Economics 526 - Mathematics for Economists

Fall 2019

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This is a course of mathematics for students in our Economics MA program. It covers powerful mathematical tools that often appear in the modern economic literature. It is essential for the students' success in other courses to master the materials taught in this course.

Hao Li and Sejin Ahn are the course TA's. Lecture notes are posted at `http://faculty.arts.ubc.ca/pschrimpf/526/526.html`. Currently these notes are from last year. The notes will be updated throughout the term. Problem sets, solutions, and grades will be posted on the course's `https://canvas.ubc.ca` page.

If you have a question for me, it is very likely that others do too. Therefore, whenever possible ask questions on the discussion board on the course's `https://canvas.ubc.ca` page. I will likely reply to your question more quickly there than on email. My email is schrimpf@mail.ubc.ca, and my office phone number is 604-822-5360.

1 Schedule

	Day(s)	**Time**	**Location**
Lecture	Monday & Wednesday	12:00pm-2:00pm	Geog 101
Tutorial	Monday	8:30am-10:00am	Buchanan B313
Office hours			
Paul	Wednesday	5:00pm-6:00pm	Iona 107
	TBA		
	TBA		
Exams			
Midterm	Wednesday, October 9th	12:00pm-2:00pm	
Final	Wednesday, November 13th	12:00pm-2:00pm	
No class			
Thanksgiving	Monday, October 14th		
Remembrance Day	Monday, November 11th		

Office hours are subject to change. Any changes in office hours will be posted on the course web page. If you cannot come to my office hours, feel free to drop by anytime or email me to schedule an appointment.

2 Course Work

The course work will consist of a series of problem sets and two exams.

2.1 Problem Sets

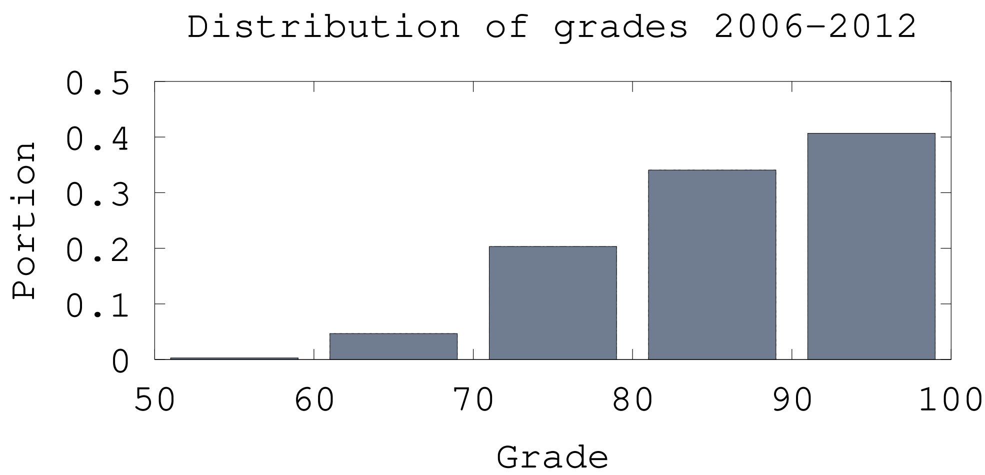
Problem sets will be due approximately weekly. Students should solve each problem set and turn in their work by the specified due date. Students are strongly encouraged to work together on problem sets, but each student should write their own solutions.

2.2 Exams

Two in-class exams will be given on October 9th and November 13th. If a student misses the first exam due to an objectively verifiable, unavoidable emergency situation, the student's grade will be determined by setting his or her first exam's grade equal to his or her second exam's grade. If a student misses the first exam for any other reasons, the student receives zero points for the first exam.

2.3 Grading

The course grade weights the problem sets by 0.15, the First exam by 0.40 and the second exam by 0.45. If a student finds an error in the grading of a problem set or exam, the student should contact the teaching assistants (for problem sets) or me (for an exam) within one week of the day the graded problem sets or exams were made available. We will then regrade the entire problem set or exam, and your total score may increase or decrease as a result. The distribution of grades in this course for the past seven years is shown below. The Faculty of Arts grading guidelines recommend that 5-25% of the class receive A's (80-100) and not more than 75% receive A's or B's (68-79). Grades may be scaled up to meet these guidelines and/or match the historical distribution of grades, but they will not be scaled down.



2.3.1 Concessions

Students must contact me via email as soon as you are aware you may need a concession related to problem sets.

For concessions related to the midterm or final exam, Arts students must contact Arts Advising as soon as you are aware you may need an in-term concession. Please review their website for concession criteria as well as process to follow. Students in other Faculties should contact their Faculty advising office for direction.

3 Textbook

There is no required textbook for this course. The lecture notes are fairly detailed and self-contained (and become more so each year). Nonetheless, additional references could be useful.

The notes were originally largely based on *Mathematics for Economists* by Simon and Blume (1994). However, over time the content has diverged quite a bit. Nonetheless, Simon and Blume remains an okay reference. Compared to the notes, Simon and Blume place less emphasis on proofs and more emphasis on examples and exercises.

Optimization in economic theory by Dixit (1990) is very readable. It lacks rigorous proofs, but does a great job of conveying the intuition for results. It also contains many economic examples. This book would be useful for the first half of the course, but not very useful for the second half.

Mathematical methods and models for economists by De la Fuente (2000) is a good middle ground between the practicality of Dixit (1990) and Simon and Blume (1994) and the abstraction of Carter (2001) and the other books listed below. I have only recently come across it and not read it in detail, but it might be the best choice for this course.

Foundations of Mathematical Economics by Carter (2001) is an excellent book, but is more difficult than the others listed above. It contains many proofs and can be quite abstract. However, it does do a good job of motivating everything with economic examples. I especially recommend it if you have a solid mathematics background or plan to continue studying economics after the MA program. Similar books that could be used instead include *Real analysis with economic applications* by Ok (2007) and *An introduction to mathematical analysis for economic theory and econometrics* by Corbae, Stinchcombe, and Zeman (2009). These books are perhaps slightly more difficult than Carter (2001). Corbae, Stinchcombe, and Zeman (2009) would be especially useful if you are interested in econometric theory because it contains an excellent chapter on measure spaces and probability (topics that will not be covered in this course).

The notes contain additional references that are relevant for specific topics.

4 Course Outline

This course has two main goals. One is to introduce some mathematical techiniques that will be useful in other courses. The second is to develop comfort with the sort of higher level mathematics that is used throughout economics.

The first half of the course focuses on optimization. Economics is largely about how to best allocate scarce resources. Consumers maximize utlity subject to a budget constraint, firms maximize profits subject to a production possibilities constraint, and the social planner maximizers welfare subject to a resource constraint. You are likely already familiar with finite dimension constrained optimization. However, you might not be familiar with optimization in infinite dimension. Infinite dimensional optimization problems appear in economics when individuals have to choose something at every instant of time or in every state of the world. The first few weeks of the course will focus on solving optimization problems, including those in inifinite dimension.

After the midterm, we will take a step back rigorously prove some of the results that we have been using. In particular, we will cover some basic real analysis and calculus on vector spaces. This part of the course is more abstract. The relevance of the mathematics that we cover to economics will not always be apparent. However, rest assured that what we cover is foundational and is essential for some areas of economics, particularly theoretical econometrics, micro, and macro.

1. Optimization
    (a) Constrained optimization
    (b) Comparative statics
    (c) The maximum principle
    (d) Dynamic programming
2. Sets and numbers
3. Linear algebra
    (a) Systems of linear equations
    (b) Matrix Algebra
    (c) Eigenvalues, eigenvectors, and definite matrices
    (d) Vector spaces
4. Calculus on vector spaces
    (a) Basic topology
    (b) Differential calculus
    (c) Inverse and implicit functions
    (d) Optimization on vector spaces
5. Convexity and the separating hyperplane theorem
6. Comparative statics without differentiability: supermodularity and topkis's theorem

It is possible that we will not have time to cover all topics, especially the last two.

5 Motivation and Goals

Look through the most recent issues of the top economics journals (the top five journals are *Econometrica, American Economic Review, Quarterly Journal of Economics, Journal of Political Economy, and Review of Economic Studies*). (i) In how many of the articles can you understand the statement of the main results? (ii) How many of the proofs could you have written yourself? (iii) How many of the proofs can you understand? (iv) How many of the proofs do you think you could figure out with the help of many hours of work and consulting additional references? I hope that after this course, the answers to these questions are (i) most, (ii) a few, (iii) half or more, (iv) most.

To be more specific, I looked at a handful of articles from the most recent (in September 2016) issues of *Econometrica* and *ReStud*. Here is what I thought.

- Tirole (2016) "From Bottom of the Barrel to Cream of the Crop: Sequential Screening With Positive Selection" is a paper that you should be able to understand quite well after this course. This paper looks at an optimal contracting problem that is somewhat similar to what we will look at with optimal control.

- Rothschild and Scheuer (2016) "Optimal Taxation with Rent-Seeking" is another paper that you should be able to understand quite well after this course. Section 2 is mostly finite dimensional optimization and may be familiar already, albeit with lots of tedious derivations. Sections 3 and 4 are related to optimal control.

- Miao and Rivera (2016) "Robust Contracts in Continuous Time" is also about optimal contracting, so it is related to what we will cover. However, this paper uses stochastic calculus, which we will not cover. I hope the course will provide enough information to understand the general idea of this paper, but understanding all the details will require further study.

- Andrews and Mikusheva (2016), "Conditional Inference With a Functional Nuisance Parameter" is a difficult econometrics paper. I hope this course will make this paper somewhat more approachable, but understanding this paper fully will still require lots of additional work.

My hope is that by the end of this course, you can look at any economics paper and, at worst, say to yourself, "with enough time and effort, I can figure this out if needed."

[0] UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious and cultural observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available here (https://senate.ubc.ca/policiesresources-support-student-success)

References

Andrews, Isaiah and Anna Mikusheva. 2016. "Conditional Inference With a Functional Nuisance Parameter." Econometrica 84 (4):1571–1612. URL http://dx.doi.org/10.3982/ECTA12868.

Carter, Michael. 2001. Foundations of mathematical economics. MIT Press.

Corbae, Dean, Maxwell B Stinchcombe, and Juraj Zeman. 2009. An introduction to mathematical analysis for economic theory and econometrics. Princeton University Press.

De la Fuente, Angel. 2000. Mathematical methods and models for economists. Cambridge University Press.

Dixit, Avinash K. 1990. Optimization in economic theory. Oxford University Press Oxford.

Miao, Jianjun and Alejandro Rivera. 2016. "Robust Contracts in Continuous Time." Econometrica 84 (4):1405–1440. URL http://dx.doi.org/10.3982/ECTA13127.

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Simon, Carl P and Lawrence Blume. 1994. Mathematics for economists. Norton New York.

Tirole, Jean. 2016. "From Bottom of the Barrel to Cream of the Crop: Sequential Screening With Positive Selection." Econometrica 84 (4):1291–1343. URL http://dx.doi.org/10.3982/ECTA12961.