Estimating Production Functions

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Economics 565

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     - Fixed effects
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Section 1

Introduction
Why estimate production functions?

- Primitive component of economic model
- Gives estimate of firm productivity — useful for understanding economic growth
  - Stylized facts to inform theory, e.g. Foster, Haltiwanger, and Krizan (2001)
  - Effect of deregulation, e.g. Olley and Pakes (1996)
  - Growth within old firms vs from entry of new firms, e.g. Foster, Haltiwanger, and Krizan (2006)
  - Effect of trade liberalization, e.g. Amiti and Konings (2007)
General references:

- Aguirregabiria (2017) chapter 2
- Ackerberg et al. (2007) section 2
- Van Beveren (2012)
Section 2

Setup
Setup

- Cobb Douglas production

\[ Y_{it} = A_{it} K_{it}^{\beta_k} L_{it}^{\beta_l} \]

- In logs,

\[ y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \]

with \( \log A_{it} = \omega_{it} + \epsilon_{it} \), \( \omega_{it} \) known to firm, \( \epsilon_{it} \) not

- Problems:

  1. Simultaneity: if firm has information about \( \log A_{it} \) when choosing inputs, then inputs correlated with \( \log A_{it} \), e.g. price \( p \), wage \( w \), perfect information

\[ L_{it} = \left( \frac{p}{w} \beta_l A_{it} K_{it}^{\beta_k} \right)^{\frac{1}{1-\beta_l}} \]

  2. Selection: firms with low productivity will exit sooner

  3. Others: measurement error, specification
Section 3

Simultaneity
Simultaneity solutions

1. IV
2. Panel data
3. Control functions
Instrumental variables

- Instrument must be
  - Correlated with $k$ and $l$
  - Uncorrelated with $\omega + \epsilon$
- Possible instrument: input prices
  - Correlated with $k, l$ through first-order condition
  - Uncorrelated with $\omega$ if input market competitive
- Other possible instruments: output prices (more often endogenous), input supply or output demand shifter (hard to find)
Problems with input prices as IV

- Not available in many data sets
- Average input price of firm could reflect quality as well as price differences
- Need variation across observations
  - If firms use homogeneous inputs, and operate in the same output and input markets, we should not expect to find any significant cross-sectional variation in input prices
  - If firms have different input markets, maybe variation in input prices, but different prices could be due to different average productivity across input markets
  - Variation across time is potentially endogenous because could be driven by time series variation in average productivity
• Have panel data, so should consider fixed effects
• FE consistent if:
  1. \( \omega_{it} = \eta_i + \delta_t + \omega^*_it \)
  2. \( \omega^*_it \) uncorrelated with \( l_{it} \) and \( k_{it} \), e.g. \( \omega^*_it \) only known to firm after choosing inputs
  3. \( \omega^*_it \) not serially correlated and is strictly exogenous
• Problems:
  • Fixed productivity a strong assumption
  • Estimates often small in practice
  • Worsens measurement error problems

\[
\text{Bias}(\hat{\beta}^FE_k) \approx -\frac{\beta_k \text{Var}(\Delta\epsilon)}{\text{Var}(\Delta k) + \text{Var}(\Delta\epsilon)}
\]
Dynamic panel: motivation 1

- General idea: relax fixed effects assumption, but still exploit panel

- Collinearity problem: Cobb-Douglas production, flexible labor and capital implies log labor and log capital are linear functions of prices and productivity (Bond and Söderbom (2005))

- If observed labor and capital are not collinear then there must be something unobserved that varies across firms (e.g. prices), but that could invalidate monotonicity assumption of control function
Dynamic panel: moment conditions

- See Blundell and Bond (2000)
- Assume $\omega_{it} = \gamma_t + \eta_i + \nu_{it}$ with $\nu_{it} = \rho \nu_{i,t-1} + e_{it}$, so

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \gamma_t + \eta_i + \nu_{it} + e_{it}$$

subtract $\rho y_{i,t-1}$ and rearrange to get

$$y_{it} = \rho y_{i,t-1} + \beta_l (l_{it} - \rho l_{i,t-1}) + \beta_k (k_{it} - \rho k_{i,t-1}) + \gamma_t - \rho \gamma_{t-1} + \eta_i (1 - \rho) + e_{it} + e_{it} - \rho e_{i,t-1}$$

\[= \eta_i^* + w_{it}\]

- Moment conditions:
  - Difference: $E[x_{i,t-s} \Delta w_{it}] = 0$ where $x = (l, k, y)$
  - Level: $E [\Delta x_{i,t-s}(\eta_i^* + w_{it})] = 0$
Dynamic panel: economic model 1

- Adjustment costs

\[ V(K_{t-1}, L_{t-1}) = \max_{l_t, K_t, H_t, L_t} P_t F_t(K_t, L_t) - P^K_t (L_t + G_t(l_t, K_{t-1})) - \]
\[ - W_t (L_t + C_t(H_t, L_{t-1})) + \]
\[ \psi E [V(K_t, L_t) | I_t] \]

\( \text{s.t. } K_t = (1 - \delta_k)K_{t-1} + I_t \)
\( L_t = (1 - \delta_l)L_{t-1} + H_t \)

Implies

\[ P_t \frac{\partial F_t}{\partial L_t} - W_t \frac{\partial C_t}{\partial L_t} = W_t + \lambda^L_t \left(1 - (1 - \delta_l) \psi E \left[ \frac{\lambda^L_{t+1}}{\lambda^L_t} | I_t \right]\right) \]
\[ P_t \frac{\partial F_t}{\partial K_t} - P^K_t \frac{\partial G_t}{\partial K_t} = \lambda^K_t \left(1 - (1 - \delta_k) \psi E \left[ \frac{\lambda^K_{t+1}}{\lambda^K_t} | I_t \right]\right) \]
Dynamic panel: economic model 2

- Current productivity shifts $\frac{\partial F_t}{\partial L_t}$ and (if correlated with future) the shadow value of future labor $E \left[ \frac{\lambda^{L}_{t+1}}{\lambda^{L}_{t}} | I_t \right]$
- Past labor correlated with current because of adjustment costs
Dynamic panel data: problems

- Problems:
  - Sometimes imprecise (especially if only use difference moment conditions)
  - Differencing worsens measurement error
  - Weak instrument issues if only use difference moment conditions but levels stronger (see Blundell and Bond (2000))
    - Level moments require stronger stationarity assumption
      - $\eta_i$ uncorrelated with $\Delta x_{it}$
Control functions

- From **Olley and Pakes (1996)** (OP)

- **Control function**: function of data conditional on which endogeneity problem solved
  - E.g. usual 2SLS $y = x\beta + \epsilon$, $x = z\pi + \nu$, control function is to estimate residual of reduced form, $\hat{\nu}$ and then regress $y$ on $x$ and $\hat{\nu}$. $\hat{\nu}$ is the control function

- Main idea: model choice of inputs to find a control function
OP assumptions

\[ y_{it} = \beta_k k_{it} + \beta_I l_{it} + \omega_{it} + \epsilon_{it} \]

1. \( \omega_{it} \) follows exogenous first order Markov process,
   \[ p(\omega_{it+1}|I_{it}) = p(\omega_{it+1}|\omega_{it}) \]

2. Capital at \( t \) determined by investment at time \( t - 1 \),
   \[ k_t = (1 - \delta)k_{t-1} + i_{it-1} \]

3. Investment is a function of \( \omega \) and other observed variables
   \[ i_{it} = I_t(k_{it}, \omega_{it}) \]

   and is strictly increasing in \( \omega_{it} \)

4. Labor variable and non-dynamic, i.e. chosen each \( t \),
   current choice has no effect on future (can be relaxed)
OP estimation of $\beta_l$

- Invertible investment implies $\omega_{it} = l_t^{-1}(k_{it}, i_{it})$

\[
y_{it} = \beta_k k_{it} + \beta_l l_{it} + l_t^{-1}(k_{it}, l_{it}) + \epsilon_{it} = \beta_l l_{it} + f_t(k_{it}, i_{it}) + \epsilon_{it}
\]

- Partially linear model
  - Estimate by e.g. regress $y_{it}$ on $l_{it}$ and series functions of $t, k_{it}, i_{it}$
  - Gives $\hat{\beta}_l, \hat{f}_t = f_t(k_{it}, i_{it})$
OP estimation of $\beta_k$

- Note: $\hat{f}_t(k_{it}, i_{it}) = \hat{\omega}_{it} + \beta_k k_{it}$
- By assumptions, $\omega_{it} = E[\omega_{it}|\omega_{it-1}] + \xi_{it} = g(\omega_{it-1}) + \xi_{it}$ with $E[\xi_{it}|k_{it}] = 0$
- Use $E[\xi_{it}|k_{it}] = 0$ as moment to estimate $\beta_k$.
  - OP: write production function as
    \[
    y_{it} - \beta_l l_{it} = \beta_k k_{it} + g(\omega_{it-1}) + \xi_{it} + \epsilon_{it} \\
    = \beta_k k_{it} + g(f_{it-1} - \beta_k k_{it-1}) + \\
    + \xi_{it} + \epsilon_{it}
    \]
    Use $\hat{\beta}_l$ and $\hat{f}_it$ in equation above and estimate $\hat{\beta}_k$ by e.g. semi-parametric nonlinear least squares
  - Ackerberg, Caves, and Frazer (2015): use
    \[
    E\left[\xi_{it}(\beta_k)k_{it}\right] = 0
    \]
Dynamic panel vs control function

• Both derive moment conditions from assumptions about timing and information set of firm
• Dealing with $\omega$
  • Dynamic panel: AR(1) assumption allows quasi-differencing
  • Control function: makes $\omega$ estimable function of observables
• Dynamic panel allows fixed effects, does not make assumptions about input demand
• Control function allows more flexible process for $\omega_{it}$
Applications

- Olley and Pakes (1996): productivity in telecom after deregulation
- Söderbom, Teal, and Harding (2006): productivity and exit of African manufacturing firms, uses IV
- Levinsohn and Petrin (2003): compare estimation methods using Chilean data
- Javorcik (2004): FDI and productivity, uses OP
- Amiti and Konings (2007): trade liberalization in Indonesia, uses OP
- Aw, Chen, and Roberts (2001): productivity differentials and firm turnover in Taiwan
- Kortum and Lerner (2000): venture capital and innovation
Section 4

Selection
Selection

- Let $d_{it} = 1$ if firm in sample.
  - Standard conditions imply $d = 1\{\omega \geq \omega^*(k)\}$
- Messes up moment conditions
  - All estimators based on $E[\omega_{it} \text{Something}] = 0$, observed data really use $E[\omega_{it} \text{Something}|d_{it} = 1]$
  - E.g. OLS okay if $E[\omega_{it}|l_{it}, k_{it}] = 0$, but even then,

$$
E[\omega_{it}|l_{it}, k_{it}, d_{it} = 1] = E[\omega_{it}|l_{it}, k_{it}, \omega_{it} \geq \omega^*(k_{it})] = \lambda(k_{it}) \neq 0
$$

- Selection bias negative, larger for capital than labor
Selection in OP model

- Estimate $\beta_l$ as above
- Write
  \[ d_{it} = 1\{\xi_{it} \leq \omega^*(k_{it}) - \rho(f_{i,t-1} - \beta_k k_{it-1}) = h(k_{it}, f_{it-1}, k_{it-1}) \} \]
- Propensity score $P_{it} \equiv \mathbb{E}[d_{it} | k_{it}, f_{it-1}, k_{it-1}]$
- Similar to before estimate $\beta_k$, from

\[
 y_{it} - \beta_l l_{it} = \beta_k k_{it} + \tilde{g}(f_{it-1} - \beta_k k_{it-1}, P_{it}) + \\
+ \xi_{it} + \epsilon_{it}
\]
Critiques and extensions

- Levinsohn and Petrin (2003): investment often zero, so use other inputs instead of investment to form control function
- Ackerberg, Caves, and Frazer (2015): control function often collinear with $l_{it}$ — for it not to be must be firm specific unobservables affecting $l_{it}$ (but not investment / other input or else demand not invertible and cannot form control function)
- Gandhi, Navarro, and Rivers (2013): relax scalar unobservable in investment / other input demand
- Wooldridge (2009): more efficient joint estimation
- Maican (2006) and Doraszelski and Jaumandreu (2013): endogenous productivity
Section 5

Ackerberg, Caves, and Frazer (2015)
Ackerberg, Caves, and Frazer (2015): contributions

  • Need $l_{it}, f_{it}(k_{it}, i_{it})$ not collinear, i.e. something causes variation in $l$, but not $k$

• Propose alternative estimator
• Relates estimator to dynamic panel (Blundell and Bond, 2000) approach
• Illustrates estimator using Chilean data

\[\textsuperscript{0}^\text{These slides are based on the working paper version Ackerberg, Caves, and Frazer (2006).}\]
Collinearity in OP 1

- OP assume $i_{it} = I_t(k_{it}, \omega_{it})$
- Symmetry, parsimony suggest $l_{it} = L_t(k_{it}, \omega_{it})$
- Then $l_{it} = L_t(k_{it}, l_{it}^{-1}(k_{it}, i_{it})) = g_t(k_{it}, i_{it})$

$$y_{it} = \beta_l l_{it} + f_t(k_{it}, i_{it}) + \epsilon_{it}$$

$l_{it}$ collinear with $f_t(k_{it}, i_{it})$
- Worse in Levinsohn and Petrin (2003)
  - Uses other input $m_{it}$ to form control function

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \omega_{it} + \epsilon_{it}$$

$$m_{it} = M_t(k_{it}, \omega_{it})$$

- Even less reason to treat labor demand differently than other input demand
Collinearity in OP 2

- Collinearity still problem with parametric input demand
- Plausible models that do not solve collinearity
  - Input price data
    - Must include in control function to preserve scalar unobservable
    - Same logic above implies $m$ and $l$ are functions of both prices, so still collinear
  - Adjustment costs in labor
    - Need to add $l_{it-1}$ to control function
- Change in timing assumptions
  - Measurement error in $l$ (but not $m$)
    - Solves collinearity, but makes $\hat{\beta}_l$ inconsistent
- Potential model change that removes collinearity
  - Optimization error in $l$ (but not $m$)
  - $m$ chosen, $l$ specific shock revealed, $l$ chosen
  - OP only: $l_{it}$ chosen at $t - 1/2$, $l_{it} = L_t(\omega_{it-1/2}, k_{it})$, $i_{it}$ chosen at $t$
ACF estimator

- Idea: like capital, labor is harder to adjust than other inputs
- Model: \( l_{it} \) chosen at time \( t - 1/2 \), \( m_{it} \) at time \( t \)
  - Implies \( m_t = M_t(k_{it}, l_{it}, \omega_{it}) \)
- Estimation:
  \[
  y_{it} = \beta_k k_{it} + \beta_l l_{it} + f_t(m_{it}, k_{it}, l_{it}) + \epsilon_{it}
  \equiv \Phi_t(m_{it}, k_{it}, l_{it})
  \]
  \[
  \hat{\omega}_{it}(\beta_k, \beta_l) = \hat{\Phi}_{it} - \beta_k k_{it} - \beta_l l_{it}
  \]

2 Moments from timing and Markov process for \( \omega_{it} \) assumptions:
\[
\omega_{it} = E[\omega_{it} | \omega_{it-1}] + \xi_{it}
\]

- \( E[\xi_{it} | k_{it}] = 0 \) as in OP
- \( E[\xi_{it} | l_{it-1}] = 0 \) from new timing assumption
- \( \hat{\xi}_{it}(\beta_k, \beta_l) \) as residual from nonparametric regression of \( \hat{\omega}_{it} \) on \( \hat{\omega}_{it-1} \)
- Can add moments based on \( E[\epsilon_{it} | I_{it}] = 0 \)
Relation to dynamic panel estimators

• Both derive moment conditions from assumptions about timing and information set of firm
• Dealing with $\omega$
  • Dynamic panel: AR(1) assumption allows quasi-differencing
  • Control function: makes $\omega$ estimable function of observables
• Dynamic panel allows fixed effects, does not make assumptions about input demand
• Control function allows more flexible process for $\omega_{it}$
Empirical example

- Chilean plant level data
- Compare OLS, FE, LP, ACF, and dynamic panel estimators
- LP and ACF using three different inputs (materials, electricity, fuel) for control function
- Results:
  - 311=food, 321=textiles, 331=wood, 381=metal
  - Expected biases in OLS and FE
  - ACF and LP significantly different
  - ACF less sensitive to which input used for control function
  - Dynamic panel closer to ACF than LP, but still significant differences
<table>
<thead>
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<th>Industry 311</th>
<th>Capital</th>
<th>SE</th>
<th>Labor</th>
<th>SE</th>
<th>Returns to Scale</th>
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<td>0.336</td>
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Note: Value is the % of bootstrap reps where ACF coeff is less than OLS, LP, or DP coef. A value either above 0.95 or below 0.05 indicates that coefficients are significantly different from each other.
Section 6

Gandhi, Navarro, and Rivers (2013)
Gandhi, Navarro, and Rivers (2013)

- Show that control function method is not nonparametrically identified when there are flexible inputs
- Propose alternate estimate that uses data on input shares and information from firm’s first order condition
- Show that value-added and gross output production functions are incompatible
- Application to Colombia and Chile
Assumptions

1. Hicks neutral productivity $Y_{jt} = e^{\omega_{jt} + \epsilon_{jt}} F_t(L_{jt}, K_{jt}, M_{jt})$
2. $\omega_{jt}$ Markov, $\epsilon_{jt}$ i.i.d.
3. $K_{jt}$ and $L_{jt}$ determined at $t - 1$, $M_{jt}$ determined flexibly at $t$
   - $K$ and $L$ play same role in the model, so after this slide I will drop $L$
4. $M_{jt} = M_t(L_{jt}, K_{jt}, \omega_{jt})$, monotone in $\omega_{jt}$
Reduced form

- Let \( h(\omega_{jt-1}) = E[\omega_{jt} \mid \omega_{jt-1}] \), \( \eta_{jt} = \omega_{jt} - h(\omega_{jt-1}) \)
- log output

\[
\begin{align*}
y_{jt} &= f_t(k_{jt}, m_{jt}) + \omega_{jt} + \epsilon_{jt} \\
&= f_t(k_{jt}, m_{jt}) + h(M^{-1}_{t-1}(k_{jt-1}, m_{jt-1})) + \eta_{jt} + \epsilon_{jt} \\
&= h_{t-1}(k_{jt-1}, m_{jt-1})
\end{align*}
\]

- Assumptions imply

\[
E[\eta_{jt} \mid k_{jt}, k_{jt-1}, m_{jt-1}, \ldots, k_{j1}, m_{j1}] = 0 = \Gamma_{jt}
\]

- Reduced form

\[
E[y_{jt} \mid \Gamma_{jt}] = E[f_t(k_{jt}, m_{jt}) \mid \Gamma_{jt}] + h_{t-1}(k_{jt-1}, m_{jt-1})
\]

(1)

- Identification: given observed \( E[y_{jt} \mid \Gamma_{jt}] \) is there a unique \( f_t, h_{t-1} \) that satisfies (3)?
Example: Cobb-Douglas 1

- Let $f_t(k, m) = \beta_k k + \beta_m m$
- Assume firm is takes prices as given
- First order condition for $m$ gives

$$m = \text{constant} + \frac{\beta_k}{1 - \beta_m} k + \frac{1}{1 - \beta_m} \omega$$

- Put into reduced form

$$E[y_{jt} | \Gamma_{jt}] = c + \frac{\beta_k}{1 - \beta_m} k_{jt} + \frac{\beta_m}{1 - \beta_m} E[\omega_{jt} | \Gamma_{jt}] + h_{t-1}(k_{jt-1}, m_{jt-1})$$  \hspace{1cm} (2)

- $\omega$ Markov and $\omega_{jt-1} = M_{t-1}^{-1}(k_{jt-1}, m_{jt-1})$ implies

$$E[\omega_{jt} | \Gamma_{jt}] = E[\omega_{jt} | \omega_{jt-1} = M_{t-1}^{-1}(k_{jt-1}, m_{jt-1})] = h_{t-1}(k_{jt-1}, m_{jt-1})$$
Example: Cobb-Douglas 2

- Which leaves

\[
E[y_{jt} | \Gamma_{jt}] = \text{constant} + \frac{\beta_k}{1 - \beta_m} k_{jt} + \frac{1}{1 - \beta_m} h_{t-1}(k_{jt-1}, m_{jt-1})
\]

(3)

from which \( \beta_k, \beta_m \) are not identified

- Rank condition fails, \( E[m_{jt} | \Gamma_{jt}] \) is collinear with \( h_{t-1}(k_{jt-1}, m_{jt-1}) \)

- After conditioning on \( k_{jt}, k_{jt-1}, m_{jt-1} \), only variation in \( m_{jt} \) is from \( \eta_{jt} \), but this is uncorrelated with the instruments
Identification from first order conditions 1

- Since \( m \) flexible, it satisfies a simple static first order condition,

\[
\rho_t = p_t \frac{\partial F_t}{\partial M} E[e^{\varepsilon_j}] e^{\omega_j}
\]

\[
\log \rho_t = \log p_t + \log \frac{\partial F_t}{\partial M}(k_{jt}, m_{jt}) + \log E[e^{\varepsilon_j}] + \omega_j
\]

- Problem: prices often unobserved, endogenous \( \omega \)

- Solution: difference from output equation to eliminate \( \omega \), rearrange so that it involves only the value of materials and the value of output (which are often observed)

\[
S_{jt} \equiv \left( \frac{\rho_t M_{jt}}{p_t Y_{jt}} \right) = \log G_t(k_{jt}, m_{jt}) + \log E[e^{\varepsilon_j}] - \varepsilon_j
\]
Identification from first order conditions 2

- Identifies elasticity up to scale, $G_t \mathcal{E}$ and $\varepsilon_{jt}$ which identify $\mathcal{E}$
- Integrating,

$$\int_{m_0}^{m_{jt}} G_t(k_{jt}, m)/m = f_t(k_{jt}, m_{jt}) + c_t(k_{jt})$$

identifies $f$ up to location
- Output equation

$$y_{jt} = \int_{m_0}^{m_{jt}} \tilde{G}_t(k_{jt}, m)/m - c_t(k_{jt}) + \omega_{jt} + \varepsilon_{jt}$$

$$-c_t(k_{jt}) + \omega_{jt} = y_{jt} - \int_{m_0}^{m_{jt}} \tilde{G}_t(k_{jt}, m)/m - \varepsilon_{jt}$$

$$\equiv \mathcal{Y}_{jt}$$
where the things on the right have already been identified

- Identify $c_t$ from

$$\mathcal{V}_{jt} = -c_t(k_{jt}) + \tilde{h}_t(\mathcal{V}_{jt-1}, k_{jt-1}) + \eta_{jt}$$
Value added vs gross production

• Value added:

\[ VA_{jt} = p_t Y_{jt} - \rho_t M_{jt} = p_t F_t(K_{jt}, M_t(K_{jt}, \omega_{jt})) e^{\omega_{jt} + \epsilon_{jt}} - \rho_t M_t(K_{jt}, \omega_{jt}) \]

• Envelope theorem implies

\[ \text{elasticity}_{e\omega}^Y \approx \text{elasticity}_{e\omega}^{VA}(1 - \frac{\rho_t M_{jt}}{p_t Y_{jt}}) \]

Problems

• Production Hicks-neutral productivity does not imply value-added Hicks-neutral productivity

• Ex-post shocks \( \epsilon_{jt} \) not accounted for in approximation
Empirical results

- Look at tables
- Value-added estimates imply much more productivity dispersion than gross (90-10) ratio of 4 vs 2
Part III

Selected applications and extensions
Estimating Production Functions

Paul Schrimpf

References


Section 7

Grieco and McDevitt (2017)
Grieco and McDevitt (2017)

Model details

- **Timing:**
  1. Quality chosen $q_{it} = q(k_{it}, \ell_{it}, x_{it}, \omega_{i,t-b})$
  2. Production occurs, $\omega_{it}$ revealed to firm
  3. Hiring chosen $\ell_{i,t+1} - \ell_{it} = h_{it} = h(k_{it}, \ell_{it}, x_{it}, \omega_{it})$

- $\omega$ follows Markov process:

\[
E[\omega_{i,t-b} | I_{i,t-b}] = E[\omega_{i,t-b} | \omega_{i,t-1}] \quad \& \quad E[\omega_{it} | I_{i,t}] = E[\omega_{it} | \omega_{i,t-b}]
\]

and $\omega_{it} = E[\omega_{it} | \omega_{i,t-1}] + \eta_{it} = g(\omega_{i,t-1}) + \eta_{it}$
Moment conditions

- Control function assumption: hiring is a monotonic function of $\omega$
  
  $$h_{it} = h(k_{it}, \ell_{it}, x_{it}, \omega_{it})$$

  so
  
  $$\omega_{it} = h^{-1}(k_{it}, \ell_{it}, x_{it}, h_{it})$$

- Substitute into production function:
  
  $$y_{it} = \alpha q_{it} + \beta_k k_{it} + \beta_\ell \ell_{it} + h^{-1}(k_{it}, \ell_{it}, x_{it}, h_{it}) + \epsilon_{it}$$

  $$y_{it} = \alpha q_{it} + \Phi(k_{it}, \ell_{it}, x_{it}, h_{it}) + \epsilon_{it}$$

- Evolution of $\omega$
  
  $$\omega_{it} = y_{it} - \alpha q_{it} - \beta_k k_{it} - \beta_\ell \ell_{it} - \epsilon_{it} = g(\omega_{i,t-1}) + \xi_{it}$$

  $$g(\Phi(k_{it-1}, \ell_{it-1}, x_{it-1}, h_{it-1}) - \beta_\ell \ell_{it-1} - \beta_k k_{it-1}) + \xi_{it}$$

- Moment conditions:
  
  $$E[\epsilon_{it}|q_{it}, k_{it}, \ell_{it}, x_{it}, h_{it}] = 0$$

  $$E[\xi_{it}|k_{it}, \ell_{it}, x_{it}, k_{it-1}, \ell_{it-1}, x_{it-1}] = 0$$
Estimation

1. Estimate, $\alpha_q$, $\Phi$ from

$$y_{it} = \alpha_q q_{it} + \Phi(k_{it}, \ell_{it}, x_{it}, h_{it}) + \epsilon_{it}$$

by semiparametric regression

2. Estimate $\beta_k$, $\beta_\ell$

- Let $\omega(\beta)_{it} = \hat{\Phi}(k_{it}, \ell_{it}, x_{it}, h_{it}) - \beta_k k_{it} - \beta_\ell \ell_{it}$
- For each $\beta$ estimate $g()$

$$y_{it} - \hat{\alpha} q_{it} - \beta_k k_{it} - \beta_\ell \ell_{it} = g(\omega(\beta)_{it-1}) + \xi_{it} + \epsilon_{it}$$

$\equiv \eta_{it}(\beta)$

by nonparametric regression

- Minimize empirical moment condition for $\eta$

$$\hat{\beta} = \arg\min\left(\frac{1}{NT} \sum_{it} k_{it} \eta_{it}(\beta)\right)^2 + \left(\frac{1}{NT} \sum_{it} \ell_{it} \eta_{it}(\beta)\right)^2$$
• Should hemoglobin level be controlled for when measuring quality?
  • Anemia (low hemoglobin) is risk-factor for infection
  • Anemia can be treated through diet, iron supplements (pills or IV), EPO, etc
    • Are dialysis facilities responsible for this treatment?
    • In 2006-2014 data average full-time dieticiens = 0.5, average part-time = 0.6
Measurement error

• Simplified setup:
  \[ y = \alpha \tilde{q} + \epsilon \]

  \( \tilde{q} \) unobserved, observe \( q = \tilde{q} + \epsilon^q \) with \( E[\epsilon^q|\tilde{q} = 0] \)

• Then \( \text{plim} \hat{\alpha}^{\text{OLS}} = \alpha \frac{\text{Var}(\tilde{q})}{\text{Var}(\tilde{q}) + \text{Var}(\epsilon^q)} \)

• If \( d = d(\tilde{q}) + \epsilon^d \) with \( E[\epsilon^d|\tilde{q}] = 0 \) and \( E[\epsilon^d \epsilon^q] = 0 \), then
  \[ \text{plim} \hat{\alpha}^{\text{IV}} = \alpha \]

• Is \( E[\epsilon^d \epsilon^q] = 0 \) a good assumption?
  • Paper argues \( E[\epsilon^d \epsilon^q] = O(1/(\text{patients per facility})) \)
• Estimation details:

Step 1: Estimate $\alpha_q$

$$y_{jt} \hat{E}[y|h_{jt}, i_{jt}, k_{jt}, \ell_{jt}, x_{jt}] = \alpha_q(q_{jt} \hat{E}[q|h_{jt}, i_{jt}, k_{jt}, \ell_{jt}, x_{jt}]) + \epsilon_{jt}$$

- Drop observations with $h_{jt} = 0$ (not invertible)
- Okay here, because selecting on $\omega$, and residual, $\epsilon_{jt}$ is uncorrelated with $\omega$
- Problematic in last step? No, see footnote 49

Step 2: Estimate $\beta_k, \beta_\ell$ from

$$y_{jt} + \hat{\alpha}_q + \beta_k k_{jt} + \beta_\ell \ell_{jt} = g(\hat{\omega}_{jt-1}(\beta)) + \eta_{jt} + \epsilon_{jt}$$

- Only have $\hat{\omega}_{jt-1}(\beta)$ when $h_{jt-1} \neq 0$, okay because $\epsilon_{jt}$ and $\eta_{jt}$ are uncorrelated with $\omega_{jt-1}$, would be problem if using $\hat{\omega}_{jt}$
- Nothing about selection — number of centers, 4270, vs center-years, 18295, implies there must be entry and exit
• Estimate implications:
  • Holding inputs constant, reducing infections by one per year requires reducing output by 1.5 patients
    • Cost of treatment $\approx$ $50,000$, so one infection $\approx$ $75,000$
  • Holding output constant, reducing infections by one per year requires hiring 1.8 more staff
    • Cost of staff $\approx$ $42,000$, so one infection $\approx$ $75,000$
• Would like to see some results related to productivity dispersion e.g.
  • Decompose variation in infection rate into: productivity variation, incentive variation, quality-quantity choices, and random shocks
  • Compare strengthening incentives vs closing least productive facilities as policies to increase quality
Section 8

Amiti and Konings (2007)
Summary:

- Effect of reducing input and output tariffs on productivity
- Reducing output tariffs affects productivity by increasing competition
- Reducing input tariffs affects productivity through learning, variety, and quality effects
- Previous empirical work focused on output tariffs; might be estimating combined effect
- Input tariffs hard to measure; with Indonesian data on plant-level inputs can construct plant specific input tariff
Methodology

• Estimate TFP using Olley-Pakes
  • Output measure is revenue ⇒ may confound productivity and markups

• Estimate relation between TFP and tariffs

\[
\log(TFP_{it}) = \gamma_0 + \alpha_i + \alpha_{tl(i)} + \gamma_1(\text{output tariff})_{tk(i)} + \\
+ \gamma_2(\text{input tariff})_{tk(i)} + \epsilon_{it} \tag{4}
\]

• \(k(i) = 5\text{-digit (ISIC) industry of plant } i\)
• \(l(i) = \text{island of plant } i\)

• Explore robustness to:
  • Different productivity measure
  • Specification of 4
  • Endogeneity of tariffs
Data and tariff measure

- Indonesian annual manufacturing census of 20+ employee plants 1991-2001, after cleaning 15,000 firms per year
- Input tariffs:
  - Data on tariffs on goods, $\tau_{jt}$, but also need to know inputs
  - 1998 only: have data on inputs, use to construct input weights at industry level, $w_{jk}$
  - Industry input tariff = $\sum_j w_{jk} \tau_{jt}$
• Look at tables
• Input tariffs have larger effect than output, $\hat{\gamma}_1 \approx -0.07$, $\hat{\gamma}_2 \approx -0.44$
• Robust to:
  • Productivity measure
  • Tariff measure
  • Including/excluding Asian financial crisis
• Less robust to instrumenting for tariffs
  • Qualitatively similar, but larger coefficient estimates
• Explore channels for productivity change
  • Markups (maybe), product switching/addition (no), foreign ownership (no), exporters (no)
Section 9

Doraszelski and Jaumandreu (2013)
Overview

- Estimable model of endogenous productivity, which combines:
  - Knowledge capital model of R&D
  - OP & LP productivity estimation

- Application to Spanish manufacturers focusing on R&D
  - Large uncertainty (20%-60% or productivity unpredictable)
  - Complementarities and increasing returns
  - Return to R&D larger than return to physical capital investment
- **Cobb-Douglas production:**

  \[ y_{it} = \beta_l l_{it} + \beta_k k_{it} + \omega_{it} + \epsilon_{it} \]

- **Controlled Markov process for productivity,** \( p(\omega_{it+1}|\omega_{it}, r_{it}) \),

  \[ \omega_{it} = g(\omega_{it-1}, r_{it-1}) + \xi_{it} \]

- Labor flexible and non-dynamic

- **Value function**

  \[
  V(k_t, \omega_t, u_t) = \max_{i,r} \Pi(k_t, \omega_t) - C_i(i, u_t) - C_r(r, u_t) + \frac{1}{1 + \rho} \mathbb{E}[V(k_{t+1}, \omega_{t+1}, u_{t+1})|k_t, \omega_t, i, r, u_t]
  \]
Model (simplified) 2

- $u$ scalar or vector valued shock
- $u$ not explicitly part of model, but identification discussion (especially p10 and footnote 6) implicitly adds it
- $u$ independent of? $k$, $l$? across time?

- Control function incorporating Cobb-Douglas assumption (and perfect competition):

$$\omega_{it} = h(l_{it}, k_{it}, w_{it} - p_{it}; \beta) = \lambda_0 + (1 - \beta_l)l_{it} - \beta_k k_{it} + (w_{it} - p_{it})$$

- Form moments based on

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + g(h(l_{it-1}, k_{it-1}, w_{it-1} - p_{it-1}; \beta), r_{it-1}) + \xi_{it} + \epsilon_{it}$$

- No collinearity because:
  - Parametric $h$
  - Variation in $k$, $r$ due to $u$

- Estimated model adds
Model (simplified) 3

- Material input instead of labor for control function
- $h$ based on imperfect competition
- Comparison to OP, LP, ACF
Results

• Look at tables and figures
• Large uncertainty (20%-60% or productivity unpredictable)
• Complementarities and increasing returns
• Return to R&D larger than return to physical capital


Estimating Production Functions
Paul Schrimpf

Grieco and McDevitt (2017)
Amiti and Konings (2007)
Doraszelski and Jaumandreu (2013)

References


