# 14.32 Recitation – Interpreting Multivariate Regression

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April 10, 2009

In lecture, we've been spending some time interpreting regression estimates. This is important and somewhat fun, so we'll do a bit more of it today.

## 1 Performance Pay and Wages

We're going to look at some results from Lemieux, MacLeod, and Parent (2009) "Performance Pay and Wage Inequality."

**Background** Over the past 30 years in the U.S. wage inequality has increased. Roughly speaking, during the 80s both low tail inequality (for example, the gap between the median wage and the 10%-tile wage) and high tail inequality (e.g. the gap between the median and the 90%-tile wage) increased. During the 90s and continuing to the present, low-tail inequality has been roughly constant, but high tail inequality has continued to increase. From the late 90s until today very high tail inequality (the gap between the 95%-tile and 80%-tile) has increased. Additionally, the gap between wages among various education levels has increased. However, even conditional on education, inequality has grown.

A number of explanations for the rise in inequality have been proposed and studied. These include the fall in the real minimum wage, declining unionization, immigration, and out-sourcing. However, the leading explanation is skill-biased technological change. The idea is that something about production has changed that has made the relationship between productivity and skills far steeper. In a competitive labor market, wages equal marginal productivity, so wages are now more tightly linked to skills and have become more unequal.

The real world is not quite so simple, and wages do not necessarily equal productivity. The problem is that productivity is generally unobserved. Firms can link wages to measures of output that are correlated with productivity. However, firms might not want to because monitoring output is costly, and if part of output is random and workers are risk averse, then the firm will have to pay the worker a higher expected wage to compensate for risk. Nonetheless, performance pay has become increasingly common. This might be because skill biased technological change has made it more important for firms to attract and reward skill workers, or it could be because technological improvements have made it easier to monitor output.

### 1.1 Theory

Motivated by the increase in performance pay, Lemieux, MacLeod, and Parent (2009) ask whether we should expect greater performance pay to lead to an increase in inequality. They begin by working out a simple model of performance contracts and worker sorting. To test the predictions of the model they are going to estimate wage regressions of the form:[1]

$$w_i^p = a^p + x_i b^p + z_j c_t^p + \epsilon_{ij}^p$$

[1] I've simplified somewhat leaving out things that we have not yet covered.

where $i$ indices workers, $j$ jobs, and $t$ time. The $p$ superscript is for performance pay jobs. $x_i$ are worker characteristics like education and experience, $z_j$ are job characteristics like industry, and $\epsilon_{ij}^p$ is an error term. Similarly, for non-performance pay jobs,

$$w_i^n = a^n + x_i b^n + z_j c_t^n + \epsilon_{ij}^n$$

The predictions of the model are:

1. Higher skilled workers should sort into performance pay jobs, and so the average wages should be higher in performance pay jobs.
2. The wage intercept is lower in performance-pay than in non-performance-pay jobs: $a^p < a^n$
3. The return to observable worker characteristics, $x_i$ ,is larger in performance-pay jobs than in non-performance- pay jobs: $b^p > b^n$.
4. The return to observable job characteristics, $z_j$ , is smaller in performance-pay jobs than in non-performance-pay jobs: $c^p < c^n$
5. The return to unobservable ability is larger in performance-pay jobs than in non-performance-pay jobs. (Fixed effects have a greater effect in performace-pay jobs).[2]

## 1.2 Results

Table I shows some summary statistics. Table II shows the increase in performance pay jobs. There are some difficulties in measuring performance pay. The many numbers in table II show that no matter how you define a job as having performance pay, they became more common.



TABLE I
SUMMARY STATISTICS: PANEL STUDY OF INCOME DYNAMICS 1976–1998

| | Non-performance-pay jobs (1) | Performance-pay jobs (2) |
|---|---|---|
| Average hourly earnings ($79) | 8.38 | 10.86 |
| Education | 12.52 | 13.39 |
| Potential experience | 19.74 | 19.61 |
| Employer tenure | 7.62 | 9.25 |
| Married | 0.72 | 0.77 |
| Unionized | 0.28 | 0.14 |
| Nonwhite | 0.13 | 0.09 |
| Paid by the hour | 0.66 | 0.31 |
| Paid a salary | 0.32 | 0.51 |
| Annual hours worked | 2,122 | 2,286 |
| Number of workers (total: 3,053) | 2,616 | 1,271 |
| Number of job matches (total: 7,442) | 5,657 | 1,785 |
| Number of observations (total: 26,146) | 16,466 | 9,680 |

*Notes*. The sample consists of male household heads aged 18–65 working in private sector wage and salary jobs. All figures in the table represent sample means. Education, potential experience, and employer tenure are measured in years. Potential experience is defined as age minus education minus 6. Performance-pay jobs are employment relationships in which part of the worker's total compensation includes a variable pay component (bonus, commission, piece rate). Any worker who reports overtime pay is considered to be in a non-performance-pay job. Workers are considered unionized if they are covered by a collective bargaining agreement.

TABLE II
CHANGES IN THE INCIDENCE OF PERFORMANCE PAY BETWEEN 1976–1979 AND 1990–1993: ROBUSTNESS TO DIFFERENT DEFINITIONS AND CONTRIBUTIONS OF VARIOUS FACTORS

| | Minimum frequency of actual payments of performance pay | | | Received PP this year (4) | Bonus only (5) |
|---|---|---|---|---|---|
| | Any (1) | 1/5 (2) | 1/2 (3) | | |
| Incidence in 1976–1979 | 37.56 | 20.79 | 9.41 | 11.95 | 30.24 |
| Change between 1976–1979 and 1990–1993 | | | | | |
| Unadjusted change | 12.92 | 6.78 | 1.51 | 4.45 | 12.36 |
| Adjusted for the number of times a job match is observed | 7.06 | 5.74 | 3.52 | 4.63 | 6.31 |
| Row (3) plus adjustments for characteristics in rows (5)–(9) | 4.57 | 3.93 | 3.00 | 3.70 | 4.79 |
| Contribution of changes in characteristics (other than the number of times the job match is observed) | | | | | |
| Total (row (3) minus row (4)) | 2.49 | 1.80 | 0.52 | 0.93 | 1.52 |
| Unions | 1.44 | 0.83 | 0.18 | 0.37 | 1.28 |
| Occupation | 0.70 | 0.64 | 0.34 | 0.37 | 0.60 |
| Industry | 0.53 | 0.72 | 0.42 | 0.44 | 0.10 |
| Other factors | −0.18 | −0.40 | −0.42 | −0.25 | −0.45 |

*Note*. All the adjustments and contributions of characteristics are computed by estimating linear probability models with a full set of dummies for periods (1976–1979, 1980–1984, 1985–1989, 1990–1993, and 1994–1998) and the number of times a job match is observed (1 to 22), as well as dummies for industry, occupation, marital status, race, union status, a cubic function in potential experience, and a quadratic function in job tenure. A total of 26,146 observations are used in all columns.

[2]This is something we haven't talked about.

TABLE III
REGRESSION ESTIMATES OF THE EFFECT OF PERFORMANCE PAY ON LOG AVERAGE HOURLY EARNINGS

| | Estimation method | | | | |
|---|---|---|---|---|---|
| | OLS | | Fixed effects | | |
| | (1) | (2) | (3) | (4) | (5) |
| Performance-pay job | 0.0873 | 0.0597 | 0.0400 | 0.0225 | — |
| | (0.0152) | (0.0166) | (0.0117) | (0.0120) | |
| Performance-pay received in current year | — | 0.0794 | — | 0.0380 | 0.0462 |
| | | (0.0167) | | (0.0084) | (0.0059) |
| Worker fixed effect | No | No | Yes | Yes | Yes |
| Job-match fixed effect | No | No | No | No | Yes |

*Notes.* A total of 26,146 observations were made. Standard errors (in parentheses) are adjusted for clustering at the job-match level. All specifications also include a full set of industry (10), occupation (8), and year (22) dummies, a cubic in potential experience, a quadratic in job tenure, years of completed schooling, calendar year average of the unemployment rate in the county of residence, and dummies for being married, for being nonwhite, and for union status. The "performance-pay job dummy" indicates if either a bonus or commission/piece rate earnings were received at any time during the employment relationship; the "performance pay received in current year" dummy indicates if a bonus or commission/piece rates earnings were received in the current year.

Table III shows the estimated coefficient on an indicator for performance pay in a regression of log wages on performance pay and the other covariates (whose coefficients are not shown).

- What is the percentage difference in wages between performance pay jobs and normal jobs?
- Does this result agree with or contradict any of the theoretical predictions above?

TABLE IV
SKILLS-RELATED WAGE DIFFERENTIALS AND PERFORMANCE-PAY (PP) JOBS

| | Sample | | | | | |
|---|---|---|---|---|---|---|
| | | | All jobs | | | |
| Estimation method | PP jobs OLS (1) | Non-PP jobs OLS (2) | OLS (3) | FE (4) | OLS (5) | FE (6) |
| Performance-pay job dummy | — | — | −0.4526 | −0.2061 | −0.2406 | 0.1414 |
| | | | (0.1019) | (0.0723) | (0.1251) | (0.0998) |
| Years of education | 0.0929 | 0.0665 | 0.0637 | 0.0167 | 0.0584 | 0.0040 |
| | (0.0071) | (0.0039) | (0.0040) | (0.0091) | (0.0047) | (0.0096) |
| Education × performance-pay job | — | — | 0.0365 | 0.0169 | 0.0217 | −0.0079 |
| | | | (0.0071) | (0.0048) | (0.0092) | (0.0071) |
| Education × 1990–1993 | — | — | — | — | 0.0161 | 0.0222 |
| | | | | | (0.0085) | (0.0056) |
| Education × performance-pay job × 1990–1993 | — | — | — | — | 0.0190 | 0.0280 |
| | | | | | (0.0137) | (0.0089) |
| Potential experience (effect at 20 years) | 0.4259 | 0.2882 | 0.3010 | 0.4545 | 0.3002 | 0.4231 |
| | (0.0535) | (0.0288) | (0.0294) | (0.1258) | (0.0294) | (0.1256) |
| Experience × performance-pay job | — | — | 0.1162 | 0.0149 | 0.1018 | −0.0278 |
| | | | (0.0584) | (0.0501) | (0.0581) | (0.0509) |
| Tenure (effect at ten years) | 0.1670 | 0.2197 | 0.2262 | 0.1158 | 0.2271 | 0.1191 |
| | (0.0268) | (0.0154) | (0.0154) | (0.0129) | (0.0154) | (0.0129) |
| Tenure × performance-pay job | — | — | −0.0666 | 0.0278 | −0.0677 | 0.0196 |
| | | | (0.0301) | (0.0237) | (0.0303) | (0.0239) |
| Number of observations | 9,680 | 16,466 | 26,146 | 26,146 | 26,146 | 26,146 |

*Notes.* Standard errors (in parentheses) are adjusted for clustering at the job-match level. All specifications also include a full set of industry (10), occupation (8), and year (22) dummies, a cubic in potential experience, a quadratic in job tenure, years of completed schooling, calendar year average of the unemployment rate in the county of residence, and dummies for being married, for race, and for union status. The reported effects of potential experience (at 20 years) and tenure (at 10 years) are the predicted levels computed using the estimated polynomial models. The models in columns (3)–(6) include interactions between the performance-pay dummy and education, a cubic in potential experience, and a quadratic in tenure. The models in columns (5) and (6) include a full set of interactions between period dummies for 1980–1984, 1985–1989, 1990–1993, and 1994–1998, education, and the performance-pay job dummy, but only the estimates for 1990–1993 are reported. The acronym FE refers to the fixed-effect method (worker fixed-effect).

Table IV estimates the wage equations of interest given above. The dependent variable is log average wage. Note that in addition to the reported coefficients, the regression include: experience, experience squared, and experience cubed; tenure and tenure squared; industry, occupation, and year dummies; county unemployment; and married, race, and union status.

- We have seen many regressions of log wages on years of education. How do the coefficients on education in these results compare to previous ones? Focus on columns (1) and (2). These regressions include far more explanatory variables than the ones we have looked at earlier. Can you explain the change in the coefficient on education in terms of omitted variables bias?
- The table reports the effect of tenure at ten years. Why does it do this instead of the coefficients on tenure and tenure squared? How is the effect of tenus at ten years computed? How is its standard error calculated?
- One of the predictions of the model is that the intercept should be lower in column (1) than in (2). Sadly, the table does not report the intercepts. Fortunately it reports something that should be very close to the difference in intercepts. What is it? Does it have the sign predicted by theory?
- Another prediction of the theory is that the return to worker characteristics should be higher at performance pay jobs. What variables in the table are worker characteristics? Do the results match the theoretical prediction?
- Should tenure count as a job or worker characteristic? What do you think a-priori? Assuming the theory above is correct, what do the results in Table IV suggest tenure should be considered?
- One of the basic facts mentioned as background above is that the return to education has increased. Column (5) includes interactions between education and dummies for 1980-1984, 1985-1989, 1990-1993, and 1994-1998. The omitted category is 1976-1979. What was the return to education in the late seventies at non-performance pay jobs? What about performance-pay jobs? How about in the nineties? What is the difference in the return to education in the early nineties and the late seventies for performance and non-performance pay jobs?

Table VII shows the contribution of performance pay to the increase in wage dispersion. It is sort of outside the scope of this class. I will talk about it if there is time, or you can read the paper.

TABLE VII

THE CONTRIBUTION OF PERFORMANCE-PAY (PP) JOBS TO THE VARIANCE OF LOG HOURLY EARNINGS

| | 1976–1979 | | | 1990–1993 | | |
|---|---|---|---|---|---|---|
| | Actual variance (1) | Variance w/o PP jobs (2) | PP jobs effect (3) | Actual variance (4) | Variance w/o PP jobs (5) | PP jobs effect (6) |
| **Within-group variance due to observables** | | | | | | |
| 1. Var(XB \| PP Jobs=1) | 0.1280 | 0.1034 | 0.0246 | 0.2037 | 0.1507 | 0.0529 |
| 2. Var(XB \| PP jobs=0) | 0.0922 | 0.0922 | 0.0000 | 0.1420 | 0.1420 | 0.0000 |
| 3. Total variance (%(PP jobs) × row (1) + (1−%(PP jobs)) × row (2)) | 0.1057 | 0.0964 | 0.0093 | 0.1706 | 0.1460 | 0.0245 |
| **Within-group variance due to unobservables** | | | | | | |
| 4. Var(e \| PP jobs=1) | 0.1220 | 0.1085 | 0.0135 | 0.1773 | 0.1583 | 0.0189 |
| 5. Var(e \| PP jobs=0) | 0.1139 | 0.1139 | 0.0000 | 0.1690 | 0.1690 | 0.0000 |
| 6. Total variance (%(PP jobs) × row (4) + (1−%(PP jobs)) × row (5)) | 0.1170 | 0.1119 | 0.0051 | 0.1728 | 0.1640 | 0.0088 |
| **Between-group variance (wage gap effect)** | | | | | | |
| 7. %(PP jobs) × (1−%(PP jobs)) × $\Delta^2$ | 0.0062 | 0.0009 | 0.0054 | 0.0217 | 0.0062 | 0.0155 |
| **Overall variance of wages** | | | | | | |
| 8. Var(XB +$e$): (row (3) + row (6) + row (7)) | 0.2290 | 0.2091 | 0.0198 | 0.3650 | 0.3163 | 0.0488 |
| Fraction of performance-pay jobs (%(PP jobs)) | | 0.3783 | | | 0.4632 | |
| Change in overall variance (col. (4) − col. (1)) | 0.1361 | | | | | |
| Change in performance-pay job effect (col. (6) − col. (3)) | 0.0290 | | | | | |
| Share of performance-pay job effect | 21.28% | | | | | |

*Notes.* Computations for the counterfactual variances (columns (2) and (5)) done by reweighting to produce a counterfactual distribution for performance-pay workers. $\Delta$ is the difference in mean wages between performance-pay and non-performance-pay workers. The samples for 1976–1979 and 1990–1993 are also adjusted by reweighting so that the distribution of the number of job matches observed is the same as in 1982–1990. The wage regression estimated to divide wages into an explained (XB) and an unexplained ($e$) component uses a more flexible specification in the explanatory variables listed in the notes to Tables III and IV. Relative to these specifications, we add a set of four education dummies that we also interact with potential experience (linear term), union status, and the race dummy. We also add a cubic in tenure, a dummy for full-time/full-year workers, and an interaction between potential experience and the race dummy. The probit used for reweighting also uses the same specification. See the text for more detail.