Legislative Bargaining and Coalition Formation*

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Abstract

The finite horizon version of a popular legislative bargaining model due to Baron and Ferejohn is investigated. With three or more rounds of bargaining a continuum of distributions are supportable as subgame perfect equilibria in Markov strategies. Allowing for history dependent strategies subgame perfect equilibria can be constructed where strictly positive shares are given to a larger group of people than necessary for a minimal winning coalition and, if players are sufficiently patient, any distribution where every agent get a strictly positive share can be supported as a subgame perfect equilibrium. In sharp contrast to these results I obtain a generic uniqueness result when allowing for differences in players' time preferences. However, the unique backwards induction equilibrium in the perturbed game is non-stationary. Hence, neither the original (symmetric) game or the perturbed game provides any guidance for equilibrium selection in the infinite game.

1 Introduction

In an influential paper, Baron and Ferejohn [2] propose a model of legislative voting with endogenous agenda setting. The model has later been generalized and applied to a wide variety of questions

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in economics and political science. For example, Baron and Ferejohn [3] use the model to analyze the role of committees, Baron [1] studies how legislative equilibria depend on characteristics on the goods provided, Chari, Jones and Marimon [7] use the model in their analysis of split ticket voting, McKelvey and Riezman [15] for an analysis of seniority in legislatures, Merlo [14] studies legislative bargaining in a stochastic environment and Eraslan [8] uses the model in a study of corporate bankruptcy.[1]

The game is a version of Rubinstein's [18] bargaining model with $n \geq 3$ agents. There are some other differences, but the crucial departure from standard bargaining models is that a proposal is implemented whenever a majority vote in favor of the proposal. Most applications use the version with an infinite time horizon for which a folk theorem applies (Baron and Ferejohn [2]) and get predictive power only by making a particular equilibrium selection, typically the stationary equilibrium.

In this paper I study the finite horizon version of Baron and Ferejohns model. I show that if there are three or more rounds of bargaining, then a continuum of divisions are supportable as subgame perfect equilibrium outcomes. In fact, *any* interior division can be supported as a subgame perfect equilibrium outcome if players are sufficiently patient and there are sufficiently many rounds of bargaining.

The basic driving force behind these results is that majority rule creates ties. Due to symmetry of the game the ex ante value of the game in the last period is the same for all players. The proposer in the second period from the end needs to bribe only a minimal winning coalition with the discounted value of the game and is therefore indifferent over which players to include in the winning coalition. This in itself implies non-uniqueness, but with three or more rounds of bargaining a more interesting multiplicity arises. If players are selected to be included in the winning majority with different probabilities, then the value of the game in the beginning of the penultimate period is different for different players. Acceptance rules will therefore differ across agents, so some players will be more expensive to bribe than other players. This makes it possible to support a continuum of divisions as equilibrium outcomes *even in strategies that are independent of payoff irrelevant history* when mixed strategies are allowed.

[1] Further examples are Calvert and Dietz [6], Leblanc et al [13], McKelvey and Riezman [16] and Eraslan and Merlo [9].

Players may also pick coalitions coalition partners based on history. It is therefore possible to reward agents for rejecting certain proposals by including them in the winning coalition and punish proposers that deviated by excluding them. This type of strategies may be used to further enlarge the set of equilibrium outcomes to include also divisions with broader support than a minimal winning coalition, and, with sufficiently many rounds of bargaining, eventually any interior allocation.

The indeterminacy of equilibria is created by ties and if the ties are broken there will be a unique equilibrium. I therefore ask whether a *natural* perturbation of the payoffs is enough to break the ties. In the paper I allow for heterogeneity in time preferences, but the analysis of the model where agents have different probabilities of making proposals is almost identical. Maybe unsurprisingly, I show that when differences in time preferences are allowed there is a unique subgame perfect equilibrium for almost all (vectors of) discount factors. However, whenever the equilibrium is unique each agent has a distinct discounted equilibrium continuation value in every period. Hence, the selection of coalition partners is determinate. Since inclusion in the winning coalition in the next period raises the value of the game in the next period players that are expensive to bribe in one period are cheap to bribe in the next, so uniqueness implies non-stationarity.

One interpretation of the generic uniqueness result is that the symmetric model is a knife-edge case. However, no matter which view one takes on this, the two models in conjunction leads to one unambiguous conclusion. The finite horizon version of the model does not provide any guidance at all for equilibrium selection in the infinite horizon game. This is in sharp contrast with standard Rubinstein style sequential bargaining where the unique backwards induction equilibrium outcome converges to the stationary equilibrium proposals when the number of rounds in the truncated game goes to infinity.

One may think of this as bad news, but the analysis provides some insights as to how sequential bargaining with majority rule (or any other rule different from unanimity rule) differs from the standard setup where unanimous agreement is required. With majority rule, being strong in the future may be a disadvantage, since it makes the player more expensive to bribe in earlier stages and therefore more likely to be excluded from the winning coalition.

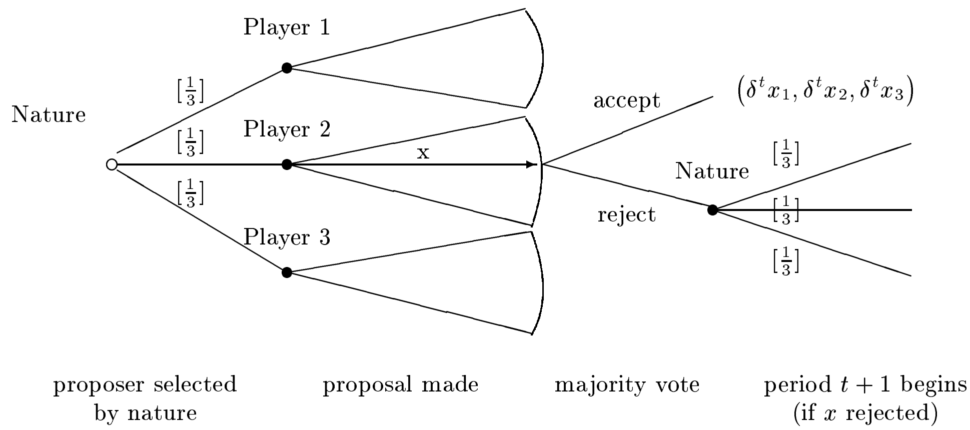

Figure 1: Sketch of One Period of the Extensive Form

2 The Symmetric Model

The model was initially suggested by Baron and Ferejohn [2] and following their interpretation of the model one may think of a body of legislators, each representing the voters of their district, who are to make a decision about how to distribute some benefits. Time progresses discretely and $t = 1, ..., T$ is used to label the periods. I let $I = \{1, .., n\}$ denote the set of players (legislators) and $X = \{x \in R^n_+ \mid \sum_{i=1}^n x^i \leq 1\}$ denote the set of potential "outcomes". Each player $i \in I$ is assumed to care only about their own share of the pie and has an instantaneous utility function $u_i : X \to R$ given by $u_i(x) = x^i$. Payoffs are discounted by the common discount factor $\delta \in (0, 1]$. The rules of the game are as follows:

Proposals: For each $t \leq T$, if the legislature has not already agreed on any proposal one randomly chosen player is *recognized* to make a proposal in the beginning of period t. Irrespective of time and history it is assumed that each player is recognized with probability $1/n$.

Voting: After a proposal $x \in X$ has been made at time t the legislature votes sequentially and according to a predetermined order between accepting and rejecting the proposal (see Figure 2). If a majority votes to accept, x is implemented and the game ends with payoffs $\delta^t x^i$ for each $i \in I$. If a majority decides to reject and $t < T$ the game goes on to the next stage. If a majority decides to reject and $t = T$ the game ends with the zero payoffs being realized for all players.

The extensive form is sketched in Figure 1 for the case with three players, where it should be noted that the voting stage really includes moves by all players. A history at the beginning of time t is given by a list $h_t = (a_1, a_2, ..., a_{t-1})$ where $a_k = (i_k, x_k, v_k^1, ..., v_k^n)$ is the "action profile" for periods $k = 1, 2, ...t \Leftrightarrow 1$, $i_k \in I$ is the identity of the proposer, $x_k \in X$ is the proposal made and $v_k^j \in \{yes, no\}$ is the vote cast by player j in period k. I let H_t denote the set of all possible (beginning of) time t histories. Since voting is sequential, each player knows not only the proposal and h_t when voting, but also all previous votes. I let h_t^i denote a history when it is is turn to vote in period t, which lists the information in h_t, the identity of the proposer, the proposal and all previous votes. The set of all such histories is denoted by H_t^i. A generic pure strategy s^i for player $i \in I$ is given by $s^i = \{p_t^i, v_t^i\}_{t=1}^T$ where $p_t^i : H_t \rightarrow X$ lists a proposal for each possible history at time t and $v_t^i : H_t^i \rightarrow \{yes, no\}$ describes the voting behavior of player i for every possible history. A generic mixed (behavioral) strategy for i is denoted σ_i and s and σ are used in the usual way to denote pure and mixed strategy profiles.

3 Examples

To gain intuition and to make the mechanics of the model clear I first consider a few simple examples. All examples assume that there are three players and no discounting. As a benchmark, I begin with constructing a symmetric Markovian equilibrium, which is the finite horizon analogue of the stationary equilibrium in the infinite game usually studied in the literature. I then show that with three rounds of bargaining, there is multiplicity in the set of divisions that are supportable as equilibrium outcomes even with history independent strategies. The result that there is a continuum of distributions supportable as (Markov perfect) equilibrium outcomes follows as a slight generalization of this example. Finally, I show that (again with three periods) there are subgame perfect equilibrium outcomes where all agents get a strictly positive share of the surplus. Hence, the model does not necessarily predict that the cake is divided by a minimal winning coalition.

3.1 The Symmetric Markovian Equilibrium

The Last Period: If the proposal in the last period is rejected, the game ends with zero payoffs for everybody. This implies that in any subgame perfect equilibrium, the proposer takes everything for herself, that is player 1 proposes $(1, 0, 0)$, player 2 $(0, 1, 0)$ and player 3 $(0, 0, 1)$, and these proposals are accepted for sure.[2]

[2]The uniqueness argument combines the fact that pivotal voters must vote in favor of a proposal that benefits them strictly under sequential voting (see Figure 2) with the usual arguments for why indifferences are resolved in

The Penultimate Period: Since each player gets 1 with probability $\frac{1}{3}$ and 0 with probability $\frac{2}{3}$ if the game continues to the final stage, the continuation value for player any player i in the beginning of the last period is $\frac{1}{3}$. Thus if player j is pivotal in the voting stage of the second period, she must accept if $x^j > \frac{1}{3}$ and reject if $x^j < \frac{1}{3}$. Since indifferences must be resolved in favor of acceptance in equilibrium, any proposal where $x^j \geq \frac{1}{3}$ for at least two agents is accepted and any other proposal is rejected in any subgame perfect equilibrium. Denote by ρ^{ij} the probability that j is offered $\frac{1}{3}$ by player i when i is recognized to propose. The continuation payoff in the beginning of the second to last period for agent i is then

$$V^i_{T-1} = \frac{1}{3}\frac{2}{3} + \frac{1}{3}\rho^{ji}\frac{1}{3} + \frac{1}{3}\rho^{ki}\frac{1}{3}. \tag{1}$$

Hence, if $\rho^{ij} = \frac{1}{2}$ for all i, j (or any configuration of "inclusion probabilities" such that $\rho^{ij} + \rho^{kj} = 1$ for all j), the continuation value of the game in the beginning of the second period is $\frac{1}{3}$ for all players.

All Earlier Periods: Consider the game with $T \geq 3$ periods and suppose that after any history of play, each player i randomizes with equal probabilities over which of the remaining players to give $\frac{1}{3}$ to and keeps $\frac{2}{3}$ for herself (that is, player 1 proposes $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$ with probability $\frac{1}{2}$ and $\left(\frac{2}{3}, 0, \frac{1}{3}\right)$ with probability $\frac{1}{2}$ and so on). I have already verified that this is consistent with equilibrium at time $T-1$. Moreover, the continuation payoff in the beginning of time $T-1$ is $\frac{1}{3}$ for each player i. Hence the same argument as in the penultimate period establishes that the voting strategy "vote to accept if and only if $x^i \geq \frac{1}{3}$" is sequentially rational at time $T-2$, implying that the proposal rule is consistent with equilibrium at time $T-2$. The continuation payoff is then again $\frac{1}{3}$ for each player, so it follows by induction that the proposed strategies are consistent with equilibrium.

It should be obvious that the playing in accordance with these voting and proposal rules in every period constitutes an equilibrium of the infinite horizon game as well. Indeed, this is what is referred to as the stationary equilibrium of the infinite horizon game, which is the equilibrium selection typically made in applications.[3]

favor of acceptance.

[3]The stationary equilibrium in the infinite horizon game and (play in all but the last period of) the symmetric Markovian equilibrium in the finite game can for general (odd) n and an arbitrary $\delta \leq 1$ be described as follows. In the voting stage, agents vote to accept if and only if they receive at least $\frac{\delta}{n}$. When making proposals, each agent picks a group of $\frac{n-1}{2}$ coalition partners by randomizing with equal probabilities over all subsets of $\frac{n-1}{2}$ players of

3.2 An Asymmetric Markovian Equilibrium

I consider the model with three periods , where, again, I for simplicity assume that there are three agents and no discounting. In the second period, let the players follow the proposal rules,

$$\begin{aligned} p_2^1(h_2) &= \left(\frac{2}{3},\frac{1}{3},0\right) \text{ after every } h_2 \in H_2 \\ p_2^2(h_2) &= \left(\frac{1}{3},\frac{2}{3},0\right) \text{ after every } h_2 \in H_2 \\ p_2^3(h_2) &= \left(\frac{1}{3},0,\frac{2}{3}\right) \text{ after every } h_2 \in H_2. \end{aligned} \tag{2}$$

That is, player 1 always selects player 2, player 2 always selects player 1 and player 3 always select player 1 as her coalition partner. The analysis of the penultimate period of the symmetric equilibrium in Section 3.1 establishes that these proposal rules are consistent with equilibrium. Given the proposal rules in (2), the continuation values of the game in the beginning of the second period for players $i = 1, 2, 3$ are,

$$\begin{aligned} V_2^1 &= \frac{1}{3}\frac{2}{3} + \frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{1}{3} = \frac{4}{9} \\ V_2^2 &= \frac{1}{3}\frac{2}{3} + \frac{1}{3}\frac{1}{3} = \frac{3}{9} \\ V_2^3 &= \frac{1}{3}\frac{2}{3} = \frac{2}{9}. \end{aligned} \tag{3}$$

The voting rule "accept if and only if $x^i \geq V_2^i$" is then sequentially rational for each player. With these acceptance rules player 3 is cheapest, player 2 is the second cheapest, and player 1 is the most expensive player to "bribe". Hence, the optimal proposals in the first period given the specified continuation play are

$$\begin{aligned} p_1^1 &= \left(\frac{7}{9},0,\frac{2}{9}\right) \\ p_1^2 &= \left(0,\frac{7}{9},\frac{2}{9}\right) \\ p_1^3 &= \left(0,\frac{1}{3},\frac{2}{3}\right), \end{aligned} \tag{4}$$

which will be accepted in equilibrium. Hence, I have constructed a subgame perfect equilibrium in Markov strategies that yields equilibrium distributions on the outcome path that is different

the remaining $n-1$. The $\frac{n-1}{2}$ coalition partners receive $\frac{\delta}{n}$ and the proposer takes $1 - \frac{(n-1)\delta}{2n}$, while all other agents receive nothing.

from the symmetric equilibrium in Section 3.1. Hence, there is multiplicity of equilibria that goes beyond the identities of the players who are selected for the winning majority.

One interesting feature of the asymmetric equilibrium is that the expected payoff is for player 1 is lower than for the other two players. At the same time, player 1 is the player who, conditionally on reaching it, has the strongest position in the second round. While this helps player 1 to take a somewhat larger share *if making a proposal in the initial period,* it is a disadvantage if any of the other players is making a proposal: the other players have less to gain by turning down proposals, so player one will simply be excluded from the winning coalition in the first period. That is, being too strong in the future is bad since it makes a player more expensive to bribe.

3.3 Indeterminacy

To see that there is indeed a continuum of divisions supportable as subgame perfect equilibrium outcomes, let π be the probability that players behave as in the asymmetric equilibrium constructed above in the second period and $(1 \Leftrightarrow \pi)$ the probability that the players behave as in the symmetric equilibrium. With this specification the ordering of the players continuation payoffs remains the same for any $\pi \in (0,1]$ and it is easy to see that by choosing π appropriately any $x^1 = (s^1, 0, 1 \Leftrightarrow s^1)$ with $s^1 \in [\frac{2}{3}, \frac{7}{9}]$ is supportable as an equilibrium outcome (realized if player one proposes in first period).

Now let $T > 3$ and consider the set of equilibria of the T-period game where all players are playing in accordance to the symmetric equilibrium in Section 3.1 form the third period and onwards. The (history independent) continuation payoffs in the beginning of period 3 is then $\frac{1}{3}$ for each player, exactly as the unique continuation payoff in the last period of the game. It follows that any division that can be supported in the game with three periods can be supported also in the game with $T > 3$ periods. It is rather clear that this logic doesn't rely on either the assumption that there are only three players or the absence of discounting. Indeed, it is easy to show:

Proposition 1 *Suppose that* $\delta > 0$, $T \geq 3$ *and* $n \geq 3$. *Then there is a continuum of divisions supportable as subgame perfect equilibria of the model.*

The proof is a direct extension of the construction above, but a sketch is given in Appendix A for completeness. Nothing in the argument hinges on majority rule per se. If q votes are required for acceptance of a proposal the exact same construction can be used to prove that there is a continuum of equilibrium outcomes for any q other than 1 and n.

3.4 History Dependent Proposal Rules

The purpose of this example is to show that the set of equilibrium outcomes is enlarged even further if history dependent strategies are allowed. In particular, equilibria where all players receive a positive share of the benefits can be supported by simple punishments and rewards.

Again, I consider the case with three agents, three periods and no discounting. For $i = 1, 2, 3$ I let $x^i \in X$ be some "target equilibrium outcome". Without loss I take the first period proposer to be player 1 and assume that, after any history of play, player 1 randomizes with equal probabilities over $(\frac{2}{3}, \frac{1}{3}, 0)$ and $(\frac{2}{3}, 0, \frac{1}{3})$ if recognized to make a proposal in the second period. Player 2 and 3 on the other hand follows the proposal rules,

$$p_2^2(h_2) = \begin{cases} \left(0, \frac{2}{3}, \frac{1}{3}\right) & \text{if } y \neq x^1 \text{ was proposed in first period} \\ \left(\frac{1}{3}, \frac{2}{3}, 0\right) & \text{if } y = x^1 \text{ was proposed in first period} \end{cases} \tag{5}$$

$$p_2^3(h_2) = \begin{cases} \left(0, \frac{1}{3}, \frac{2}{3}\right) & \text{if } y \neq x^1 \text{ was proposed in first period} \\ \left(\frac{1}{3}, 0, \frac{2}{3}\right) & \text{if } y = x^1 \text{ was proposed in first period} \end{cases}$$

Since some permutation of $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$ with the proposer getting $\frac{2}{3}$ is proposed after any history of play it follows from the analysis in Section 3.1 that these proposal rules together with the uniquely determined backwards induction continuation strategies implies Nash equilibrium play in any subgame from the beginning of the second period and on. Now, for a history h_2 where x^1 was proposed in the first period and rejected, the continuation payoffs in the beginning of period 2 are

$$\begin{aligned} V_2^1(h_2) &= \frac{1}{3}\frac{2}{3} + \frac{2}{3}\frac{1}{3} = \frac{8}{18} \\ V_2^j(h_2) &= \frac{1}{3}\frac{2}{3} + \frac{1}{3}\frac{1}{2}\frac{1}{3} = \frac{5}{18} \text{ for } j = 2, 3, \end{aligned} \tag{6}$$

while if $y \neq x^1$ was proposed in the first period the continuation payoffs are

$$\begin{aligned} V_2^1(h_2) &= \frac{1}{3}\frac{2}{3} = \frac{2}{9} \\ V_2^j(h_2) &= \frac{1}{3}\frac{2}{3} + \frac{1}{3}\frac{1}{3} + \frac{1}{3}\frac{1}{2}\frac{1}{3} = \frac{7}{18} \text{ for } j = 2, 3. \end{aligned} \tag{7}$$

Hence, players 2 and 3 will both reject any deviant proposal y in the first period such that $y^j < \frac{7}{18}$, while it is sequentially rational to accept the "target proposal" x^1 if $x^{1j} \geq \frac{5}{18}$. Hence any x^1 such that $x^{11} \geq \frac{11}{18}$ and $x^{1j} \geq \frac{5}{18}$ for one $j \neq 1$ is supportable as an equilibrium outcome when 1 is recognized to make a proposal in the first period.

One example of a proposal that can be supported in equilibrium is thus $\left(\frac{11}{18}, \frac{5}{18}, \frac{2}{18}\right)$. Hence, three periods are sufficient to generate an equilibrium outcome with broader support than a minimal winning coalition.[4] Another example is $\left(\frac{11}{18}, \frac{5}{18}, 0\right)$, which shows that there are inefficient equilibria.

3.5 Concave Utility Functions

The equilibrium in Section 3.1, which is the equilibrium that is typically analyzed, is Pareto efficient, since any proposal where all resources are spent is Pareto efficient. However, extensions of the model have been studied where the players have concave utility functions, either because of risk aversion (see Harrington [11]) or, if the game is interpreted as one where legislators seek funds for local public goods to their districts, due to diminishing returns in spending on local public goods (see Baron [1]). For such preferences equilibria of the form constructed in this example actually have an efficiency rationale since the variability in the share received by any particular agent can be reduced by conditioning play on history as in (5).

Again I assume that there are three agents, three periods and no discounting. To get an example with closed form solutions I simply assume that $u^i(x) = \sqrt{x^i}$ for $i = 1, 2, 3$. This particular choice is convenient since the continuation value in the beginning of the last period is $\frac{1}{3}$. Assuming that agents accept when indifferent this implies that any proposal will be accepted where $\sqrt{x^i} \geq \frac{1}{3}$ for at least 2 agents, so the proposer in the second period will in equilibrium propose to keep $\frac{8}{9}$ and give $\frac{1}{9}$ to one of the other agents. If all agents randomize with equal probabilities over the choice of coalition partner the continuation payoff is $\frac{1}{3}\sqrt{\frac{8}{9}} + \frac{1}{3}\sqrt{\frac{1}{9}} = \frac{1}{9}\left(\sqrt{8}+1\right)$, so in the first period the proposer must give $\left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2$ to one agent and keep the rest for themselves. Assuming again that the choice of coalition partner is a fair coin flip, the ex ante value of the game is

$$V_S = \frac{1}{3}\sqrt{1 \Leftrightarrow \left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2} + \frac{1}{3}\sqrt{\left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2} \tag{8}$$

If instead proposals in the second period is done in the same fashion as in (5), but with $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$ replaced by $\left(\frac{8}{9}, \frac{1}{9}, 0\right)$ and similar for the other permutations, then, for a history h_2 where x^1 was

[4] Minimal winning coalitions have appeared as either an essential assumption or the key prediction in most formal models of coalition formation. See for example Riker [17], Shepsle [19] or, more recently, Baron and Ferejohn [2], [3]. For an interesting strategic model where winning majorities are oversized, see Groseclose and Snyder [10].

proposed in the first period and rejected, the continuation payoffs in the beginning of period 2 are

$$\begin{aligned} V_2^1(h_2) &= \frac{1}{3}\sqrt{\frac{8}{9}} + \frac{2}{3}\sqrt{\frac{1}{9}} = \frac{1}{9}\left(\sqrt{8}+2\right) \\ V_2^j(h_2) &= \frac{1}{3}\sqrt{\frac{8}{9}} + \frac{1}{3}\frac{1}{2}\sqrt{\frac{1}{9}} = \frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right) \text{ for } j=2,3. \end{aligned} \tag{9}$$

while if $y \neq x^1$ was proposed in the first period the continuation payoffs are

$$\begin{aligned} V_2^1(h_2) &= \frac{1}{3}\sqrt{\frac{8}{9}} = \frac{1}{9}\sqrt{8} \\ V_2^j(h_2) &= \frac{1}{3}\sqrt{\frac{8}{9}} + \frac{1}{3}\sqrt{\frac{1}{9}} + \frac{1}{3}\frac{1}{2}\sqrt{\frac{1}{9}} = \frac{1}{9}\left(\sqrt{8}+\frac{3}{2}\right) \text{ for } j=2,3, \end{aligned} \tag{10}$$

Hence, players 2 and 3 will reject any deviant proposal y such that $y^j < \left(\frac{1}{9}\left(\sqrt{8}+\frac{3}{2}\right)\right)^2$ in the first period, while it is sequentially rational to accept the "target proposal" x^1 if $x^{1j} \geq \left(\frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right)\right)^2$. Hence any x^1 such that $x^{11} \geq 1 \Leftrightarrow \left(\frac{1}{9}\left(\sqrt{8}+\frac{3}{2}\right)\right)^2$ and $x^{1j} \geq \left(\frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right)\right)^2$ for one $j \neq 1$ is supportable as an equilibrium outcome when 1 is recognized to make a proposal in the first period.

The labeling was arbitrary, so the same conclusion holds for all agents. Now, suppose that agent i proposes

$$x^{ij} = \begin{cases} 1 \Leftrightarrow \left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2 & \text{for } j = i \\ \left(\frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right)\right)^2 & \text{for } j = i+1 \text{ (or } i+1 \Leftrightarrow 3 \text{ if } i+1>3) \\ \left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2 \Leftrightarrow \left(\frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right)\right)^2 & \text{for } j = i+2 \text{ (or } i+2 \Leftrightarrow 3 \text{ if } i+2>3) \end{cases} . \tag{11}$$

where $i+1$ should be interpreted as agent 1 if i. The ex ante value of the game is then

$$V_A = \frac{1}{3}\sqrt{1 \Leftrightarrow \left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2} + \frac{1}{3}\sqrt{\left(\frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right)\right)^2} + \frac{1}{3}\sqrt{\left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2 \Leftrightarrow \left(\frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right)\right)^2} \tag{12}$$

Straightforward algebra shows that

$$\begin{aligned} V_A \Leftrightarrow V_S &= \frac{1}{3}\sqrt{\left(\frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right)\right)^2} \Leftrightarrow \frac{1}{3}\sqrt{\left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2} \\ &\quad + \frac{1}{3}\sqrt{\left(\frac{1}{9}\left(\sqrt{8}+1\right)\right)^2 \Leftrightarrow \left(\frac{1}{9}\left(\sqrt{8}+\frac{1}{2}\right)\right)^2} \\ &= \frac{1}{3}\frac{1}{9}\left(\sqrt{8}+\frac{1}{2} \Leftrightarrow \sqrt{8}+1+\sqrt{\left(\sqrt{8}+1\right)^2 \Leftrightarrow \left(\sqrt{8}+\frac{1}{2}\right)^2}\right) \\ &= \frac{1}{3}\frac{1}{9}\left(\sqrt{\sqrt{8}+\frac{3}{4}} \Leftrightarrow \frac{1}{2}\right) > 0, \end{aligned}$$

since the square-root is greater than one. Hence, all agents prefer the asymmetric equilibrium ex ante. The idea should be obvious. All agents get the same expected share in both equilibria, but the variability is smaller in the asymmetric equilibrium than in the symmetric. Since preferences are strictly concave in the share, this means that everyone is strictly better off in the equilibrium with history dependent strategies.

4 A General Result

The ideas in the examples can be used further to increase the set of divisions supportable in equilibrium as the time horizon gets longer. The next proposition provides conditions for when all *interior* distributions can be supported as a subgame perfect equilibrium:

Proposition 2 *Suppose that* $\max\left\{\frac{n+1}{2(n-1)}, \sqrt{\frac{2n}{(n-1)(n-2)}}\right\} \leq \delta \leq 1$ *and that there is an odd number of players* $n \geq 5$. *Then, for any distribution of benefits* x *such that* $x^i > 0$ *for all* $i \in I$ *there is a* $T < \infty$ *such that for each* $t \geq T$ *there is an equilibrium where player* $i \in I$ *proposes* x *if recognized to make a proposal in the first period of the game with* t *periods and* x *is accepted by a majority.*

The proof, which is by construction, is in Appendix B . The idea is to discipline voters by punishing an appropriately chosen group of voters if equilibrium proposals are rejected and to discipline proposers by rewarding an appropriately chosen group of voters for rejecting a deviant proposal, and to make sure that the deviant proposer is excluded in the next period unless proposing again. Since it is always possible to make sure that a group of $\frac{n+1}{2}$ agents get nothing in the next period unless they are proposing it is rather easy to create incentives for agents to vote to accept proposals even if the share is small. The harder task is to create incentives for agents to propose a division even if their own share is very small. In the proof this is done by "matching" proposals in the sense of giving the $\frac{n+1}{2}$ agents *who get the least under the deviation* the shares offered by the deviant proposer (or if this is too little to make it sequentially rational to accept, the lowest share that is consistent with acceptance). Recursive use of these punishment and reward strategies successively erodes "proposal power" and since the chance of being a proposer in future periods is the source of "power" for the agents who are not currently proposers, this makes it possible to construct an equilibrium where agents are willing to propose and vote to accept proposals in spite of getting an arbitrarily small share.[5]

[5]While the language above is similar to repeated game language it is worth noting that "to reward" means to give tomorrow more than the discounted value of what is offered today. In other words, since the game ends as soon as

It is important to notice that the source of multiplicity is different than in the well-known example of multiplicity in a $n \Leftrightarrow$person version of the Rubinstein [18] bargaining model discussed in Herrero [12] and Sutton [20]. Their example relies critically on the infinite horizon and if the game is truncated after t periods uniqueness is restored. *If majority rule is replaced by unanimity,* the same thing is true for the model in this paper *if majority rule is replaced by unanimity.* Then the unique backward induction equilibrium is for the proposer to give $\frac{\delta}{n}$ to other $n \Leftrightarrow 1$ players and keep $1 \Leftrightarrow \frac{(n-1)\delta}{n}$. In contrast, anything strictly in between dictatorship of the proposer and unanimity results in multiple equilibria, so the indeterminacy comes from the voting.

The reason that distributions on the boundary can not always be supported is simple: with positive probability the first period proposer will propose in every period, so the first period proposer can not propose a first period division that gives her less than $\left(\frac{\delta}{n}\right)^T > 0$ if this proposal would be accepted since by proposing to take 1 and give nothing to all the others in each period otherwise would be a profitable deviation (this must be accepted in last period by any subgame perfect strategies). This is in contrast with the infinite horizon game, where all subgames starting with a proposal by some agent look the same. Here proposals on the boundary can be supported by always rejecting anything but the equilibrium proposal with appropriately constructed off the equilibrium path proposal rules.

The strategies in the proof were chosen mainly for simplicity and there are several places where more efficient (and more complicated) constructions can be used to ensure faster erosion of proposal power. However, the bound $\delta \geq \frac{n+1}{2(n-1)}$ (which is the relevant bound with $n \geq 7$) is still tight. In order to defeat the proposal where the proposer takes ϵ and all the others get $\frac{1-\epsilon}{n-1}$ at least $\frac{n+1}{2}$ players must prefer the proposal to be rejected. The lowest continuation payoff in a group of $\frac{n+1}{2}$ agents can not be higher than $\frac{2}{n+1}$, so if $\frac{1-\epsilon}{n-1} > \frac{2\delta}{n+1}$ there are no *feasible* continuation strategies that induces incentives to reject the proposal. Hence, $\delta \geq \frac{n+1}{2(n-1)}$ is a necessary condition for a limit folk theorem to apply.

For the restriction on the number of players, the bound is tight and the reason why $n = 3$ does not work is rather simple: if the proposer gives $\delta + \epsilon$ to one player, that player must accept since

a proposal is accepted continuation play needs to be more flexible than in typical repeated game arguments. Hence, the proof resembles proof of the limit folk theorem in Benoit and Krishna [4] only to the extent that multiplicity in the end is used to successively enlarge.

she could not get more than 1 in the next period.

5 Heterogeneous Time Preferences

The examples and the proof of Proposition 2 exploits that the symmetry of the game makes it possible to punish and reward players in the second to last period without any cost for the proposer.

I now ask whether allowing discount factors to vary across players is enough to break all (relevant) ties. The analysis applies with trivial modifications for the model with differences in the probability of making proposals. The case with payoff externalities would presumably be similar as long as externalities are asymmetric.[6] I show that equilibria are now unique for almost all choices of discount factors. However, the uniqueness does not help in selecting equilibria. In the most interesting case, when discount factors are close and substantially different from zero, the unique equilibrium will be highly non-stationary. The identities of the (unique) set of agents included in the winning coalition will then depend crucially on how many periods there are left in the game.

5.1 Generic Uniqueness

Consider the case with three players with $\delta_1 > \delta_2 > \delta_3$. Then if players 1 or 2 is to propose in the second stage from the end they will propose to give $\frac{\delta_3}{3}$ to player 3 since player 3 will be the cheapest player to bribe. For the same reason player 3 would select player 2. Now the unique equilibrium continuation values in the beginning of the penultimate period are $V_{T-1}^1 = \frac{1}{3}\left(1 \Leftrightarrow \frac{\delta_3}{3}\right)$, $V_{T-1}^2 = \frac{1}{3}\left(1 \Leftrightarrow \frac{\delta_3}{3}\right) + \frac{1}{3}\frac{\delta_2}{3}$ and $V_{T-1}^3 = \frac{1}{3}\left(1 \Leftrightarrow \frac{\delta_2}{3}\right) + \frac{2}{3}\frac{\delta_3}{3}$. So the question whether the proposals at the third stage from the end are unique or not is a question whether $\delta_1 V_{T-1}^1 \neq \delta_2 V_{T-1}^2 \neq \delta_3 V_{T-1}^3$ or not. Clearly, most choices of $(\delta_1, \delta_2, \delta_3)$ lead to distinct discounted continuation values.

It is intuitive that the discounted continuation values are distinct for most choices of discount factors also with 4 or more periods and there seems to be no reason to suspect that n players should be any different from 3 players. The next result establishes that this is indeed the case.

Proposition 3 *For any finite T there exists a set $D^T \subset [0,1]^n$ with full Lebesgue measure such that if $(\delta_1, ..., \delta_n) \in D^T$, then there is a unique equilibrium proposal in each period and for every*

[6] This model has some interest since it can be used to give an interpretation of parties. See Calvert and Dietz [6] for an analysis of (stationary) equilibria in such a model.

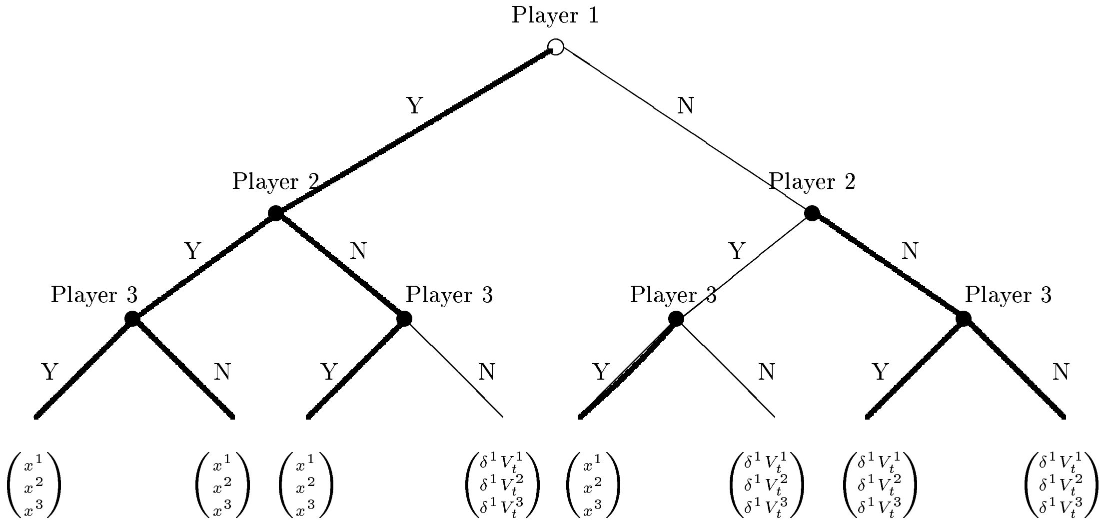

Figure 2: Voting with Unique Continuation Payoffs when $x^1 > \delta^1 V_t^1, x^2 < \delta^2 V_t^2$ and $x^3 > \delta^3 V_t^3$

$i \in I$ which will be accepted for sure in any subgame perfect equilibrium.

The proof is in Appendix C, but an informal sketch is instructive. As should be clear from Figure 2, if continuation values in the beginning of the next period are unique, a proposal is accepted if and only if a majority gets more than their respective continuation values must be accepted (recall that ties must be broken in favor of acceptance).[7] Hence, if the set of the "$\frac{n-1}{2}$ remaining agents with the lowest discounted continuation payoffs" is uniquely defined, then the unique equilibrium proposal when i is recognized is to give these $\frac{n-1}{2}$ agents their discounted continuation payoffs.

A sufficient condition for uniqueness is thus that in each period $t \geq 2$ and each player i there is a unique equilibrium continuation payoff V_t^i satisfying $\delta^i V_t^i \neq \delta^j V_t^j$ for all $j \neq i$. If this is true at time t, then the optimal proposal at time $t \Leftrightarrow 1$ is to select the $\frac{n-1}{2}$ cheapest players to bribe, give these players $\delta^i V_t^i$ and keep the rest. Hence, if the players are relabeled so that $\delta^1 V_t^1 < \delta^1 V_t^1 < .... < \delta^n V_t^n$, then agents $i = 1, ..., \frac{n-1}{2}$ are always included in the winning coalition no matter who the proposer is, agent $\frac{n+1}{2}$ is included if $i \leq \frac{n-1}{2}$ is proposing and agents $i > \frac{n+1}{2}$ are never included in the winning coalition. Hence, the continuation values at time $t \Leftrightarrow 1$ can be written in terms of the continuation values at time t and parameters of the model as

[7] Figure 2 should also make it clear that there is always a trivial multiplicity of equilibria in terms of voting strategies, which is why Proposition 3 is stated in terms of equilibrium proposals.

$$
V_{t-1}^{i}=\begin{cases}\frac{1}{n}\left(1+\delta^{i}V_{t}^{i}\Leftrightarrow\sum_{j=1}^{\frac{n+1}{2}}\delta^{j}V_{t}^{j}\right)+\frac{n-1}{n}\delta^{i}V_{t}^{i} & \text{for } i<\frac{n+1}{2}\\ \frac{1}{n}\left(1\Leftrightarrow\sum_{j=1}^{\frac{n-1}{2}}\delta^{j}V_{t}^{j}\right)+\frac{(n-1)}{2n}\delta^{i}V_{t}^{i} & \text{for } i=\frac{n+1}{2}\\ \frac{1}{n}\left(1\Leftrightarrow\sum_{j=1}^{\frac{n-1}{2}}\delta^{j}V_{t}^{j}\right) & \text{for } i>\frac{n+1}{2}\end{cases}. \tag{13}
$$

Now, think of δ^1 as a free variable and fix $\delta^2, .., \delta^n$. By repeated use of (13) one may express the continuation value for each agent as

$$
V_{t-1}^{i}=\alpha^{i}+\beta^{i1}\delta^{1}+\beta^{i2}\left(\delta^{1}\right)^{2}+.....+\beta^{in-1}\left(\delta^{1}\right)^{T-t-1}, \tag{14}
$$

Hence, any failure of uniqueness must correspond to some δ^1 that solves a polynomial. The relevant polynomial will depend on δ^1, but only to the extent that the ranking matters. Since there are finitely many ways to rank a finite set of objects there are finitely many polynomials to consider and each has a finite number of roots. Finally, the coefficients of each polynomial are continuously differentiable in $(\delta^2, ..., \delta^n)$, so discount factors where uniqueness can fail belongs to a finite collection smooth manifolds of dimension $n \Leftrightarrow 1$, a set of measure zero.

5.2 Implications for Equilibrium Selection in the Infinite Game

As is well-known and mentioned previously in the paper, majority rule is not needed to get a folk theorem in the infinite horizon game. Even in Rubinstein style bargaining with three or more players anything can be supported as a subgame perfect equilibrium in the infinite horizon game. However, here the (unique) stationary equilibrium may be viewed as a limit of unique backwards induction equilibria of truncated games (see Sutton [20]).

Here, the situation is similar, with one important difference. In Rubinstein bargaining the equilibrium proposals converge, while this can not be true in the model with majority rule if the equilibrium is unique except under exceptional and rather uninteresting circumstances. The idea is much the same as the example in Section 3.2, where the player that got favorable treatment in the second period was at a disadvantage since she was too expensive to bribe in the first period. This logic is reflected in (13) in that the continuation value of the agents with low indices consist of one part which is due to the share taken if the agent is recognized to propose at time $t \Leftrightarrow 1$ *and* in addition to this they get the discounted value of the continuation payoff at time t for sure if any other agent is called to propose at time $t \Leftrightarrow 1$. Agents with high indices on the other hand, only get a positive share if they are called to propose at time $t-1$. Unless there are very large (and systematic) differences in time preferences this means that the low index agents will be the agents that are the most expensive to bribe in period $t-2$. Hence, the identity of the agents that are included in the winning coalition will change in every period, so the proposals do not converge. The conclusion of this is that the perturbed model is as silent on equilibrium selection for the infinite horizon game as the symmetric model.

Of course, there are alternative criteria for equilibrium selection and other types of arguments have been proposed in favor of the stationary equilibrium, the most appealing perhaps being the simplicity. Indeed, Baron and Kalai [4] shows that the stationary equilibrium is the simplest equilibrium of the game in the sense that it requires the smallest number of states in order for an automaton to play it among all equilibrium strategies.

6 Concluding Remarks

For the finite horizon version of the popular model of legislative bargaining due to Baron and Ferejohn [2] I have shown that there is a continuum of divisions supportable as an equilibrium outcome for any (common) discount factor as long as there are three or more rounds of bargaining. Moreover, if the players are sufficiently patient *any* distribution of benefits where all agents receive a strictly positive share can be supported as an equilibrium outcome if there are sufficiently many rounds of bargaining. I also generalized the model to allow different players to have different time preferences and showed that this perturbed model has a unique equilibrium for almost all choices of (vectors of) discount factors.

A natural response to the generic uniqueness is to view the symmetric game as special and dismiss the model completely since the results are so wildly different from the more generic perturbed model. However, there are interesting circumstances when this is not so obvious. In particular, a natural line of research is to model representative democracy (as in Chari, Jones and Marimon [7]) by assuming that citizens elect candidates and thereby choose the preferences for the legislators who play a legislative bargaining game after the election. Since the representatives of half the districts are excluded for sure from the winning coalition (unless proposing) if the equilibrium in the legislative game is unique, a reasonable conjecture for such a set up is that preferences of the elected legislators corresponds to the "non-generic" symmetric case in equilibrium.

A Appendix:Proof of Proposition 1

Proof. Arguing as in Section 3.1 one shows that there are (symmetric) equilibrium strategies such that the continuation value is $\frac{1}{n}$ in the beginning of period 3 for all players after any history of play (see Baron and Ferejohn [2] for details). Given these equilibrium continuation strategies, each agent must in equilibrium bribe $\frac{n-1}{2}$ others with exactly $\frac{\delta}{n}$ and keep the rest proposing in period 2. Suppose each player selects the $\frac{n-1}{2}$ remaining players with the lowest indices with probability π and randomizes with equal probabilities over the remaining agents with probability $(1 \Leftrightarrow \pi)$. The value of the game in the beginning of the second period is then

$$V_2^i(\pi) = \begin{cases} \frac{1}{n}\left(1 \Leftrightarrow \frac{(n-1)\delta}{2n}\right) + \frac{(n-1)}{n}\left(\pi\frac{\delta}{n} + (1 \Leftrightarrow \pi)\frac{1}{2}\frac{\delta}{n}\right) & \text{for } i < \frac{n+1}{2} \\ \frac{1}{n}\left(1 \Leftrightarrow \frac{(n-1)\delta}{2n}\right) + \frac{(n-1)}{n}\frac{1}{2}\frac{\delta}{n} & \text{for } i = \frac{n+1}{2} \\ \frac{1}{n}\left(1 \Leftrightarrow \frac{(n-1)\delta}{2n}\right) + \frac{(n-1)}{n}(1 \Leftrightarrow \pi)\frac{1}{2}\frac{\delta}{n} & \text{for } i > \frac{n+1}{2} \end{cases}. \tag{A1}$$

Given any $\delta > 0$, $n \geq 3$ and any $\pi > 0$ we see that from (A1) that $V_2^{\frac{n+1}{2}}(\pi) < V_2^i(\pi)$ for $i \in \left\{1, ..., \frac{n-1}{2}\right\}$ and $V_2^i(\pi) < V_2^{\frac{n+1}{2}}(\pi)$for $i \in \left\{\frac{n+3}{2}, ..., n\right\}$. Hence if agent $i \in \left\{1, ..., \frac{n-1}{2}\right\}$ is called to propose in the first period she will propose to keep $1 \Leftrightarrow \delta \sum_{i=\frac{n+3}{2}}^{n} V_2^j(\pi)$ and give $\delta V_2^j(\pi)$ to agents $j = \frac{n+3}{2}, ..., n$. Depending on choice of π, $V_2^j(\pi)$ can take on any value between $\frac{1}{n}\left(1 \Leftrightarrow \frac{(n-1)\delta}{2n}\right)$ and $\frac{1}{n}$, implying that there is a continuum of equilibrium proposals. The argument goes through with obvious modifications for other proposers. ■

B Appendix:Proof of Proposition 2

For the purposes of the proofs of Propositions 2 and 3, and only for these two proofs, I abuse previously used notation and use t to indicate the period with time counted backwards with $t = 1$ being the last period. A generic proposal is denoted $x = (x^1, ..., x^n)$ or, if more than one proposal has to be considered at the same time, $y = (y^1, ..., y^n)$. Subscripts denote time and superscripts on *specific shares* of a given proposal are used to indicate who is the receiver of a certain share. When specifying *proposal rules*, however, an index is needed also for the proposer. I use the notation $p_t^i : H_t \to X$ to denote a generic *proposal rule* at time t for agent i. To index the particular shares under a specific proposal rule I use two superscripts where the first is the identity of the proposer and the second is the identity of the receiver: i.e., $p_t^{ij}(h_t)$ denotes the share received by agent j if i is the proposer at time t after history h_t.

Lemma B1 *Let $c_t, d_t \geq 0$ and suppose that for each $i \in I$ and each $J \subset I \backslash \{i\}$ such that $|J| = \frac{n-1}{2}$ there is an equilibrium in the game with t periods where i proposes x_t given by*

$$x_t^j = \begin{cases} c_t & \text{to } j = i \\ d_t & \text{to all } j \in J \\ 0 & \text{to all } j \notin J \cup \{i\} \end{cases} \tag{B1}$$

and that x_t is accepted by a majority in the first period. Then, for any feasible proposal x such that 1) $x^i \geq \frac{\delta}{n}$ for one agent i, 2) $x^j \geq \frac{\delta}{n} c_t$ for $\frac{n-1}{2}$ agents $j \neq i$, there is an equilibrium in the game with $t+1$ periods in which x is accepted if proposed.

Proof. Consider some x satisfying 1) and 2) and relabel the players so that $x^1 \geq \frac{\delta}{n}$ and $x^j \geq \frac{\delta c_t}{n}$ for $j = 2, ..., \frac{n+1}{2}$. After any history h_t when x was proposed and rejected at time $t+1$, let the proposer i at time t follow the rule:

$$\text{if } i \leq \frac{n+1}{2} \text{ let } p_t^{ij}(h_t) = \begin{cases} c_t & \text{to } j = i \\ d_t & \text{to all } j > \frac{n+1}{2} \\ 0 & \text{to all other agents } j \leq \frac{n+1}{2} \end{cases} \tag{B2}$$

$$\text{if } i > \frac{n+1}{2} \text{ let } p_t^{ij}(h_t) = \begin{cases} c_t & \text{to } j = i \\ d_t & \text{to all other } j > \frac{n+1}{2} \text{ and } j = 1 \\ 0 & \text{to all agents } j = 2, ..., \frac{n+1}{2} \end{cases} .$$

Only permutations of (B1) are proposed under (B2), so the proposal rule is consistent with subgame perfect equilibrium play in all subgames from time t by assumption. The continuation payoff in the beginning of period t (before the identity of the proposer is drawn) is then:

$$\begin{aligned} V_1 &= \tfrac{1}{n} c_t + \tfrac{n-1}{2n} d_t \leq \tfrac{1}{n} c_t && \text{for } j = 1 \\ V_L &= \tfrac{1}{n} c_t && \text{for } j = 2, ... \tfrac{n+1}{2} \\ V_H &= \tfrac{1}{n} c_t + \tfrac{n-1}{n} d_t \leq \tfrac{2-c_t}{n} && \text{for } j = \tfrac{n+3}{2}, ..., n, \end{aligned} \tag{B3}$$

where the inequalities follow since $d_t \leq \frac{2(1-c_t)}{n-1}$ for (B1) to be feasible. Since the continuation values in the beginning of time t are independent of voting behavior and since $x^1 \geq \frac{\delta}{n} \geq \delta V_1$ and

$x^j \geq \delta V_L = \delta c_t / n$ the voting rule "vote to accept independently of voting history" is sequentially rational for agent $1, ..., \frac{n+1}{2}$. Hence there are strategies consistent with equilibrium play in every subgame following the proposal x in the beginning of time $t+1$ such that x is accepted. ■

Lemma B2 *Suppose that for each $i \in I$ and each feasible proposal x_t such that:*

$$\begin{aligned} x_t^i \geq & \quad c_t && \textit{for agent } i \\ x_t^j \geq & \quad \tfrac{\delta}{n} && \textit{for some agent } j \textit{ (possibly } j = i\textit{)} \\ x_t^k \geq & \quad d_t && \textit{for at least } \tfrac{n-1}{2} \textit{ agents } k \neq i \end{aligned} \tag{B4}$$

there is an equilibrium in the game with t periods where x_t^i is proposed and accepted by a majority if i is recognized in the first period. Then, for any proposal y such that there exists a set J of $\frac{n+1}{2}$ agents $j \neq i$ such that $\sum_{j \in J} \max\{d_t, \frac{y^j}{\delta}\} < 1 \Leftrightarrow c_t$, there are sequentially rational continuation strategies such that 1) y is rejected if proposed by i at time $t+1$, 2) the discounted expected payoff for the proposer i is $\frac{\delta c_t}{n}$.

Proof. Without loss, relabel the agents so that agent n is the proposer at time $t+1$ and $y^1 \leq y^2 \leq ... \leq y^{n-1}$. Given this labeling there exists a group J of $\frac{n+1}{2}$ agents different from the proposer such that $\sum_{j \in J} \max\{d_t, \frac{y^j}{\delta}\} < 1 \Leftrightarrow c_t$ if and only if

$$\sum_{j=1}^{\frac{n+1}{2}} \max\{d_t, \frac{y^j}{\delta}\} < 1 \Leftrightarrow c_t. \tag{B5}$$

Suppose that each $i \in I$ follows the proposal rule,

$$\text{If } i \leq \frac{n+1}{2} \text{ let } p_t^{ij}(h_t) = \begin{cases} 1 + \max\{d_t, \frac{y_i}{\delta}\} \Leftrightarrow \sum_{j=1}^{\frac{n+1}{2}} \max\left\{d_t, \frac{y^j}{\delta}\right\} & \text{to } j = i \\ \max\left\{d_t, \frac{y^j}{\delta}\right\} & \text{to all other } j \leq \frac{n+1}{2} \\ 0 & \text{to all other agents} \end{cases}$$

$$\text{If } i > \frac{n+1}{2} \text{ let } p_t^{ij}(h_t) = \begin{cases} c_t & \text{to } j = i \\ \max\left\{d_t, \frac{y^j}{\delta}\right\} & \text{to all } j \leq \frac{n-1}{2} \\ 1 \Leftrightarrow \sum_{j=1}^{\frac{n-1}{2}} \max\{d_t, \frac{y^j}{\delta}\} \Leftrightarrow c_t & \text{to } j = \frac{n+1}{2} \\ 0 & \text{to all other agents} \end{cases}, \tag{B6}$$

if recognized to make a proposal at time t. Consider first the case when $i \leq \frac{n+1}{2}$ is proposing at time t. Clearly $p^{ij}(h_t) \geq 0$ for all j ($p^{ii}(h_t) > 0$ follows from (B5)) and $\sum_j p^{ij}(h_t) = 1$, so the (B6) specifies a feasible proposal. From (B5) it follows that

$$p^{ii}(h_t) = 1 + \max\left\{d_t, \frac{y_i}{\delta}\right\} \Leftrightarrow \sum_{j=1}^{\frac{n+1}{2}} \max\{d_t, \frac{y^j}{\delta}\} > c_t, \tag{B7}$$

so the first inequality in (B4) is satisfied. Since the shares sum to unity and only $\frac{n+1}{2}$ agents get a positive share $p^{ij}(h_t) \geq \frac{2}{n+1} > \frac{\delta}{n}$ for at least one $j \in I$, so the second inequality in (B4) is also satisfied.. Finally, $p^{ij}(h_t) \geq d_t$ for all agents in $\left\{1, ..., \frac{n+1}{2}\right\} \setminus \{i\}$ (i.e.$\frac{n-1}{2}$ agents), so $p_t^i(h_t)$ satisfies the three conditions in (B4), so (B6) is consistent with subgame perfect equilibrium play where $p^i(h_t)$ is accepted at time t after all histories consistent with (B5). Similarly, if $i > \frac{n+1}{2}$ is recognized in period t , then $p_t^{ii}(h_t) = c_t$, immediately establishing the first condition in (B4). Secondly, $\frac{n+3}{2}$ agents split 1 and $n > 3$, so there must be some j such that $p^{ij}(h_t) \geq \frac{2}{n+3} \geq \frac{1}{n} > \frac{\delta}{n}$, so the second condition in (B4) holds as well. Finally $p_t^{ij}(h_t) \geq d_t$ for agents $1, ..\frac{n+1}{2}$ (where (B5) is used to derive the conclusion for agent $\frac{n+1}{2}$), so also in this case are all conditions in (B4) satisfied, implying that $p_t^i(h_t)$ is consistent with subgame perfection after all relevant histories. In both cases, each agent $j \leq \frac{n+1}{2}$ receives a share $p^{ij}(h_t) \geq y^j/\delta$ *independently of who is the proposer at time* t and $p^{ii}(h_t) > y^j/\delta$, so with probability $\frac{1}{n}$ they receive strictly more. The discounted expected payoff if y is rejected is thus strictly larger than y^j for $j = 1, ..., \frac{n+1}{2}$. Hence, the voting strategy "always vote to reject y" is sequentially rational for $j = 1, ..., \frac{n+1}{2}$, implying that there is an equilibrium where y is rejected if proposed for any y satisfying (B5). The proposer at time $t+1$ (n) gets c_t if proposing again at time t and 0 otherwise, so her discounted expected payoff from proposal y is $\frac{\delta c_t}{n}$, which completes the proof. ■

Lemma B3 *Suppose that the assumptions of Lemma B2 are satisfied. Then, for any proposal* y *such that* $y^i < \frac{\delta}{n}$ *for one agent* i *and* $y^j < \delta\frac{(2-c_t)}{n}$ *for* $\frac{n-1}{2}$ *agents* $j \neq i$ *there are sequentially rational continuation strategies such that 1)* y *is rejected if proposed by* i *at time* $t+1$*, 2) the discounted expected payoff for* i *is* $\frac{\delta c_t}{n}$

Proof. As in Lemma B2 I relabel the agents so that agent n is the proposer at time $t+1$ and $y^1 \leq y^2 \leq ... \leq y^n$ and index the proposer at time t as agent i. If y is rejected at time $t+1$ I assume that the proposer at time t, i, follows the rule,

$$\text{if } i \quad \leq \quad \frac{n+1}{2} \text{ she proposes } p_t^{ij}\left(h_t\right) = \begin{cases} c_t & \text{to herself} \\ \frac{2(1-c_t)}{n-1} & \text{to all other } j \leq \frac{n+1}{2} \\ 0 & \text{to all other agents} \end{cases} \tag{B8}$$

$$\text{if } i \quad > \quad \frac{n+1}{2} \text{ she proposes } p_t^{ij}\left(h_t\right) = \begin{cases} c_t & \text{herself} \\ \frac{2(1-c_t)}{n-1} & \text{to } j = 2, ..., \frac{n+1}{2} \\ 0 & \text{to all other agents} \end{cases},$$

and that $p_t^i\left(h_t\right)$ is always accepted, which by assumption is consistent with subgame perfect equilibrium play from time t on (because (B4) are satisfied). If y is rejected, the continuation payoff in the beginning of period t is $\frac{1}{n}$ for $j=1$ and $\frac{(2-c_t)}{n}$ for $j = 2, .., \frac{n+1}{2}$. Hence it is consistent with sequential rationality for agent $1, ...\frac{n+1}{2}$ to vote reject y for all voting histories (since $y^1 < \frac{\delta}{n}$ and $y^j < \delta\frac{(2-c_t)}{n}$ for $j = 2, ..., \frac{n+1}{2}$). Again, the proposer at time $t+1$, n, gets c_t if called to propose at time t and 0 otherwise, so the discounted expected payoff for the proposer is $\frac{\delta c_t}{n}$. ■

Lemma B4 *Suppose that the assumptions of Lemma B2 are satisfied. Then, for any proposal y such that $y^j < \delta\left(\frac{1}{n} + \frac{(1-c_t)}{n}\frac{(n-1)}{(n+1)}\right)$ for $\frac{n+1}{2}$ agents j different from the proposer there are sequentially rational continuation strategies such that 1) y is rejected if proposed at time $t+1$, 2) the discounted expected payoff for the proposer is $\frac{\delta c_t}{n}$*

Proof. Again, relabel the agents so that n is the proposer at $t+1$ and $y^1 \leq y^2 \leq ...y^{n-1}$. If $i \leq \frac{n+1}{2}$ is recognized in period t the proposal rule is still in accordance with (B8), but if $i > \frac{n+1}{2}$ proposes at time t she follows the rule

$$p_t^{ij}\left(h_t\right) = \begin{cases} c_t & \text{to the proposer at time } t \\ \frac{2(1-c_t)}{n-1} & \text{to } \frac{n-1}{2} \text{ randomly selected agents from } \left\{1, .., \frac{n+1}{2}\right\} \\ 0 & \text{to all other agents} \end{cases}. \tag{B9}$$

Given this proposal rule the probability for an agent $j \leq \frac{n+1}{2}$ to be included in the winning majority when agent $i > \frac{n+1}{2}$ is called to propose at time t is $\frac{n-1}{n+1}$, so continuation value of the game for agents $1, ..., \frac{n+1}{2}$ in the beginning of period t may is

$$V_H = \frac{1}{n}c_t + \frac{(n \Leftrightarrow 1)}{2n}\frac{2\left(1 \Leftrightarrow c_t\right)}{(n \Leftrightarrow 1)} + \frac{(n \Leftrightarrow 1)}{2n}\frac{(n \Leftrightarrow 1)}{(n+1)}\frac{2\left(1 \Leftrightarrow c_t\right)}{(n \Leftrightarrow 1)} = \frac{1}{n} + \frac{\left(1 \Leftrightarrow c_t\right)}{n}\frac{(n \Leftrightarrow 1)}{(n+1)}. \tag{B10}$$

Hence $y^j < \delta V_H$ for agents $1, ..., \frac{n+1}{2}$ so it is consistent with sequential rationality for agents $1, ..., \frac{n+1}{2}$ to vote to reject y independently of previous votes. The outcome of such voting strategies is that y is rejected and, again, the proposer at time $t+1$, n, gets c_t if called to propose again at time t and nothing otherwise, so the discounted expected payoff is $\frac{\delta c_t}{n}$. ■

For ease of stating the next result I define the functions

$$\begin{aligned} G(c,d) &\equiv 1 \Leftrightarrow \delta\left(1 \Leftrightarrow c \Leftrightarrow \frac{(n \Leftrightarrow 1)}{2} d\right) \Leftrightarrow \frac{(n \Leftrightarrow 3)\,\delta}{2n}(2 \Leftrightarrow c) \\ H(c) &\equiv 1 \Leftrightarrow \delta(1 \Leftrightarrow c) \Leftrightarrow \frac{(n \Leftrightarrow 3)\,\delta}{2}\left(\frac{1}{n} + \frac{n \Leftrightarrow 1}{n+1}\frac{(1 \Leftrightarrow c)}{n}\right). \end{aligned} \tag{B11}$$

Lemma B5 *Suppose that the assumptions of Lemma B2 are satisfied and also suppose that* $c_t \leq 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$ *and* $d_t \leq \frac{\delta}{n}$. *Then, for any* $i \in I$ *and any feasible proposal* x *such that*

$$\begin{aligned} x^i &\geq \max\left\{\frac{\delta c_t}{n}, G(c_t, d_t), H(c_t)\right\} \\ x^j &\geq \frac{\delta}{n} \qquad \text{for some } j \in I \text{ (possibly } j = i) \\ x^k &\geq \frac{\delta c_t}{n} \qquad \text{for at least } \tfrac{n-1}{2} \text{ agents } k \neq i \end{aligned} \tag{B12}$$

there is a subgame perfect equilibrium in the game with $t+1$ *periods in which* x *is proposed and accepted by a majority if* i *is recognized in the first period.*

Proof. Consider some x satisfying (B12). By assumption, any proposal satisfying (B4) is an equilibrium outcome of the t period game, implying that any perturbation of the proposal (B1) is an equilibrium proposal in the game with t periods. Since (B12) imply that $x^j \geq \frac{\delta}{n}$ for one j and $x_k \geq \frac{\delta c_t}{n}$ for at least $\frac{n-1}{2}$ agents $k \neq j$ Lemma B1 implies that there are sequentially rational continuation strategies such that x is accepted if proposed at time $t+1$. It remains to check whether equilibrium strategies can be constructed so that i is willing to propose x if recognized at time $t+1$. I let y be an arbitrary deviation from x and relabel the agents so that n is the proposer at time $t+1$ and $y^1 \leq y^2 \leq ...y^{n-1}$.

CASE 1: First suppose that y is such that any of the conditions

$$\sum_{j=1}^{\frac{n+1}{2}} \max\{d_t, \frac{y^j}{\delta}\} < 1 \Leftrightarrow c_t \tag{B13}$$

$$y^1 < \frac{\delta}{n} \text{ and } y^{\frac{n+1}{2}} < \frac{(2 \Leftrightarrow c_t)\,\delta}{n} \tag{B14}$$

$$y^{\frac{n+1}{2}} < \delta\left(\frac{1}{n} + \frac{(1 \Leftrightarrow c_t)}{n}\frac{(n \Leftrightarrow 1)}{(n+1)}\right) \tag{B15}$$

is satisfied. Lemma B2 established that there are sequentially rational so that y is rejected if proposed if (B13) is satisfied and Lemma B3 and B4 guarantees that the same thing is true under condition (B14) and (B15) respectively. In all three cases continuation play can be constructed so that the expected discounted payoff for n (the proposer at time $t+1$) is $\frac{\delta c_t}{n}$. I assume continuation play after a deviant y satisfying (B13), (B14) or (B15) is such that n receives $\frac{\delta c_t}{n}$ and since $x \geq \frac{\delta c_t}{n}$ it follows that no such deviation is profitable.

CASE 2: Next consider some y where

$$\sum_{j=1}^{\frac{n+1}{2}} \max\{d_t, \frac{y^j}{\delta}\} \geq 1 - c_t \text{ and } y^{\frac{n+1}{2}} \geq \frac{(2-c_t)\,\delta}{n}. \tag{B16}$$

Since $\delta\frac{(2-c_t)}{n} > \delta\left(\frac{1}{n} + \frac{(1-c_t)}{n}\frac{(n-1)}{(n+1)}\right)$ neither condition (B13), (B14) or (B15) is satisfied. I assume that continuation play after a rejection of y is in accordance to any sequentially rational strategies where agent n gets an expected discounted payoff of $\frac{\delta c_t}{n}$ (for example, the strategies in Lemmas B2, B3 and B4 all work). It follows that the deviation must be accepted in order to be profitable, generating a payoff x^n to the proposer at time $t+1$. Since $c_t \leq 1 - \frac{(n-1)\delta}{2n}$ it follows that $2 - c_t \geq 1 + \frac{(n-1)\delta}{2n} > 1$, so $y^{\frac{n+1}{2}} \geq \delta\frac{(2-c_t)}{n} > \frac{\delta}{n} \geq d_t$. This together with the first inequality in (B16) implies that

$$\begin{aligned} 1 - c_t &\leq \sum_{j=1}^{\frac{n+1}{2}} \max\{d_t, \frac{y^j}{\delta}\} = \sum_{j=1}^{\frac{n-1}{2}} \max\{d_t, \frac{y^j}{\delta}\} + \frac{y^{\frac{n+1}{2}}}{\delta} \leq \frac{n-1}{2}d_t + \frac{1}{\delta}\sum_{j=1}^{\frac{n+1}{2}} y^j \\ &\Rightarrow \sum_{j=1}^{\frac{n+1}{2}} y^j \geq \delta\left(1 - \frac{n-1}{2}d_t - c_t\right). \end{aligned} \tag{B17}$$

Moreover, since $y^1 \leq y^2 \leq .... \leq y^{n-1}$, the second inequality in (B16) implies that $\sum_{j=\frac{n+3}{2}}^{n-1} y^j \geq \frac{(n-3)\delta}{2n}(2-c_t)$. Hence, a proposal that satisfies (B16) gives the proposer no more than

$$y^n \leq 1 - \sum_{j=1}^{\frac{n+1}{2}} y^j - \sum_{j=\frac{n+3}{2}}^{n-1} y^j \leq 1 - \delta\left(1 - \frac{n-1}{2}d_t - c_t\right) - \frac{(n-3)\,\delta}{2n}(2-c_t) = G(c_t, d_t) \leq x^n. \tag{B18}$$

Hence, there is no profitable deviation satisfying (B16).

CASE 3: The final possibility is that y satisfies

$$\sum_{j=1}^{\frac{n+1}{2}} \max\{d_t, \frac{y^j}{\delta}\} \geq 1 - c_t \text{ and } y^1 \geq \frac{\delta}{n} \text{ and } y^{\frac{n+1}{2}} \geq \delta\left(\frac{1}{n} + \frac{(1-c_t)}{n}\frac{(n-1)}{(n+1)}\right). \tag{B19}$$

Again I let continuation strategies be such that the proposer has no incentive to propose a proposal that is rejected, so that in order for y to be a profitable deviation, $y^n > x^n$. By assumption, $d_t \leq \frac{\delta}{n}$, so $\frac{\delta}{n} \leq y^1 \leq y^j$ for all $j \geq 1$. Thus, $\sum_{j=1}^{\frac{n+1}{2}} \max\{d_t, \frac{y^j}{\delta}\} = \frac{1}{\delta}\sum_{j=1}^{\frac{n+1}{2}} y^j$. Moreover, the third condition in (B19) implies that $\sum_{j=\frac{n+3}{2}}^{n-1} y^j \geq \frac{(n-3)\delta}{2}\left(\frac{1}{n} + \frac{(1-c_t)}{n}\frac{(n-1)}{(n+1)}\right)$, so a proposal satisfying (B19) gives the proposer a share no larger than

$$y^n \leq 1 \Leftrightarrow \sum_{j=1}^{\frac{n+1}{2}} y^j \Leftrightarrow \sum_{j=\frac{n+3}{2}}^{n-1} y^j \leq 1 \Leftrightarrow \delta\left(1 \Leftrightarrow c_t\right) \Leftrightarrow \frac{(n \Leftrightarrow 3)\,\delta}{2}\left(\frac{1}{n} + \frac{(1 \Leftrightarrow c_t)}{n}\frac{(n \Leftrightarrow 1)}{(n+1)}\right) = H\left(c_t\right) \leq x^n. \quad \text{(B20)}$$

A proposal that does not satisfy either (B16) or (B19) necessarily satisfies either (B13), (B14) or (B15), so this concludes the proof. ■

Lemma B6 *If $\delta \geq \frac{n+1}{2(n-1)}$, then $H(c) < c$ for all $c \in [0,1]$*

Proof. Since $1 \Leftrightarrow c \geq 0$ and $\frac{1}{n} + \frac{(n-1)}{(n+1)}\frac{(1-c)}{n} \geq 0$ it follows from direct inspection of the definition of $H(c)$ in (B11) that $H(c)$ is decreasing in δ for any c. For any $\delta \geq \frac{n+1}{2(n-1)}$ and $c \in [0,1]$ we therefore know that

$$\begin{aligned} H(c) &\leq 1 \Leftrightarrow \frac{n+1}{2(n \Leftrightarrow 1)}(1 \Leftrightarrow c) \Leftrightarrow \frac{(n \Leftrightarrow 3)(n+1)}{4(n \Leftrightarrow 1)}\left(\frac{1}{n} + \frac{n \Leftrightarrow 1}{n+1}\frac{(1 \Leftrightarrow c)}{n}\right) \qquad \text{(B21)} \\ &= c\left(\frac{2n(n+1) + (n \Leftrightarrow 1)(n \Leftrightarrow 3)}{4n(n \Leftrightarrow 1)}\right) < c\left(\frac{2(n+1) + (n \Leftrightarrow 3)}{4(n \Leftrightarrow 1)}\right) = c\frac{(3n \Leftrightarrow 1)}{(3n \Leftrightarrow 1) + (n \Leftrightarrow 3)} < c \end{aligned}$$

since $n \geq 5$. ■

Lemma B7 *Suppose that $\sqrt{\frac{2n}{(n-1)(n-2)}} \leq \delta \leq 1$, $c_t \leq 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$ and $d_t \leq \frac{\delta}{n}\left(1 \Leftrightarrow \frac{(n-1)\delta}{2n}\right)$. Then, $G(c_t, d_t) < 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$*

Proof. G is increasing in both arguments, so

$$\begin{aligned} G(c_t, d_t) &\leq G\left(1 \Leftrightarrow \frac{(n \Leftrightarrow 1)\,\delta}{2n}, \frac{\delta}{n}\left(1 \Leftrightarrow \frac{(n \Leftrightarrow 1)\,\delta}{2n}\right)\right) \qquad \text{(B22)} \\ &= 1 \Leftrightarrow \delta\left(\frac{(n \Leftrightarrow 1)\,\delta}{2n} \Leftrightarrow \frac{(n \Leftrightarrow 1)}{2}\frac{\delta}{n}\left(1 \Leftrightarrow \frac{(n \Leftrightarrow 1)\,\delta}{2n}\right)\right) \Leftrightarrow \frac{(n \Leftrightarrow 3)\,\delta}{2n}\left(1 + \frac{(n \Leftrightarrow 1)\,\delta}{2n}\right) \\ &= 1 \Leftrightarrow \delta\left(\left(\frac{(n \Leftrightarrow 1)\,\delta}{2n}\right)^2\right) \Leftrightarrow \frac{(n \Leftrightarrow 3)\,\delta}{2n} \Leftrightarrow \frac{\delta^2 (n \Leftrightarrow 3)(n \Leftrightarrow 1)}{4n^2} \end{aligned}$$

For $\delta = 1$, this gives $G(c_t, d_t) \leq 1 \Leftrightarrow \frac{(n-1)}{2n} + \frac{1}{n}\left(\frac{5n-n^2-2}{4n}\right) < 1 \Leftrightarrow \frac{(n-1)}{2n}$, where the second inequality follows since $5n \Leftrightarrow n^2 \Leftrightarrow 2 < 0$ for $n \geq 5$. Hence, the claim is true for $\delta = 1$. For the case with $\sqrt{\frac{2n}{(n-1)(n-2)}} \leq \delta < 1$, we observe that $\delta \geq \sqrt{\frac{2n}{(n-1)(n-2)}}$ implies that

$$\begin{aligned} \left(\frac{(n \Leftrightarrow 1)\,\delta}{2n}\right)^2 &\geq \frac{2n}{(n \Leftrightarrow 1)(n \Leftrightarrow 2)}\frac{(n \Leftrightarrow 1)^2}{4n^2} = \frac{(n \Leftrightarrow 1)}{2n(n \Leftrightarrow 2)} \text{ and} \qquad \text{(B23)} \\ \frac{\delta^2 (n \Leftrightarrow 1)(n \Leftrightarrow 3)}{4n^2} &\geq \frac{2n(n \Leftrightarrow 1)(n \Leftrightarrow 3)}{4n^2 (n \Leftrightarrow 1)(n \Leftrightarrow 2)} = \frac{(n \Leftrightarrow 3)}{2n(n \Leftrightarrow 2)}, \end{aligned}$$

Combining (B22) and (B23) we find that

$$G(c_t, d_t) \leq 1 \Leftrightarrow \delta \left(\frac{(n \Leftrightarrow 1)}{2n(n \Leftrightarrow 2)} \right) \Leftrightarrow \frac{(n \Leftrightarrow 3)\,\delta}{2n} \Leftrightarrow \frac{(n \Leftrightarrow 3)}{2n(n \Leftrightarrow 2)} = \tag{B24}$$

$$= 1 \Leftrightarrow \frac{(n \Leftrightarrow 1)\,\delta}{2n} + \frac{(\delta \Leftrightarrow 1)}{n} \frac{(n \Leftrightarrow 3)}{2(n \Leftrightarrow 2)} < 1 \Leftrightarrow \frac{(n \Leftrightarrow 1)\,\delta}{2n},$$

since $\delta < 1$. ■

Lemma B8 *For any feasible x such that $x^i \geq 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$ for agent i and $x^j \geq \frac{\delta}{n}\left(1 \Leftrightarrow \frac{(n-1)\delta}{2n}\right)$ for $\frac{n-1}{2}$ agents $j \neq i$ there is an equilibrium in the game with 3 periods in which i proposes x if recognized in the first period and x is accepted by a majority.*

Proof. A standard backwards induction argument (see Proposition 1 in Baron and Ferejohn [2]) establishes that in any equilibrium each agent i selects a set $J \subset \{1, ..., n\} \setminus \{i\}$ with $|J| = \frac{n-1}{2}$ of agents to "bribe" and proposes

$$p_2^{kj}(h_2) = \begin{cases} 1 \Leftrightarrow \frac{(n-1)\delta}{2n} & \text{for } j = i \\ \frac{\delta}{n} & \text{for all } j \in J \\ 0 & \text{for all } j \notin J \cup \{i\} \end{cases}, \tag{B25}$$

if i is called to make a proposal at $t = 2$. This proposal is accepted in any equilibrium. Since the proposer is indifferent between any J, $J' \subset \{1, ..., n\} \setminus \{i\}$, the assumptions of Lemma B1 are satisfied at $t = 2$ for $c_2 = 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$ and $d_2 = \frac{\delta}{n}$. Consider an arbitrary x satisfying the inequalities in the statement of the lemma. Assume that continuation play is such that x is accepted if proposed, which, by Lemma B1, is consistent with sequential rationality in all subgames following the proposal of x. After any other proposal $y \neq x$ assume that each agent randomizes with equal probabilities over all other agents to select $\frac{n-1}{2n}$ agents to receive $\frac{\delta}{n}$ and keeps $1 \Leftrightarrow \frac{(n-1)\delta}{2n}$. Clearly, this is consistent with (B25) and the continuation payoff in the beginning of period 2 is then $\frac{1}{n}$ for all agents after a history when $y \neq x$ was proposed in period 3. Hence, if some $y \neq x$ is proposed at time 3 the voting rule "accept $y \neq x$ if and only if $y^j \geq \delta/n$" is sequentially rational for all agents $j \in I$, implying that the proposer must give at least $\frac{(n-1)\delta}{2n}$ to the other players to get a proposal $y \neq x$ accepted. Thus $y^i \leq 1 \Leftrightarrow \frac{(n-1)\delta}{2n} \leq x^i$ for any proposal that would be accepted and the discounted expected payoff from a proposal that is rejected in equilibrium is $\frac{\delta}{n} < 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$. Hence, this constitutes an equilibrium where x is proposed and accepted at time t. Since x is an arbitrary allocation satisfying the inequalities in the claim this ends the proof. ■

Lemma B9 *Suppose that* $\delta \geq \max\left\{\frac{n+1}{2(n-1)}, \sqrt{\frac{2n}{(n-1)(n-2)}}\right\}$ *and let the sequences* $\{c_t\}_{t=3}^{\infty}$ *and* $\{d_t\}_{t=3}^{\infty}$ *be given by* $c_3 = 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$, $d_3 = \frac{\delta}{n}\left(1 \Leftrightarrow \frac{(n-1)\delta}{2n}\right)$ *and* $c_t = \max\left\{\frac{\delta c_{t-1}}{n}, G\left(c_{t-1}, d_{t-1}\right), H\left(c_{t-1}\right)\right\}$ *and* $d_t = \frac{\delta}{n}c_{t-1}$ *for each* $t \geq 4$. . *Then,* $0 < c_t < c_{t-1}$ *and* $0 < d_t \leq d_{t-1}$ *for each* $t > 4$.

Proof. $\delta \geq \sqrt{\frac{2n}{(n-1)(n-2)}}$, so, by Lemma B7, $G(c_3, d_3) < 1 \Leftrightarrow \frac{(n-1)\delta}{2n} = c_3$. Lemma B6 guarantees that $H(c_3) < c_3$since $\delta \geq \frac{n+1}{2(n-1)}$ and since $0 < \frac{\delta c_3}{n} < c_3$ it follows that $0 < c_4 = \max\left\{\frac{\delta c_3}{n}, G\left(c_3, d_3\right), H\left(c_3\right)\right\} < c_3$ and $0 < d_4 = \frac{\delta}{n}c_3 = \frac{\delta}{n}\left(1 \Leftrightarrow \frac{(n-1)\delta}{2n}\right) = d_3$,so the claim is true for $t = 4$. Next, suppose that $0 < c_t < c_{t-1}$ and $0 < d_t \leq d_{t-1}$ for some $t > 4$. The function G is strictly increasing in both arguments, so $G\left(c_t, d_t\right) < G\left(c_{t-1}, d_{t-1}\right)$ and since $G\left(c_{t-1}, d_{t-1}\right) \leq \max\left\{\frac{\delta c_{t-1}}{n}, G\left(c_{t-1}, d_{t-1}\right), H\left(c_{t-1}\right)\right\} = c_t$ it also follows that $G\left(c_t, d_t\right) < c_t$. Lemma B6 guarantees that $H\left(c_t\right) < c_t$ and $0 < \frac{\delta c_t}{n} < c_t$ since $c_t > 0$. Taken together this means that $0 < c_{t+1} = \max\left\{\frac{\delta c_t}{n}, G\left(c_t, d_t\right), H\left(c_t\right)\right\} < c_t$ and since $0 < c_t < c_{t-1}$ it also follows that $d_{t+1} = \frac{\delta}{n}c_t < \frac{\delta}{n}c_{t-1} = d_{t-1}$. The result follows by induction. ■

Lemma B10 *Suppose that* $\delta \geq \max\left\{\frac{n+1}{2(n-1)}, \sqrt{\frac{2n}{(n-1)(n-2)}}\right\}$ *and let the sequences* $\{c_t\}_{t=3}^{\infty}$ *and* $\{d_t\}_{t=3}^{\infty}$ *be defined as in Lemma B9. Then, for each* $t \geq 3$*, each* $i \in I$ *and any feasible proposal* x_t *such that; 1)* $x_t^i \geq c_t$ *for agent* i*, 2)* $x_t^j \geq \frac{\delta}{n}$ *for one agent* j *(possibly* $j = i$*) 3)* $x_t^k \geq d_t$ *for at least* $\frac{n-1}{2}$ *agents* $k \neq i$*, there exists an equilibrium in the game with* t *periods in which* x_t *is proposed and accepted in the first period if agent* i *is recognized to make a proposal.*

Proof. Lemma B8 establishes the claim for $t = 3$. Suppose the claim is true for some $t \geq 4$. By application of Lemma B9, $c_t < 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$ and $d_t < \frac{\delta}{n}$. Hence Lemma B5 applies, so the claim holds at time $t + 1$. The result follows by induction. ■

Lemma B11 *Suppose that* $\max\left\{\frac{n+1}{2(n-1)}, \sqrt{\frac{2n}{(n-1)(n-2)}}\right\} \leq \delta \leq 1$. *Then,* $c_t \to 0$ *and* $d_t \to 0$ *as* $t \to \infty$ *for the sequences* $\{c_t\}_{t=3}^{\infty}$ *and* $\{d_t\}_{t=3}^{\infty}$ *constructed in Lemma B9.*

Proof. Lemma B9 implies that $\{c_t\}_{t=3}^{\infty}$ is monotonically decreasing on $[0, c_3]$, where $c_3 = 1 \Leftrightarrow \frac{(n-1)\delta}{2n}$. Hence there must be a limit c^* satisfying $c^* = \max\left\{\frac{\delta c^*}{n}, G\left(c^*, \frac{\delta c^*}{n}\right), H\left(c^*\right)\right\}$. Suppose for contradiction that $c^* > 0$. Since $H\left(c\right) < c$ and $\frac{\delta c}{n} < c$ for all $c \in [0, 1]$ the only possibility for $c^* > 0$ is if $0 < c^* = G\left(c^*, \frac{\delta c^*}{n}\right) \leq c_3$. After some algebra, the equation $c = G\left(c, \frac{\delta c}{n}\right)$ can be rewritten as

$$c\left(\frac{(n \Leftrightarrow 1)\,\delta}{2n}(\delta+3) \Leftrightarrow 1\right) = \frac{(2n \Leftrightarrow 3)\,\delta}{n} \Leftrightarrow 1, \tag{B26}$$

where $\delta \geq \frac{n+1}{2(n-1)}$ and $n \geq 5 \Rightarrow \frac{(2n-3)\delta}{n} \Leftrightarrow 1 > 0$. It follows from (B26) that $\frac{(n-1)\delta}{2n}(\delta+3) \Leftrightarrow 1 > 0$ if $c^* > 0$ and the limit must be the (unique) fixed point to (B26), i.e.,

$$c^* = \frac{\frac{(2n-3)\delta}{n} \Leftrightarrow 1}{\frac{(n-1)\delta}{2n}(\delta+3) \Leftrightarrow 1} = \frac{4\frac{\left(n-\frac{3}{2}\right)}{n-1}\frac{(n-1)\delta}{2n} \Leftrightarrow 1}{\frac{(n-1)\delta}{2n}(\delta+3) \Leftrightarrow 1} = \frac{4\frac{\left(n-\frac{3}{2}\right)}{n-1}K \Leftrightarrow 1}{K(\delta+3) \Leftrightarrow 1}, \tag{B27}$$

where $K = \frac{(n-1)\delta}{2n}$. In order c^* to be a possible limit to the sequence $\{c_t\}_{t=3}^{\infty}$ it must be that $c^* \leq c_3$, or

$$\begin{aligned}\frac{\frac{4\left(n-\frac{3}{2}\right)}{n-1}K \Leftrightarrow 1}{K(\delta+3) \Leftrightarrow 1} \leq 1 \Leftrightarrow K \quad &\Leftrightarrow \quad \frac{4\left(n \Leftrightarrow \frac{3}{2}\right)}{n \Leftrightarrow 1} \leq (\delta+3) \Leftrightarrow (K(\delta+3) \Leftrightarrow 1) \\ &\Leftrightarrow \quad K(\delta+3) \Leftrightarrow \delta \leq \frac{2}{n \Leftrightarrow 1}\end{aligned} \tag{B28}$$

Substituting back $K = \frac{(n-1)\delta}{2n}$ and using the condition that $\delta \geq \sqrt{\frac{2n}{(n-1)(n-2)}}$ we find that

$$\begin{aligned}\frac{(n \Leftrightarrow 3)\,\delta}{2n} + \frac{1}{n \Leftrightarrow 2} &= \frac{(n \Leftrightarrow 3)\,\delta}{2n} + \frac{(n \Leftrightarrow 1)\,2n}{2n\,(n \Leftrightarrow 1)\,(n \Leftrightarrow 2)} \leq \frac{(n \Leftrightarrow 3)\,\delta}{2n} + \frac{(n \Leftrightarrow 1)\,\delta^2}{2n} \\ &= \frac{(n \Leftrightarrow 1)\,\delta}{2n}(\delta+3) \Leftrightarrow \delta = K(\delta+3) \Leftrightarrow \delta \leq \frac{2}{n \Leftrightarrow 1} \\ &\Rightarrow \frac{(n \Leftrightarrow 3)\,\delta}{2n} \leq \frac{2}{n \Leftrightarrow 1} \Leftrightarrow \frac{1}{n \Leftrightarrow 2} = \frac{2\,(n \Leftrightarrow 2) \Leftrightarrow (n \Leftrightarrow 1)}{(n \Leftrightarrow 1)\,(n \Leftrightarrow 2)} = \frac{(n \Leftrightarrow 3)}{(n \Leftrightarrow 1)\,(n \Leftrightarrow 2)} \\ &\Rightarrow \delta \leq \frac{2n}{(n \Leftrightarrow 1)\,(n \Leftrightarrow 2)} < \sqrt{\frac{2n}{(n \Leftrightarrow 1)\,(n \Leftrightarrow 2)}},\end{aligned}$$

which establishes that any $c^* > 0$ solving (B27) must be such that $c^* > c_3$. Hence $c^* = 0$ is the only possible limit for $\{c_t\}_3^{\infty}$. It follows immediately that $\lim_{t\to\infty} d_t = \frac{\delta}{n}\lim_{t\to\infty} c_t = 0$. ■

Proof of Proposition: Let x be such that $x^j > 0$ for all j. First consider a history h_{t-1} where x was proposed and rejected at time t. Then suppose that every $i \in I$ follows the proposal rule

$$p_{t-1}^{ij}(h_{t-1}) = \begin{cases} 1 \Leftrightarrow (n \Leftrightarrow 1)\,c_{t-1} & j = 1 \\ c_{t-1} & \text{to all other agents} \end{cases} \tag{B29}$$

Since $d_t < c_t$ for each $t \geq 3$ and since $c_t \to 0$ as $t \to \infty$ Lemma B10 implies that (B29) is supportable as an equilibrium proposal independently of the identity of the proposer for t sufficiently large. If x is rejected, the discounted expected payoff for agent $j = 2, ..., n$ is $\delta c_{t-1} \to 0$ as $t \to \infty$. Hence there is some t large enough so that the voting rule "vote to accept x independently of the voting history" is sequentially rational. Hence, there are sequentially rational continuation strategies such that x is accepted if proposed in the first period. For histories where the proposer proposed $y \neq x$, relabel the agents such that n is the deviant proposer at time t and $y^1 \leq y^2 \leq ... \leq y^{n-1}$. If $y^n = 0$, suppose that the next period proposal is in accordance with (B29). The proposer then either get a payoff of zero (if the deviant proposal is accepted) or δc_{t-1} (if the deviant proposal is rejected), and for t large enough $x^n > \delta c_{t-1}$ implying that no such deviation is profitable. If $y^n > 0$ on the other hand, assume that every $i \in I$ follows the proposal rule

$$p_{t-1}^{ij}(h_{t-1}) = \begin{cases} \frac{y_i}{\delta} + c_{t-1} & \text{to } j = 1, ..., \frac{n-1}{2} \\ \max\left\{\frac{y_i}{\delta} + c_{t-1}, \frac{\delta}{n}\right\} & \text{to } j = \frac{n+1}{2} \\ c_{t-1} & \text{to all other agents} \end{cases}. \tag{B30}$$

at time $t \Leftrightarrow 1$. Suppose first that $\frac{y_j}{\delta} < \frac{\delta}{n}$. In this case $\sum_{j=1}^{\frac{n+1}{2}} \frac{y^j}{\delta} < \frac{(n-1)\delta}{2n}$ and since $c_{t-1} \to 0$ as $t \to \infty$ the proposal rule (B30) is feasible for large enough t. If $\frac{y_j}{\delta} \geq \frac{\delta}{n}$, then we note that $y^n > 0$ implies that $\sum_{j=1}^{\frac{n+1}{2}} y^j < \frac{n+1}{2(n-1)}$ since $\frac{2}{n+1} \sum_{j=1}^{\frac{n+1}{2}} y^j \leq \frac{1}{n-1} \sum_j y^j \leq \frac{1-y^n}{n-1}$. Again the proposal is feasible for t large enough and in both cases Lemma B10 implies that (B30) is supportable as an equilibrium proposal independently of the identity of the proposer for t sufficiently large. Moreover, each agent $j \leq \frac{n+1}{2}$ receives a share $p^{ij}(h_t) > y^j/\delta$ *independently of who is the proposer at time* t. Hence, each $j \leq \frac{n+1}{2}$ has a strict incentive to reject y, so the voting strategy "always vote to reject y" is sequentially rational for $j = 1, ..., \frac{n+1}{2}$, implying that for t large enough any $y \neq x$ is rejected in equilibrium. The expected discounted payoff for the proposer is $\frac{\delta}{n} c_{t-1}$ if y is rejected and since $\frac{\delta}{n} c_{t-1} \to 0$ as $t \to \infty$ there is some $T < \infty$ such that there exists an equilibrium where each i is willing to propose x if recognized in the first period of the game with $t \geq T$ periods and where x is accepted by a majority. ■

C Appendix: Proof of Proposition 3

Proof of Proposition 3: In the last stage the unique equilibrium proposal is that the proposer takes everything for herself, so uniqueness holds trivially in the one stage (ultimatum) game for all δ. Now suppose that there is a unique equilibrium in the game with $T \Leftrightarrow 1$ periods, but that uniqueness fails when there are T periods. Uniqueness of equilibria in the game with $T \Leftrightarrow 1$ periods implies that there is a vector $(V^1_{T-1}, ..., , V^n_{T-1})$ of continuation values such that $V^i_{T-1} = V^i(h_{T-1} \mid \sigma_{|T-1})$ for every agent i and any subgame perfect equilibrium σ (in the game with $T \Leftrightarrow 1$ or more rounds). But then (recall Figure 2 on page 5.1) a proposal where a majority gets strictly more than their respective discounted continuation values is accepted and any proposal where a majority gets strictly less is rejected if proposed at time T. For the usual reasons, acceptances are resolved in favor of acceptance in equilibrium, so if for each i the set $I \backslash \{i\}$ can be partitioned into two sets with $\frac{n-1}{2}$ agents each where any agent in one of the sets has a strictly lower discounted continuation value than all agents in the other set, then the equilibrium is unique also in the game with T periods. Hence, a necessary condition for *failure* of uniqueness is that $\delta^i V^i_{T-1} = \delta V^j_{T-1}$ for some i, j. If there is a unique equilibrium continuation V^i_{T-1}at time $T \Leftrightarrow 1$ for all $i \in I$ there is a unique equilibrium continuation payoff V^i_t for $i \in I$ and each $t < T \Leftrightarrow 1$. If agents are labeled so that $\delta V^1_t < \delta V^2_t < ... < \delta V^n_t$ then $(V^1_{t+1}, ..., , V^n_{t+1})$ can be expressed in terms $(V^1_t, ..., , V^n_t)$ according to

$$V^i_{t+1} = \begin{cases} \frac{1}{n}\left(1 + \delta^i V^i_t \Leftrightarrow \sum_{j=1}^{\frac{n+1}{2}} \delta^j V^j_t\right) + \frac{n-1}{n}\delta^i V^i_t & \text{for } i < \frac{n+1}{2} \\ \frac{1}{n}\left(1 \Leftrightarrow \sum_{j=1}^{\frac{n-1}{2}} \delta^j V^j_t\right) + \frac{(n-1)}{2n}\delta^i V^i_t & \text{for } i = \frac{n+1}{2} \\ \frac{1}{n}\left(1 \Leftrightarrow \sum_{j=1}^{\frac{n-1}{2}} \delta^j V^j_t\right) & \text{for } i > \frac{n+1}{2} \end{cases} . \tag{C1}$$

Now, pick any $i \in I$ and treat $\delta^{-i} = (\delta^1, .., \delta^{i-1}, \delta^{i+1}, .., \delta^n)$ as a vector of constants. We can then express the continuation payoff at time $t + 1$ in the form

$$V^j_{t+1} = a^j_t + b^{ji}_t \delta^i V^i_t + \sum_{k \neq i} b^{jk}_t V^k_t \tag{C2}$$

Now, repeated substitution from (C2) gives

$$\begin{aligned} V^j_T &= a^j_{T-1} + b^{ji}_{T-1}\delta^i V^i_{T-1} + \sum_{k\neq i} b^{jk}_{T-1} V^k_{T-1} \\ &= a^j_{T-1} + b^{ji}_{T-1}\delta^i \left(a^i_{T-2} + b^{ii}_{T-2}\delta^i V^i_{T-2} + \sum_{k\neq i} b^{ik}_{T-2} V^k_{T-2}\right) \\ &\quad + \sum_{k\neq i} b^{jk}_{T-2}\left(a^k_{T-2} + b^{ki}_{T-2}\delta^i V^i_{T-2} + \sum_{j\neq i} b^{kj}_{T-2} V^j_{T-2}\right) \end{aligned} \tag{C3}$$

Collecting terms in (C3) we see that the value at time T can be written in the form $V^j_T = \widetilde{a}^j + \widetilde{b}^{j1}\delta^i + \widetilde{b}^{j2}\left(\delta^i\right)^2 V^i_{T-2} + \sum_{k\neq i} \widetilde{b}^{jk}_{T-2} V^k_{T-2}$. Continuing the process recursively all the way back to $t = 1$ we can eventually express the continuation value in the T period game for any agent $j \in I$ as a polynomial of degree $T \Leftrightarrow 1$ in δ^i

$$V_T^j = \alpha^j + \beta^{j1}\delta^i + \beta^{j2}\left(\delta^i\right)^2 + ... + \beta^{jT-1}\left(\delta^i\right)^{T-1}. \tag{C4}$$

I now claim that $\alpha^j > 0$ for all j. The reason for this is most easily understood from the procedure used to generate the polynomial. The coefficients in (C2) corresponds with some ranking of the agents, say P_t, at time t. Hence (C4) can be generated by setting $V_1^j = \frac{1}{n}$ for all j and picking a sequence of rankings $P_{T-1}, ..., P_1$ and assume that in each period $t \leq T \Leftrightarrow 1$ each agents bribe the $\frac{n-1}{2}$ lowest ranked agents according to the ranking P_t with their "continuation value" and keeps the rest. While this process is consistent with equilibrium only for particular rankings it generates a well-defined sequence of "values" that sum to unity in every period. But values summing to unity means that $V_T^j > 0$ for all j (by use of (C1)). In particular this means that the formula (C4) evaluated at $\delta^i = 0$ must be non-zero, which establishes that $\alpha^j > 0$. Next note that since the coefficients in (C2) depend on the rankings, they do depend on δ^i, *but only to the extent that the ranking is changed.* There are $n!$ ways to order n agents and there are $T \Leftrightarrow 1$ recursions on (C2), implying that there is at most $(n!)^{T-1}$ sets of n polynomials to consider[8]. Now, if $\delta^i V_T^i = \delta^j V_T^j$, then we can use (C4) to write this as

$$\begin{aligned} 0 = \delta^j V_T^j \Leftrightarrow \delta^i V_T^i &= \delta^j \alpha^j + \left(\delta^j \beta^{j1} \Leftrightarrow \delta^i \alpha^i\right)\delta^i + \left(\delta^j \beta^{j2} \Leftrightarrow \beta^{i1}\right)\left(\delta^i\right)^2 + ... \\ &+ \left(\delta^j \beta^{jT-1} \Leftrightarrow \beta^{iT-2}\right)\left(\delta^i\right)^{T-1} \Leftrightarrow \beta^{jT-1}\left(\delta^i\right)^T. \end{aligned} \tag{C5}$$

Since (C5) has at least one coefficient different from zero there can be at most T roots to the polynomial. The set of polynomials to consider is bounded by $(n!)^{T-1}$ and there are $n \Leftrightarrow 1$ agents different from i, so given any δ^{-i} the number of values for δ^i that generates $\delta^i V_T^i = \delta^j V_T^j$ for some $j \neq i$ is bounded by $T(n \Leftrightarrow 1)(n!)^{T-1}$, a finite number. Moreover, each coefficient of each polynomial is a sum of products of terms from the set $\left\{\delta^1, ..., \delta^{i-1}, \delta^{i+1}, ..., \delta^n, 1, \frac{n-1}{2n}, \frac{1}{n}\right\}$, so the coefficients are continuously differentiable in δ^{-i}. Hence, $X_T^i = \left\{\delta \in \Delta | \delta^i V_T^i = \delta^j V_T^j \text{ for some } j \in I \setminus \{i\}\right\}$ is a $n \Leftrightarrow 1$ dimensional smooth manifold, a set with Lebesgue measure zero. Agent i was arbitrarily chosen, so we conclude that the set of discount factors such that *some* pair of discounted continuation values coincide, $X_T = \cup_{i \in I} X_T^i$ is a also negligible set. $\delta \in X$ is a necessary condition for uniqueness to

[8] While it doesn't matter for the argument, observe that this is a gross overstament of the number of polynomials to consider. In particular, it only matters whether an agent is the median, cheaper than the median or more expensive than the median, so the number of relevant possibilities in each stage is no more than $n\frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!}$.

fail at time T so the critical values for which the $T\Leftrightarrow$period game has a non-unique equilibrium is a subset of $\cup_{t=1}^{T} X_t$, again a negligible set. We conclude that the set of δ for which there is not a unique equilibrium has measure zero. ■

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