References

- Review paper: Berry and Haile (2015) – summarizes Berry and Haile (2009), Berry and Haile (2014), and Berry, Gandhi, and Haile (2013)
- Alternative approach : Bonnet, Galichon, and Shum (2017), Chiong, Hsieh, and Shum (2017)

Motivation

- Can numerically check local parametric identification
- Parametric identification not enough
  - Functional form assumptions in BLP are somewhat arbitrary and mostly chosen for convenience
  - Do not want our conclusions to be driven by arbitrary assumptions
- Non parametric identification shows what assumptions are essential for results

Section 1

Market level data: Berry and Haile (2014)

Market level data: Berry and Haile (2014)

Model

- Market characteristics $\chi_t = (x_t, p_t, \xi_t)$, x_t exogenous, p_t endogenous
- Random utilities with distribution $F_v(v_{i1t}, \ldots, v_{iJt} | \chi_t)$
- Shares

$$s_{jt} = \sigma_j(\chi_t) = \mathrm{P}(\arg\max_k v_{ikt} = j | \chi_t)$$

Rationale for setup

- Parametric models:
  - Logit random utility:

$$u_{ijt} = x_{jt}\beta - \alpha p_{jt} + \xi_{jt} + \epsilon_{ijt}$$

implies

$$x_{jt}^{(1)} + \tilde{\xi}_{jt} = \frac{1}{\beta^{(1)}}(\ln(s_{jt} - \ln(s_{0t})) + \frac{\alpha}{\beta^{(1)}} p_{jt} - \frac{1}{\beta^{(1)}} x_{jt}^{(-1)} \beta^{(-1)}$$

  - BLP implies:

$$x_{jt}^{(1)} + \tilde{\xi}_{jt} = \frac{1}{\bar{\theta}^{(1)}} \left(\delta_j(s_t, p_t, \theta) - x_{jt}^{(-1)} \bar{\theta}^{(-1)} \right)$$

- In each case:

$$x_{jt}^{(1)} + \xi_{jt} = \text{function of} s_t, p_t, x_t^{(-1)}$$

Assumptions

1. **Index restriction**: partition $x_{jt} = (x_{jt}^{(1)}, x_{jt}^{(2)})$, define

$$\delta_{jt} = x_{jt}^{(1)} + \xi_{jt}$$

then

$$F_v(\cdot|\chi_t) = F_v(\cdot|\delta_t, x_t^{(2)}, p_t)$$

2. **Connected substitutes**: (see Berry, Gandhi, and Haile (2013))
   1. $\sigma_k(\delta_t, p_t, x_t^{(2)})$ is nonincreasing in δ_{jt} and $-p_{jt}$ for $k \neq j$
   2. Among any subset $\mathcal{K} \subseteq \mathcal{J}$, $\exists j, k \in \mathcal{K}$ and $j \notin \mathcal{K}$ s.t. $\sigma_k(\delta_t, p_t, x_t^{(2)})$ is decreasing in δ_{jt} and $-p_{jt}$
3. **IV exogeneity** $\mathrm{E}[\xi_{jt}|z_t, x_t] = 0$
4. **Rank condition / completeness**: $\forall B(s_t, p_t)$ if $\mathrm{E}[B(s_t, p_t)|z_t, x_t] = 0$ a.s., then $B(s_t, p_t) = 0$ a.s.

Identification of demand 1

- If 1-4, then ξ_{jt} and $\sigma_j(\chi_t)$ are identified [Theorem 1]
- Connected substitute (1) $\Rightarrow$ demand invertible

$$\delta_{jt} = \sigma^{-1}(s_t, p_t)$$

- IV & rank condition:

$$\mathrm{E}[x_t^{(1)} - \sigma^{-1}(s_t, p_t) | z, x] = 0$$

Implications for instruments 1

- 2J endogenous variables (s, p), so need at least $2J$ instruments
  - J instruments from exogenous characteristic $x^{(1)}$
  - Need J instruments that shift price and are excluded from demand
  - Need variation in price and variation in share conditional on price
- Types of instruments that have been used:
  - "BLP instruments" = characteristics of competing products
    - Needed, but not sufficient (without more restrictions)
  - Cost shifters:
    - "Hausman instruments" = price of same good in other markets
    - Consumer characteristics in nearby markets (e.g. Fan (2013))

Implications for instruments 2

- Functional form restrictions can reduce needed number of instruments, e.g. if

$$\delta_{jt} = x_{jt}^{(1)} - \alpha p_{jt} + \xi_{jt}$$

then only need 1 instrument for price

Marginal costs

- More assumptions:
  1. $\sigma_j(\delta_t, p_t)$ continuously differentiable wrt p_t
  2. Known form of competition so know ψ_j such that

$$mc_{jt} = \psi_j(s_t, M_t, \{\frac{\partial \sigma}{\partial p}\}, p_t)$$

- Then mc_{jt} is identified [Theorem 3]
- If want to do counterfactuals that change quantities, need to know marginal cost function, not just $mc_{jt} =$ marginal cost at observed quantity
  - If $mc_{jt} = \tilde{c}_j(Q_{jt}, w_{jt}) + \omega_{jt}$ and have instruments y_{jt} such that $\mathrm{E}[\omega|w, y] = 0$, then c_j and ω_{jt} identified [Theorem 4]
- If ψ_j is unknown, then with stronger assumptions about marginal cost function and cost instruments, can still identify ω_{jt} [Theorem 5]
  - Can then test for different forms of ψ_j [Theorem 9]
  - Require index restriction on c_j,

$$mc_{jt} = c_j(Q_{jt}, w_{jt}^{(1)}\gamma_j + \omega_{jt}, w_{jt}^{(2)})$$

Simultaneous equations approach 1

- Use demand and supply equations together and can replace completeness conditions with regularity conditions about demand and supply equations
- Will need no external instruments (but do need exclusions)
- Index assumptions imply

$$\underbrace{x_{jt} + \xi_{jt}}_{=\delta_{jt}} = \sigma^{-1}(s_t, p_t) \quad (1)$$

$$\underbrace{w_{jt} + \omega_{jt}}_{=\kappa_{jt}} = \pi^{-1}(s_t, p_t) \quad (2)$$

- Assume $x, w \perp\!\!\!\perp \xi, \omega$ and $\text{supp}(x, w) = \mathbb{R}^{2J}$

Simultaneous equations approach 2

- Assume conditions (including for each δ, κ there is unique s, p) such that can make change of variables so

$$f_{s,p}(s_t, p_t | x_t, w_t) = \\ = f_{\xi,\omega}\left(\sigma^{-1}(s_t, p_t) - x_t, \pi^{-1}(s_t, p_t) - w_t\right) |\mathcal{J}(s_t, p_t)| \tag{3}$$

  where $\mathcal{J}(s,p)$ = Jacobian wrt s, p of (1) and (2)
- Then can identify ξ, σ, ω [Theorems 6 and 7]
  - Integrating (3) wrt x, w identifies $|\mathcal{J}(s,p)|$, then dividing gives $f_{\xi,\omega}$
  - Integrating $f_{\xi,\omega}$ give F_ξ
  - Location normalization: assume $\sigma^{-1}(s^0, p^0) - x^0) = 0$, then know $F_\xi(0)$

Simultaneous equations approach 3

- For other s, p, can find x^* such that $F_\xi(\sigma^{-1}(s,p) - x^*) = F_\xi(0)$, i.e. $\sigma^{-1}(s,p) = x^*$, so σ^{-1} identified
- (1) identifies ξ_{jt}
- Same argument for ω

Micro data: Berry and Haile (2009)

Model 1

- Consumer i, markets t, products $j \in \mathcal{J}_t$
- Consumer observations $z_{it} = (z_{i1t}, \ldots, z_{iJ_t t})$
- Observed product characteristics $x_t = (x_{1t}, \ldots, x_{J_t t})$ (includes prices, may include product dummies)
- Scalar product unobservable $\xi_{jt}(z_{it})$
- Random utility function $u_{it} : \mathcal{X} \to \mathbb{R}$
    - Formally, there's a probability space, $(\Omega, \mathcal{F}, \mathbb{P})$ and

$$v_{ijt} = u_{it}(x_{jt}, \xi_{jt}(z_{it}), z_{ijt}) = u(x_{jt}, \xi_{jt}(z_{it}), z_{ijt}, \omega_{it})$$

      where $\omega_{it} \in \Omega$ and u is measurable in ω_{it} with $\omega_{it} \perp\!\!\!\perp (x_{jt}, z_{it}, \xi_{jt}(z_{it}))$
    - E.g. random coefficients

$$u(x_{jt}, \xi_{jt}(z_{it}), z_{ijt}, \omega_{it}) = x_{jt}\theta_{it} + z_{ijt}\gamma + \xi_{jt} + \epsilon_{ijt}$$

$$\omega_{it} = (\theta_{it}, \epsilon_{i1t}, \ldots, \epsilon_{iJ_t t})$$

Model 2

- Allows distribution of θ and ϵ to depend on z, x, ξ

- Special regressor: $z_{ijt}^{(1)} \in \mathbb{R}$ s.t.

$$v_{ijt} = \phi(\omega_{it}) z_{ijt}^{(1)} + \tilde{\mu}\left(x_{jt}, \xi_{jt}(z_{it}^{(2)}), z_{ijt}^{(2)}, \omega_{it}\right)$$

  Restrictions:

  1. invariance of $\xi_{jt}(z_{it})$ to $z_{it}^{(1)}$
  2. Additive separability
  3. $\tilde{\mu}$ monotonic in ξ_{jt}

  Identifies mapping from choice probabilities to utilities

- Henceforth, all argument conditional on $z_{it}^{(2)}$, so leave out of notation
- Normalizations
  - $\xi_{jt} \sim U(0, 1)$
  - Location of utilities

$$v_{i0t} = 0$$

Model 3

- Scale of utilities $\phi_{it} = 1$,

$$v_{ijt} = z_{ijt}^{(1)} + \underbrace{\mu(x_{jt}, \xi_{jt}(z_{it}^{(2)}), z_{ijt}^{(2)}, \omega_{it})}_{\equiv \mu_j(x_{jt}, \xi_{jt}, \omega_{it})}$$

Identification definition 1

- Data: $(t, y_{it}, \{x_{jt}, \tilde{w}_{jt}, z_{ijt}\}_{j \in \mathcal{J}_t})$
- Conditional choice probabilities

$$p_{ijt} = \mathrm{P}\left(y_{it} = j | t, \{x_{kt}, \tilde{w}_{kt}, z_{ikt}\}_{k \in \mathcal{J}_t}\right)$$

- Full identification of random utility model: means that for any given conditional choice probabilities there is a unique distribution of ξ_{jt} (given $z_{ijt}^{(2)}$) and a conditional distribution of utilities $\{v_{ijt}\}_{j \in \mathcal{J}_t}$ given $\{x_{jt}, \tilde{w}_{jt}, z_{ijt}\}_{j \in \mathcal{J}_t}$ that generates the given choice probabilities
- Identification of demand: for any conditional choice probabilities there is a unique distribution of ξ_{jt} (given $z_{ijt}^{(2)}$) and unique structural choice probabilities

$$\rho_j\left(\{x_{jt}, \xi_{jt}, z_{ijt}\}_{j \in \mathcal{J}_t}\right) = \mathrm{P}\left(y_{it} = j | \{x_{kt}, \xi_{kt}, z_{ikt}\}_{k \in \mathcal{J}_t}\right)$$

that match the choice probabilities

Assumptions

1. Large support: $\operatorname{supp}\{z_{ijt}\}_{j\in\mathcal{J}_t}|\{x_{jt}\}_{j\in\mathcal{J}_t} = \mathbb{R}^{|\mathcal{J}_t|-1}$
2. Independence of instruments: $\xi_{jt} \perp\!\!\!\perp (w_{jt}, z_{ijt}) \forall j, t$
3. Rank condition: the quantile version of bounded completeness from Chernozhukov and Hansen (2005) applies to $D_j(x_{jt}, \xi_{jt})$

Identification proof 1

- $\mu_{ijt} = \mu(x_{jt}, \xi_{jt}(z_{it}^{(2)}), z_{ijt}^{(2)}, \omega_{it})$
- Large support:

$$\lim_{z_{ikt}^{(1)} \to -\infty \forall k \neq j} \mathrm{P}\left(y_{it} = j | t, \{x_{kt}, \tilde{w}_{kt}, z_{ikt}\}_{k \in \mathcal{J}_t}\right) =$$

$$= \lim_{z_{ikt}^{(1)} \to -\infty \forall k \neq j} \mathrm{P}\begin{pmatrix} z_{ijt} + \mu_{ijt} \geq z_{ikt} + \mu_{ikt} \forall k \neq j \cap \\ \cap z_{ijt} + \mu_{ijt} \geq 0 | \\ | t, \{x_{kt}, \tilde{w}_{kt}, z_{ikt}\}_{k \in \mathcal{J}_t} \end{pmatrix}$$

$$= \mathrm{P}\left(z_{ijt} + \mu_{ijt} \geq 0 | t, \{x_{kt}, \tilde{w}_{kt}, z_{ikt}\}_{k \in \mathcal{J}_t}\right)$$

Identification proof 2

- Independence of z_{ijt} and (ξ_{jt}, ω_{it})

$$
\begin{aligned}
\mathrm{P}\left(z_{ijt} + \mu_{ijt} \geq 0 | t, \{x_{kt}, z_{ikt}\}_{k \in \mathcal{J}_t}\right) = & \mathrm{P}\left(z_{ijt} + \mu_{ijt} \geq 0 | t, x_{jt}, z_{ijt}\right) \\
= & 1 - F_{\mu_{ijt}|t}(-z_{ijt}|x_{jt}, t)
\end{aligned}
$$

averaging over x_{jt} identifies $F_{\mu_{ijt}|t}$, and so identifies conditional quantiles, e.g.

$$
\delta_{jt} \equiv \mathrm{median}[\mu_j(x_{jt}, \xi_{jt}, \omega_{it})|t]
$$

Define

$$
\mathrm{median}[\mu_j(x_{jt}, \xi_{jt}, \omega_{it})|x_{jt}, \xi_{jt}] = D_j(x_{jt}, \xi_{jt})
$$

- Independence of w_{jt} and ξ_{jt}, and $\xi_{jt} \sim U(0,1)$ implies that

$$
\mathrm{P}\left(\delta_{jt} \leq D_j(x_{jt}, \tau) | w_{jt}\right) = \tau
$$

Identification proof 3

- Nonparametric IV quantile regression of Chernozhukov and Hansen (2005) shows that with the bounded completeness condition D_j is unique identified, and so is $\xi_{jt} = D_j^{-1}(x_{jt}, \delta_{jt})$
- Joint distribution of μ from

$$p_{i0t} = \mathrm{P}(z_{ijt} + \mu_{ijt} \leq 0 \forall j \neq 0 | t, z_{it})$$

- $F_{\mu|t} = F_{\mu|x_t, z_{it}, \xi_t}$

Further remarks

- To summarize if (1)-(3) then the random utility model is identified
- If large support fails, then can still identify demand
- Identifying random utility model is not exactly the same as identifying random coefficients

$$v_{ijt} = x_{jt}\theta_{it} + z_{ijt}\gamma + \xi_{jt} + \epsilon_{ijt}$$

  - This paper identifies $F_{v|x,z,\xi}$, further conditions needed for distribution of θ, ϵ
  - Given results in this paper, we can treat v_{ijt} as observed and use standard results to identify distribution of coefficients (see conclusion of Berry and Haile (2009) for references)
  - Most economic quantities that we might care about depend on the random utility model not the random coefficients

Section 2

Bonnet, Galichon, and Shum (2017)

Bonnet, Galichon, and Shum (2017)

Yogurts choose consumers? identification of random utility models via two-sided matching

- Inversion of demand (market shares to mean utilities) for nonadditive models
- Connection between demand and two-sided matching
- Algorithm for computing identified set

Setup

- Products $\mathcal{J}_0$
- Nonadditive random utility model for consumers

$$u_{\epsilon_i} = \max_{j \in \mathcal{J}_0} \mathcal{U}_{\epsilon_i, j}(\delta_j)$$

  with $\epsilon_i \in \Omega$ with distribution P known
- $\mathcal{U}_{\epsilon_i}$ and distribution of ϵ_i known
- Econometrician observes market shares

$$s_j = \sigma_j(\delta) = \mathrm{P}\left(\mathcal{U}_{\epsilon j}(\delta_j) \geq \max_{j'} \mathcal{U}_{\epsilon j'}(\delta_j')\right)$$

- Will characterize the identified set for δ given s

$$\sigma^{-1}(s) = \{\delta \in \mathbb{R}^{|\mathcal{J}_0|} : \sigma(\delta) = s\}$$

- All results could be conditional on individual covariates

Examples

- Additive model:
$$\mathcal{U}_{\epsilon j}(\delta_j) = \delta_j + \epsilon_j$$
and could have $\delta_j = x_j \beta$ as further restriction
- Risk aversion
$$\mathcal{U}_{\epsilon j}(\delta_j) = \mathrm{E}_\eta \left[\frac{(\delta_j - p_j + \eta_j)^{1-\epsilon}}{1-\epsilon} \right]$$
- Income shock
$$\mathcal{U}_{\epsilon j}(\delta_j) = \max_x V(x) + \delta_j \; : \; x'p \leq y_j + \epsilon_i$$

Matching game

- Preferences and feasibility described by transfers :
  - $f_{\epsilon j}(u) =$ transfer needed by consumer ϵ for utility u when matched with product j (increasing function)
  - $g_{\epsilon j}(\delta) =$ transfer needed by product j to reach utility δ (increasing function)
- Equilibrium consists of :
  - Matching probibility distribution π on $\Omega \times \mathcal{J}_0$
  - Realized utilities u_ϵ and δ_j

  such that
  - No blocking pair : $f_{\epsilon j}(u_\epsilon) + g_{\epsilon j}(v_j) \geq 0$ for all ϵ, j
  - Feasibility : if $\pi(\epsilon, j) > 0$, then $f_{\epsilon j}(u_\epsilon) + g_{\epsilon j}(v_j) = 0$

Equivalence

Theorem 1 : the following are equivalent

1. $\delta \in \sigma^{-1}(s)$
2. In matching game with $f_{\epsilon j}(u) = u$ and $g_{\epsilon_j}(-\delta) = -\mathcal{U}_{\epsilon_j}(\delta)$
   Let $u_\epsilon = \max_{j \in \mathcal{J}_0} \mathcal{U}_{\epsilon,j}(\delta_j)$, and $v_j = -\delta_j$
   Then $\exists \pi$ such that $(\pi, u, -\delta)$ is an equilibrium

Implications

- $\sigma^{-1}(s)$ is a connected lattice
  - Partially order : $\delta \leq \delta'$ iff $\delta_j \leq \delta_j$ for all j
  - $\delta \wedge \delta' \equiv$ greatest lower bound = coordinate wise min, $\delta \vee \delta'$ exist

  $\Rightarrow$ Has minimal and maximal elements
- Minimal and maximal elements = equilibrium matchings most preferred by consumers and products, respectively
- Modified versions of algorithms from matching literature can compute min and max

Combining with BLP framework

- This paper identifies set of mean utilities, still need assumption about how mean utilities relate to product characteristics to identify demand over characteristics
- Assume $\delta_j = p_j \alpha + \xi_j$ (or similar)
- Use algorithm to compute δ_{min} and δ_{max}
- Identified set for α :

$$\{\alpha : \mathrm{E}[\delta - p_j \alpha | Z] = 0 \text{ for some } \delta \in [\delta_{min}, \delta_{max}]\}$$

Section 3

Other identification results

Other identification references

- Berry, Gandhi, and Haile (2013) show connected substitutes is sufficient for invertibility of demand
- Fox and Gandhi (2011) identification of demand for any dimension ξ
- Chiappori and Komunjer (2009) identification through conditional independence instead of special regressor
- Fox et al. (2012) shows random coefficients logit (without endogeneity) is identified

Section 4

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