Endogenous Comparative Advantage[*]

Andrea Moro[§] Peter Norman[¶]

February 26, 2004

Abstract

A stylized model of trade between identical countries is developed, where the only departure from standard neoclassical theory is that worker skills are imperfectly observable. This creates an informational externality since firms take aggregate investments into consideration when making inference about individual workers. The interaction between the informational externality and price effects generates a force in favor of specialization. Equilibria where comparative advantages in different industries arise endogenously exist even when the autarky model has a unique equilibrium.

[*]This paper was previously circulated under the title "Endogenous Inequality in a Trade Model with Private Information". We thank Hal Cole, Lutz Hendricks, Pat Kehoe, Tim Kehoe, Narayana Kocherlakota, John Knowles, Rody Manuelli, Andrew Postlewaite, Paul Segerstrom, Ananth Seshadri, Robert Staiger, Kjetil Storesletten, Scott Taylor and Fabrizio Zilibotti for helpful comments, suggestions and discussions. We have also benefited from feedback from seminar participants at several conferences and institutions. Support from NSF Grants #SES-0003520 and #SES-0001717 is gratefully acknowledged. The research for this paper was done partly when Peter Norman was visiting IIES Stockholm and he is grateful for their hospitality. The usual disclaimer applies.

[§]Department of Economics, University of Minnesota. Email: `amoro@atlas.socsci.umn.edu`

[¶]Department of Economics, University of Wisconsin-Madison. Email: `Pnorman@facstaff.wisc.edu`

1 Introduction

A cursory look at countries, regions, and cities reveal large disparities in productivity. Moreover, highly productive areas often tend to be associated with clustering of knowledge intensive industries. This has led many economists to believe that *localized* knowledge spillovers are at play. Consequently, knowledge spillovers have become a central feature in the endogenous growth literature, modern international trade, and in recent developments of economic geography.

Scientific advances and innovations can be copied, so spillover effects seem endemic to the accumulation of knowledge. What is less clear is the scope of the externality. Social ties and network effects have been suggested as possible explanations for weakening diffusion with geographic distance. However, explicit modelling of such considerations is in its infancy.[1] Direct measurements of such effects are difficult given the basic premise that local spillovers are generated by *tacit* knowledge.[2]

This paper develops a stylized model of trade, where the source of the localized spillover is workers' private information about their skills. The technological frontier is common knowledge, but effects similar to localized knowledge spillovers nevertheless occur in equilibrium due to an informational externality that arises when rational firms estimate the worker skills using Bayes rule. As a result, comparative advantages can arise purely as an equilibrium phenomenon.

The model is a minimal deviation from neoclassical theory. Two countries have access to the same technology. A "high-tech" good requires skilled labor to produce, whereas a "low-tech" good can be produced by both skilled and unskilled labor. There are no trade frictions or technological externalities and workers decide on whether to acquire the skill necessary in the high tech sector based on costs and benefits from doing so. Countries are endowed with identical distributions of workers, so there is no exogenous source that creates comparative advantages.

Our only assumption that departs from traditional neoclassical theory is that skills cannot be

[1] Eeckhout and Jovanovic [5], construct a model of imperfect knowledge spillovers with solid micro foundations. However, there is no spatial dimension in this model.

[2] Some indirect approaches to measuring the geographic localization of knowledge have been proposed. Notably, Jaffe et al [11] argue that patent citations can be taken as a proxy for knowledge spillovers. Hence, if patent violators tend to be clustered near the patent holder, this is taken as evidence for localized knowledge.

perfectly observed. Rational firms therefore use prior knowledge about the distribution of skills when evaluating workers. In equilibrium, the prior is given by the aggregate skill level in the population. Given a noisy measurement of the skills of an individual worker, the firms' assessment of the likelihood that the worker is skilled is monotonically increasing in the aggregate skill level. Aggregate skills therefore act like a public good, and create a derived *informational externality.*

If the high tech sector is more intensive in skilled labor, the informational externality creates a force in favor of specialization. As a result, equilibria emerge where one country has more skilled workers and a higher standard of living than the other. In equilibrium, a comparative advantage for the rich country in the high tech sector arises endogenously.

The two crucial assumptions are i) the *asymmetric effect of human capital on productivity* in the two sectors, and, ii) the *imperfect observability* of skills, which generates the informational externality. To isolate the interplay between these assumptions, the rest of the model deviates as little as possible from textbook trade theory.

In equilibrium, workers are paid the expected value of their contributions to output. High signal workers are more likely to perform well in a high tech firm, implying that the equilibrium wage is increasing in the signal. Wages for both high and low signal workers also depend on aggregate investments, and incentives to acquire skills are supported by differences in expected wages for high and low skilled workers. The informational externality therefore creates a feedback between aggregate skills and incentives for skill acquisition.

Trade and inequalities are driven by the interaction between the informational externality and price effects. The price effects are straightforward. An increase in the skill level abroad shifts production of the high tech good towards the foreign country. Since skills are more abundant after the change, the wage differential between workers with high and low signals decreases, thereby reducing incentives at home. The same reduction in the relative price of skilled labor operates also in the foreign country. But, unlike the country where the skill level is held fix, the increase in the skill level also affects the inference made by the firms, and thereby wages, directly. Incentives are therefore affected asymmetrically in the two countries when investments change in one country.

The price effects are present also with perfect information, but factor price equalization would then imply that wages depend only on the investment decision. Nationality is then irrelevant, so, if the informational asymmetry is removed, the model has a unique equilibrium with no gainful trade. Asymmetric information is thus crucial for the price effects to work "as if" there is a negative cross country externality in human capital investments.

Under some restrictions on parameters, there is a unique autarky equilibrium, but equilibria where countries specialize nevertheless arise under free trade. There is also a symmetric equilibrium replicating autarky, with no gainful trade and no inequality, but this equilibrium may be unstable under free trade. Our model thus opens up the possibility that cross country income differences is an inevitable aspect of free trade. This outcome *may* however be Pareto superior to the symmetric equilibrium, so although the model is compatible with uneven development, it does not follow that the model is a justification for trade barriers to protect infant industries.

There is no systematic advantage for large economies in our model. Almost all analysis considers a world consisting of two countries, but equilibria of the two-country model can be reinterpreted as an equilibrium of a n country extension, where countries are partitioned into a "North" and a "South". In this reinterpretation, the size of an individual nation is irrelevant. Only the relative size of the South to the North matters. The model is thus consistent with a world where there is no particular relationship between size and development.

Being static and highly stylized, our model has some obvious drawbacks compared with models considered in (for example) the endogenous growth literature. However, we believe that the explicit micro foundation of the externality considered in this paper is an important virtue. As has been pointed out by many others (for example Lucas [14]), it is a serious drawback that it is unclear whether cities, regions, industries, countries, language groups, trading blocks, or something else is the most relevant unit for the scope of localized knowledge spillovers.[3] In contrast, the derived informational externality of the model studied in this paper is necessarily local, being limited in scope by barriers to labor mobility.

[3]It should be stressed that there are circumstances where this arbitrariness with local spillovers is less problematic. In particular, in the emerging literature on the role of cities there are many well established empirical regularities that provide meaningful restrictions on the way local externalities enter (see for example Lucas and Rossi-Hansberg [15] or Eeckhout [4]).

2 The Model

Two countries, labeled by $j = h, f$, are populated by a continuum of agents, where λ^h and $\lambda^f = 1 - \lambda^h$ denote the mass of agents in each country. All agents are price takers, and the production technology is a simplified version of a standard $2 \times 2 \times 2$ trade model, but with factors being workers with and without human capital. In one ("high tech") sector, output depends on the number of workers *with human capital.* In the other ("low tech") sector only the number of workers employed is relevant. The model is closed by a stylized version of a model of human capital acquisition and informational technology from Moro and Norman [18].

2.1 Preferences and Human Capital Investments

Agents have preferences over two consumption goods, x_1 and x_2, and a binary investment choice. All agents have identical preferences over the two goods, but differ in their attitudes towards the investment decision. This is modelled by assuming that the workers of each country are distributed on interval $[\underline{c}, \overline{c}]$ according to a distribution function G, where $c \in [\underline{c}, \overline{c}]$ is interpreted as the utility cost (or gain if negative) of making the human capital investment.[4] The utility of an agent c consuming the bundle (x_1, x_2) is thus $u(x_1, x_2) - c$ if the agent invests and $u(x_1, x_2)$ otherwise, where u is a homothetic and strictly quasi-concave function representing the (common) preferences over the two goods.

In the remainder of the paper we call workers who invested in human capital *qualified* workers, and workers who did not *unqualified.*

2.2 Production Technology

The two consumption goods are produced solely from qualified and unqualified labor, denoted q and n respectively, in accordance with production functions $y_1(\cdot)$ and $y_2(\cdot)$ given by,

$$y_1(q, n) = q \tag{1}$$

$$y_2(q, n) = q + n, \tag{2}$$

[4]We will often assume that $\underline{c} < 0$. The rationale is that if an arbitrarily small fraction of workers like to make the investment even if there are no monetary gains, this eliminates "nuisance equilibria" with zero investments.

All workers are thus perfect substitutes in industry 2, whereas only qualified workers contribute to the production of good 1. [5]

2.3 Information Technology

A crucial assumption is that human capital investments are observed with noise. This part of the model is set up to make the firms' signal extraction problem as simple as possible. After the investments, nature assigns each worker a signal $\theta \in \{g, b\}$. For simplicity we assume that

$$\Pr[g|\text{worker qualified}] = \Pr[b|\text{worker unqualified}] = \eta > \frac{1}{2}. \tag{3}$$

The only reason for the restriction that $\eta > 1/2$ is that it orders the signals so as to make g "good news" and b "bad news".

2.4 What Are Good Empirical Analogues of the Signals?

There is a large empirical literature that seeks to relate human capital with growth and development. Usually, years of schooling is taken as the (or one) measure of human capital. From the point of view of the firms' problem, it also seems quite reasonable to consider schooling as an imperfect measure of worker productivity. That is, *anything* that is correlated with productivity is useful for the signal extraction problem, and schooling is presumably one indicator of productivity. The problem with thinking of the signals in our model as formal education is that workers directly choose how many years of schooling to invest in. In our model the actual choice is unobservable.[6]

Our preferred interpretation of the setup is as a model where all workers have the same level of formal schooling. The human capital investment can then be thought of as costly effort while in

[5] This extreme technology is for simplicity only. Qualitatively, we need two sectors with different factor intensities, just like in the Hecksher-Ohlin model with fixed factor endowments.

[6] That is, when signals are chosen we would get a *signaling model,* with a labor market similar to Spence [24]. Our model is a *signal extraction model.* We believe that several qualitative insights from our model could be generated also from a signaling model. One consideration that led us to stick with the current formulation is that the signaling setup generates a huge multiplicity of equilibria, where off-the equilibrium path beliefs can be set arbitrarily. In general, we also have multiple equilibria, but, unlike the signalling model, everything is on the equilibrium path. This means that there is a unique equilibrium wage scheme associated with any fixed behavior by the workers. The source of multiplicity is therefore strategic complementarities rather than choice of off-the-equilibrium path beliefs.

school, and the most natural interpretation of the signal is as an aggregate of grades from school, scores on performance tests, letters of recommendation, etc. Firms do make use of various performance tests and spend resources on job interviews. In our view, the only reasonable interpretation of this is that there is an unobservable component of productivity that firms cannot discern by looking at variables such as education and experience.

Obviously, worker productivity is also influenced by years of formal schooling. However, for those workers with the same level of formal schooling we believe that the signal extraction problem in our model, while stylized, is a quite realistic description. The same informational issues would remain also in a richer model with an actual choice also on the length of schooling.

3 Equilibrium Characterization

Our notion of equilibrium is analogous with a competitive equilibrium in a perfect information environment, but the informational asymmetry makes the treatment of the "labor supply" somewhat non-standard. For clarity, Section 3.1 therefore provides a detailed definition of equilibrium. We then show in Proposition 1 that, for *fixed* investments, versions of the welfare theorems hold: the equilibrium is characterized by a planning problem (where the informational asymmetry is built into the feasible set). This allows us to appeal to simple graphs in the analysis that follows.

3.1 Conditions for Equilibrium

Consider first a agent with realized wage w deciding on how to allocate her earnings between the two goods given prices $p = (p_1, p_2)$. Define the (ex post) maximized utility of the worker as

$$\begin{aligned} v(w,p) \; = \quad & \max_{x_1,x_2} u\left(x_1, x_2\right) \\ & \text{subject to } p_1 x_1 + p_2 x_2 \leq w. \end{aligned} \tag{4}$$

By strict quasi-concavity of $u\left(x_1, x_2\right)$, the optimization problem in (4) has a unique solution, and, with the usual notational abuse, we denote the demand functions by $x_1(w,p), x_2(w,p)$.

Firms cannot observe if a worker is qualified or not, so a labor demand is a map $l : \{g, b\} \rightarrow R_+$. Associated with any fraction of qualified workers, π, and a given labor demand l, the corresponding quantities of qualified and unqualified workers are

$$\begin{aligned} q &= l(g)\,\mu(g,\pi) + l(b)\,\mu(b,\pi) \\ n &= l(g)\,(1-\mu(g,\pi)) + l(b)\,(1-\mu(b,\pi)), \end{aligned} \tag{5}$$

where $\mu(\theta,\pi)$ denotes the posterior probability that a worker is qualified given prior π, that is

$$\mu(g,\pi) \equiv \frac{\eta\pi}{\eta\pi + (1-\eta)(1-\pi)} \qquad \mu(b,\pi) \equiv \frac{(1-\eta)\,\pi}{(1-\eta)\,\pi + \eta(1-\pi)}. \tag{6}$$

To get tractable market clearing conditions we assume that a strong law of large numbers applies and treat q and n in (5) both as expected and realized inputs of labor.

Without loss of generality there is a representative firm in each sector and each country, which takes the *wage schedule* $w^j : \{g,b\} \to R_+$ and output price p_i as given.[7] Using the production function (1) and (5), the profit maximization problem for a sector 1 firm may be written as

$$\max_l p_1\left(l(g)\,\mu\left(g,\pi^j\right) + l(b)\,\mu\left(b,\pi^j\right)\right) - w_g^j l(g) - w_b^j l(b), \tag{7}$$

where $\mu\left(\theta,\pi^j\right)$ is the posterior probability of being qualified defined in (6). For sector 2, where qualified and unqualified workers are equally productive, the profit maximization problem is

$$\max_l p_2\,(l(g) + l(b)) - w_g^j l(g) - w_b^j l(b). \tag{8}$$

Agents have rational expectations about the wages and prices, but face uncertainty about the realization of the signal. The expected utility for an agent with investment cost c is

$$\eta v(w_g^j,p) + (1-\eta)\,v(w_b^j,p) - c \text{ if agent } c \text{ invests, and} \tag{9}$$

$$(1-\eta)\,v(w_g^j,p) + \eta v(w_b^j,p) \text{ if if agent } c \text{ does not invest.} \tag{10}$$

[7]The caveat is that the informational asymmetry would disappear if (qualified) workers could start their own firms. We rule this and other contractual solutions to the informational asymmetry out by assumption. One way to justify this is to assume that there is some minimum efficient scale for production and that only aggregate output, and not the performance of individual workers, can be observed. Risk aversion or limited liability issues are alternative routes to rule out first best solutions to the contracting problem, but lead more complicated wage schemes. If the informational asymmetry could be *fully* contracted away, the model would have a unique symmetric equilibrium with no gainful trade.

If (9) exceeds (10), the worker is better off investing. Investing in human capital is thus optimal for all workers with $c \leq (2\eta - 1)\,(v(w_g^j, p) - v(w_b^j, p))$, and the implied fraction of investors is[8]

$$\pi^j = G\Big((2\eta - 1)\,(v(w_g^j, p) - v(w_b^j, p))\Big). \tag{11}$$

To sum up: optimal consumption plans are defined in (4), the problems (7) and (8) describe the profit maximization problems for each sector, and (11) summarizes the individually optimal human capital investments. What remains to describe are the market clearing conditions.

Factor market clearing simply requires that the aggregate demand for workers with each signal equals the mass of agents who draw the signal. That is, let $l_i^j = (l_i^j(g), l_i^j(b))$ be a labor demand scheme in industry j and country i and write the labor market clearing conditions as

$$\begin{aligned} l_1^j(g) + l_2^j(g) &= \eta\pi^j + (1-\eta)(1-\pi^j) \\ l_1^j(b) + l_2^j(b) &= (1-\eta)\,\pi^j + \eta(1-\pi^j). \end{aligned} \tag{12}$$

Finally, for the product market equilibrium conditions it is convenient to let x_i^j be the output in industry j and country i. That is

$$\begin{aligned} x_1^j &= l_1^j(g)\,\mu\left(g, \pi^j\right) + l_1^j(b)\,\mu\left(b, \pi^j\right) \\ x_2^j &= l_2^j(g) + l_2^j(b), \end{aligned} \tag{13}$$

which allows us to write the product market clearing conditions for the world market as

$$\sum_{j=h,f} \lambda^j \left(x_i^j - \underbrace{\left[\eta\pi^j + (1-\eta)(1-\pi^j)\right]}_{\#\text{agents with wage } w_g^j} x_i(w_g^j, p) - \underbrace{\left[(1-\eta)\,\pi^j + \eta(1-\pi^j)\right]}_{\#\text{agents with wage } w_b^j} x_i(w_b^j, p) \right) = 0 \tag{14}$$

Our definition of equilibrium is then:

Definition 1 *A Competitive Equilibrium consists of output prices p^*, wages w^{j*}, labor demands l_i^{j*}, outputs x_i^{j*}, and fractions of qualified workers π^{j*} for each country $j = h, f$ and industry $i = 1, 2$, satisfying:*

[8] If a fraction π^j become qualified, the workers that invest must be those with cost $c \leq G^{-1}\left(\pi^j\right)$. To reduce notation we therefore omit the trivial individual investment rules from the definition of equilibrium.

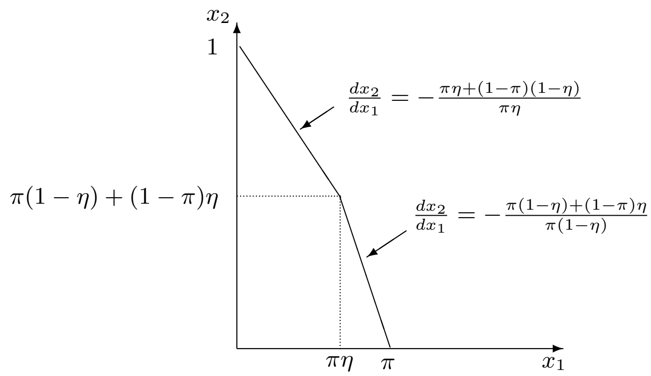

Figure 1: Per Capita Production Possibilities in a Country

(1) l_1^{j*} solves (7) and l_2^{j*} solves (8) given $p_i = p_i^*$ and x_1^{j*} and x_2^{j*} are the associated profit maximizing outputs in $j = h, f$

(2) the product market clearing conditions in (14) are satisfied.

(3) the factor market clearing conditions in (12) are satisfied.

(4) π^{j*} satisfies (11) given $p = p^*$ and wages $w^j = w^{j*}$for $j = h, f$

3.2 The Production Possibilities Set

A useful way to represent the technology is to construct the *production possibilities set.* The set of feasible output plans in a country depends on π and we let $X(\pi)$ denote the (per capita) production possibilities set in a country. The set $X(\pi)$ is depicted graphically in Figure 1. To understand the figure, first observe that $(x_1, x_2) = (0, 1)$ if all workers are producing good 2, and that $(x_1, x_2) = (\pi, 0)$ if all workers are producing good 1, since a fraction π of the workers are productive in sector 1. Moreover, if all signal g workers are in sector 1 ($\pi\eta$ of these $\pi\eta + (1-\pi)(1-\eta)$ workers are productive) and all signal b workers (a total of $\pi(1-\eta) + (1-\pi)\eta$ such workers) are in sector 2, then the outputs are given by the point at the kink in the graph.

The *world production possibilities* set is given by $X^W(\pi^h, \pi^f) = \lambda^h X(\pi^h) + \lambda^f X(\pi^f)$. By convexity of $X(\pi)$, this set is also convex. Moreover, $X^W(\pi, \pi) = X(\pi)$ if $\pi^h = \pi^f = \pi$, which means that autarky is equivalent to the restriction that $\pi^h = \pi^f$.

3.3 A Planning Characterization of Continuation Equilibria

We refer to a situation where all equilibrium conditions except (4), the condition that investments are chosen optimally, are fulfilled, as a *continuation equilibrium.*[9] From a first best point of view, a continuation equilibrium is inefficient: qualified and unqualified workers with the same signal are treated symmetrically, resulting in a misallocation of workers to jobs. However, if the symmetric treatment of workers with the same signal is viewed as a fundamental property of the environment, then the equilibrium allocation is (constrained) efficient *conditional on the investment behavior.* This allows us to describe aggregate equilibrium allocations as solutions to the planning problem,

$$\max_{(x_1,x_2)\in X^W\left(\pi^h,\pi^f\right)} u\left(x_1,x_2\right), \tag{15}$$

where $X^W\left(\pi^h,\pi^f\right)$ is defined in Section 3.2. While the normative implications of this are of some interest, the main value of this result is that it allows us to appeal to intuitive graphs in the analysis that follws:

Proposition 1 *Suppose that $u\left(x_1,x_1\right)$ is homothetic. Then:*

1. *The aggregate world consumption in any continuation equilibrium is a solution to (15)*

2. *Suppose that $\left(x_1^*,x_2^*\right)$ solves (15), $\left(p_1^*,p_2^*\right)$ is a normal to a hyperplane that separates the set of bundles such that $u\left(x_1,x_2\right)\geq u\left(x_1^*,x_2^*\right)$ and $X^W\left(\pi^h,\pi^f\right)$, and that $w_g^{j*}=\max\left\{p_1^*\mu\left(g,\pi^j\right),p_2^*\right\}$ and $w_b^{j*}=\max\left\{p_1^*\mu\left(b,\pi^j\right),p_2^*\right\}$in each country j. Then, then these prices, wages and aggregate consumptions are part of a continuation equilibrium.*[10]

The proof is in the appendix. Proposition 1 immediately implies:

Corollary 1 *Given any $\left(\pi^h,\pi^f\right)$ there is a unique continuation equilibrium up to a re-normalization of the prices.*

[9]This term is mainly due to lack of a better alternative. Due to the workers being non-atomic it does not make a difference whether investments are made before or simultanously with the wage posting.

[10]The allocation of workers in each country is somewhat complicated to describe in general, but is implicitly pinned down as the (almost always) unique worker allocation that can produce the equilibrium bundle.

4 A Parametric Specification

In the remainder of the paper we will restrict attention to the case where

$$u(x_1, x_2) = x_1^{\alpha} x_2^{1-\alpha}, \tag{16}$$

unless otherwise stated. We will also for the most part assume that the distribution of investment costs, G, is uniform.

After deriving an expression for the incentives to invest in terms of continuation equilibrium prices, we first consider the autarky benchmark. Proposition 2 establishes some sufficient conditions for uniqueness of the autarky equilibrium. Section 4.5 demonstrates how trade changes incentives, and shows that a unique autarky equilibrium (which is stable in autarky) may be unstable if the economies are open. In Section 4.5.1, we construct an example where countries specialize, and where only the rich country is better off than in autarky. The point with the next example, in Section 4.5.2, is that both countries may gain relative autarky, despite one country being richer than the other "for no good reason". Section 4.6 explains how equilibria of may be reinterpreted as equilibria in an extension with n countries. Finally, Section 5.2 explains why the effects highlighted in this paper would be strengthened by the introduction of mobile capital.

4.1 Incentives to Invest

Given the Cobb-Douglas preferences in (16) the relevant individual demand functions are

$$x_1(p, w) = \frac{\alpha w}{p_1} \qquad x_2(p, w) = \frac{(1-\alpha)\, w}{p_2}, \tag{17}$$

implying that the maximized continuation utility for a worker that earns wage w is

$$v(w, p) = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_1^{\alpha} p_2^{1-\alpha}} w \tag{18}$$

We set $p_2 = 1$ and, with some abuse of notation, denote by $p\left(\pi^h, \pi^f\right), w_g^j\left(\pi^h, \pi^f\right)$, and $w_b^j\left(\pi^h, \pi^f\right)$ (by Corollary 1) the unique continuation equilibrium prices given this normalization.

A qualified worker earns $w_g^j\left(\pi^h, \pi^f\right)$ with probability η and $w_b^j\left(\pi^h, \pi^f\right)$ with probability $1-\eta$. Symmetrically, an unqualified worker earns $w_g^j\left(\pi^h, \pi^f\right)$ with probability $1-\eta$ and $w_b^j\left(\pi^h, \pi^f\right)$ with probability η. Computing the expectation of $v(w, p)$ in (18) *conditional on investment* and

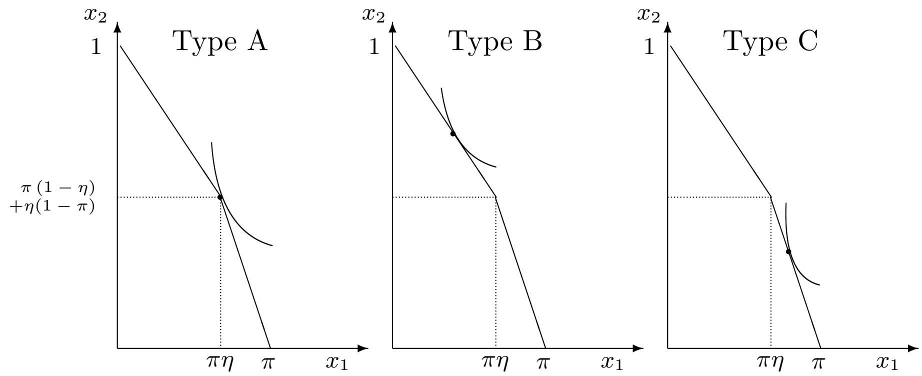

Figure 2: Three "Types" of Continuation Equilibria

subtracting from this the expectation of $v(w,p)$ *conditional on not investing* we get the *gross benefits of investment* for an agent in country j, denoted $B^j\left(\pi^h,\pi^f\right)$, which is given by

$$\begin{aligned} B^j\left(\pi^h,\pi^f\right) &= E\left\{v\left(w,p\right)|\text{qualified}\right\} - E\left\{v\left(w,p\right)|\text{unqualified}\right\} \\ &= \frac{(2\eta-1)(w_g^j\left(\pi^h,\pi^f\right) - w_b^j\left(\pi^h,\pi^f\right))}{\left(p\left(\pi^h,\pi^f\right)\right)^\alpha}\alpha^\alpha(1-\alpha)^{1-\alpha}. \end{aligned} \tag{19}$$

Using condition 4 in Definition 1 we see that any $\left(\pi^h,\pi^f\right)$ such that $\pi^j = G\left(B^j\left(\pi^h,\pi^f\right)\right)$ for $j=h,f$ gives an equilibrium fraction of investors in each country. All that remains to calculate full equilibria is to derive expressions for the continuation equilibrium prices.

4.2 Continuation Equilibria in Autarky

As a benchmark, we first consider a closed economy. Suppressing the country index, we write π for the proportion of qualified workers. By consulting Figure 2 we conclude that the there are three possible "types" of continuation equilibria;[11]

Type A equilibria (allocation of workers "according to signals") Diagrammatically, this is when the tangency is at the kink of the feasible set. That is, all workers with signal b (g) are working in the low (high) tech sector. Outputs are then $x_1 = \eta\pi$ and $x_2 = (1-\eta)\pi + \eta(1-\pi)$,

[11] Calculations are straightforward, but tedious. See Moro and Norman [19].

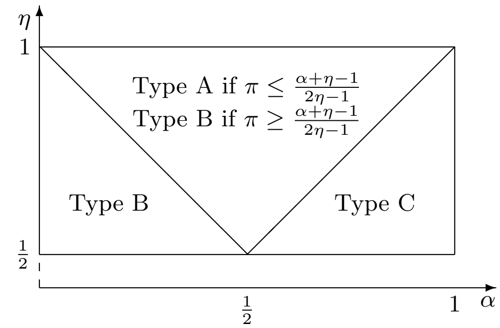

Figure 3: Types of autarky equilibria in the (α, η) space

so the demands in (17) pin down the price of the high tech good as

$$p(\pi) = \frac{\alpha}{1-\alpha} \frac{(1-\eta)\pi + \eta(1-\pi)}{\eta\pi}. \tag{20}$$

Candidate equilibrium wages are obtained by observing that zero profits is necessary for profit maximization. Since $p_2 = 1$, this immediately gives $w_b(\pi) = 1$. The high tech firm sells $\eta\pi$ units at price $p(\pi)$ and hires $\eta\pi + (1-\eta)(1-\pi)$ workers with signal g. Zero profits Sector 1 therefore implies that

$$w_g(\pi) = p(\pi) \frac{\pi\eta}{\pi\eta + (1-\eta)(1-\pi)} = p(\pi)\mu(g,\pi), \tag{21}$$

which has the interpretation that the wage equals the expected value of output. Finally, we have to check that a high tech firm has no incentive to hire a worker with signal b, and that a low tech firm has no incentive to hire a worker with signal g. These conditions give rise to inequalities that determine the region where a Type A continuation equilibrium exists (shown in Figure 3).

Type B equilibria (mixing of good signals) In terms of Figure 2, this corresponds to a tangency to the left of the kink. Workers with signal g earn the same wage in each sector, and, since all workers in the low tech sector are paid 1, it follows immediately that $w_g(\pi) = w_b(\pi) = 1$. All that remains is therefore to determine the region where this is an equilibrium. To do this, one first observes that, for the high tech firm to make a zero profit, it must be that $p(\pi) = 1/\mu(g,\pi)$. The price of the high tech good in units of the low tech good is thus determined on the "supply side" in this case. The tangency condition from the planning problem therefore determines the *outputs* that consumers are willing to purchase at these prices, so this type of equilibrium requires

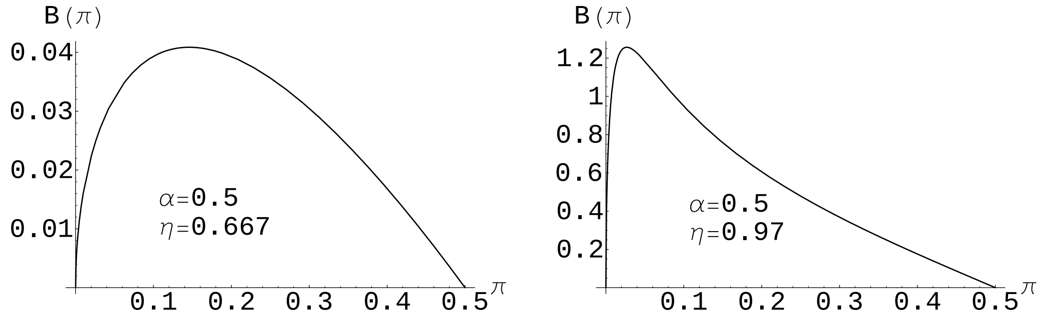

Figure 4: Gross incentives to invest under autarchy

that there exists $\gamma \in (0, 1]$ such that

$$\frac{\alpha}{(1-\alpha)\,p\,(\pi)} = \frac{\overbrace{\gamma\eta\pi}^{x_1}}{\underbrace{(1-\gamma)\,(\eta\pi + (1-\eta)\,(1-\pi)}_{x_2 \text{ produced by } g\text{-workers}} + \underbrace{(1-\eta)\,\pi + \eta\,(1-\pi)}_{x_2 \text{ produced by } b\text{-workers}}}. \tag{22}$$

Figure 3 shows the relevant region.

Type C equilibria (mixing of bad signals) This occurs if and only if $\alpha > \eta$, that is when the demand for the high tech good is very strong. Since no example that follow has an autarky equilibrium of this form we refer the reader to Moro and Norman [19] for details.

4.3 Equilibrium investments in Autarky

A closed form expression for the incentives to invest as a function of π is obtained by substituting the wages and prices derived in Section 4.2 into (19). If $\alpha \leq \eta$, this function may be written as,

$$B\,(\pi) = \Phi \max\left\{(2\eta - 1)\left(\frac{\pi\eta}{\pi(1-\eta) + (1-\pi)\,\eta}\right)^{\alpha}\left(\frac{\alpha - (\pi\eta + (1-\pi)(1-\eta))}{\pi\eta + (1-\pi)(1-\eta)}\right), 0\right\}, \tag{23}$$

where $\Phi = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Figure 4 plots $B\,(\pi)$ for two different sets of parameter values. One can show analytically that $B\,(\pi)$ is single-peaked, but not necessarily concave (see the example to the right). Any π such that $\pi = G\,(B\,(\pi))$ is an equilibrium fraction of investors. Since $G\,(B\,(\pi))$ is continuous and takes values on $[0, 1]$, existence follows trivially. The fixed point condition is illustrated in Figure 5, where $\eta = 2/3$, $\alpha = 1/2$ and G is uniform over $[\underline{c}, \overline{c}]$, with $\overline{c} - \underline{c} = 0.2$. Changes in $\underline{c}$ correspond to shifts in the cost distribution. If $\underline{c} < 0$ (in case some workers prefer to

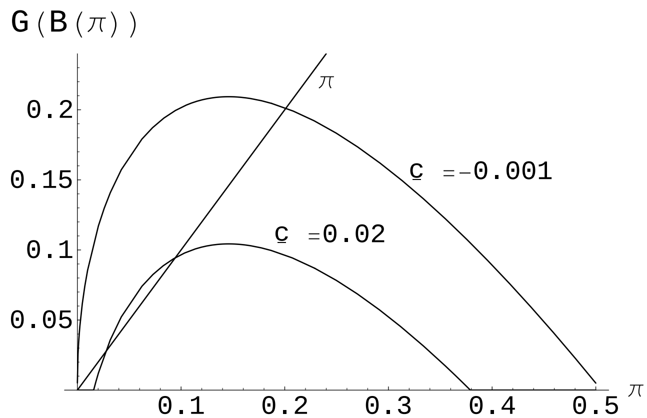

Figure 5: Equilibrium fixed point maps for two values of $\underline{c}$, with $\eta = 2/3, \alpha = 1/2$

invest at a zero wage difference) the equilibrium is unique. For $\underline{c} = 0$, there is a trivial equilibrium with no investments and an equilibrium with $\pi > 0$. As $\underline{c}$ gets slightly larger there are three equilibria, whereas if $\underline{c}$ is sufficiently large, only an equilibrium with no investment remains.

4.4 Uniqueness of Autarky Equilibria

A useful feature of this parametrization is that there are simple sufficient conditions for when the autarky equilibrium is unique. While not being of much interest in itself, this result facilitates comparisons between trade and autarky.[12]

Proposition 2 *If $G(\cdot)$ is concave and $\underline{c} < 0$, then there is a unique autarky equilibrium.*

The interpretation of the condition $\underline{c} < 0$ is that the investment in itself provides utility to some (arbitrarily small proportion of) workers. This condition arises because the proof exploits that $G(B(\pi))$ cannot intersect the 45 degree line from below.

4.5 Equilibria in the Trade Regime

We now assume that h and f trade on a frictionless world market. The number of potential forms of continuation equilibria now swells to 9: in each country the allocation of workers may be like

[12] With multiple autarky equilibria we would either have to make an equilibrium selection or make set-wise comparisons, which would obscure the economics of the model.

in any of the three types of autarky equilibria (however, mixing in both countries is a knife-edge possibility). To reduce the number of cases we therefore set $\eta = 2/3$, $\alpha = 1/2$, and $\lambda^h = \lambda^f = 1/2$ in the analysis that follows. With these parameter values the continuation equilibrium can be of three different forms. If countries are labeled so that $\pi^h \leq \pi^f$ the possibilities are:

Type	A^T	B^T	C^T
$p\left(\pi^h, \pi^f\right)$	$\frac{4-\pi^f-\pi^h}{2(\pi^f+\pi^h)}$	$\frac{1+\pi^h}{2\pi^h}$	$\frac{2-\pi^f}{\pi^f}$
$w_g^h\left(\pi^h, \pi^f\right)$	$p\left(\pi^h, \pi^f\right)\frac{2\pi^h}{1+\pi^h}$	1	1
$w_b^h\left(\pi^h, \pi^f\right)$	1	1	1
$w_g^f\left(\pi^h, \pi^f\right)$	$p\left(\pi^h, \pi^f\right)\frac{2\pi^f}{1+\pi^f}$	$p\left(\pi^h, \pi^f\right)\frac{2\pi^f}{1+\pi^f}$	$p\left(\pi^h, \pi^f\right)\frac{2\pi^f}{1+\pi^f}$
$w_b^h\left(\pi^h, \pi^f\right)$	1	1	1
Exists when	$\pi^h \leq \pi^f \leq \frac{\pi^h(3-2\pi^h)}{1+2\pi^h}$	$\frac{\pi^h(3-2\pi^h)}{1+2\pi^h} \leq \pi^f \leq \frac{4\pi^h}{1+3\pi^h}$	$\pi^f \geq \frac{4\pi^h}{1+3\pi^h}$

Table 1: Continuation Equilibria Under International Trade

Type A^T Equilibria (according to signals in both countries) This is the obvious analogue to equilibria of Type A in the autarky model. If investments in both countries are near a proportion when this occurs in autarky, the continuation equilibrium is of this form.

Type B^T Equilibria (according to signals in f, mixing of good signals in h) In analogy with Type B equilibria in autarky, the equilibrium price is then determined from an indifference condition in the allocation of workers with signal g in country h

Type C^T Equilibria (mixing of bad signals in f, all in low skill sector in h) This is just like a Type C equilibria in autarky, with some exogenous extra output of the low skilled good.

The equilibrium characterization for the relevant continuation equilibria is summarized in Table 1. It is understood that $\pi^h \leq \pi^f$, so Table 1 does provide a unique continuation equilibrium for any possible $\left(\pi^h, \pi^f\right) \neq (0, 0)$ by reversing the roles of the countries when necessary. Figure 6 shows the different regions of investment behavior that is relevant for each type of equilibrium.

The most illuminating way to use the continuation equilibria in Table 1 and (19) is to express

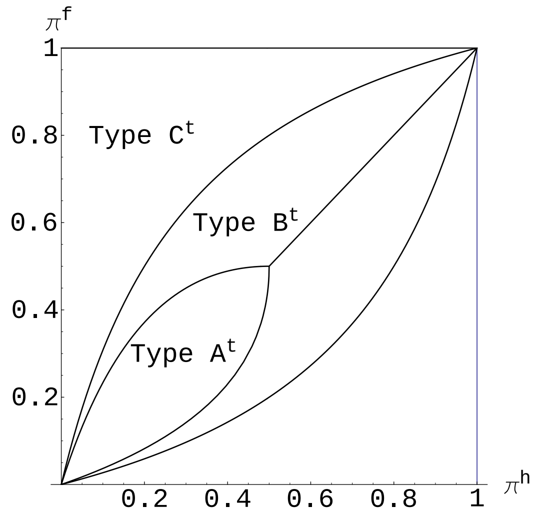

Figure 6: Types of asymmetric equilibria, with $\eta = 2/3, \alpha = 1/2$

the benefit to invest for a worker in country f as

$$B^f(\pi^h, \pi^f) = \frac{1}{6}\left(\sqrt{p\left(\pi^h, \pi^f\right)}\mu\left(g, \pi^f\right) - \frac{1}{\sqrt{p\left(\pi^h, \pi^f\right)}}\right), \tag{24}$$

where $\mu\left(g, \pi\right) = 2\pi/(1+\pi)$ and

$$p\left(\pi^h, \pi^f\right) = \begin{cases} \frac{2-\pi^f}{\pi^f} & \pi^f \geq \frac{4\pi^h}{(1+3\pi^h)} \\ \frac{1+\pi^h}{2\pi^h} & \frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \leq \pi^f \leq \frac{4\pi^h}{(1+3\pi^h)} \\ \frac{4-\pi^f-\pi^h}{2\left(\pi^f+\pi^h\right)} & \pi^h \leq \pi^f \leq \frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \end{cases}. \tag{25}$$

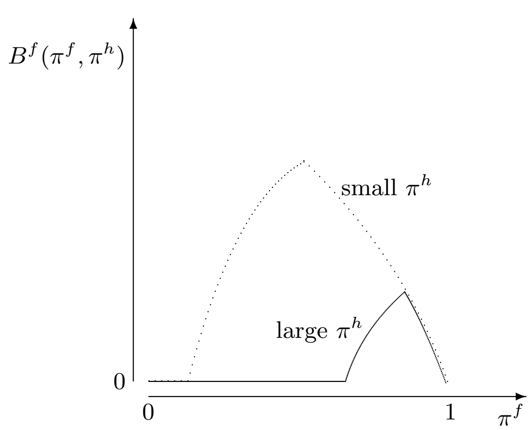

Figure 7: Incentives to invest in country f at different values of π^h

Expression (24) shows that incentives are strictly increasing in the price of the high tech good. Moreover, as is easy to check from (25), the equilibrium price is strictly decreasing in π^h in the range where some workers in h are in the high tech sector. Hence, a decrease in investments at home improves incentives abroad and an increase in investments in the foreign country reduces incentives at home. In reduced form, this is like a negative cross-country externality in human capital acquisition (see Figure 7 which shows how incentives in f are affected by π^h). These effects create equilibria where countries specialize as rich countries exporting the high-tech good and poor countries exporting the low tech good, also when the autarky equilibrium is unique.

An Asymmetric Equilibrium May Be the Only Stable Outcome A symmetric equilibrium, replicating autarky, always exists in the trade regime. However, for many parameterization, this equilibrium is destabilized when the economy is opened up for international trade.[13]

Assume that $\underline{c} < 0$, so that there is a unique autarky equilibrium, which we denote by π^A. It is then immediate that π^A must be stable since $G(B(\pi))$ must intersect the 45^o line from above. It also follows that $(\pi^h, \pi^f) = \left(\pi^A, \pi^A\right)$ is an equilibrium when the countries are allowed to trade.

We want to analyze the effects of small deviations from the symmetric equilibrium. Consider the change in relative price first. When $\pi^h = \pi^f = \pi$ the price of the high tech good is $p\left(\pi, \pi\right) = (4 - \pi - \pi)/2(\pi + \pi) = \left(2 - \pi\right)/\pi$. Differentiation of (25) gives

$$\begin{aligned} \frac{d}{d\pi} p(\pi, \pi) &= \frac{-1}{\left(\pi\right)^2} \qquad \text{(relevant under autarky)} \\ \frac{\partial}{\partial \pi^f} p(\pi^h, \pi^f) &= \frac{-2}{\left(\pi^h + \pi^f\right)^2} \qquad \text{(relevant with trade).} \end{aligned} \tag{26}$$

Evaluating each expression at (π^A, π^A) we have that

$$\left.\frac{d}{d\pi} p(\pi, \pi)\right|_{\pi = \pi^A} - \left.\frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f}\right|_{\pi^h = \pi^f = \pi^A} = \frac{-1}{\left(\pi^A\right)^2} - \frac{-2}{4\left(\pi^A\right)^2} = \frac{-1}{2\left(\pi^A\right)^2} < 0\ . \tag{27}$$

An increase in investments thus have a larger negative impact on the price in autarky, as intuition would suggest. Autarky is equivalent to the trade regime with the added restriction that $\pi^h =$

[13]Since the model lacks real time, "stability" is a somewhat ad hoc criterion that corresponds to the seemingly myopic adjustment dynamic where $\pi^j_{t+1} = G(B^j(\pi^j_t, \pi^k_t))$, $j, k = h, f$, $j \neq k$ (or the natural continuous analogue). Embedding the model in an OLG framework one obtains a dynamic system like this if one assumes that employers can not differentiate between workers of different cohorts.

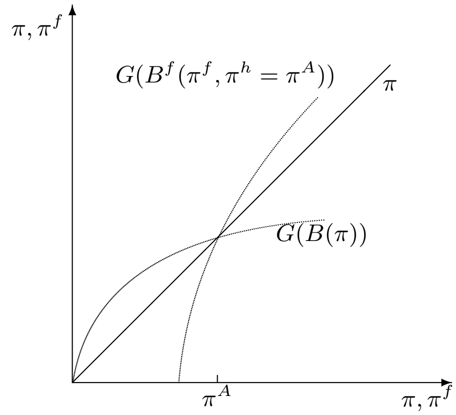

Figure 8: Best responses under trade and autarky, at the autarky equilibrium

$\pi^f = \pi$. We can thus use (24) for a comparison of the regimes. In the autarky case, we restrict the two arguments of B^f to be equal, while the second argument is unrestricted in the open economy case. Differentiating, we obtain

$$\left.\frac{dB^j(\pi,\pi)}{d\pi}\right|_{\pi=\pi^A} = \underbrace{\frac{\sqrt{p^A}}{6}\left.\frac{d\mu(g,\pi)}{d\pi}\right|_{\pi=\pi^A}}_{\text{“information effect”}} + \underbrace{\frac{1}{\sqrt{p^A}}\left(\mu\left(g,\pi^A\right)+\frac{1}{2p^A}\right)\left.\frac{dp(\pi,\pi)}{d\pi}\right|_{\pi=\pi^A}}_{\text{“price effect”}} \tag{28}$$

$$\left.\frac{\partial B^f\left(\pi^h,\pi^f\right)}{\partial \pi^f}\right|_{\substack{\pi^h=\pi^A\\ \pi^f=\pi^A}} = \underbrace{\frac{\sqrt{p^A}}{6}\left.\frac{d\mu\left(g,\pi^f\right)}{d\pi^f}\right|_{\pi^f=\pi^A}}_{\text{“information effect”}} + \underbrace{\frac{1}{\sqrt{p^A}}\left(\mu\left(g,\pi^A\right)+\frac{1}{2p^A}\right)\left.\frac{\partial p(\pi^h,\pi^f)}{\partial \pi^f}\right|_{\substack{\pi^h=\pi^A\\ \pi^f=\pi^A}}}_{\text{“price effect”}}, \tag{29}$$

where p^A is shorthand notation for $p(\pi^A, \pi^A)$. In each case, the effect on incentives is decomposed as a positive "information effect" and a negative "price effect". The information effect in (28) is the same as in (29), but, by (27), the price effect is stronger in autarky, so the slope of $B^f\left(\pi^f, \pi^h = \pi^A\right)$ exceeds the slope of the autarky benefits of investment $B(\pi)$, when evaluating both functions at π^A (see Figure 8). Hence, it is possible that $G(B^f\left(\pi^f, \pi^h = \pi^A\right))$ intersects the 45^o line from below at $\pi^f = \pi^A$ even if $G(B(\pi))$ intersects from above. Since the curve $G(B^f\left(\pi^f, \pi^h = \pi^A\right))$ intersecting the 45^o line from below is a *sufficient* condition for local instability this shows that the autarky equilibrium may be destabilized by opening up for trade.[14]

[14] Examples are easy to find. When c is uniformly distributed on $[0, 2]$, the unique (non-trivial) autarky equilibrium is $\pi = .0067$. The equilibrium where $\pi^f = \pi^h = 0.067$ is unstable under trade, while an asymmetric equilibrium with $\pi^f = .0283$, $\pi^h = 0$ is stable.

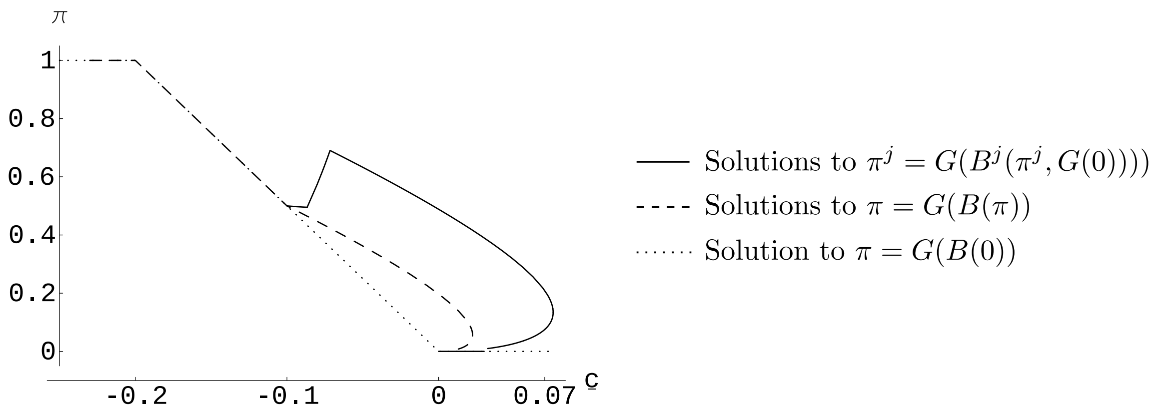

Figure 9: Equilibrium investments under trade with $\eta = 2/3, \alpha = 1/2$

Numerical Illustration The simplest asymmetric equilibrium is when the poor country, which we label as country h, is fully specialized in the low-tech sector. In such an equilibrium, the wage gap in h is zero, so the fraction of qualified workers in h is pinned down as $\pi^h = G(0)$. Moreover, since the equilibrium under consideration must be either of type B^T or C^T, the calculation of the incentives in f is straightforward, and the proportion of qualified workers in f solves a single variable fixed point equation similar to the autarky case, but with some "exogenous" extra production of x_2. Once π^f is obtained from this condition it only remains to check that firms in h have no incentives to hire workers signal g to produce the high tech good.

In Figure 9 we illustrate how shifts in the cost distribution affect the possibility for asymmetric equilibria. The calculations assume that G is uniform over $[\underline{c}, \overline{c}]$, where $\overline{c} - \underline{c} = 0.2$, where $\underline{c}$ is treated as a variable and other parameters are held fix. The solid line represents what would be equilibrium investments in one country if there are no incentives to invest in the other country. The dotted line is the fraction that is willing to invest when the wage gap is zero, and the line in between represents equilibrium investments in autarky (or if symmetry is imposed). It cannot be seen in the figure, but it can be checked that $\pi = G(B(0))$ is a best response given that the other country invests in accordance with the solid line, so one country investing in accordance with the solid and the other in accordance with the dotted line is an equilibrium.

Both curves bend backwards, so there is a range with multiple equilibria also in the autarky model (when $\underline{c} \geq 0$). However, for $\underline{c} \in [-0.1, 0)$ there are asymmetric equilibria in the trade regime, and a unique autarky equilibrium. There is also a range to the right where there are non-trivial asymmetric trade equilibria, despite the unique autarky equilibrium being a trivial no investment equilibrium.

4.5.1 Example 1: Specialization May be Beneficial Only to the Rich Country

Table 2 displays a parametrization where all country f citizens are better off in the asymmetric trade equilibrium than in the unique autarky equilibrium, and where all country h citizens are worse off in the asymmetric trade equilibrium than under autarky.[15]

$\eta = \frac{2}{3}, \alpha = \frac{1}{2}$, $c \sim U[-0.02, , 0.18]$	Trade, Country h	Trade, Country f	Autarky
Equilibrium Investment	$\pi^h = 0.1$	$\pi^f = 0.548$	$\pi = .269$
Per Capita Production	$y_1^h = 0$ $y_2^h = 1$	$y_1^f = 0.463$ $y_2^f = 0.226$	$y_1 = 0.179$ $y_2 = 0.577$
Per Capita Consumption	$x_1^h = 0.189$ $x_2^h = 0.5$	$x_1^f = 0.274$ $x_2^f = 0.726$	$x_1 = y_1$ $x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = 0.090$	$B(\pi, \pi) = 0.034$
Gross expected utility	0.307	0.446	0.321
Expected utility net of inv. cost	0.308	0.427	0.319
Expected utility if invest	$0.307 - c$	$0.487 - c$	$0.346 - c$
Expected utility if don't invest	0.307	0.397	0.313
Wages	$w_g^h = 1$ $w_b^h = 1$	$w_g^f = 1.875$ $w_b^f = 1$	$w_g = 1.364$ $w_b = 1$
Expected Wage	1	1.452	1.154
Prices	$p_1 = 2.648$		$p_1 = 3.216$

Table 2: Trade and autarky equilibria in Example 1

Notice that the total world output of both goods is higher in the asymmetric equilibrium (see the

[15]Although some agents change their investment behavior in the comparison across equilibria, this does not complicate Pareto comparisons. The crucial fact is that (in the example) both qualified and unqualified workers gain (loose) in country f (h). All workers in the rich country have the option to invest as in the autarky equilibrium, so revealed preference implies that all workers gain. Similarily, in the poor country all workers have the option to invest as in the trade equilibrium when in autarky, so again, by revealed preference, all workers are better off in autarky.

second row of the table). While prohibitive trade barriers would make country h better off (ignoring that it might take a generation to change the distribution of skills), it is also true that a transfer from f to h is sufficient to make both countries better off relative to the autarky equilibrium. That is, despite the countries being identical, there are some productive gains from specialization.

4.5.2 Example 2: Specialization May Make Both Countries Better Off

We now consider an example where trade makes both countries better off. For maximal simplicity we rig this example so that the "free rider problem" in human capital investments is so severe the unique equilibrium under autarky is the trivial equilibrium. However, with trade, the existence of the other country means that, for any investment π^f in country f, the price of good 1 is higher than without trade under the assumption that there are no investments in the other country. Hence, trade allows a new market to emerge that would not operate without trade.

In Table 3 we summarize one such example where the market for good 1 can only operate with international trade. Here, there are actually multiple trade equilibria and the numbers in the table is for the equilibrium with the largest fraction of investors in the country producing good 1.[16]

Consumers are happier when consuming both goods than when consuming only one good. Hence since a new market opens up trade is beneficial for both countries.

Pareto Improving Inequality The example above is extreme, but illustrates a more general point: specialization through trade may be viewed as an imperfect "solution" to the informational problem in the model[17]. In the example, there is no way for a market to open unless the rewards for getting into the market are large enough. These rewards are bigger if only one country enters the market: the same "kick" from the local informational externality is generated at a smaller negative price effect. Specialization thus reduces the problem of underinvestment in human capital.

Even in less extreme cases, both countries may gain from specializing. As is illustrated in Figure 10 it is *always* true that the production possibilities set expand when moving from a situation where both countries invest at the same rate to an asymmetric investment profile for a constant total quantity of investors in the world. In the figure, the frontier to the left with the kink at point A is

[16] There is also an equilibrium with $\pi^h = 0$, $\pi^f = 0.0157$. However, unlike the equilibrium in Table 3 this is unstable.

[17] For a detailed elaboration on this point in the context of discrimination, see Norman [20].

$\eta = \frac{2}{3}, \alpha = \frac{1}{2}$, $c \sim U[.04, .24]$	Trade, Country h	Trade, Country f	Autarky
Equilibrium Investment	$\pi^h = 0$	$\pi^f = 0.353$	$\pi = 0$
Production	$y_1^h = 0$ $y_2^h = 1$	$y_1^f = 0.284$ $y_2^f = 0.323$	$y_1 = 0$ $y_2 = 1$
Consumption	$x_1^h = 0.107$ $x_2^h = 0.5$	$x_1^f = 0.177$ $x_2^f = 0.823$	$x_1 = y_1$ $x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = 0.111$	$B(\pi, \pi) = 0$
Gross average utility	0.232	0.381	0
Avg. utility net of inv. cost	0.232	0.355	0
Expected utility if invest	$0.232 - c$	$0.452 - c$	$0 - c$
Expected utility if don't invest	0.232	0.342	0
Wages	$w_g^h = 1$ $w_b^h = 1$	$w_g^f = 2.433$ $w_b^h = 1$	$w_g = -$ $w_b = 1$
Expected Wage	1	1.647	1
Prices	$p_1 = 4.660$		$p_1 = -$

Table 3: Trade and autarky equilibria in Example 2

some symmetric investment profile, whereas the frontiers with kinks at B and C corresponds with an asymmetric investment profile. Assuming that countries are of equal size, the total number of investors in the world is unchanged, but the world production possibilities set is nevertheless larger (the frontier with kinks at D, A and E in the graph to the right). To understand this, note that the efficient way of increasing x_1 starting from the vertical intercept is to first only use workers from the country with investments $\pi + k$ with good signals, so initially the slope of the world production possibilities set must be the same as the set to the left with kink at C. The graph is drawn for the case where it is better to use high signal workers from the low investment country than low signal workers from the high investment country in sector 1, but the result is fully general.

4.6 The Irrelevance of Size of a Country

Since this is a general equilibrium model with large countries, changes of the relative size of the countries will in general affect the *asymmetric* equilibria due to price effects. The nature of such

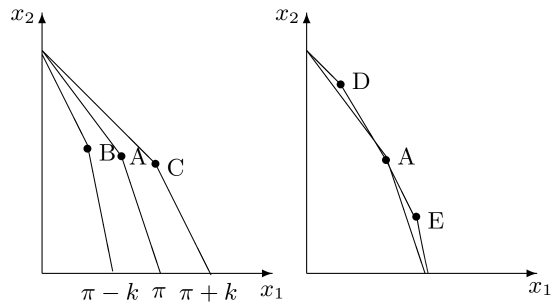

Figure 10: Specialization Expands the World Production Possibilities

changes depends on the parametrization. For example, if the example in Section 4.5.2 is extended to allow for different country sizes there is a critical size such that the country must fully specialize in the low-tech industry if it exceeds this critical size. Reducing the size of the country from $\frac{1}{2}$ on the other hand only improves incentives. Hence, there are circumstances where *the only asymmetric equilibrium* is that the small country becomes rich. It is also possible to set up examples that go the other way, where only the big country can end up on top (see Appendix A.3).

However, these scale effects are not really "country-scale-effects". Instead, we prefer to think of them as scale effects that have to do with the relative size of the North to the South. To understand this, suppose that there are n countries indexed by $j \in \{1, ..., n\}$. Let λ^j denote the size of country j and consider an equilibrium in this model where the set of countries is partitioned into the sets P and R and where $\pi^j = \pi^p$ for all $j \in P$ and $\pi^j = \pi^r$ for all $j \in R$. Finally let $\lambda^p = \sum_{j \in P} \lambda^j$ and $\lambda^r = \sum_{j \in R} \lambda^j$. This is an equilibrium if and only if (π^p, π^r) is an equilibrium in the two-country model with countries of sizes (λ^p, λ^r). There may of course be other equilibria as well, but at least for this form of equilibrium the size of the *individual country* is irrelevant and the relevant scale effect can be interpreted in our preferred manner.

A "development miracle" can therefore be interpreted as a country which manages to re-coordinate from being part of the developing world to being part of the developed world. The model cannot explain how such a re-coordination is achieved, but, if the economy is small, the effects on the rest of the world are negligible. In contrast, a simultaneous recoordination of a significant fraction of the "South" may lead to large enough relative price changes so that it is not worth the while as long as there is no change in the "North". Obviously, the model is too stylized for direct policy recomendations, but this nevertheless suggests that it may be misguided to use a few small successful countries as a model for all developing countries.

5 Discussion

5.1 The Skill Premium

As discussed in Section 2.4, we have some reservations about identifying the signals as the level of formal education. Nevertheless, we will for now ignore this issue and identify w_g^j as the wage of a worker with "high education", and w_b^j as the "low education" wage.

It is known that the "skill premium", the ratio w_g^j/w_b^j, is higher in poor countries than in rich countries (see for example Hendricks [10]). This is *not* inconsistent with the model: unlike the standard neoclassical growth model, what matters for incentives is the *difference* between w_g^j and w_b^j rather than the ratio (see (19)).[18] Given that the rich country (f) is fully specialized in the "high tech" sector, which allows w_b^f to exceed w_b^h, the model is consistent with the conventional skill premium for poor countries to be larger than the one in rich countries.

Table 4 displays such an example where the skill premium in the poor country measured as a ratio exceeds that of the rich country. For simplicity, the example is constructed by assuming that the rich country is small, which assures that the poor country will be near the autarky allocation. This allows us to set π^f to an arbitrary value (high enough to assure full specialization), which we rationalize in equilibrium by an appropriate choice of a uniform distribution.[19]

Relying on full specialization may seem extreme, but the restriction to two sectors is made for tractability rather than realism. The basic economics of the model should be present also in a model with more than two sectors, and with n sectors it seems rather plausible that some sectors are shut down in some countries.

[18] Indeed, the wage differential tends to be larger in richer countries, which is neither surprising, nor support for our model. Our argument in this section is only that our model has nothing to say about the skill premium as conventionally measured.

[19] The distribution must also rationalize the particular autarky equilibrium in the poor country as well. However, the benefit to invest in the poor country (0.0378) and the benefit to invest in the rich country (0.0384) give us two points of the cumulative: $G(0.0378) = \pi^h = 1/12$ and $G(0.0384) = \pi^f = 5/6$, so this is pure back-calculation.

$\eta = \frac{2}{3}, \alpha = \frac{1}{2}$, $c \sim U[0.03774, 0.03855]$	Country h	Country f
Equilibrium Investment	$\pi^f = 5/6$	$\pi = 1/12$
Gross incentives to invest	$B^f(\pi^h, \pi^f) = 0.0384$	$B(\pi, \pi) = 0.0378$
Wages	$w_g^f = 115/11$ $w_b^f = 115/14$	$w_g = 23/13$ $w_b = 1$
Wage difference	2.24	10/13=0.769
Wage ratio	14/11=1.273	23/13=1.769
Price	$p_1 = \frac{23}{2}$	

Table 4: Trade and autarky equilibria in Example 1

To sum up, the existing empirical literature measures the skill premium as a ratio for a good reason: the neoclassical growth model has implications about ratios and is silent about differences. Our model, on the other hand, has implications about differences and is silent on ratios. This is not a technical detail, but comes directly from the economics of the model. An individual compares her expected utility from investing with the expected utility if not investing, which means that the difference rather than the ratio of expected utilities is what is relevant.[20]

Notice that this does not mean that are model is empirically vacuous. Checking whether incentives to acquire skills line up as they should in accordance with the theory is something that can be implemented with the right data. Moreover, the model has other implications. For example, a robust implication is that, in a rich country, the wage of workers in the export sector should exceed the wage of workers in the sector where the country is a net importer. The opposite relation should hold in a poor country.[21]

[20]It may be argued that this is also driven by the fact that the cost is an additively separable utility cost rather than a time cost. This is correct *under the assumption that firms cannot condition wages on age.* In the seemingly more plausible case where age can be used for inference, differences again matter. We also note that the date for graduation usually is to a large extent predetermined, and that if the time cost is foregone time for working extra at a given wage (w_b^j for example), then we are back in the additively separable case.

[21]Firm level data is difficult to obtain for many developing countries, but using industry data from the World Bank Trade and Production database one can compute export and domestic by weighting industry wages by the shares of value of exports. This calculation led to numbers consistent with our model. See Moro and Norman [19].

5.2 Complementarities Between Capital and Human Capital

A natural extension is to introduce (regular) capital into the production technology. We believe that this would be interesting for analyzing the role of foreign capital and capital flight from poor countries. Moreover, it could also be helpful for quantitative purposes. The main reason that we do not introduce capital in this paper is that it adds little to the economics of the model.

To understand this, suppose initially that capital cannot flow between countries. While there is now an additional dimension to the problem in that capital must be allocated across sectors, qualitative results from the model without capital continue to hold. Now, consider an equilibrium where one country specializes in the high-tech industry and the other in the low-tech industry. Assuming equal initial endowments of capital, the return on capital in the high tech country is higher than in the other country, so capital would flow from the poor to the rich country. This in turn would increase (decrease) the marginal product of labor in the high tech sector in the rich (poor) country, thereby improving (reducing) incentives to invest in the rich (poor) country. Mobile capital would thus only strengthen the incentives to specialize.[22]

5.3 Relationship with the Existing Literature

Increasing Returns and Agglomeration Models While our underlying assumptions are very different, the model shares many features with trade models with increasing returns (Eithier [7], and Krugman [12]). In particular, versions that are usually referred to as "agglomeration models" (Krugman and Venables [13], Matsuyama [16], Puga and Venables [21]) are similar along several dimensions. These models, as is ours, are capable of generating a core-periphery pattern in equilibrium between fundamentally identical countries.

Agglomeration models can sustain a concentration of (high income) manufacturing because production costs decrease in the size of the manufacturing industry. Manufactured goods are inputs in the production of manufactures, implying that being close to other producers of manufactured goods saves on transportation costs. This creates incentives to concentrate production. If transportation costs are neither too small or too large, there are equilibria where manufacturing is concentrated in one country. As a consequence, this country becomes richer than the other country.

[22] Details are available on request from the authors.

While our model is considerably less complicated and closer to the neoclassical benchmark, there is a close similarity in how a pecuniary externality interacts with local market conditions. However, there are also crucial differences. Agglomeration models predict a positive relation between size and development: the larger is the home market, the more room there is for a wide array of manufactures. Our model has no such implications (see Section 4.6).[23] Moreover, the division into rich and a poor countries may in our model Pareto dominate autarky. We are not aware of any increasing returns model with this feature.

Alternative Mechanisms for Human Capital Externalities Our paper is related in spirit with Acemoglu [1], who provides an alternative microfoundation for human capital externalities. The mechanism considered by Acemoglu is the interaction between non-contractible investments in human and physical capital and costly search, which generates a pecuniary externality since optimal investment strategies depend on aggregate investments on the other side of the market.

Trade and Asymmetric Information Recently, models of asymmetric information have gained popularity in the literature on international trade, and have been used to analyze a variety of issues. For example, Razin and Sadka [22] use an informational asymmetry to model the role of foreign direct investments, Casella and Rauch [3] derives a role for minority groups in international trade from an informational friction, and McCalman [17] considers the impact of asymmetric information in bargaining about trade agreements.

More closely related to this paper are Grossman and Maggi [9] and Grossman [8], who consider an essentially competitive environment with imperfect observability of talent. However, for their purposes it is sufficient to consider how trade is affected by exogenous differences in the distribution of talent, so they completely ignore the incentive issues for *acquiring* skills that are central in our model. To our knowledge, the only paper that considers asymmetric information about skills in an environment where individual workers actually invest in their skills is Eicher [6]. He consider a model that is significantly richer than ours in many ways, but the treatment of the informational asymmetry is more reduced form than the model considered here.

[23] There is disagreement about the facts. Our view is that the lack of country scale effects is a virtue, but Alesina *et al* [2] argues that scale effects can be detected in the data.

6 Summary and Concluding Remarks

We have shown that it is possible to generate endogenous comparative advantages between identical countries in an essentially neoclassical model. The model generates specialization and income differentials due to an informational externality that arises because workers are better informed than firms about abilities to perform different jobs. This microfoundation necessarily implies that the scope of the externality is local, where the local market is defined by barriers to labor mobility. An important difference from the existing literature is that, while the two country modeled studied is not completely scale neutral, there are no scale effects that are driven by the size of the home market. The two country model can be reinterpreted as equilibria of a n country model where countries cluster in two groups in terms of level of development. Equilibria of this model are neutral with respect to the size of individual countries, so the model is consistent with a world with no particular relationship between size and the level of development.

A Appendix

A.1 Proof of Proposition 1

Proof. (Part 1) Consider an arbitrary equilibrium. Let $x^* = (x_1^*, x_2^*)$ denote the world production, where $x_i^* = \lambda^h x_i^{h*} + \lambda^f x_i^{f*}$ and x_i^{j*} denotes the production of good i in country j in equilibrium. Also let $l_i^{j*}(\theta)$ denote the corresponding input of workers with signal g in economy j and sector i. By profit maximization

$$p_i^* x_i^{j*} - \sum_{\theta \in g,b} w_\theta^{j*} l_i^{j*}(\theta) \geq p_i^* x_i^{j\prime} - \sum_{\theta \in g,b} w_\theta^{j*} l_i^{j\prime}(\theta) \tag{A1}$$

for any alternative plan $(x_i^{j\prime}, l_i^{j\prime}(\cdot))$. Adding over the two sectors and imposing the market clearing conditions on the labor market we conclude that

$$\begin{aligned} & \sum_{i=1,2} p_i^* x_i^{j*} - w_g^{j*}\left(\eta\pi^j + (1-\eta)(1-\pi^j)\right) - w_b^{j*}\left((1-\eta)\pi^j + \eta(1-\pi^j)\right) \\ \geq \quad & \sum_{i=1,2} p_i^* x_i^{j\prime} - w_g^{j*}\left(l_1^j(g) + l_2^j(g)\right) - w_b^{j*}\left(l_1^j(b) + l_2^j(b)\right) \end{aligned} \tag{A2}$$

for all possible alternative production plans (feasible as well as non-feasible in the aggregate). Now for any feasible alternative allocation

$$\begin{aligned} l_1^j(g) + l_2^j(g) &\le \eta\pi^j + (1-\eta)(1-\pi^j) \\ l_1^j(b) + l_2^j(b) &\le (1-\eta)\pi^j + \eta(1-\pi^j), \end{aligned} \tag{A3}$$

implying that $\sum_{i=1,2} p_i^* x_i^{j*} \ge \sum_{i=1,2} p_i^* x_i^{j\prime}$ for any feasible alternative $(x_1^{j\prime}, x_2^{j\prime})$. Since this must hold in each country we conclude that $p^*x^* \ge p^*x'$ for any alternative feasible world production vector $x' = (x_1', x_2')$. Moreover, in order for (x_1^{j*}, x_2^{j*}) to be profit maximizing it must be that

$$\sum_{i=1,2} p_i^* x_i^{j*} - w_g^{j*}\left(\eta\pi^j + (1-\eta)(1-\pi^j)\right) + w_b^{j*}\left((1-\eta)\pi^j + \eta(1-\pi^j)\right) = 0. \tag{A4}$$

Finally, since u is homothetic it follows from standard arguments that if $(x_1^{j*}(w), x_2^{j*}(w))$ solves the utility maximization problem for a worker with income w , then $(\frac{w'}{w}x_1^{j*}(w), \frac{w'}{w}x_2^{j*}(w))$ solves the utility maximization problem for a worker with income w'. Consider the program

$$\begin{aligned} &\max_{x_1,x_2} u(x_1,x_2) \\ \text{s.t } p_1^*x_1 + p_2^*x_2 &\le p_1^*x_1^* + p_2^*x_2^* = p_1^*\sum_{j=h,f}\lambda^j x_1^{j*} + p_2^*\sum_{j=h,f}\lambda^j x_2^{j*}, \end{aligned} \tag{A5}$$

where the variables with the star-superscript refers to equilibrium variables. It is straightforward to show that the aggregate consumption bundle of any equilibrium must be a solution to (A5), the basic reasoning being that the problem gets the relative consumptions of x_1 and x_2 right and that $p_1^*x_1^* + p_2^*x_2^*$ is the aggregate world income. We thus conclude that if x^* is an equilibrium world consumption plan it must solve (A5). Since the set $X^W(\pi^h, \pi^f)$ is contained in the "budget set" of representative and $x^* \in X^W(\pi^h, \pi^f)$ it follows that x^* must be a solution to (15).

(Part 2) Let x^* solve (15) and let $V = \{x \in R_+^2 | u(x) > u(x^*)\}$. Quasi-concavity implies that V is a convex set. The set $X^W(\pi^h, \pi^f)$ is also convex (see Page 9). Moreover, $V \cap X^W(\pi^h, \pi^f) = \emptyset$, so the separating hyperplane theorem (Theorem 11.3. in Rockafellar [23]) implies that there exists some p^* such that $p^*x \ge p^*x^*$ for all $x \in V$ and $p^*x \le p^*x^*$ for every $x \in X^W(\pi^h, \pi^f)$. Let the wages be given by

$$\begin{aligned} w_g^{j*} &= \max\left\{p_1^*\mu(g,\pi^j), p_2^*\right\} \\ w_b^{j*} &= \max\left\{p_1^*\mu(b,\pi^j), p_2^*\right\} \end{aligned} \tag{A6}$$

and let the allocation of workers be as in the planning solution. Observe in particular that if $p_1^* \mu\left(\theta, \pi^j\right) > p_2^*$, then no worker with signal θ is employed in sector 2 in the allocation that produces x^*. This is most easily seen in the differentiable case, where the optimality condition to (15) implies that

$$\frac{\partial u\left(x^*\right)}{\partial x_1^*} / \frac{\partial u\left(x^*\right)}{\partial x_2^*} = \frac{p_1^*}{p_2^*} > \frac{1}{\mu\left(g, \pi^j\right)}. \tag{A7}$$

But, $\frac{1}{\mu(\theta,\pi^j)}$ is the cost of producing an extra unit of good 1 by giving up some country j workers with good signal currently in production of good 2, so we conclude that as if representative consumer would be better off if some of these workers would be switched to the production of good 1, contradicting optimality of x^* if $p_1^* \mu\left(\theta, \pi^j\right) > p_2^*$ and some of the j workers are assigned to sector 2. A symmetric argument holds for when the inequality is reversed. Hence, if $l_1^{*j}(\theta) > 0$, then $p_1^* \mu\left(\theta, \pi^j\right) = \max\left\{p_1^* \mu\left(\theta, \pi^j\right), p_2^*\right\} = w_\theta^{j*}$, implying that the profit from hiring any quantity workers with signal θ is zero in sector 1, whereas if $l_1^{*j}(\theta) = 0$, then $p_1^* \mu\left(\theta, \pi^j\right) \leq \max\left\{p_1^* \mu\left(\theta, \pi^j\right), p_2^*\right\} = w_\theta^{j*}$, so no gain can be earned from hiring a positive quantity. The argument for sector 2 is symmetric, which leads us to conclude that the outputs and (implicit) allocation of workers in the solution to (15) are consistent with profit maximizing behavior given the prices and wages constructed. ■

A.2 Proof of Proposition 2

Proof. We only prove the result for the case with $\alpha \leq \eta$. The case with $\alpha > \eta$ proceeds along the same lines, but the calculations are different. We first consider a uniform cost distribution and then generalize to concave distributions by use of a linear approximation. Since $G\left(B\left(0\right)\right) > 0$ by the assumption that $\underline{c} < 0$ and since $G\left(B\left(\pi\right)\right) > G\left(B\left(1\right)\right) = G(B\left(0\right))$ for all $\pi \in (0,1)$ an equilibrium $\pi^* > 0$ must exist. If $\pi^* = 1$ uniqueness is trivial (then $G\left(B\left(\pi\right)\right) > G\left(B\left(1\right)\right) \geq 1$ for all $\pi < 1$, implying that there is no other equilibrium) so we assume $\pi^* < 1$ in any equilibrium. For brevity we let $\phi = 2\eta - 1 > 0$ and define

$$b\left(\pi\right) \equiv \left(\frac{\pi}{\eta - \phi\pi}\right)^{\alpha} \left(\frac{\alpha}{\phi\pi + (1-\eta)} - 1\right). \tag{A8}$$

By straightforward algebra on the expression in (19) on can check that $B\left(\pi\right) = \Phi\phi\eta^{\alpha} b(\pi)$ for all π such that $B\left(\pi\right) \geq 0$. With c distributed uniformly on $[\underline{c}, \overline{c}]$ we then have that $G\left(B\left(\pi\right)\right) = Qb\left(\pi\right) + L$

for some positive constants Q and L and any $B(\pi) \in [\underline{c}, \overline{c}]$.[24] By a direct calculation we have that

$$\begin{aligned}
b'(\pi) &= \alpha\left(\frac{\pi}{\eta-\phi\pi}\right)^{\alpha-1}\left(\frac{\alpha}{\phi\pi+(1-\eta)}-1\right)\frac{\eta}{(\eta-\phi\pi)^2}-\left(\frac{\pi}{\eta-\phi\pi}\right)^{\alpha}\frac{\phi\alpha}{(\phi\pi+(1-\eta))^2} \\
&= b(\pi)\frac{\alpha\eta}{\pi(\eta-\phi\pi)}-b(\pi)\frac{\phi\alpha}{(\phi\pi+(1-\eta))^2}\frac{\phi\pi+(1-\eta)}{\alpha-\phi\pi-(1-\eta)} \\
&= b(\pi)\,\alpha\left(\frac{\eta}{\pi(\eta-\phi\pi)}-\frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right). \qquad \text{(A9)}
\end{aligned}$$

A sufficient condition for uniqueness is that $\frac{d}{d\pi}G(B(\pi^*)) < 1$ in any equilibrium π^* and we will show by direct computation that this holds in any equilibrium. We drop the $*$-superscript and note that for any equilibrium π (satisfying $\pi = G(B(\pi)) = Qb(\pi) + L$) we have:

$$\begin{aligned}
\frac{d}{d\pi}G(B(\pi)) &= Qb'(\pi) = Qb(\pi)\,\alpha\left(\frac{\eta}{\pi(\eta-\phi\pi)}-\frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right) \qquad \text{(A10)} \\
&= \alpha(\pi-L)\left(\frac{\eta}{\pi(\eta-\phi\pi)}-\frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right) \\
&< \alpha\pi\left(\frac{\eta}{\pi(\eta-\phi\pi)}-\frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right),
\end{aligned}$$

where the equality comes from the fact that we are evaluating the derivative at an equilibrium point and the inequality since $L > 0$ and the bracketed expression needs to be strictly positive for $B(\pi) > 0$. But, the expression is increasing in α and $\alpha \leq \eta$, so

$$\begin{aligned}
&\alpha\pi\left(\frac{\eta}{\pi(\eta-\phi\pi)}-\frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right) \qquad \text{(A11)} \\
&\leq \eta\pi\left(\frac{\eta}{\pi(\eta-\phi\pi)}-\frac{\phi}{(\phi\pi+(1-\eta))(\eta-\phi\pi-(1-\eta))}\right)
\end{aligned}$$

$$\begin{aligned}
&= \frac{\eta^2}{(\eta-\phi\pi)}-\frac{\eta\pi}{(\phi\pi+(1-\eta))(1-\pi)} \quad \Big/ \quad \phi\pi+(1-\eta) < \phi+(1-\eta) = 2\eta-1+1-\eta = \eta \;\Rightarrow\; \tfrac{1}{\phi\pi+(1-\eta)} > \tfrac{1}{\eta} \quad \Big/ \\
&< \frac{\eta^2}{(\eta-\phi\pi)}-\frac{\pi}{(1-\pi)} = \frac{\eta^2}{(\eta-(2\eta-1)\pi)}-\frac{\pi}{(1-\pi)},
\end{aligned}$$

We now observe that

$$\frac{d}{d\eta}\left(\frac{\eta^2}{(\eta-(2\eta-1)\pi)}\right) = \frac{\eta(\eta+2\pi(1-\eta))}{(\eta-(2\eta-1)\pi)^2} > 0, \qquad \text{(A12)}$$

[24] Where in terms of the parameters of the model, $Q = \frac{1}{\overline{k}-\underline{k}}\Phi\phi\eta^{\alpha}$ and $L = -\frac{\underline{k}}{\overline{k}-\underline{k}} > 0$ given that $\underline{c} < 0$.

since $0 < \eta, \pi < 1$. Thus

$$\frac{\eta^2}{(\eta - (2\eta - 1)\pi)} - \frac{\pi}{(1-\pi)} \leq \frac{1}{(1-\pi)} - \frac{\pi}{(1-\pi)} = \frac{1-\pi}{1-\pi} = 1. \tag{A13}$$

Combing (A11) and (A13) we get

$$\alpha\pi\left(\frac{\eta}{\pi(\eta - \phi\pi)} - \frac{\phi}{(\phi\pi + (1-\eta))(\alpha - \phi\pi - (1-\eta))}\right) < 1, \tag{A14}$$

which combined with (A10) establishes the claim for the case with a uniform distribution. For a general concave distribution we first note that for any $c > \underline{c}$ there exists some $c^* \in [\underline{c}, c]$ such that $G(c) = G'(c^*)\,c$. Hence it follows that the equilibrium must satisfy $\pi = G(B(\pi)) = G'(c^*)\,B(\pi)$ for some $c^* \leq B(\pi)$. Since G is concave it follows that $G'(B(\pi)) \leq G'(c^*)$, so

$$\frac{d}{d\pi}G(B(\pi)) = G'(B(\pi))\,B'(\pi) \leq G'(c^*)B'(\pi) = G'(c^*)\phi\eta^{\alpha}b'(\pi) = Qb'(\pi), \tag{A15}$$

for $Q = G'(c^*)\phi\eta^{\alpha} > 0$. At this point one can proceed as with a uniform distribution with $L = 0$ ■

A.3 Scale Effects in the Two Country Model

We construct examples supporting the claim that scale effects may go either way. The most tractable way to do so is to look at the extreme case where λ^h is near zero. This simplifies the analysis since equilibria can then can be calculated by solving two separate (but different) one-dimensional fixed point problem. Consider the incentives to invest in a country with fraction of investors π under the "small open economy" assumption that the price (of good 1) is fixed at p. The equilibrium wages in the small open economy are still determined so as to generate zero profits. Hence, $w_g^O = \max\{p\mu(g,\pi), 1\}$ and $w_g^O = \max\{p\mu(b,\pi), 1\}$. The gross incentive to invest in the small open economy, denoted $B^O(\pi; p)$, is thus (using (19)),

$$B^O(\pi; p) = \frac{(2\eta - 1)\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p^{\alpha}}\max\{p\mu(g,\pi) - \max\{p\mu(b,\pi), 1\}, 0\}. \tag{A16}$$

One can show that, if p^A is well defined (i.e., whenever the autarky equilibrium is non-trivial), then

$$\begin{aligned} B^h\left(\pi^h, \pi^A\right) &\rightarrow B^O\left(\pi^h, p = p^A\right) \text{ for all } \pi^h \in [0,1] \text{ as } \lambda^h \rightarrow 0 \qquad \text{(A17)} \\ B^f\left(\pi^h, \pi^A\right) &\rightarrow B\left(\pi^f\right) \text{ for all } \pi^f \in [0,1] \text{ as } \lambda^h \rightarrow 0, \end{aligned}$$

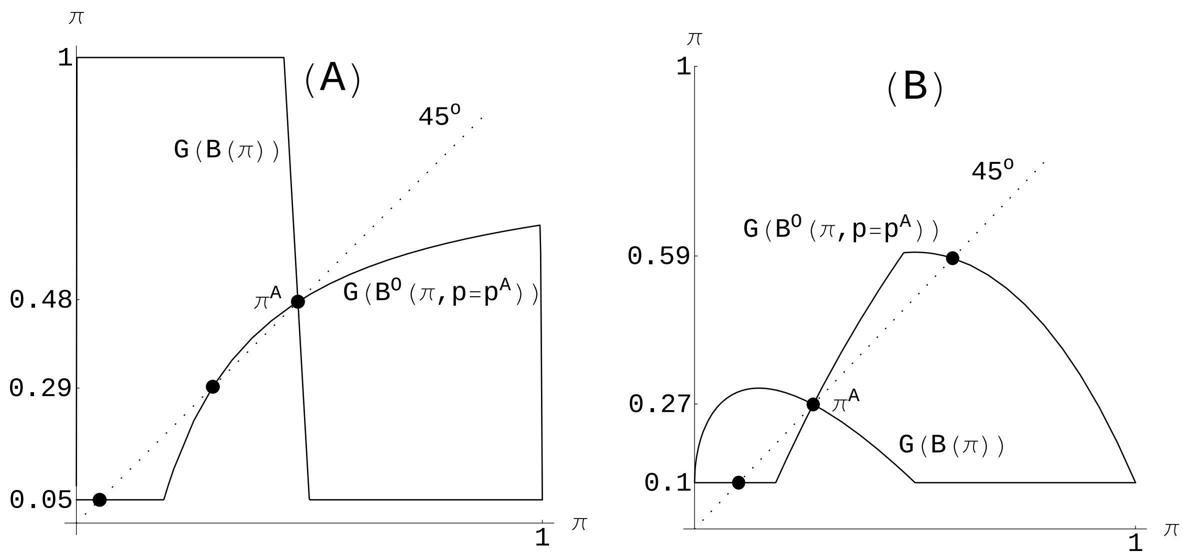

Figure 11: Equilibrium fixed point maps in a small open economy

Assume parameters such that the autarky equilibrium is unique, and call it π^A. Let p^A denote the associated autarky price. If $\pi = \pi^A$, then $B^O\left(\pi^A; p = p^A\right) = B\left(\pi^A\right)$, implying that π^A solves

$$\pi = G\left(B^O\left(\pi; p = p^A\right)\right). \tag{A18}$$

While both (A18) and the autarky fixed point equation have π^A as a common solution, incentives diverge for other π since in autarky benefit price change as π changes whereas there are no such price effects in (A16). Equation (A18) will therefore in many cases have solutions different from π^A. Now, if π^O solves (A18) and if $\frac{d}{d\pi}\big|_{\pi=\pi^A}\left[\pi - G\left(B\left(\pi\right)\right)\right] \neq 0$ and $\frac{d}{d\pi}\big|_{\pi=\pi^O}\left[\pi - G\left(B^O\left(\pi; p = p^A\right)\right)\right] \neq 0$, then, for λ^h small enough, there exists an equilibrium $\left(\pi^{h*}, \pi^{f*}\right)$ in the trade model near $\left(\pi^O, \pi^A\right)$.[25]

We computed two examples to show that scale effects are indeterminate. Figure 11 (A) (computed using $\alpha = 1/2$, $\eta = .97$, $\underline{c} = -0.005$, $\bar{c} = .995$) illustrates the case where only the big country can be rich. There is a unique symmetric equilibrium at $\pi^A = 0.48$. It is evident that (A16) intersects the 45^0 line only below π^A. There are two asymmetric equilibria where the big country invests at $\pi^h = \pi^A$ and the small country at $\pi^f = 0.05$ or 0.29. Figure 11 (B) was computed with the

[25]The slope condition for the autarky equilibrium is satisfied under the conditions of Proposition 2. Its role is that if the equilibrium was at a tangency with the 45^0 line, the slightest effect from abroad could eliminate the equilibrium.

parameters as in Numerical Example 1. Here both $\left(\pi^h, \pi^f\right) = (.27, 0.1)$ and $\left(\pi^h, \pi^f\right) = (.27, 0.59)$ are equilibria, so the small country can be either richer or poorer than the big country. Finally, when the unique autarky equilibrium is at $\pi^A = 0$ and if the large country is large enough, then only the small country can be richer. Taken together, these three cases imply that there may be scale effects in favor of either the larger or the smaller economy, and that sometimes the equilibrium selection matters.

A.4 Stability Analysis

We show here that $\partial G(B^f(\pi))/\partial \pi^f > 1$ is a sufficient condition for instability. Consider the Jacobian of the difference equation system evaluated at the autarky equilibrium $\pi = (\pi^A, \pi^A)$:

$$\begin{bmatrix} G'\left(B\left(\pi\right)\right) \left.\frac{\partial(B^f(\pi))}{\partial \pi^f}\right|_{\pi=(\pi^A,\pi^A)} & G'\left(B\left(\pi\right)\right) \left.\frac{\partial(B^f(\pi))}{\partial \pi^h}\right|_{\pi=(\pi^A,\pi^A)} \\ G'\left(B\left(\pi\right)\right) \left.\frac{\partial(B^h(\pi))}{\partial \pi^f}\right|_{\pi=(\pi^A,\pi^A)} & G'\left(B\left(\pi\right)\right) \left.\frac{\partial(B^h(\pi))}{\partial \pi^h}\right|_{\pi=(\pi^A,\pi^A)} \end{bmatrix}. \tag{A19}$$

At the autarky equilibrium both the "cross derivatives" and the "own derivatives" are identical, so (dropping the common factor $G'\left(B\left(\pi\right)\right)$ the characteristic polynomial can be written as:

$$\begin{aligned} & \left(\left.\frac{\partial(B^f(\pi))}{\partial \pi^f}\right|_{\pi=(\pi^A,\pi^A)} - \lambda \right)^2 - \left(\left.\frac{\partial(B^f(\pi))}{\partial \pi^h}\right|_{\pi=(\pi^A,\pi^A)} \right)^2 \qquad \text{(A20)} \\ = \; & \lambda^2 + 2\lambda \left.\frac{\partial(B^f(\pi))}{\partial \pi^f}\right|_{\pi=(\pi^A,\pi^A)} + \left(\left.\frac{\partial(B^f(\pi))}{\partial \pi^f}\right|_{\pi=(\pi^A,\pi^A)} \right)^2 - \left(\left.\frac{\partial(B^f(\pi))}{\partial \pi^h}\right|_{\pi=(\pi^A,\pi^A)} \right)^2 \end{aligned}$$

with roots:

$$\lambda_{1,2} = \left.\frac{\partial(B^f(\pi))}{\partial \pi^f}\right|_{\pi=(\pi^A,\pi^A)} \pm \left.\frac{\partial(B^f(\pi))}{\partial \pi^h}\right|_{\pi=(\pi^A,\pi^A)} \tag{A21}$$

The system is unstable if at least one of the roots is greater than one, therefore a sufficient condition is that the own derivative is greater than one.

References

[1] Acemoglu, Daron "A Microfoundation For Social Increasing Returns In Human Capital Accumulation," *Quarterly Journal of Economics,* August 1996, 779-803.

[2] Alesina, Alberto, Spolaore, Enrico, and Romain Wacziarg, "Economic Integration and Political Disintegration," *American Economic Review*, 90(5), December 2000, 1276-1296.

[3] Alessandra, Casella and James E. Rauch, "Anonymous Market and Group Ties in International Trade" *Journal of International Economics,* 58 , October 2002, 19-47.

[4] Eeckhout, Jan, "Gibrat's Law for (all) Cities," *Manuscript,* University of Pennsylvania, December 2003.

[5] Eeckhout, Jan, and Boyan Jovanovics, "Knowledge Spillovers and Inequality" *American Economic Review*, **92 (**2), December 2002, 1290-1307.

[6] Eicher, Theo, S. "Training, Adverse Selection and Appropriate Technology: Development and Growth in a Small Open Economy," *Journal of Economic Dynamics and Control*, 23, April 1999, 727-746.

[7] Ethier, Wilfred, "Decreasing Costs in International Trade and Frank Graham's argument for Protection." *Econometrica,* **50,** 1982, 1243-1268.

[8] Grossman, Gene , "The Distribution of Talent and the Pattern and Consequences of International Trade," forthcoming in the *Journal of Political Economy*, 2004.

[9] Grossman, Gene and Giovanni Maggi , "Trade and Diversity," *American Economic Review*, **90 (**5), December 2000.

[10] Hendricks, Lutz, "How Important is Human Capital for Development? Evidence from Immigrant Earnings", *American Economic Review* 2002, 92(1), 2002, 198-219.

[11] Jaffe, Adam B, Manuel Trajtenberg, and Rebecca Henderson, "Geographic Localization of Knowledge Spillovers as Evidenced by Patent Citations," *Quarterly Journal of Economics,* August 1993, 577-598.

[12] Krugman, Paul, "Scale Economics, Product Differentiation, and the Pattern of Trade," *American Economic Review,* 70, December 1980, 950-959.

[13] Krugman, Paul and Anthony Venables, "Globalization and the Inequality of Nations," *Quarterly Journal of Economics,* November 1995, 857-880.

[14] Lucas, Robert, E., Jr., "On the Mechanics of Economic Development," *Journal of Monetary Economics,* January 1988, **22,** 3-22.

[15] Lucas, Robert, E., Jr., and Esteban Rossi-Hansberg, "The Internal Structure of Cities," *Econometrica,* **70 (**4), 2002, 1445-1476.

[16] Matsuyama, Kiminori, "Why Are There Rich and Poor Countries? Symmetry-Breaking in the World Economy," *J. of the Japanese and Int. Economies*, 10(4), December 1996, 419-39

[17] McCalman, Phillip, "Multilateral Trade Negotiations and the Most Favoured Nation Clause," *Journal of International Economics*, 57(1), June 2002, 151-76

[18] Moro, Andrea and Peter Norman, "A General Equilibrium Model of Statistical Discrimination", *Journal of Economic Theory*, 114(1), January 2004, 1-30.

[19] Moro, Andrea and Peter Norman "Supplement to Endogenous Comparative Advantage," *Mimeo February 2004* (available upon request or directly from the authors' web page).

[20] Norman, Peter, "Statistical Discrimination and Efficiency", *Review of Economic Studies,* 70(3), July 2003, 615-627.

[21] Puga, Diego and Anthony Venables, "Agglomeration and Economic Development: Import substitution vs. Trade Liberalisation," *Economic Journal* **109**, April 1999, 292-311

[22] Razin, Assaf, and Efraim Sadka, "Gains from FDI inflows with incomplete information" *Economics Letters* , 78 (1), Issue 1 , January 2003, Pages 71-77

[23] Rockafellar, R. Tyrrell, *Convex Analysis*, Princeton Landmarks in Mathematics, Princeton, New Jersey, 1997.

[24] Spence, Michael A."Job Market Signaling," *Quarterly Journal of Economics,* 87(3), August 1973, 355-374.