Auctions

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References

- Reviews:
  - Hendricks and Porter (2007) (working paper version Hendricks and Porter (2000))
  - Reiss and Wolak (2007) section 8 relates auctions to other structural models
  - Athey and Haile (2007) focuses on identification
  - Klemperer (1999) and Klemperer (2004) for theory

Section 1

Introduction

Introduction 1

- Auctions widely used
  - Historical:
    - Babylonian wives
    - Greek mines & slaves
    - Roman war booty, debtors' goods, etc
  - Governments:
    - Finance: treasury bills, foreign exchange
    - Procurement
    - Privatization
    - Natural resources: oil, gas, fishing, timber, pollution, wireless spectrum
  - Private commerce:
    - Art
    - Houses
    - eBay
- Purpose of auctions: efficiently allocate goods and maximize seller revenue when there is asymmetric information about the value of the good

Introduction 2

- Rich theory of auctions - results depends on:
  - Nature of information & values: independent or affliated (correlated)
  - Single or multi-unit
  - Risk aversion
- Close connection between theoretic models and estimable empirical models
- Empirical work on auctions:
  - Rich environment allows us to estimate a lot (mainly bidders' values) under plausible assumptions
  - Informs auction design (Klemperer, 2002)
    - Private or common values
    - Independent or affiliated values
    - Collusion

Section 2

Theory

Auction theory

- Reference: Klemperer (2004) and references therein
- Very brief overview, will return to some of these issues as needed when we look at empirical papers
- Begin with simplest case: single unit auction with independent private values and risk neutrality
- Generalize to allow
    - Common values
    - Affliated values
    - Risk averse buyers/seller/both
    - Multi-unit

Values and information 1

- Use notation of Athey and Haile (2007)
- Bidders $i = 1, \ldots, n$
- Uncertain value of u_i
- Private information: scalar signal x_i
    - Assume $\mathrm{E}[u_i | x_i, x_{-i}]$ increasing in x_i
    - Usually normalize $\mathrm{E}[u_i | x_i] = x_i$
- Private values $\equiv \mathrm{E}[u_i | x_1, \ldots, x_n] = \mathrm{E}[u_i | x_i]$
- Common values $\equiv \mathrm{E}[u_i | x_1, \ldots, x_n]$ increasing in x_j for all i, j
- Pure common values $\equiv u_i = u_j \; \forall \; i, j$
- Independent private values (IPV) $\equiv x_i \perp\!\!\!\perp x_j$
- Affiliated private values $\sim$ non-independent private values
    - Affiliated means $f(x \vee y) f(x \wedge y) \geq f(x) f(y)$
    - Affiliated implies non-negatively correlated

Values and information 2

- Affiliated implies $\frac{f(x_i|x_j)}{f(x_i'|x_j)}$ for $x_i > x_i'$ is increasing in x_j (monotone likelihood ratio)

Common types of auctions 1

	First-price	Second-price
Open bid		Ascending/English
Sealed bid	Descending/Dutch	Vickrey

- Equivalence of these forms of auctions is for IPV single-unit auctions
- Bidding with risk neutral independent private values:
    - Open second price = ascending/English : bid x_i
    - Sealed first price = descending/Dutch: bids increasing in x_i

Revenue equivalence theorem 1

Theorem

In a single unit auction with risk neutral bidders, assume identically distributed independent values with the distribution of x_i strictly increasing and atomless. Then any auction mechanism in which

1. *the bidder with the highest x_i always wins, and*
2. *any bidder with the lowest-feasible signal expects zero surplus,*

yields the same expected revenue (and results in each bidder making the same expected payment as a function of her signal).

- Vickrey (1961), Myerson (1981), Rogers and Samuelson (1981)
- Empirical implications:
  - IPV is important – Do we believe it? Is is testable?

Revenue equivalence theorem 2

- If we believe IPV, is 2 satisfied?
  - Can meet 2 by choosing optimal reserve price
  - Optimal reserve price is independent of number of bidders

Relaxing assumptions 1

- Risk aversion:
  - Risk averse buyers
    - Revenue equivalence no longer holds
    - Expected revenue of sealed first price > expected revenue of sealed second price
  - Risk averse seller:
    - Revenue equivalence holds
    - Seller prefers sealed first price to sealed second price
- Affiliated private values
  - Optimal auction extracts full surplus, but is unlike any observed auction
  - Ascending > sealed second > sealed first
  - Optimal reserve price decreases with number of bidders
- Non-identically (non-symmetric) distributed x_i
  - Revenue maximizing auction not necessarily allocatively efficient (i.e. bidder with highest x_i might not win)
- Common values

Relaxing assumptions 2

- "Winner's curse" winner likely to have signal that is higher than value
- Endogenous entry of bidders
- Collusion
- Multi-unit: few general results on efficiency or revenue

Section 3

Identification

Identification 1

- Reference: Athey and Haile (2007)
- Identification: given the observed distribution of bids is there a unique distribution of information and values that could have generated it?
- Data:
  - Bidders $\mathcal{N} = \{1, ..., n\}$
  - Bids b_i, $b = (b_1, ..., b_n) \sim G_b(\cdot; \mathcal{N})$
  - If do not observe identities of bidders or only observe winning bid, then observe some order statistics

$$b^{(k:n)} = \text{ } k\text{th order statistic from } n \text{ observations}$$

  $b^{(k:n)}$ is the kth smallest bid

    - $b^{(n:n)}$ is maximum bid
    - In ascending auction $b^{(n:n)} = x^{(n-1:n)}$ is the winning bid

  $G_b^{(k:n)}$ is the CDF of the $b^{(k:n)}$

- Model:

Identification 2

- Uncertain value of u_i (will assume private values throughout this section)
  - Private information: scalar signal x_i
  - Normalization $\mathrm{E}[u_i|x_i] = x_i$, with risk neutrality, wlog can assume $u_i = x_i$
  - $x, u \sim F_{x,u}(\cdot; \mathcal{N})$ is common knowledge
  - Assumptions (e.g. IPV) restrict $F_{x,u} \in \mathbb{F}$
  - Bayesian Nash equilibrium with bidding strategies $\beta_i(x_i; \mathcal{N})$ gives mapping from model to data, $\gamma \in \Gamma$
- Formal definition of identification: $(\mathbb{F}, \Gamma)$ is identified if for all $F, \tilde{F} \in \mathbb{F}$ and $\gamma, \tilde{\gamma} \in \Gamma$, $\gamma(F) = \tilde{\gamma}(\tilde{F})$ implies $(F, \gamma) = (\tilde{F}, \tilde{\gamma})$

Ascending 1

- Private values: bidders exit at u_i
- Winning bid $= u^{(n-1):n}$
- u_i i.i.d. implies

$$F_u^{(k:n)}(s) = \frac{n!}{(n-k)!(k-1)!} \int_0^{F_u(s)} t^{k-1}(1-t)^{n-K} dt$$

  so observing only the number of bidders and the winning bid identifies F_u
- If u_i not identically distributed (i.e. "asymmetric independent private values"') then need to observe winning bid, identity of the winner, and set of bidders to identify F_u
- Without independence F_u is not identified (Athey and Haile, 2002)

Ascending 2

- Partial identification: Haile and Tamer (2003)
    - Assume:
        1. Bidders do not bid more than they are willing to pay.
        2. Bidders do not allow an opponent to win at a price they are willing to beat.
    - Can estimate bounds on F_u, see Haile and Tamer (2003)

First price sealed bid 1

- Key result due to Guerre, Perrigne, and Vuong (2000) (and refined by Li, Perrigne, and Vuong (2002), Campo, Perrigne, and Vuong (2003))
-

Theorem

1. *Suppose all bids are observed in first-price sealed-bid auctions. Then the symmetric affiliated private values model is identified.*
2. *Suppose all bids and bidder identitites are observed in first-price sealed-bid auctions. Then the asymmetric affiliated private values model is identified.*

- Proof: (with bids and identities observed)

First price sealed bid 2

- Bidder i's problem:

$$\max_b \mathrm{E}\left[u_i - b | x_i, \max_{j \in \mathcal{N}_{-i}} b_j \leq b\right] \mathrm{P}\left(\max_{j \in \mathcal{N}_{-i}} b_j \leq b | x_i\right)$$

- Value conditional on highest competing bid $= m_i$

$$\tilde{v}_i(x_i, m_i; \mathcal{N}) = \mathrm{E}\left[u_i | x_i, \max_{j \in \mathcal{N}_{-i}} b_j = m_i\right]$$

- Note that $G_{m_i|b_i}(m_i|b_i; \mathcal{N}) = \mathrm{P}\left(\max_{j \neq i} b_j \leq m_i | b_i, \mathcal{N}\right)$ is observed
- Increasing strategies implies conditioning on b_i is same as conditioning on x_i
- Rewrite bidder's problem:

$$\max_b \int_{-\infty}^{b} [\tilde{v}_i(x_i, m_i; \mathcal{N}) - b] g_{m_i|b_i}(m_i | \beta_i(x_i; \mathcal{N}); \mathcal{N}) dm_i$$

First price sealed bid 3

- First order condition:

$$\tilde{v}_i\left(x_i, \beta_i(x_i;\mathcal{N});\mathcal{N}\right) = b_i + \frac{G_{m_i|b_i}(b_i|b_i;\mathcal{N})}{g_{m_i|b_i}(b_i|b_i;\mathcal{N})} \equiv \xi_i(b_i;\mathcal{N})$$

  RHS observable
- Private values implies

$$\begin{aligned}\tilde{v}_i\left(x_i, \beta_i(x_i;\mathcal{N});\mathcal{N}\right) =& \mathrm{E}\left[u_i | x_i, \max_{j\in\mathcal{N}_{-i}} \beta_j(x_j;\mathcal{N}) = \beta_i(x_i;\mathcal{N})\right] \\ =& \mathrm{E}[u_i|x_i] \text{ (private values)} \\ =& x_i \text{ (normalization)} \\ =& u_i \text{ (risk neutrality)}\end{aligned}$$

- So F_u identified from observed distribution of $\xi_i(b_i;\mathcal{N})$

- Extensions:

First price sealed bid 4

- Incomplete bid data: often only observe $b^{(n:n)}$ results depend on independent or affiliated values, whether we observe identity of winner, see Athey and Haile (2007) section 3.3
- Unobserved heterogeneity: Krasnokutskaya (2011)
- Risk aversion: Guerre, Perrigne, and Vuong (2009)

Section 4

Estimation

Estimation 1

- Data: auction with $n = \underline{n}, \ldots, \overline{n}$ bidders, each observed $T_n \to \infty$ times
- Focus on first-price sealed-bid auctions
- Parametric: parametrically specify $F_{u,x}(\cdot; \theta)$, estimate θ by MLE or GMM
    - Examples: Paarsch (1992a), Donald and Paarsch (1993), Laffont, Ossard, and Vuong (1995) , Paarsch (1997)
    - Challenge: equilibrium bid function only has closed form for some distributional families
- 2-step semi parametric: estimate distribution of bids non-parametrically but make parametric assumption about distribution of values (or vice-versa)

Step 1 : use bids to estimate

$$\hat{u}_i = b_i + \frac{\hat{G}_{m_i|b_i}(b_i|b_i; \mathcal{N})}{\hat{g}_{m_i|b_i}(b_i|b_i; \mathcal{N})}$$

Estimation 2

Step 2 : estimate $F_u(\hat{u};\theta)$ by MLE or GMM

- Examples: Jofre-Bonet and Pesendorfer (2003), Athey, Levin, and Seira (2011), Campo et al. (2011)

- Non parametric:

Step 1 : use bids to estimate

$$\hat{u}_i = b_i + \frac{\hat{G}_{m|b}(b_i|b_i;\mathcal{N})}{\hat{g}_{m|b}(b_i|b_i;\mathcal{N})}$$

E.g. kernel estimates

- $\hat{G}_{m|b}(b|b;n) = \frac{1}{nT_n h_G}\sum_{i=1}^{n}\sum_{t=1}^{T} K\left(\frac{b-b_{it}}{h_G}\right) 1\{m_{it} < b, n_t = n\}$
- $\hat{g}_{m|b}(b|b;n) = \frac{1}{nT_n h_g^2}\sum_{i=1}^{n}\sum_{t=1}^{T} K\left(\frac{b-b_{it}}{h_g}, \frac{b-m_{it}}{h_g}\right)$

Step 2 : Estimate F_u

- E.g. kernel estimate

$$\hat{f}_u(u_1,\ldots,u_n) = \frac{1}{T_n h_f^n}\sum_{t=1}^{T} K_f\left(\frac{u_1-\hat{u}_{1t}}{h_f},\ldots,\frac{u_n-\hat{u}_{nt}}{h_f}\right) 1\{n_t = n\}$$

Estimation 3

- Data-driven choice of bandwidth is an open question
- Inference for $\hat{f}_u$ is tricky because $\hat{u}_{it}$ non parametrically estimated
  - Asymptotic distribution of $\hat{f}_u$ is not known
  - Marmer and Shneyerov (2012) gives alternative non parametric estimator and proves asymptotic normality

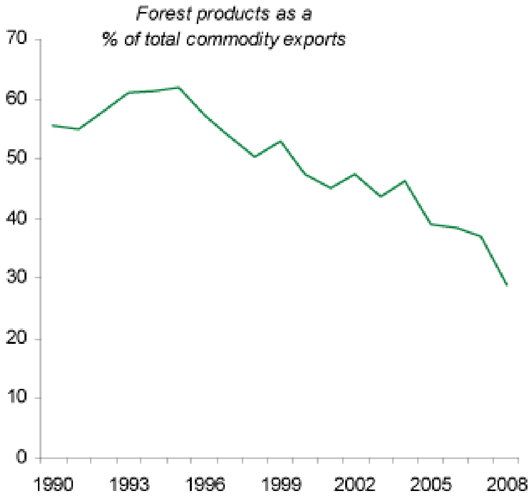
Section 5

Applications

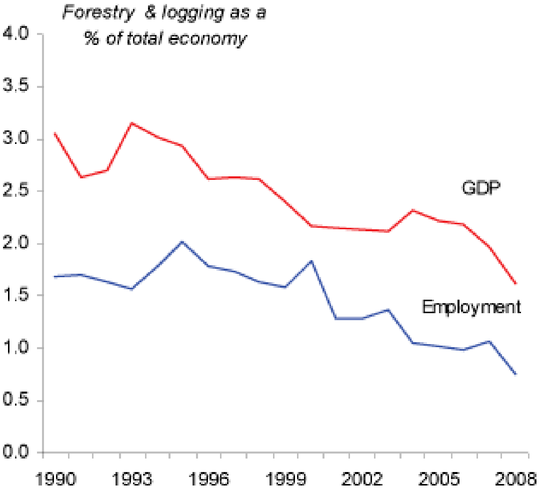
Applications: timber auctions

- In many countries (U.S., Canada, France, Russia, etc) auctions are used to allocate logging rights on government owned land
- Variation across countries and over time in auction format
- Trade dispute between U.S. and Canada
- Foresty $\approx$ 30% of BC exports and 2% of BC GDP in 2009

Foresty in BC



Foresty in BC



Questions

- Modeling:
  - Common or private values (e.g. Paarsch (1992b))
  - Resale (e.g. Haile (2001))
  - Risk aversion (e.g. Lu and Perrigne (2008))
- Auction design:
  - Reserve price (e.g. Paarsch (1997), Haile and Tamer (2003))
  - Format (e.g. Athey, Levin, and Seira (2011))
- Collusion:
  - E.g. Baldwin, Marshall, and Richard (1997), List, Millimet, and Price (2007), Price (2008)

Paarsch (1997)

- BC timber auctions 1984-1987
  - Small Business Forest Enterprise Program (SBFEP) auctions rights to timber on Crown land to independent loggers
  - Combination of ascending (English) and first-price sealed-bid auctions (choice between seems to be random)
  - Only uses ascending auctions in estimation
- Main question: what is the optimal reserve price?

Paarsch (1997) - method

- Method:
    - Estimate IPV model of ascending timber auction
    - Use model estimates and assumptions about Crown valuation of timber to calculate optimal reserve price

Paarsch (1997) - issues

- Issues:
  - Why is IPV a good assumption?
    - Forest service provides information about common component of value: volume and type of timber, terrain, roads
    - Private information from inspection, idiosyncratic costs (labor, capital, transportation)
    - Knowledge of others' bids unlikely to provide any information about each private value
  - Auctions have a reserve price, so observed bids are a selected sample of valuations
    - Parametrically specify distributions and estimate by MLE

Paarsch (1997) - model

- Valuations:

$$v = \left((\sum_{j=1}^{k} p_j \lambda_j) - a \right) q$$

- j indexes species, prices p_j, portions λ_j
- Harvesting cost $a \sim F(a)$

$$a = \gamma_{q1} + \gamma_{q2} q + \gamma_{q3} q^2 + \gamma_{q0} q^{-1} + \gamma_{qd} d$$

with $F_\gamma(c) = 1 - \exp(-\delta_1 c^{\delta_2})$

Table 4
Sample descriptive statistics – English auctions. Sample size = 129, January 1987 CPI = 1.0

Variable	Mean	SD	Minimum	Maximum
Winning Bonus Bid	6.89	7.01	0.00	28.16
'Average' upset	2.39	1.59	0.30	10.07
'Average' stumpage	9.29	7.36	0.30	31.87
'Average' price	46.89	· 6.69	34.14	67.49
Actual bidders	3.29	2.00	1.00	9.00
Potential bidders	92.39	31.88	27.00	185.00
Total cruised volume	10140.04	9720.55	130.00	53300.00
Conversion factor[a]	126.75	91.95	0.00	217.10
Haul distance	37.80	28.37	1.00	136.00

[a] Conversion factors apply only to interior sales. Zeros apply to coastal sales, of which there are 44.

Table 2
Maximum likelihood estimates: Weibull γ_{ql} specification

Specification	(4.5a)	(4.5b)	(4.6a)	(4.6b)
γ_0	3891.8712	3786.8913	3762.3312	3973.1016
	(859.1012)	(795.0515)	(1802.1011)	(1581.2231)
γ_{q2}	−0.0002	−0.0002	−0.0001	−0.0001
	(0.0002)	(0.0001)	(0.0002)	(0.0001)
γ_{q3}	0.0000	0	0.0000	0
	(0.0000)	—	(0.0000)	—
γ_{dq}	−0.1132	−0.1139	0.0502	0.0505
	(0.2306)	(0.2307)	(0.0788)	(0.0735)
δ_1	0.3686	0.3661	6.9335	6.7643
	(0.0432)	(0.0428)	(1.6236)	(1.6864)
δ_2	3.3230	3.3524	4.3119	4.0827
	(0.1748)	(0.1551)	(0.6511)	(0.4999)
LLF	−1319.1231	−1319.1875	372.8813	372.7174

The estimates for δ_1 and δ_2 are for cost in hundreds of dollars. White (1982) standard errors are presented in parentheses beneath each estimate.

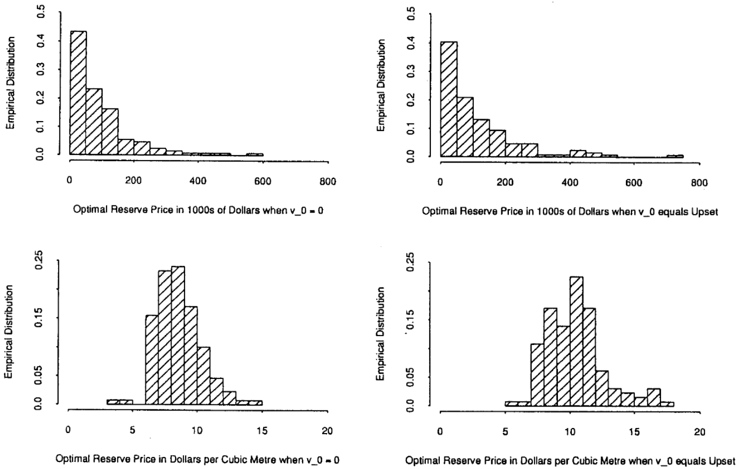


Fig. 3. Histograms of optimal Reserve price, per site and per cubic metre of timber Harvested.

Paarsch (1997) - results

- Results:
  - Optimal reserve price > observed reserve price
  - \$2.39 per cubic metre = observed reserve price
  - \$8.59 per cubic metre = optimal reserve if government value of timber is 0
  - \$10.43 per cubic metre = optimal reserve if government value of timber is observed reserve price

Athey, Levin, and Seira (2011)

- Compares open and sealed bid timber auctions
    - Revenue, welfare, collusion
- Data: 1982-1990 Idaho-Montana border and California
- Basic data facts:
    - Sealed bids induce more participation by small firms (loggers)
    - Large firms (mills) equally likely to enter either
    - Sealed bid auctions more likely to be won by loggers
    - Winning bids 10% higher in sealed bid auctions
- Construct model of auction participation to explain basic findings

Athey, Levin, and Seira (2011) - model

- Model features:
  - Participation cost of acquiring information
  - Heterogeneous value distributions
  - Possible collusion
- Model properties:
  - Sealed bid auctions favor weaker bidders
  - No clear implication for revenue (depends on model primitives)

Athey, Levin, and Seira (2011) - estimation

- Estimation of model:
  - Use sealed bid auctions and Guerre, Perrigne, and Vuong (2000) method
  - Take estimates and use to predict open auctions (in model with & without collusion) and compare with data

Athey, Levin, and Seira (2011) - results

- Some evidence of collusion by mills in Idaho-Montana border region (predictions of competitive model do not fit as well)
- Welfare calculations:
  - Sealed bid auctions raise more revenue and distort the allocation away from efficiency and in favor of loggers, but the effects are small (less than 1%)
  - Mild degree of cooperative bidding by the mills at open auctions—the behavioral assumption most consistent with the observed outcomes in the Northern forests—results in much more substantial revenue differences (on the order of 5–10%)

Haile (2001)

- Auction followed by resale opportunity
  - Seller effect: auction winner has option to sell the contract, so value of winning higher
  - Buyer effect: auction loser could buy later, so value of winning lower
  - More bidders ⇒ more competition among buyers in resale market ⇒ higher seller effect, lower buyer effect
  - Private use value, but endogenous common willingness to pay from resale value

Haile (2001) - data

- Context & data: U.S. Forest Service timber auctions
  - English auctions
  - Bid on price per-unit, pay based on amount harvested
  - Resale $\approx$ subcontracting of harvesting and/or milling, some transfers
  - 1981 legislative changes affecting resale
    - Pre-1981: on average 55 months to harvests
    - Post-1981: 33 months (so less motive to subcontract), transfers mostly forbidden
    - Data 1974-1989

Haile (2001) - model

- Theoretic Model:
  1. Initial auction by Forest Service
  2. All bidders learn use values, winner re-auctions
  - Willingness to pay in first stage $\geq$ use value because of resale option
  - Resale option $\Rightarrow$ other players' bids in first stage affect willingness to pay
  - Key result: each bidders' willingness to pay in first stage increases with the number of bidders

Haile (2001) - econometric specification

- Econometric specification:
  - Oral English auction, so only observe winning bid = 2nd highest willingness to pay

$$\log b_t^{(n:n)} = \underbrace{w_t\theta + h_t}_{\text{common use value}} + \underbrace{\Omega_{2t}}_{\text{info from other bids}} + \underbrace{\epsilon_t^{(n-1:n)}}_{\text{private value}}$$

  - Instruments z_t independent of h, Ω, ϵ, so

$$\mathrm{E}[\log b_t^{(n:n)} - w_t\theta | z_t] = \mathrm{E}[\epsilon_t^{(n-1:n)} | n_t]$$

  - Assume $\epsilon \sim N(0, \sigma_t^2)$
  - w_t = auction characteristics, number of bidders
  - z_t = auction characteristics, number of nearby
  - Estimate by GMM

Haile (2001) - results

- Results:
  - Estimate for full sample, separately for pre & post 1981
  - Significant difference in $\frac{\partial v}{\partial n}$ pre and post
    - $\frac{\partial v}{\partial n} \approx 50$ before 1981
    - $\frac{\partial v}{\partial n} \approx 20$ after 1981
  - Interpretation: resale important determinant of bids
  - Robust to: assumed distribution of ϵ, information in others' bids, region

Detecting collusion

- Baldwin, Marshall, and Richard (1997)
- List, Millimet, and Price (2007)
- Price (2008)

Baldwin, Marshall, and Richard (1997)

- Data: ascending auctions in U.S. Pacific NW 1975-1981
  - Allegations of collusion
- Question: was there collusion?
  - Collusion $\Rightarrow$ low prices
  - High supply (many auctions) $\Rightarrow$ low prices
- Method: estimate model with and without collusion and with and without supply

Baldwin, Marshall, and Richard (1997)

- Results: evidence of collusion
  - Adding collusion or supply increase likelihood substantially
  - After adding collusion, allowing supply effects as well does not increase likelihood
  - Loss in revenue from collusion: 7.9%

List, Millimet, and Price (2007)

- Question: is there collusion in BC timber auctions?
- Motivation: auctions are central to current resolution to US-Canada Softwood Lumber Dispute
  - Ongoing dispute since 1820s
    - Canada exports a lot of lumber to U.S.
    - Canadian forests 94% publicly owned, U.S. $\approx$ 27%
  - First-price sealed-bid auctions for some plots, use to estimate
$$\text{winning bid} = x\beta + e$$
  - Non-auctioned plots:
$$\text{price} = x\hat{\beta}$$
  - Collusion in auctions would distort prices in many plots

List, Millimet, and Price (2007)

- Data: 1996-2000 Small Business Forest Enterprise Program (SBFEP) auctions in BC
  - First-price sealed-bid
  - MoF announces upset rate (reserve price) and estimated volume of timber (NCV)
  - Bidders evaluate plot, submit bids
  - Identities and bids of all bidders revealed

List, Millimet, and Price (2007)

- Model:
  - Assume IPV η conditional on characteristics Z
  - Bidding function $b_i = \phi(Z_i, \eta_i)$
  - Collusion (vs no-collusion) implies:
    1. Cartel members make systematically lower bidders and have different bidding function than non-members
    2. Cartel member bids likely correlated conditional on Z

List, Millimet, and Price (2007)

- Empirical approach: infer treatment assignment given outcomes
  - Collusion indicator D_{it}
  - Bids:
  $$y_{it} = D_{it}[X_{it}\beta_1] + (1 - D_{it})[X_{it}\beta_0] + \epsilon_{it}$$
  - Collusion implies:
    - $\beta_1 \neq \beta_0$, and $X_{it}\beta_1 \leq X_{it}\beta_0$
    - $\text{Cov}(\epsilon_{it}, \epsilon_{jt} | D_i = 0) = 0$, but $\text{Cov}(\epsilon_{it}, \epsilon_{jt} | D_i = 1, D_j = 1) \neq 0$
- Problem: D_{it} is unobserved
  - Assume constant over time, $D_{it} = D_i$
  - Use proxy $\tilde{D}_{it}$ (geographic proximity of bidders)

List, Millimet, and Price (2007) - results

- Results:
  - Fixed effects: evidence of collusion
  - Proxy: mixed evidence (not always expected sign, limited statistical significance)
- Price (2008) focuses on geographic proximity, finds "Firms located within the same town as one another submit bids that are on average 12.77 percent below predicted levels. As the distance between firms increases, the difference between estimated and predicted bids declines. In fact, there is no discernable difference in the bids submitted by suspected cartel pairs located 21–100 miles from each other and the control group of bids submitted in auctions that are assumed competitive."

Comparison

- Baldwin, Marshall, and Richard (1997)
  - Highly structured model - strong behavioral and parametric assumptions
  - More precise results
  - Estimates of magnitude
- List, Millimet, and Price (2007)
  - Less structured model - weaker behavioral assumptions, no distributional assumptions, so perhaps more credible
  - Less precise results
  - No estimate of magnitude

Schurter (2017)

"Identification and Inference in First-Price Auctions with Collusion"

- Main idea : collusive bidders' and competitives bidders' strategies depend on competitiveness of auction in different ways
  - Under no collusion, GPV estimates of bidder valuations should be independent of exogenous shifts in auction competitiveness
    - Number of bidders, reserve price, etc
  - With collusion, GPV estimates of valuations will depend on auction competitiveness
- Construct test to identify which bidders are colluding
- Given set of collusive bidders, modification of GPV identifies distribution of valuations

Setting and data

- BC SBFEP 1996-2000
- Same as List, Millimet, and Price (2007)
- Instrument : reserve price
    - 1999 : change from reserve prices set based on appraised price of timber, a Ministry revenue target, and silviculture & development costs to reserve prices equal to 70% of appraised value

Inference with an incomplete model of English auctions — Haile and Tamer (2003)

- Motivation:
  - Idealized theoretical English auction: bid rises continuously, bidders hold down button until bid exceeds value
  - Actual English auctions: bidders call out bids
- Assume:
  1. Bidders do not bid more than they are willing to pay.
  2. Bidders do not allow an opponent to win at a price they are willing to beat.
- Identifies bounds on CDF of values

Model

- Bidders $i \in \{1, \ldots, M\}$
- Value V_i independently $\sim F_0$, support $[\underline{v}, \overline{v}]$
- Seller value v_0
- Minimum bid increment Δ
- Reserve price r
- N of M bidders participate
- Value distribution conditional on participating ($v \geq r$)

$$F(v) = \frac{F_0(v) - F_0(r)}{1 - F_0(r)}$$

- Bids B_j, order statistics $B_{j:n}$ with CDF $G_{j:n}$

Upper bound

- Assumption 1, $b_i \leq v_i$, implies
  - $G(v) \geq F(v)$
  - $b_{i:n} \leq v_{i:n}$
  - $G_{i:n}(v) \geq F_{i:n}(v)$
- Identical distributions implies $F(v) = \phi(F_{i:n}(v); i, n)$
- Upper bound

$$F(v) \leq F_U(v) \equiv \min_{n,i} \phi(G_{i:n}(v) : i, n)$$

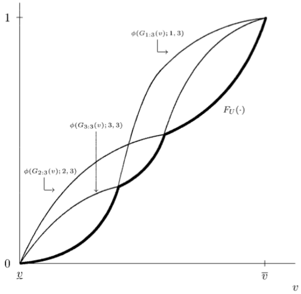


Fig. 1

Lower bound

- Assumption 2: $v_i \leq v_i^u \equiv \begin{cases} \bar{v} & b_i = b_{n:n} \\ b_{n:n} + \Delta & b_i < b_{n:n} \end{cases}$
- Implies $v_{n-1:n} \leq b_{n:n} + \Delta$, i.e. $F_{n-1:n}(v) \leq G^{\Delta}_{n:n}(v)$
- Lower bound:

$$F(v) \geq F_L(v) \equiv \max_n \phi(G^{\Delta}_{n:n}(v) : n-1, n)$$

Estimation 1

- Observe T auctions, subscript by t
- Plug in empirical CDFs

$$\hat{G}_{i:n}(v) = \frac{1}{T_n}\sum_{t=1}^{T} \mathbf{1}[n_t = n, b_{i:n} \leq v]$$

$$\hat{G}^{\Delta}_{n:n}(v) = \frac{1}{T_n}\sum_{t=1}^{T} \mathbf{1}[n_t = n, b_{n:n} + \Delta_t \leq v]$$

$$\hat{F}_U(v) = \min_{n,i} \phi(\hat{G}_{i:n}(v) : i, n)$$

$$\hat{F}_L(v) = \max_{n} \phi(\hat{G}^{\Delta}_{n:n}(v) : n-1, n)$$

- Uniformly consistent as $T_n \to \infty$, $T_n/T \to \lambda_n \in (0,1)$

Estimation 2

- Large finite sample bias from min & max, so smooth instead

$$\mu(y_1, \ldots, y_J; \rho_T) = \sum_{j=1} y_j \frac{e^{y_j \rho_T}}{\sum_{k=1}^{J} e^{y_k \rho_T}}$$

$\min = \lim_{\rho_T \to -\infty}$ and $\max = \lim_{\rho_T \to \infty}$

  - $|\rho_T| / \log \sqrt{T} \to \infty$ for consistency
  - Inference is tricky: see Hirano and Porter (2012) and Chernozhukov, Lee, and Rosen (2013)

Optimal reserve price

- Assume $(p - v_0)[1 - F_0(p)]$ is strictly pseudo-concave in p
- Optimal reserve price solves (assuming observed $r < p^*$

$$p^* \in \arg\max_p (p - v_0)[1 - F(p)]$$

- Let

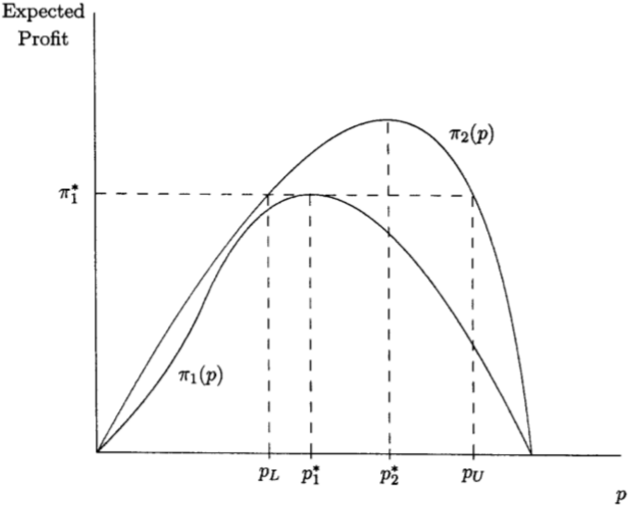
$$\pi_1(p) = (p - v_0)[1 - F_U(p)]$$
$$\pi_2(p) = (p - v_0)[1 - F_L(p)]$$

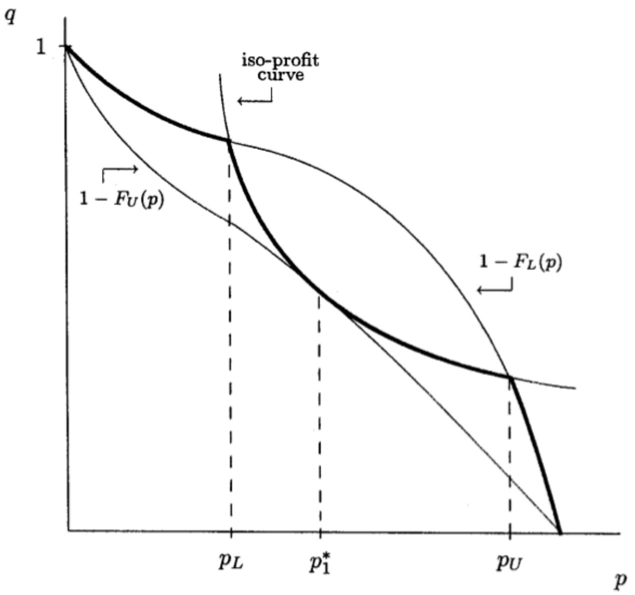
note:

$$\pi_1(p) \leq (p - v_0)[1 - F(p)] \leq \pi_2(p)$$

- Bounds:

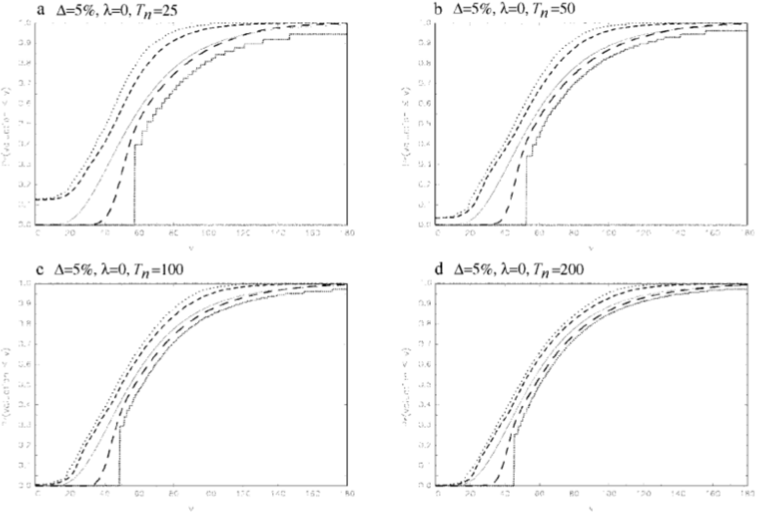
$$p_L \equiv \sup\{p < \arg\max \pi_1(\tilde{p}) : \pi_2(p) < \sup \pi_1(\tilde{p})\}$$
$$p_U \equiv \inf\{p > \arg\max \pi_1(\tilde{p}) : \pi_2(p) < \sup \pi_1(\tilde{p})\}$$

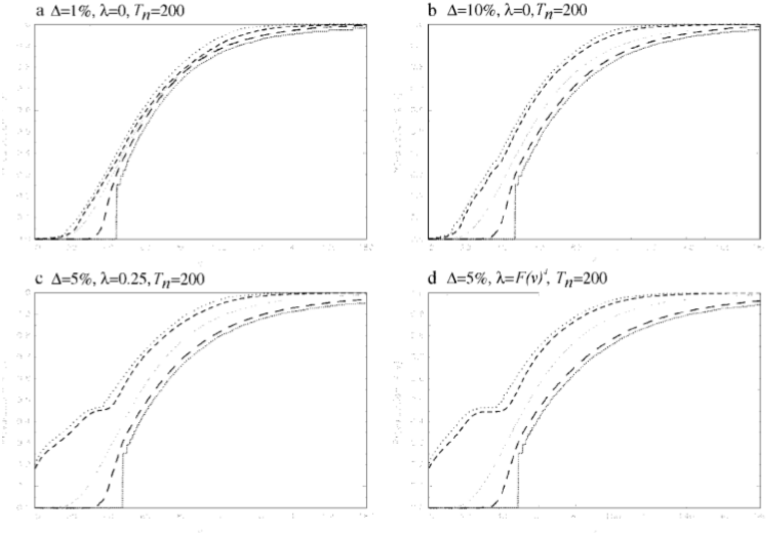

Expected Profit
$\pi_2(p)$
π_1^*
$\pi_1(p)$
p_L
p_1^*
p_2^*
p_U
p


q
1
iso-profit curve
1 − F_U(p)
1 − F_L(p)
p_L
p*_1
p_U
p

Simulations

- Bidders randomly selected, either drop out or raise bid by Δ
- With probability λ bid uniformly between $b + \Delta$ and valuation


a Δ=5%, λ=0, T_n=25
b Δ=5%, λ=0, T_n=50
c Δ=5%, λ=0, T_n=100
d Δ=5%, λ=0, T_n=200


a Δ=1%, λ=0, T_n=200
b Δ=10%, λ=0, T_n=200
c Δ=5%, λ=0.25, T_n=200
d Δ=5%, λ=$F(v)^4$, T_n=200

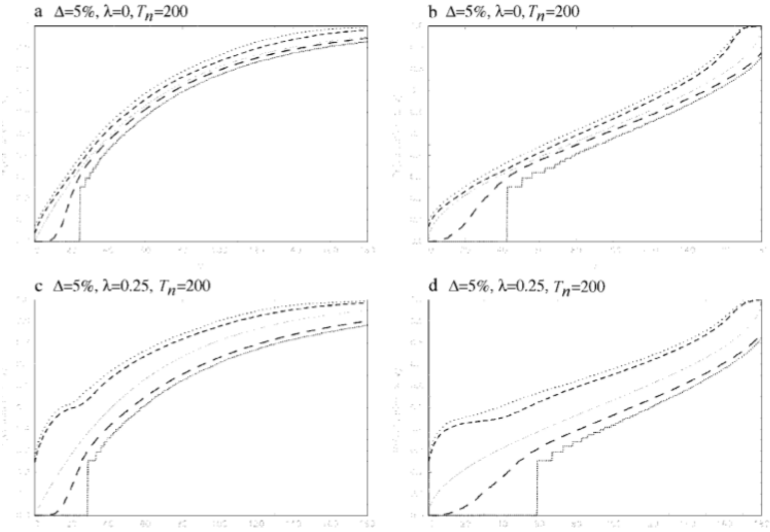

a Δ=5%, λ=0, T_n=200
b Δ=5%, λ=0, T_n=200
c Δ=5%, λ=0.25, T_n=200
d Δ=5%, λ=0.25, T_n=200

TABLE 1
MONTE CARLO SIMULATIONS: OPTIMAL RESERVE PRICE

Lognormal Parameters	$\mu=4,\ \sigma=.5$	$\mu=3,\ \sigma=1.0$	$\mu=5,\ \sigma=.25$
True p^*	42.1	27.2	112.6
$F(p^*)$	.30	.62	.13
Mean estimated bounds			
$[\hat{p}_L, \hat{p}_U]$	[28.4, 67.7]	[17.2, 50.3]	[82.9, 152.8]
90% confidence interval	[27.1, 70.3]	[15.2, 58.0]	[80.3, 157.3]

Forest Service timber auctions

- Data: Washington & Oregon, 1982-1990

TABLE 3
SUMMARY STATISTICS

	Mean	Standard Deviation	Minimum	Maximum
Number of bidders	5.7	3.0	2	12
Year	1985.2	2.6	1982	1990
Species concentration	.68	.23	.24	1.0
Manufacturing costs	190.3	43.0	56.7	286.5
Selling value	415.4	61.4	202.2	746.8
Harvesting cost	120.2	34.1	51.1	283.1
Six-month inventory*	1,364.4	376.5	286.4	2,084.3
Zone 2 dummy	.88		0	1

* In millions of board feet.

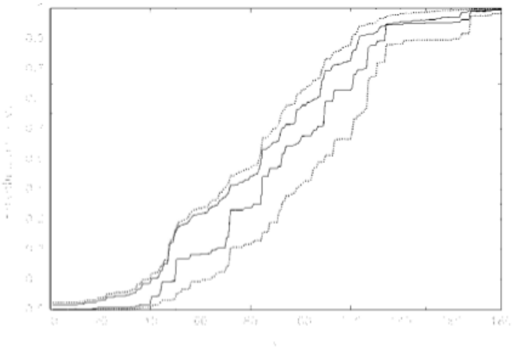


FIG. 10.—U.S. Forest Service timber auctions. Solid curves are estimated bounds, and dotted curves are bootstrap confidence bands.

TABLE 4
SIMULATED OUTCOMES WITH ALTERNATIVE RESERVE PRICES

	Reserve Price					
	p_L		$(p_L + p_U)/2$		p_U	
	Distribution of Valuations					
	F_L	F_U	F_L	F_U	F_L	F_U
Reserve price when $v_0 = \$20$	62.40		86.02		109.65	
Change in profit	6.96	−2.78	6.67	−2.74	1.74	−18.57
Pr(no bids)	.00	.02	.07	.12	.19	.41
Reserve price when $v_0 = \$40$	74.93		92.29		109.65	
Change in profit	7.64	−.61	7.61	−1.14	6.30	−10.04
Pr(no bids)	.03	.05	.11	.18	.19	.41
Reserve price when $v_0 = \$60$	85.67		103.39		121.11	
Change in profit	9.23	1.92	12.04	3.14	7.21	−6.05
Pr(no bids)	.07	.12	.15	.28	.35	.58
Reserve price when $v_0 = \$80$	98.20		112.34		126.48	
Change in profit	13.65	7.63	15.03	6.82	10.44	.96
Pr(no bids)	.13	.24	.28	.46	.46	.72
Reserve price when $v_0 = \$100$	111.09		122.54		134.00	
Change in profit	20.09	15.94	21.65	16.87	17.00	14.30
Pr(no bids)	.28	.45	.45	.60	.67	.80
Reserve price when $v_0 = \$120$	144.74		156.01		167.29	
Change in profit	32.06	31.31	33.72	31.64	31.56	28.87
Pr(no bids)	.84	.86	.84	.89	.88	.97

NOTE.—Profit and reserve price figures are given in 1983 dollars per MBF. See text for additional details.

TABLE 5

FOREST SERVICE TIMBER AUCTIONS: SEMIPARAMETRIC MODEL OF BIDDER VALUATIONS (Modified Minimum Distance Estimates)

	Interval Estimate	95% Bootstrapped Confidence Interval
Constant	[8.8, 15.12]	[2.33, 18.15]
Species concentration	[13.19, 13.64]	[11.14, 16.54]
Manufacturing cost	[−.85, −.81]	[−1.02, −.79]
Selling value	[.61, .71]	[.57, .96]
Harvesting cost	[−.54, −.51]	[−.59, −.48]
Six-month inventory	[−.026, −.025]	[−.030, −.021]
Number of bidders	[.81, 1.23]	[.66, 1.24]

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