10 Durable Goods Monopoly

Many goods are durable: cars, refrigerators, light bulbs, art, computers etc. all last for some time. This “durability” introduces some quite interesting issues concerning the market power (or lack thereof) for a monopolist. The novel aspect is that the customers that are available at any given time, are those customers who choose not to buy in the previous periods (ignoring the fact that some durable goods have a tendency to break). Presumably, those that remains have a relatively low willingness to pay, so this suggests that the price should decline over time. However, with a declining price, potential buyers have an incentive to wait for lower prices in the future, so the monopolist “competes with itself” in the sense that the temptation to lower the price in the future erodes the market power today.

This is referred to as a “time inconsistency problem”, which is something that often arises in strategic situations where there is a time dimension. We will talk more formally about “time inconsistency” when we do game theory. For now, I will appeal to intuitive reasoning for how to write down the “right problem” for the monopolist.

One can show that if the monopolist can revise its price “often enough”, then a durable good monopolist will sell at a price close to the marginal cost: that is, a monopolist generates an approximately competitive outcome. To show this would be beyond the scope of intermediate micro material. Instead, I will simply show that, in a model with two periods, the monopolist does worse if it cannot shy away from the temptation to set a new price in the second period.

10.1 Model

1. There are two periods, indexed by $t = 1, 2$

2. Both consumers and the monopolist discounts future utility and profits respectively at a common discount factor $\delta < 1$

3. Monopolist sells an indivisible good
4. The (direct) demand is given by \( D(p) = 1 - p \). For interpretation, it is a good idea to think of \( D(p) \) as the probability that the valuation for the good is less than \( p \).

5. For simplicity, we let the constant marginal cost be \( c = 0 \).

### 10.2 Static Benchmark (Commitment)

First consider the case where the monopolist commits to never changing the price. Since consumers discount their utilities, nobody will then buy anything at time \( t = 2 \). The best (common) price the monopolist can charge is then the solution to

\[
\max_p p(1 - p)
\]

This problem has solution \( p^* = \frac{1}{2} \), and the associated quantity sold is \( q^* = D(p^*) = 1 - p^* = \frac{1}{2} \), so the monopolist makes a profit \( \pi^* = p^*q^* = \frac{1}{4} \) if it somehow can promise never to change the price in the future.

### 10.3 Selling in Both Periods (Non-Commitment)

We will discuss the general principles in a week or two: for now, just note that once the first period is over, then (if the monopolist has not entered a binding agreement) the monopolist will do whatever is optimal in the second period. For this reason, we will begin the analysis with looking at the second period.

Consider the second period, and assume that in the first period the monopolist sold \( q_1 \) units. In second period, the demand facing the monopolist is

\[
D_2(p, q_1) = 1 - q_1 - q_2
\]

Second period problem is thus

\[
\max_{q_2} q_2 (1 - q_1 - q_2),
\]
which has a solution (which depends on $q_1$) given by

\[
q_2 (q_1) = \frac{1 - q_1}{2},
\]
\[
p_2 (q_1) = \frac{1 - q_1}{2}.
\]

The second period profit is thus

\[
\pi_2 (q_1) = \frac{(1 - q_1)^2}{4}
\]

Now, in the first period a consumer with willingness to pay $v$ will buy the product if

\[
v - p_1 \geq 0 \text{ and } \quad v - p_1 \geq \delta (v - p_2)
\]

Clearly, the price in the second period will be below the price in the first period, so we can ignore the inequality. Now, if $q_1$ is the quantity sold, it will be bought by the $q_1$ agents with the highest valuation, so the agents with valuation

\[
1 - q_1 \leq v \leq 1
\]

are the ones that buy in the first period, and

\[
(1 - q_1) - p_1 = \delta ((1 - q_1) - p_2),
\]

since the agent with valuation $v = 1 - q_1$ must be indifferent between buying in period 1 and buying in period 2. But we know that when the monopolist re-optimizes in the second period, then

\[
p_2 (q_1) = \frac{1 - q_1}{2},
\]

so, the indifference condition becomes

\[
(1 - q_1) - p_1 = \delta ((1 - q_1) - p_2 (q_1)) = \delta \left( (1 - q_1) - \frac{1 - q_1}{2} \right)
\]

\[
= \frac{\delta}{2} (1 - q_1) \Rightarrow
\]

\[
p_1 = \left( 1 - \frac{\delta}{2} \right) (1 - q_1).
\]
The first period profit maximization problem for the monopolist is thus
\[
\max_{q_1} q_1 (1 - q_1) \left(1 - \frac{\delta}{2}\right) + \delta \frac{(1 - q_1)^2}{4},
\]
which gives first order condition
\[
(1 - 2q_1) \left(1 - \frac{\delta}{2}\right) - \frac{\delta}{2} (1 - q_1) = 0
\]
\[
\Leftrightarrow \quad q_1 \left(2 - \delta - \frac{\delta}{2}\right) = 1 - \frac{\delta}{2} - \frac{\delta}{2} = 1 - \delta
\]
\[
q_1^* = \frac{2(1 - \delta)}{4 - 3\delta}
\]
Now, when we have the solution in terms of the first period quantity we notice:

1. Previously, we expressed the second period price, quantity sold and profit as
\[
q_2 (q_1) = \frac{1 - q_1}{2}
\]
\[
p_2 (q_1) = \frac{1 - q_1}{2}
\]
\[
\pi_2 (q_1) = \left(\frac{1 - q_1}{2}\right)^2.
\]
By evaluating these expressions at \(q_1^*\) we get the values that will be chosen after the best first period choice as
\[
q_2^* = q_2 (q_1^*) = \frac{1 - q_1^*}{2} = \frac{1 - \frac{2(1 - \delta)}{4 - 3\delta}}{2} = \frac{4 - 3\delta - 2(1 - \delta)}{4 - 3\delta}
\]
\[
p_2^* = p_2 (q_1^*) = \frac{1 - q_1^*}{2} = \frac{2 - \delta}{4 - 3\delta}
\]
\[
\pi_2^* = \pi_2 (q_1^*) = \left(\frac{1 - q_1^*}{2}\right)^2 = \frac{1}{4} \left(\frac{2 - \delta}{4 - 3\delta}\right)^2
\]

2. Next, we observe that the indifference condition for buying in first versus second period was
\[
p_1 = \left(1 - \frac{\delta}{2}\right) (1 - q_1)
\]
so, the price charged at period 1 is
\[
p_1^* = \left(1 - \frac{\delta}{2}\right) \left(1 - \frac{2(1 - \delta)}{4 - 3\delta}\right)
\]
\[
= \left(\frac{2 - \delta}{2}\right) \left(\frac{4 - 3\delta - 2(1 - \delta)}{4 - 3\delta}\right) = \frac{(2 - \delta)^2}{2(4 - 3\delta)}.
\]
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3. Hence, the discounted value of the monopolists profit is (I write it as a function of \( \delta \) as that is a key parameter)

\[
\Pi^*(\delta) = \pi_1^* + \delta \pi_2^* = p_1^* q_1^* + \delta p_2^* q_2^*
\]

\[
= \frac{(2 - \delta)^2}{2(4 - 3\delta)} \frac{2(1 - \delta)}{4 - 3\delta} + \delta \frac{1}{4} \left( \frac{2 - \delta}{4 - 3\delta} \right)^2
\]

\[
= \frac{1}{4} \left( \frac{2 - \delta}{4 - 3\delta} \right)^2 [4(1 - \delta) + \delta]
\]

4. Observe that

\[
\Pi^*(0) = \frac{1}{4} \left( \frac{2}{4} \right)^2 = \frac{1}{4}
\]

\[
\Pi^*(1) = \frac{1}{4} \left( \frac{2 - 1}{4 - 3} \right)^2 [4(1 - 1) + 1]
\]

\[
= \frac{1}{4} \left( \frac{1}{1} \right)^2 [1] = \frac{1}{4},
\]

that is:

- when \( \delta = 0 \), nobody cares about the next period so the problem becomes like the static problem

- when \( \delta = 1 \), the monopolist charges \( \frac{(2-1)^2}{2(4-3)} = \frac{1}{2} \) (or higher) in the first period, so nobody actually buys anything in the first period. The second period is then just like the only period in the static model.

- For intermediate values of \( \delta \) we have that \( 0 < \delta^2 < \delta \), implying that

\[
\left( \frac{2 - \delta}{4 - 3\delta} \right)^2 [4(1 - \delta) + \delta] = \frac{(2 - \delta)^2}{4 - 3\delta} = \frac{4 - 4\delta + \delta^2}{4 - 3\delta} < \frac{4 - 4\delta + \delta}{4 - 3\delta} = 1
\]

Hence

\[
\Pi^*(\delta) = \frac{1}{4} \left( \frac{2 - \delta}{4 - 3\delta} \right)^2 [4(1 - \delta) + \delta] < \frac{1}{4}.
\]

That is, the option to sell to those customers who don’t buy at time 1 unambiguously hurts the monopolist.
10.4 Discussion

The conclusion that not selling in the second period is better for the monopolist is often considered counter intuitive: flexibility, which is often times considered valuable, is bad. The point is that consumers are not going to be fooled. If the monopolist will do what is optimal in the second period, potential buyers will understand this, so the monopolist is better off if it can somehow commit not to change the price in the second period even though the second period incentive obviously is to change the price to sell more units.