*+1)} \right] < \varepsilon \left[ \frac{1}{2} \ln \left( \frac{K+1}{2} \right) - k^* \right] \end{aligned} \quad (23)$$

Define  $\tilde{k}(K)$  as the integer part of  $\frac{1}{2} \ln \left( \frac{K+1}{2} \right) + 1$ . Since  $\frac{1}{2} \ln \left( \frac{K+1}{2} \right) - k^*$  is negative for  $k^* \geq \tilde{k}(K)$  this implies that type  $\mathbf{l}|\theta_{k^*}^i = m$  is worse off from participating in the Groves mechanism than from the status quo outcome  $d^0 = (d_2^0, \dots, d_K^0)$  for every  $k^* \geq \tilde{k}(K)$ . For brevity, denote this subset of types with a strict incentive to veto the Groves mechanism by  $\Theta_V^i$ . That is

$$\Theta_V^i = \{ \theta^i | \theta_{k^*}^i = m \text{ for some } k^* \geq k^*(K) \text{ and } \theta_k^i = l \text{ for all } k \neq k^* \} \quad (24)$$

We note that, for any  $k^* \geq k^*(K)$

$$\begin{aligned} \Pr[\theta^i \in \Theta_V^i | \theta_{k^*}^i = m] &= \prod_{k \neq k^*} \Pr[\theta_k^i = l] > \Pr[\theta^i = \mathbf{l}] = \prod_{k=2}^K \Pr[\theta_k^i = l] = \prod_{k=2}^K \frac{k(k+2)}{(k+1)^2} \\ &= \left( \frac{2 \times 4}{3^2} \right) \times \left( \frac{3 \times 5}{4^2} \right) \times \dots \times \frac{(K-1)(K+1)}{K^2} \times \frac{K(K+2)}{(K+1)^2} \\ &= \frac{2}{3} \left[ \frac{K+2}{K+1} \right] > \frac{2}{3} \end{aligned} \quad (25)$$

In words, conditional on  $\theta_{k^*}^i = m$ , the probability that all other coordinates are  $l$ s is obviously (slightly) larger than the unconditional probability that the type  $\mathbf{l}$  is realized. For  $k \geq \tilde{k}(K)$  we can therefore calculate

an upper bound on the expected surplus conditional on  $\theta_k^i = l$  and conditional on that agent  $i$  is aware that any agent  $\theta^i \in \Theta_V^i$  will veto the mechanism.

$$\begin{aligned}
\mathbb{E}[s_k(\theta_k)|l, \text{ veto by } \Theta_V^i] &= \Pr[\theta_k^i = m] [1 - \Pr[\theta^i \in \Theta_V^i | \theta_k^i = m]] s_k(lm) + \Pr[\theta_k^i = h] s_k(lh) \quad (26) \\
&= \mathbb{E}[s_k(\theta_k)|l] - \Pr[\theta_k^i = m] \Pr[\theta^i \in \Theta_V^i | \theta_k^i = m] s_k(lm) \\
&= \underbrace{\frac{k+1}{2} + \frac{\varepsilon}{2(k+1)}}_{=\mathbb{E}[s_k(\theta_k)|l] \text{ by (8)}} - \underbrace{\frac{1}{2(k+1)^2}}_{=\Pr[\theta_k^i=m]} \Pr[\theta^i \in \Theta_V^i | \theta_k^i = m] \underbrace{\left[ \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) \right]}_{=s_k(lm) \text{ by (2)}} \\
/(25)/ &< \frac{k+1}{2} + \frac{\varepsilon}{2(k+1)} - \frac{1}{2(k+1)^2} \frac{2}{3} \left[ \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) \right] \\
&= \frac{k+1}{2} + \frac{\varepsilon}{2(k+1)} - \frac{1}{12k} - \frac{\varepsilon}{3(k+1)} = \frac{k+1}{2} + \frac{\varepsilon}{6(k+1)} - \frac{1}{12k}
\end{aligned}$$

The interim expected payoff of type  $\mathbf{1} = (l, \dots, l)$  conditional on vetoes from types in  $\Theta_V^i$  is thus

$$\begin{aligned}
U_K^i(\mathbf{1} | \text{ veto by } \Theta_V^i) &= \sum_{k=2}^{\tilde{k}(K)-1} \mathbb{E}[s_k(\theta_k)|l] + \sum_{k=\tilde{k}(K)}^K \mathbb{E}[s_k(\theta_k)|l, \text{ veto by } \theta^i \in \Theta_V^i] - \frac{1}{2} \sum_{k=2}^K \mathbb{E}[s_k(\theta_k)] \\
&= \frac{\varepsilon}{2} \sum_{k=2}^{\tilde{k}(K)-1} \frac{1}{k+1} + \frac{\varepsilon}{6} \sum_{k=\tilde{k}(K)}^K \frac{1}{k+1} - \frac{1}{12} \sum_{k=\tilde{k}(K)}^K \frac{1}{k} \quad (27)
\end{aligned}$$

where,

$$\sum_{k=2}^{\tilde{k}(K)-1} \frac{1}{k+1} = \sum_{k=2}^{\tilde{k}(K)-1} \left[ \int_k^{k+1} \frac{1}{k+1} dz \right] < \sum_{k=2}^{\tilde{k}(K)-1} \left[ \int_k^{k+1} \frac{1}{z} dz \right] = \int_2^{\tilde{k}(K)} \frac{1}{z} dz = \ln \left( \frac{\tilde{k}(K)}{2} \right) \quad (28)$$

$$\sum_{k=\tilde{k}(K)}^K \frac{1}{k+1} = \sum_{k=\tilde{k}(K)}^K \left[ \int_k^{k+1} \frac{1}{k+1} dz \right] < \sum_{k=\tilde{k}(K)}^K \left[ \int_k^{k+1} \frac{1}{z} dz \right] = \int_{\tilde{k}(K)}^{K+1} \frac{1}{z} dz = \ln \left( \frac{K+1}{\tilde{k}(K)} \right) \quad (29)$$

$$\sum_{k=\tilde{k}(K)}^K \frac{1}{k} = \sum_{k=\tilde{k}(K)}^K \left[ \int_k^{k+1} \frac{1}{k} dz \right] > \sum_{k=\tilde{k}(K)}^K \left[ \int_k^{k+1} \frac{1}{z} dz \right] = \ln \left( \frac{K+1}{\tilde{k}(K)} \right) \quad (30)$$

Hence,

$$U_K^i(\mathbf{1} | \text{ veto by } \Theta_V^i) = \frac{\varepsilon}{2} \sum_{k=2}^{\tilde{k}(K)-1} \frac{1}{k+1} + \frac{\varepsilon}{6} \sum_{k=\tilde{k}(K)}^K \frac{1}{k+1} - \frac{1}{12} \sum_{k=\tilde{k}(K)}^K \frac{1}{k} \quad (31)$$

$$\begin{aligned}
&< \frac{\varepsilon}{2} \ln \left( \frac{\tilde{k}(K)}{2} \right) - \frac{1-2\varepsilon}{6} \ln \left( \frac{K+1}{\tilde{k}(K)} \right) \\
&= \ln \tilde{k}(K) \left[ \frac{1+\varepsilon}{6} \right] - \frac{\varepsilon}{2} \ln 2 - \ln(K+1) \left[ \frac{1-2\varepsilon}{6} \right] \quad (32)
\end{aligned}$$

But,  $\tilde{k}(K) \leq \frac{1}{2} \ln \left( \frac{K+1}{2} \right) + 1$ , so

$$\ln(K+1) \geq 2\tilde{k}(K) + \ln 2 - 2$$

Since  $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$  and since  $\lim_{K \rightarrow \infty} \tilde{k}(K) = \infty$  it follows that the term with  $\ln(K+1)$  eventually dominates in expression (31). Consequently, whenever  $\varepsilon < \frac{1}{2}$  there exists  $\bar{K}$  such that  $U^i(\mathbf{1}|\text{veto by } \theta^i \in \Theta_V^i) < 0$  for any  $K \geq \bar{K}$ .<sup>3</sup> In (25) we calculated the probability that  $\theta^i = \mathbf{1}$  to be

$$\Pr[\theta^i = \mathbf{1}] = \frac{2}{3} \left[ \frac{K+2}{K+1} \right].$$

We conclude that;

**Claim 4** *Suppose that type  $\mathbf{1} = (l, \dots, l)$  expects that all types in  $\Theta_V^i$  will veto play of the Groves mechanism. Then, there exists some  $\bar{K}$  such that type  $\mathbf{1} = (l, \dots, l)$  have a strict incentive to veto the Groves mechanism given any economy  $K \geq \bar{K}$ .*

For our next recursion, define the set

$$\Theta_W^i = \{\theta^i | \theta_k^i \in \{l, m\} \text{ for every } k\} \quad (33)$$

By virtue of Claim 4 types in  $\Theta_V^i$  and type  $\mathbf{1}$  will use their veto power. For issue  $k < \tilde{k}(K)$  we have that the expected surplus conditional on a veto in the set  $\Theta_V^i$  is the same as the expected surplus with no veto being cast, so we have that,

$$\mathbb{E}[s_k(\theta_k) | m, \text{ veto by } \mathbf{1} \text{ and } \Theta_V^i] < \mathbb{E}[s_k(\theta_k) | m] < \mathbb{E}[s_k(\theta_k) | l] = \mathbb{E}[s_k(\theta_k) | l, \text{ veto by } \Theta_V^i], \quad (34)$$

where the first inequality holds because a veto is always a loss of surplus in the example, and the second comes from direct comparison between (8) and (9). For  $k \geq \tilde{k}(K)$  we first note that

$$\begin{aligned} \mathbb{E}[s_k(\theta_k) | m, \text{ veto by } \mathbf{1} \text{ and } \Theta_V^i] &< \Pr[\theta_k^i = l] \underbrace{[1 - \Pr[\theta^i = \mathbf{1} | \theta_k^i = l]]}_{< \frac{1}{3}} s_k(ml) \\ &+ \Pr[\theta_k^i = m] \underbrace{[1 - \Pr[\Theta_V^i | \theta_k^i = m]]}_{< \frac{1}{3}} s_k(mm) < \frac{1}{3} \mathbb{E}[s_k(\theta_k) | m], \end{aligned} \quad (35)$$

and that,

$$\begin{aligned} &\mathbb{E}[s_k(\theta_k) | l, \text{ veto by } \Theta_V^i] \quad (36) \\ &= \mathbb{E}[s_k(\theta_k) | l] - \Pr[\theta_k^i = m] \underbrace{[\Pr[\Theta_V^i | \theta_k^i = m]]}_{< 1} s_k(lm) > \mathbb{E}[s_k(\theta_k) | l] - \Pr[\theta_k^i = m] s_k(lm) \\ &= \frac{k+1}{2} + \frac{\varepsilon}{2(k+1)} - \frac{1}{2(k+1)^2} \left[ \frac{1}{4} \frac{(k+1)^2}{k} + \varepsilon(k+1) \right] = \frac{k+1}{2} - \frac{1}{8k} \\ &> \frac{1}{3} \frac{k+1}{2} > \frac{1}{3} \mathbb{E}[s_k(\theta_k) | m] > \mathbb{E}[s_k(\theta_k) | m, \text{ veto by } \mathbf{1} \text{ and } \Theta_V^i] \end{aligned}$$

Combining (34) and (36) we have that

$$\mathbb{E}[s_k(\theta_k) | m, \text{ veto by } \mathbf{1} \text{ and } \Theta_V^i] < \mathbb{E}[s_k(\theta_k) | l, \text{ veto by } \Theta_V^i]$$

---

<sup>3</sup>We already have the restriction  $\varepsilon \leq \frac{1}{24}$  in order for the surplus in (2) to be valid for every  $k \geq 2$ .

for any  $k$ , which implies that

$$\begin{aligned} U_K^i(\theta^i | \text{veto by } \Theta_V^i \text{ and } \mathbf{1}) &= \sum_{k=2}^K \mathbb{E} [s_k(\theta_k) | \theta_k^i, \text{ veto by } \mathbf{1} \text{ and } \Theta_V^i] \\ &< \sum_{k=2}^K \mathbb{E} [s_k(\theta_k) | l, \text{ veto by } \Theta_V^i] = U_K^i(\mathbf{1} | \text{veto by } \Theta_V^i) \end{aligned} \quad (37)$$

for any  $K$  and any type  $\theta^i \in \Theta_W^i$ . By claim 4 type  $\mathbf{1}$  has a strict incentive to veto when  $K \geq \bar{K}$ , so

**Claim 5** *There exists some  $\bar{K}$  such that every type  $\theta^i \in \Theta_W^i$  vetoes the Groves mechanism in any equilibrium for any  $K \geq \bar{K}$ .*

For the final step in the argument consider a type with  $\theta_k^i = h$  for  $k = 2, \dots, \hat{K} - 1$  and  $\theta_k^i = l$  for  $k = \hat{K}, \dots, K$  and denote such a type by  $(\mathbf{h}_{\hat{K}-1}, \mathbf{1}_{K-\hat{K}})$ . It is immediate that

$$\mathbb{E} [s_k(\theta_k) | h, \text{ veto by } \Theta_W^i] < \mathbb{E} [s_k(\theta_k) | h] = (k+1)^3 + \frac{(k+1)}{2}. \quad (38)$$

Moreover, since  $\Pr[\theta^i \in \Theta_W^i | \theta_k^i = m] > \frac{2}{3}$  we can repeat the exact same steps as in calculation (23) to conclude that, for any  $k \geq 2$ ,<sup>4</sup>

$$\begin{aligned} \mathbb{E} [s_k(\theta_k) | l, \text{ veto by } \Theta_W^i] &= \Pr[\theta_k^i = m] [1 - \Pr[\theta^i \in \Theta_W^i | \theta_k^i = m]] s_k(lm) + \Pr[\theta_k^i = h] s_k(lh) \\ &\quad \text{/same steps as in (23)/} \\ &< \frac{k+1}{2} + \frac{\varepsilon}{6(k+1)} - \frac{1}{12k}. \end{aligned} \quad (39)$$

Hence,

$$\begin{aligned} &U_K^i(\mathbf{h}_{\hat{K}-1}, \mathbf{1}_{K-\hat{K}} | \text{veto by } \Theta_W^i) \\ &< \sum_{k=2}^{\hat{K}-1} \left[ (k+1)^3 + \frac{(k+1)}{2} \right] + \sum_{k=\hat{K}+1}^K \left[ \frac{k+1}{2} + \frac{\varepsilon}{6(k+1)} - \frac{1}{12k} \right] - \sum_{k=2}^K \frac{k+1}{2} \\ &= \sum_{k=2}^{\hat{K}-1} (k+1)^3 + \sum_{k=\hat{K}}^K \left[ \frac{\varepsilon}{6(k+1)} - \frac{1}{12k} \right] \end{aligned} \quad (40)$$

But, by the calculations in (29) and (30) we know that  $\sum_{k=\hat{K}}^K \frac{1}{k+1} < \ln\left(\frac{K+1}{\hat{K}}\right) < \sum_{k=\hat{K}}^K \frac{1}{k}$ , which implies that

$$U_K^i(\mathbf{h}_{\hat{K}-1}, \mathbf{1}_{K-\hat{K}} | \text{veto by } \theta^i \in \Theta_V^i) < \sum_{k=2}^{\hat{K}-1} (k+1)^3 - \frac{1-2\varepsilon}{12} \ln\left(\frac{K+1}{\hat{K}}\right) \equiv G(\hat{K}, K) \quad (41)$$

It is easy to check that:

1.  $G$  is strictly increasing in  $\hat{K}$ , and;

---

<sup>4</sup>The difference is that (23) is only applicable for  $k \geq \tilde{k}(K)$ .

2.  $G(2, K) < 1$  (assuming that  $\varepsilon < \frac{1}{2}$ , which we do since  $\varepsilon < \frac{1}{24}$  for the surplus function (2) to be the relevant one)
3.  $G(K + 1, K) > 0$ .

Together, this implies that there exists a unique integer  $\widehat{K}(K)$  such that  $G(\widehat{K}, K) < 0$  for every  $K \leq \widehat{K}(K)$  and  $G(\widehat{K}, K) \geq 0$  for  $K > \widehat{K}(K)$ . Moreover,  $\widehat{K}(K) \rightarrow \infty$  as  $K \rightarrow \infty$  since otherwise there exists some  $\overline{K}$  such that  $G(\overline{K}, K) \geq 0$  for all  $K$ , which cannot be the case, since  $\lim_{K \rightarrow \infty} G(\overline{K}, K) = -\infty$  for any fixed  $\overline{K}$ .

To complete the argument we observe that if  $(\theta_{\widehat{K}(K)-1}^i, \mathbf{1}_{K-\widehat{K}(K)})$  is any type profile such that  $\theta_k^i = l$  for  $k = \widehat{K}(K), \dots, K$ , then the right hand side of (41) is an upper bound for the interim expected payoff also for type  $(\theta_{\widehat{K}(K)-1}^i, \mathbf{1}_{K-\widehat{K}(K)})$  conditional on types in  $\Theta_W^i$  vetoing the mechanism, implying that

$$U_K^i(\theta_{\widehat{K}(K)-1}^i, \mathbf{1}_{K-\widehat{K}(K)} | \text{veto by } \theta^i \in \Theta_V^i) < G(\widehat{K}(K), K) \leq 0.$$

Since  $\widehat{K}(K) \rightarrow \infty$ , calculation (19) implies that  $\Pr\left[(\theta_{\widehat{K}(K)}^i, \dots, \theta_K^i) = \mathbf{1}_{K-\widehat{K}(K)}\right] \rightarrow 1$  as  $K \rightarrow \infty$ . We can thus conclude that:

**Claim 6** *The probability that a veto is cast approaches 1 as  $K \rightarrow \infty$ .*