Review

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Section 1

Asymptotics

Asymptotics

- Idea: use limit of distribution of estimator as $n \to \infty$ to approximate finite sample distribution of estimator
- W_n converges in probability to θ if for every $\epsilon > 0$,

$$\lim_{n\to\infty} \mathrm{P}\left(|W_n - \theta| > \epsilon\right) = 0$$

  denote by $\operatorname{plim} W_n = \theta$ or $W_n \xrightarrow{p} \theta$
- Law of large numbers: if $y_1, ..., y_n$ are not too dependent and $\mathrm{Var}(y_i) < \infty$, then $\bar{y} \xrightarrow{p} \mathrm{E}[Y]$
  - $\operatorname{plim} g(W_n) = g(\operatorname{plim} W_n)$ if g is continuous (continuous mapping theorem (CMT))
  - If $W_n \xrightarrow{p} \omega$ and $Z_n \xrightarrow{p} \zeta$, then (Slutsky's lemma)
    - $W_n + Z_n \xrightarrow{p} \omega + \zeta$
    - $W_n Z_n \xrightarrow{p} \omega\zeta$
    - $\frac{W_n}{Z_n} \xrightarrow{p} \frac{\omega}{\zeta}$

Asymptotics

- W_n is a consistent estimate of θ if $W_n \xrightarrow{p} \theta$
- W_n converges in distribution to W, written $W_n \xrightarrow{d} W$, if $\lim_{n\to\infty} F_n(x) = F(x)$ for all x where F is continuous
- Central limit theorem: Let $\{y_1, ..., y_n\}$ be not too dependent with mean μ and variance σ^2 then $Z_n = \sqrt{n}\frac{\bar{y}_n - \mu}{\sigma}$ converges in distribution to a standard normal random variable
  - If $W_n \xrightarrow{d} W$, then $g(W_n) \xrightarrow{d} g(W)$ for continuous g (continuous mapping theorem (CMT))
  - Slutsky's theorem: If $W_n \xrightarrow{d} W$ and $Z_n \xrightarrow{p} c$, then (i) $W_n + Z_n \xrightarrow{d} W + c$, (ii) $W_n Z_n \xrightarrow{d} cW$, and (iii) $W_n/Z_n \xrightarrow{d} W/c$

Section 2

OLS

OLS

$$y_i = \beta_0 + \beta_1 x_{1,i} + \beta_2 x_{2,i} + \cdots + \beta_k x_{k,i} + \epsilon_i$$

- Assumptions:

MLR.1 (linear model)

MLR.2 (independence) $\{(x_{1,i}, x_{2,i}, y_i)\}_{i=1}^n$ is an independent random sample

MLR.3 (rank condition) no multicollinearity: no $x_{j,i}$ is constant and there is no exact linear relationship among the $x_{j,i}$

MLR.4 (exogeneity) $\mathrm{E}[\epsilon_i | x_{1,i}, \ldots, x_{k,i}] = 0$

MLR.5 (homoskedasticity) $\mathrm{Var}(\epsilon_i | X) = \sigma_\epsilon^2$

MLR.6 $\epsilon_i | X \sim N(0, \sigma_\epsilon^2)$

- Unbiased if 1-4
- Consistent under 1, 3 and (2') observations are not too dependent and (4') $\mathrm{E}[\epsilon_i x_{j,i}] = 0$

OLS

- Asymptotically normal under 1, 2', 3, 4'
- No need to assume 5, just use heteroskedasticity robust standard errors
  - If observations are dependent through time or through clustering must modify standard errors
- Interpretation of coefficients: β_j is effect of $x_{j,i}$ holding the other x's constant
- Estimates satisfy OLS first order conditions

$$\sum_{i=1}^{n} \left(y_i - \hat{\beta}_0 - \hat{\beta}_1 x_{1,i} - \cdots - \hat{\beta}_k x_{k,i} \right) x_{j,i} = 0 \text{ for } j = 1, \ldots, k$$

or

$$\sum_{i=1}^{n} \hat{\epsilon}_i x_{j,i} = \text{ for } j = 1, 2, \ldots, k$$

- Can also write estimates using partitioned regression

OLS

- Regress $x_{1,i}$ on other regressors

$$x_{1,i} = \hat{\gamma}_0 + \hat{\gamma}_2 x_{2,i} + \cdots + \hat{\gamma}_k x_{k,i} + \tilde{x}_{1,i}$$

  where $\tilde{x}_{1,i}$ is the OLS residual
- Then

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n \tilde{x}_{1,i} y_i}{\sum_{i=1}^n \tilde{x}_{1,i}^2}$$

Section 3

IV

IV

- Model

$$y_i = \beta_0 + \beta_1 x_{1,i} + \cdots + \beta_k x_{k,i} + \beta_{k+1} w_{1,i} + \cdots + \beta_{k+r} w_{r,i} + \epsilon_i \quad (1)$$

with instruments $z_{1,i}, \ldots, z_{m,i}$

- Assumptions:
  - IV.1 Linearity: (1) holds
  - IV.2 Independent observations
  - IV.3 Relevance (rank condition): $m \geq k$ and (loosely speaking) each $x_{j,i}$ is correlated with some $z_{l,i}$
  - IV.4 Exogeneity: $\mathrm{E}[w_{s,i}\epsilon_i] = 0$ for $s = 1, .., r$ and $\mathrm{E}[z_{l,i}\epsilon_i] = 0$ for $l = 1, \ldots, m$
- Estimate by two stage least squares
  1. Regress x's on z's and w's
  2. Regress y on $\hat{x}$'s and w's

  - Should check relevance in the first stage

IV

  - Reduced form is regression of y on z and w
  - IV estimate $\simeq$ reduced form divided by first stage
- If IV.1-IV.4, then 2SLS is consistent and asymptotically normal
- Need to use IV instead of OLS when we don't believe $\mathrm{E}[x\epsilon] = 0$
  - i.e. the model we want to estimate is not the population regression for the data we have
  - instead we want a causal effect or parameters from an economic model
- Comparing OLS and IV estimates