1 Appendix B: Calculations For Example 5.1 in To Bundle or Not to Bundle

We want to show that if the monopolist sets a price $p_B < 2$, then, if α is sufficiently small, the profit is lower than the maximized profit under separate sales, $\Pi_1 + \Pi_2 = 2 - \alpha$. We begin with a simple observation:

Claim 11 *It must be the case that $p_B > 2 - \alpha$ in order for the profit under bundling to exceed $2 - \alpha$.*

This is obvious, since p_B would be the profit if the consumer would buy for sure.

Next, observe that for $p < 1$ we have that

$$\begin{aligned} 1 - G_B(p) &= \Pr\left[\frac{\theta_1 + \theta_2}{2} \geq p\right] \\ &= \underbrace{(1-\alpha)^2}_{\Pr[(\theta_1,\theta_2)=(1,1)]} + \underbrace{2(1-\alpha)\alpha}_{\substack{\Pr[\theta_1=1\cap\theta_2\neq 1] \\ +\Pr[\theta_1\neq 1\cap\theta_2=1]}} \left[1 - \tilde{F}(2p-1)\right] + \underbrace{\alpha^2}_{\Pr[\theta_1\neq 1\cap\theta_2\neq 1]} [1 - G(p)] \end{aligned}$$

where $\tilde{F}$ is the CDF of the underlying uniform distribution over $[0,2]$ and

$$G(p) = \begin{cases} \frac{p^2}{2} & \text{on } [0,1] \\ 1 - \frac{(2-p)^2}{2} & \text{on } [1,2]. \end{cases}$$

By Claim 11, we can restrict our attention to values of $p_B \in (2-\alpha, 2)$, which is equivalent to restricting to per-good average price $p \in \left(1 - \frac{\alpha}{2}, 1\right)$. Since $\alpha \in [0,1]$, $\left(1 - \frac{\alpha}{2}, 1\right)$ is a subset of $\left(\frac{1}{2}, 1\right)$. For any $\frac{1}{2} < p < 1$, the monopolist's profit from selling bundle at a bundle-price of $2p$ receives profit:

$$2p[1 - G_B(p)] = 2p\left[(1-\alpha)^2 + 2(1-\alpha)\alpha\left(\frac{3-2p}{2}\right) + \alpha^2\left(1 - \frac{p^2}{2}\right)\right].$$

On the other hand, the monopolist's profit from selling the two goods at price p for each receives profit

$$2p[1 - F(p)] = 2p\left(1 - \frac{\alpha}{2}p\right).$$

Define

$$\begin{aligned}
\Delta(p) &= G_B(p) - F(p) \\
&= 1 - \left[(1-\alpha)^2 + 2(1-\alpha)\alpha\left(\frac{3-2p}{2}\right) + \alpha^2\left(1-\frac{p^2}{2}\right)\right] - \frac{\alpha}{2}p \\
&= (1-\alpha)^2 + 2\alpha(1-\alpha) + \alpha^2 - \left[(1-\alpha)^2 + 2(1-\alpha)\alpha\left(\frac{3-2p}{2}\right) + \alpha^2\left(1-\frac{p^2}{2}\right)\right] - \frac{\alpha}{2}p \\
&= \alpha\left[(1-\alpha)(2p-1) + \alpha\frac{p^2}{2} - \frac{1}{2}p\right].
\end{aligned}$$

Note that $2p\Delta(p)$ measures the difference in the monopolist's profit between selling each good separately at a price of p for each good and selling the bundle at a price of $2p$. Thus if $\Delta(p)$ is positive, then the monopolist increases its profit by selling the goods separately at half the price of the bundled good; and if $\Delta(p)$ is negative, then the profit under bundling is higher.

Next, we show that $\Delta(p)$ is monotonic on $\left(\frac{1}{2},1\right)$. Differentiating $\Delta(p)$ we have that

$$\begin{aligned}
\frac{d\Delta(p)}{dp} &= \alpha\left[2(1-\alpha) + \alpha p - \frac{1}{2}\right] = \alpha\left[\frac{3}{2} + \alpha(p-2)\right] \\
&> \alpha\left[\frac{3}{2} + \alpha\left(\frac{1}{2}-2\right)\right] = \alpha\frac{3}{2}(1-\alpha) > 0.
\end{aligned}$$

Hence;

Claim 12 *$\Delta(p)$ is strictly increasing on $\left(\frac{1}{2},1\right)$.*

We know from Claim 11 that we only need to consider $p_B > 2-\alpha$, which corresponds to an average price $p > 1-\frac{\alpha}{2}$. Evaluating $\Delta(p)$ at $p = 1-\frac{\alpha}{2}$ we have that

$$\begin{aligned}
\Delta\left(1-\frac{\alpha}{2}\right) &= \alpha\left\{(1-\alpha)\left[2\left(1-\frac{\alpha}{2}\right)-1\right] + \alpha\left[1-\frac{\left(2-\left(1-\frac{\alpha}{2}\right)\right)^2}{2}\right] - \frac{1}{2}\left(1-\frac{\alpha}{2}\right)\right\} \\
&= \alpha\left\{(1-\alpha)^2 + \alpha\left[1-\frac{\left(1+\frac{\alpha}{2}\right)^2}{2}\right] - \frac{1}{2}\left(1-\frac{\alpha}{2}\right)\right\}
\end{aligned}$$

Hence,

$$\begin{aligned}
\Delta\left(1-\frac{\alpha}{2}\right) &\geq 0 \Leftrightarrow \\
(1-\alpha)^2 + \alpha\left[1-\frac{\left(1+\frac{\alpha}{2}\right)^2}{2}\right] - \frac{1}{2}\left(1-\frac{\alpha}{2}\right) &\geq 0 \Leftrightarrow \\
(1-\alpha) - \alpha(1-\alpha) + \alpha - \alpha\frac{\left(1+\frac{\alpha}{2}\right)^2}{2} - \frac{1}{2}\left(1-\frac{\alpha}{2}\right) &\geq 0 \Leftrightarrow \\
\frac{1}{2} - \alpha(1-\alpha) - \alpha\frac{\left(1+\frac{\alpha}{2}\right)^2}{2} + \frac{\alpha}{4} &\geq 0 \Leftrightarrow
\end{aligned}$$

We conclude:

Claim 13 $\Delta\left(1-\frac{\alpha}{2}\right)>0$ *for* α *sufficiently small.*

To sum up:

1. Claim 11 shows that bundling at $p_B < 2-\alpha$ is dominated by separate sales;

2. Claim 12 shows that bundling at any price on the interval $(2-\alpha, 2)$ leads to lower sales than separate sales, provided that α is small enough.

3. Claim 13 shows that bundling at $p_B = 2-\alpha$ also leads to lower sales than separate sales if α is small enough.

Together, this implies that for α is sufficiently small, there exists no price $p_B < 2$ for the bundled good that gives a higher payoff than $\Pi_1 + \Pi_2$.