Network formation

Paul Schrimpf

UBC
Vancouver School of Economics
565

April 3, 2025

Network formation

- Network formation: model of which nodes are connected
- Goal: parsimonious, tractable, and estimable model that matches features of observed networks
- Types of models
  - Random network models: specify $\mathrm{P}(i \& j$ connect$|$other connections, node characteristics$)$
  - Strategic network formation: specify payoffs $u_i(G, \cdot)$ and equilibrium concept (e.g. pairwise stability)
    - G is pairwise stable if for each link neither player would be better off without it, and there are no two players would both be better off by adding a link
    - Payoffs could come from a subsequent game on the network

Section 1

Hsieh, König, and Liu (2024)

“Endogenous Technology Spillovers in R&D Collaboration Networks” Hsieh, König, and Liu (2024)

- Previously titled “Network Formation with Local Complements and Global Substitutes: The Case of R&D Networks” Hsieh, König, and Liu (2017) (these slides originally based on this older version)
- Estimable model of R&D network formation and production
- Estimate for chemical firms
- Examine key firms and R&D collaboration subsidies

Model 1

- Profits

$$\pi_i(q,G) = \eta_i q_i - \nu q_i^2 - b q_i \sum_{j \neq i} q_j + \rho \sum \sum_{j=1}^{n} a_{ij} q_i q_j - \zeta d_i$$

  where
  - A is collaboration network
  - $\rho \geq 0$ local complementarity
  - $b > 0$ global substitutability
  - d_i = number of collaborators
- Potential function

$$\Phi(q,G) = \sum_{i=1}^{n} (\eta_i q_i - \nu q_i^2) - \frac{b}{2} \sum_i \sum_{j \neq i} q_i q_j + \frac{\rho}{2} \sum_i \sum_j a_{ij} q_i q_j - \zeta m$$

  is such that
  - $\Phi(q, G \oplus (i,j)) - \Phi(q,G) = \pi_i(q, G \oplus (i,j)) - \pi_i(q,G)$
  - $\Phi(q_i', q_{-i}, G) - \Phi(q,G) = \pi_i(q_i', q_{-i}, G) - \pi_i(q,G)$

Model 2

- Equilibrium:
  - "Natural" equilibrium concepts (e.g. pairwise stable links + Nash in q) difficult to characterize and typically not unique
  - Instead, introduce time and stochastic move opportunities, solve for unique stationary distribution of q, G

Network formation process 1

- Continuous time
- $q \in \mathcal{Q}$ a discrete and bounded set
- State of model $\omega_t = (q_t, G_t)$
- Move opportunities
  1. Quantity adjustment, arrival rate χ firm i chooses q to maximize profits with some error

$$P(\omega_{t+\Delta t} = (q, q_{-it}, G_t) | \omega_t = (q_t, G_t)) = \chi \frac{e^{\vartheta \pi_i (q, q_{-it}, G_t)}}{\int_Q e^{\vartheta \pi_i (q', q_{-it}, G_t)} dq'} \Delta t + o(\Delta$$

Network formation process 2

2. Link formation, arrival rate τ , (i, j) choose whether to link

$$P(\omega_{t+\Delta t} = (q_t, G_t \oplus (i,j)) | \omega_t = (q_t, G_t)) = \tau \frac{e^{\vartheta \Phi(q, G_t \oplus (i,j))}}{e^{\vartheta \phi(q, G_t \oplus (i,j))} + e^{\vartheta \phi(q, G_t)}} \Delta t$$

- Linking if $\pi_i(q, G_t \oplus (i,j)) - \pi_i(q, G_t) + \epsilon_{i,j,t} > 0$ and $\pi_j(q, G_t \oplus (i,j)) - \pi_j(q, G_t) + \epsilon_{i,j,t} > 0$
- Difference in π equal for i and j, and $= \Phi(q, G \oplus (i,j)) - \Phi(q, G)$

Network formation process 3

3. Link removal, arrival rate ξ, (i,j) choose whether to remove link

$$P(\omega_{t+\Delta t} = (q_t, G_t \ominus (i,j)) | \omega_t = (q_t, G_t)) = \xi \frac{e^{\vartheta \Phi(q, G_t \ominus (i,j))}}{e^{\vartheta \Phi(q, G_t \ominus (i,j))} + e^{\vartheta \Phi(q, G_t)}} \Delta$$

Stationary distribution

- Model is continuous time, discrete state Markov chain
- Stationary distribution:

$$\mu^{\vartheta}(q, G) = \frac{e^{\vartheta(\Phi(q,G) - m\log(\xi/\tau))}}{\sum_{G' \in \mathcal{G}^n} \int_{Q^n} e^{\vartheta(\Phi(q,G') - m' \log(\xi/\tau))} dq'}$$

  where
  - Potential function

$$\Phi(q, G) = \sum_{i=1}^{n} (\eta_i q_i - \nu q_i^2) - \frac{b}{2} \sum_i \sum_{j \neq i} q_i q_j + \frac{\rho}{2} \sum_i \sum_j a_{ij} q_i q_j - \zeta m$$

    is such that
    - $\Phi(q, G \oplus (i,j)) - \Phi(q, G) = \pi_i(q, G \oplus (i,j)) - \pi_i(q, G)$
    - $\Phi(q_i', q_{-i}, G) - \Phi(q, G) = \pi_i(q_i', q_{-i}, G) - \pi_i(q, G)$
- Propositions 2-3 characterize stationary distribution

Average degree and output

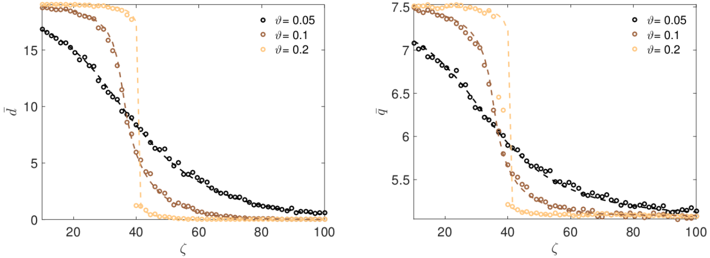


Figure 1: The average degree $\bar{d}$ (left panel) and the average output $\bar{q}$ (right panel) as a function of the linking cost ζ for varying values of $\vartheta \in \{0.05, 0.1, 0.2\}$ with $n = 20$ firms and $\tau = \xi = \chi = 1$, $\eta = 300$, $\rho = 1$, $b = 1$ and $\nu = 20$. Dashed lines indicate the theoretical predictions of Equations (10) and Equation (12) in Proposition 2, respectively.

Output and degree distributions

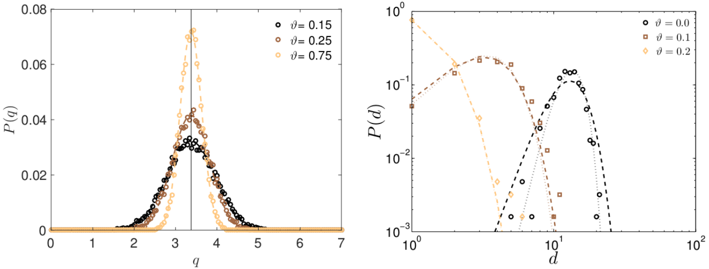


Figure 3: (Left panel) The stationary output distribution $P(q)$ for $n = 50$, $\eta = 150$, $b = 0.5$, $\nu = 10$, $\rho = 1$, $\vartheta \in \{0.1, 0.25, 0.75\}$ and $\zeta = 60$. Dashed lines indicate the normal distribution $\mathcal{N}(q^*, \sigma^2)$ of part(i) of Proposition 2). (Right panel) The stationary degree distribution $P(k)$ for the same parameter values. The dashed lines indicate the solution in Equation (11) of Proposition 2.

Output and degree distributions with Pareto productivity

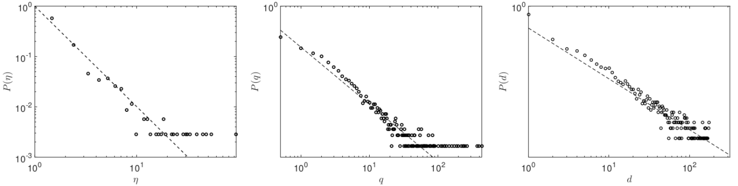


Figure 5: The distribution $P(\eta)$ of η following a Pareto distribution with exponent 2 (left panel), the resulting stationary output distribution $P(q)$ (middle panel) and the degree distribution $P(d)$ (right panel) from a numerical simulation of the stochastic process of Definition 1. Dashed lines indicate a power-law fit. Observe that $P(\eta)$ and $P(q)$ exhibit a power law tail with the same exponent, consistent with part (iii) of Proposition 3. The parameters used are $n = 350$, $\nu = 0.95$, $b = 0.75$, $\rho = 2$ and $\zeta = 75$.

Welfare

- Proposition 5: with homogenous firms, efficient G is either complete or empty depending on ζ (link cost)

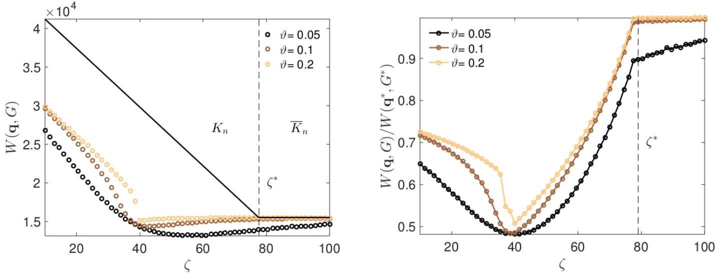


Figure 6: (Left panel) Welfare $W(\mathbf{q}, G)$ as a function of the linking cost ζ for varying values of $\vartheta \in \{0.05, 0.1, 0.2\}$ with $n = 20$ firms and $\tau = \xi = \chi = 1$, $\eta = 300$, $\rho = 1$, $b = 1$ and $\nu = 20$. The solid line indicates welfare in the efficient graph of Proposition 4 (which is either complete or empty). (Right panel) The ratio of welfare relative to welfare in the efficient graph.

Data

- CATI and SDC alliance database for R&D collaborations
- Compustat and Orbis for other firm information
- PATSTAT for patents

R&D Network

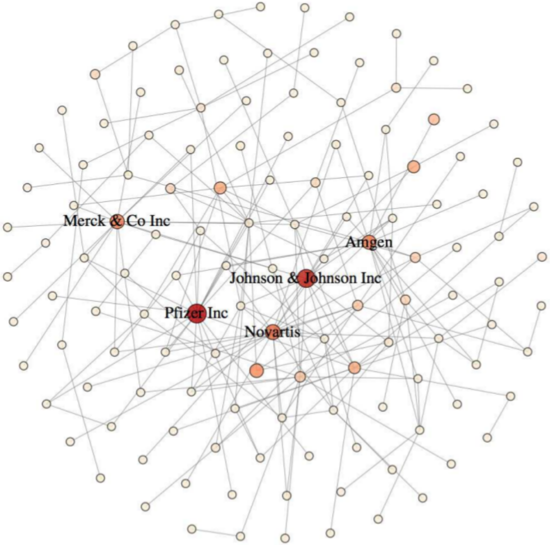


Figure 7: The largest connected component in the observed network of R&D collaborations for firms in the sector SIC-28 in the year 2006. The shade and size of a node indicates its R&D expenditures. The five largest firms in terms of their R&D expenditures are mentioned in the graph.

R&D Network

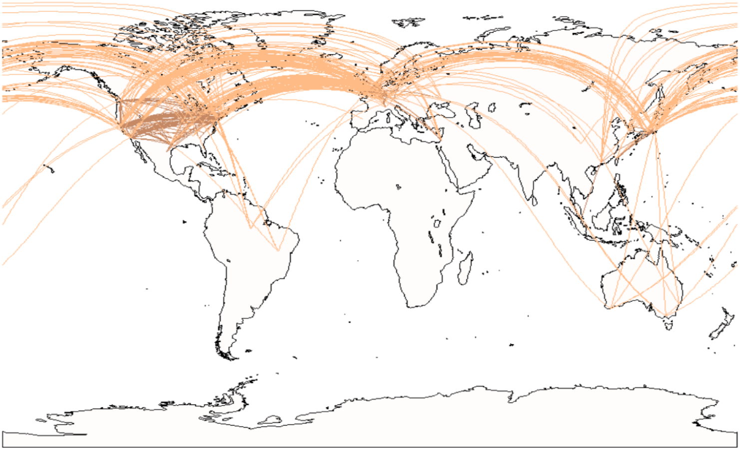


Figure F.8: The locations (at the city level) and collaborations of the firms in the combined CATI-SDC database.

Data

Table 1: Descriptive statistics.

		Log R&D Expenditure			Productivity			Log # of Patents		
Sample	# of firms	mean	min	max	mean	min	max	mean	min	max
Full	1201	9.6496	2.5210	15.2470	1.6171	0.0002	20.2452	4.9320	0.0000	11.8726
SIC-28	351	9.6416	3.2109	15.2470	1.3385	0.0002	10.1108	4.7711	0.0000	11.8014
SIC-281	27	9.5288	7.5464	11.2266	2.0951	0.8124	4.5133	6.9610	2.3026	9.9499
SIC-282	22	10.1250	7.5123	12.1022	2.4637	0.1667	5.7551	6.7015	2.9957	10.3031
SIC-283	259	9.4797	3.2109	15.2470	1.0326	0.0002	6.5232	4.1962	0.0000	10.8752
SIC-284	12	11.0216	8.7933	13.2439	1.4869	0.6021	2.6405	7.7903	3.9890	10.9748
SIC-285	5	11.0548	9.8144	13.2205	1.5160	1.2591	1.7099	8.4910	7.1325	10.3017
SIC-286	8	9.3278	6.0924	11.3144	3.9443	1.1249	10.1108	3.6924	0.6931	6.6174
SIC-287	8	8.8004	6.1510	12.8862	1.8069	0.0672	2.7076	3.9510	0.6931	10.6792
SIC-289	10	9.0683	6.2913	10.5094	1.5494	0.0760	2.9324	5.3012	0.6931	9.8807

Note: The logarithm of a firm's R&D expenditures (by thousand dollars) measures its R&D effort. A Firm's productivity is measured by the ratio of sales to employment. The logarithm of the number of patents is used as a control variable in the linking cost function [cf. e.g. Hanaki et al., 2010].

Data

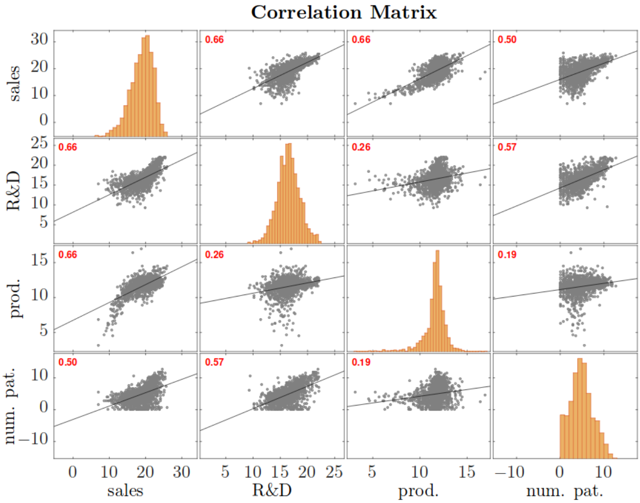
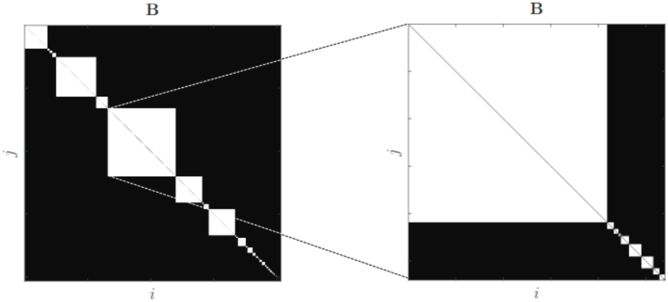


Figure F.5: Correlation scatter plot for sales, productivity, R&D expenditures and the patent stocks.

Competition Network



	20	33	87	37	73	35	38	36	28
20	0	0	1	1	0	0	1	0	3
33	0	1	0	4	0	2	1	3	1
87	1	0	0	0	1	2	4	3	14
37	1	4	0	17	5	2	7	2	1
73	0	0	1	5	4	17	7	17	6
35	0	2	2	2	17	9	5	26	2
38	1	1	4	7	7	5	6	13	25
36	0	3	3	2	17	26	13	29	3
28	3	1	14	1	6	2	25	3	141

	281	282	283	284	285	286	287	289
281	1	2	13	0	0	0	0	0
282	2	1	1	0	0	0	0	0
283	13	1	121	0	2	0	0	0
284	0	0	0	0	0	0	0	0
285	0	0	2	0	0	0	0	0
286	0	0	0	0	0	0	0	0
287	0	0	0	0	0	0	0	0
289	0	0	0	0	0	0	0	0

Figure 8: (Top left panel) The empirical competition matrix **B** across all 2-digit SIC sectors. The largest sector is the SIC-28 sector with 351 firms, which comprises 29.22% of all firms in the sample. (Top right panel) The empirical competition matrix **B** across all 3-digit SIC sectors within the SIC-28 sector. The largest sector is the SIC-283 "drugs" sector with 259 firms, which comprises 73.78% of all firms in the SIC-28 sector. (Bottom left panel) The number of R&D collaborations across all 2-digit SIC sectors. The sector SIC-28 has 141 within sector R&D collaborations. (Bottom right panel) The number of R&D collaborations within the sector SIC-28. The sector SIC-283 has 121 within sector R&D collaborations.

Estimation

- MLE using stationary distribution?

$$\mu^{\vartheta}(q,G) = \frac{e^{\vartheta(\Phi(q,G) - m\log(\xi/\tau))}}{\sum_{G'\in\mathcal{G}^n}\int_{\mathcal{Q}^n} e^{\vartheta(\Phi(q,G') - m'\log(\xi/\tau))} dq'}$$

  no, denominator too hard to compute
- Use MCMC instead
  - Still difficult, reports results from 3 different algorithms

Estimates

Table 2: Estimation results of the full sample and the SIC-28 sector

		Full sample	SIC-28 subsample		
		LP	LP	DMH	AEX
R&D Spillover	(ρ)	0.0355***	0.0386***	0.0408***	0.0458***
		(0.0008)	(0.0015)	(0.0021)	(0.0010)
Substitutability	(b)	0.0002***	0.0001**	0.0002***	0.0002***
		(0.0000)	(0.0001)	(0.0001)	(0.0000)
Prod.	(δ_1)	0.2099***	0.4475***	0.3769***	0.3787***
		(0.0127)	(0.0457)	(0.0509)	(0.0424)
Sector FE	(δ_2)	Yes	Yes	Yes	Yes
Linking Cost					
Constant	(γ_0)	13.1415***	13.2627***	14.4023***	14.3366***
		(0.1336)	(0.3507)	(1.1547)	(0.1180)
Same Sector	(γ_1)	-2.1458***	-1.9317***	-1.9648***	-1.8579***
		(0.1053)	(0.2551)	(0.5749)	(0.3972)
Same Country	(γ_2)	-0.8841***	-0.4186***	-0.6359*	-0.6555***
		(0.1030)	(0.1591)	(0.3903)	(0.1907)
Diff-in-Prod.	(γ_3)	0.0231	-1.2698***	-1.4300**	-1.3255***
		(0.0554)	(0.2937)	(0.6450)	(0.1436)
Diff-in-Prod. Sq.	(γ_4)	-0.0014	0.3276***	0.4023**	0.4505***
		(0.0044)	(0.0876)	(0.1910)	(0.0563)
Patents	(γ_5)	-0.0943***	-0.0783***	-0.1176**	-0.0410**
		(0.0053)	(0.0150)	(0.0562)	(0.0210)
Sample size		1,201		351	

Note: The dependent variable is log R&D expenditures. The parameters $\boldsymbol{\theta} = (\rho, b, \boldsymbol{\delta}^\top, \boldsymbol{\gamma}^\top, \varkappa)$ correspond to Equation (24), where $\psi_{ij} = \boldsymbol{\gamma}^\top \mathbf{c}_{ij}$ and $\eta_i = \mathbf{X}_i \boldsymbol{\delta}$ (cf. Section 3.2). We make 50,000 MCMC draws where we drop the first 2,000 draws during a burn-in phase and keep every 20th of the remaining draws to calculate the posterior mean (as point estimates) and posterior standard deviation (shown in parenthesis). All cases pass the convergence diagnostics provided by Geweke [1992] and Raftery and Lewis [1992]. The MCMC draws for ρ and b are shown in

Patent Similarity

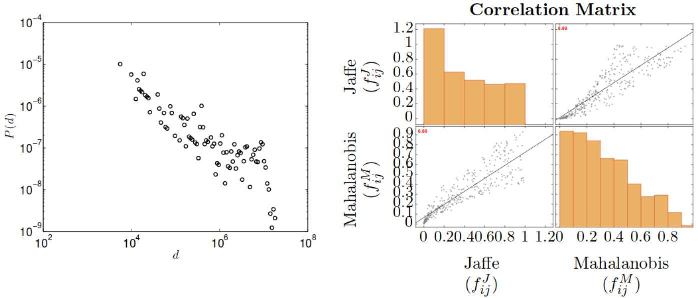


Figure F.9: (Left panel) The distance distribution, $P(d)$, across collaborating firms in the combined CATI-SDC database. (Right panel) Correlation plot for the Jaffe (f_{ij}^J) and the Mahalanobis (f_{ij}^M) technology proximity metrics across pairs of firms $1 \leq i, j \leq n$.

Heterogeneous spillovers

Table 3: Homogeneous versus heterogeneous spillovers

		Homogeneous		Jaffe		Mahalanobis	
		DMH	Logit	DMH	Logit	DMH	Logit
R&D Spillover	(ρ)	0.0396***	0.0356***	0.0524***	0.0070	0.0275***	0.0038**
		(0.0019)	(0.0030)	(0.0090)	(0.0042)	(0.0042)	(0.0019)
Substitutability	(b)	0.0002***	-	0.0001***	-	0.0001***	-
		(0.0001)	-	(0.0001)	-	(0.0001)	-
Prod.	(δ_1)	0.3696***	-	0.4367***	-	0.4372***	-
		(0.0526)		(0.0556)		(0.0612)	
Sector FE	(δ_2)	Yes	-	Yes	-	Yes	-
Linking Cost							
Constant	(γ_0)	13.5645***	12.8064***	13.5182***	11.4667***	14.3226***	11.4501***
		(0.6067)	(0.5075)	(0.2966)	(0.4764)	(0.5195)	(0.4859)
Same Sector	(γ_1)	-2.0559***	-1.7129***	-1.8892***	-2.0271***	-2.8818***	-2.0253***
		(0.4247)	(0.2681)	(0.3261)	(0.2547)	(0.7106)	(0.2609)
Same Country	(γ_2)	-0.3782	-0.3677**	-0.6871***	-0.4679***	-0.9134***	-0.4674***
		(0.3267)	(0.1781)	(0.3082)	(0.1740)	(0.3905)	(0.1669)
Diff-in-Prod.	(γ_3)	-0.8575*	-1.2679***	-3.3302***	-1.3288***	-3.1080***	-1.3145***
		(0.3881)	(0.3116)	(0.4379)	(0.2981)	(0.6717)	(0.3106)
Diff-in-Prod. Sq.	(γ_4)	0.2655**	0.3046**	0.9665***	0.3187***	0.9984***	0.3167***
		(0.1270)	(0.0936)	(0.1916)	(0.0889)	(0.2880)	(0.0929)
Patents	(γ_5)	-0.0909**	-0.0384	-0.2128***	-0.2340***	-0.1957***	-0.2310***
		(0.0449)	(0.0295)	(0.0336)	(0.0269)	(0.0534)	(0.0270)
Cyclic Triangles	($\varkappa$)	-1.6277***	-1.5486***	-3.5815***	-2.2637***	-3.0555***	-2.2509***
		(0.4095)	(0.1753)	(0.3898)	(0.1587)	(0.4338)	(0.1537)

Note: The dependent variable is log R&D expenditures. The parameters $\boldsymbol{\theta} = (\rho, b, \boldsymbol{\delta}^\top, \boldsymbol{\gamma}^\top, \varkappa)$ correspond to Equation (24), where $\psi_{ij} = \boldsymbol{\gamma}^\top \mathbf{c}_{ij}$, $\varphi_{ij} = \varkappa t_{ij}$ and $\eta_i = \mathbf{X}_i \boldsymbol{\delta}$ (cf. Section 3.2). The estimation results are based on 351 firms from the SIC-28 sector. We make 50,000 MCMC draws where we drop the first 2,000 draws during a burn-in phase and keep every 20th of the remaining draws to calculate the posterior mean (as point estimates) and posterior standard deviation (shown in parenthesis). All cases pass the convergence diagnostics provided by Geweke [1992] and Raftery and Lewis [1992]. The asterisks ***(**,*) indicate that its 99% (95%, 90%) highest posterior density range does not cover zero. Heterogeneous spillovers are captured

Model fit

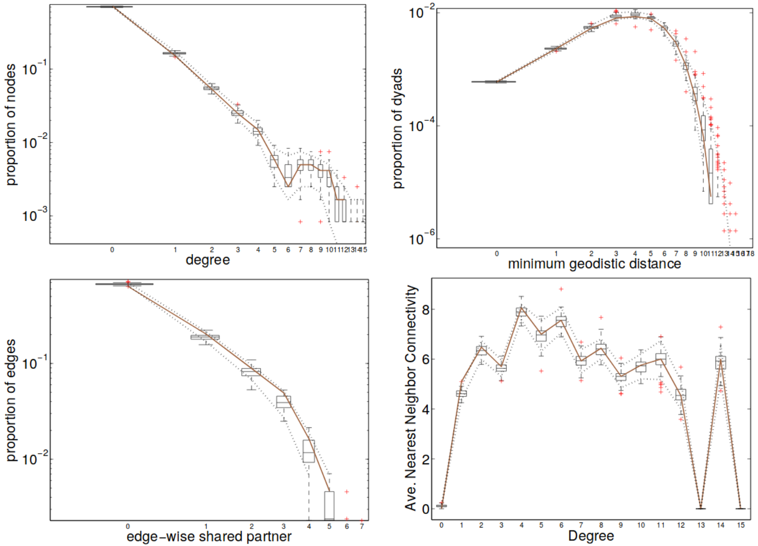


Figure 9: Goodness-of-fit statistics.

Key firms

Table 4: Key player ranking for firms in the chemicals and allied products sector (SIC-28).

Firm	Mkt. Sh. [%][a]	Patents	Degree	ΔW [%][b]	ΔW_{F} [%][c]	ΔW_{N} [%][d]	SIC	Rank
Pfizer Inc.	2.7679	78061	15	-1.8764	-1.7943	-0.3843	283	1
Novartis	2.0691	18924	15	-1.7369	-1.8271	-0.3273	283	2
Amgen	0.8193	6960	13	-1.6272	-1.4240	-0.4753	283	3
Bayer	3.8340	133433	10	-1.3781	-1.2910	-0.3445	280	4
Merck & Co. Inc.	1.2999	52847	10	-1.0182	-1.1747	-0.2892	283	5
Dyax Corp.	0.0007	227	6	-0.7709	-0.6660	-0.3289	283	6
Medarex Inc.	0.0028	168	9	-0.7452	-0.8749	-0.3847	283	7
Exelixis	0.0057	58	7	-0.7293	-0.8603	-0.3686	283	8
Xoma	0.0017	648	7	-0.6039	-0.6863	-0.2254	283	9
Genzyme Corp.	0.1830	1116	3	-0.5904	-0.2510	-0.2987	283	10
Johnson & Johnson Inc.	3.0547	1212	7	-0.5368	-0.8556	-0.3520	283	11
Abbott Lab. Inc.	1.2907	11160	3	-0.5162	-0.1867	-0.3543	283	12
Infinity Pharm. Inc.	0.0011	44	4	-0.4623	-0.5155	-0.2724	283	13
Curagen	0.0023	174	3	-0.4335	-0.4388	-0.3742	283	14
Cell Genesys Inc.	0.0001	236	5	-0.4133	-0.4629	-0.2450	283	15
Solvay SA	1.2445	22689	3	-0.4048	-0.3283	-0.2480	280	16
Takeda Pharm. Co. Ltd.	0.6445	19460	7	-0.3934	-0.7817	-0.3818	283	17
Daiichi Sankyo Co. Ltd.	0.4590	14	5	-0.3691	-0.5581	-0.3377	283	18
Maxygen	0.0014	252	3	-0.3455	-0.3013	-0.2268	283	19
Compugen Ltd.	0.0000	246	5	-0.3130	-0.5251	-0.3202	283	20

[a] Market share in the primary 3-digit SIC sector in which the firm is operating.
[b] The relative welfare loss due to exit of a firm i is computed as $\Delta W = \left(\mathbb{E}_{\mu^\vartheta}[W_{-i}(\mathbf{q},G)] - W(\mathbf{q}^{\mathrm{obs}},G^{\mathrm{obs}})\right)/W(\mathbf{q}^{\mathrm{obs}},G^{\mathrm{obs}})$, where $\mathbf{q}^{\mathrm{obs}}$ and G^{obs} denote the observed R&D expenditures and network, respectively.
[c] ΔW_{F} denotes the relative welfare loss due to exit of a firm assuming a fixed network of R&D collaborations.
[d] ΔW_{N} denotes the relative welfare loss due to exit of a firm in the absence of a network of R&D collaborations.

Mergers

Table 5: Merger ranking for firms in the chemicals and allied products sector (SIC-28).

Firm i	Firm j	Mkt. Sh. i [%][a]	Mkt. Sh. j [%]	Pat. i	Pat. j	d_i	d_j	ΔW [%][b]	ΔW_{F} [%][c]	ΔW_{N} [%][d]	SIC	Rank
WELFARE LOSS												
Daiichi Sankyo Co. Ltd.	Schering-Plough Corp.	0.4590	0.6057	14	52847	5	1	-0.6036	0.0476	-0.2386	283	1
MorphoSys AG	Daiichi Sankyo Co. Ltd.	0.0038	0.4590	20	14	4	5	-0.5976	0.0132	-0.3948	283	2
Vical Inc.	Cephalon	0.0008	0.1005	170	810	1	1	-0.5639	0.3903	-0.3111	283	3
Galapagos NV	Medarex Inc.	0.0025	0.0028	30	168	2	9	-0.5581	0.1017	-0.3253	283	4
Galapagos NV	Coley Pharm. Group Inc.	0.0025	0.0012	30	125	2	1	-0.5409	0.2329	-0.3935	283	5
Infinity Pharm. Inc.	Alnylam Pharm. Inc.	0.0011	0.0015	44	114	4	3	-0.5339	0.0484	-0.3309	283	6
Icagen	Biosite Inc.	0.0005	0.0177	423	182	1	3	-0.5261	0.3587	-0.3244	283	7
Clinical Data Inc.	Renovis	0.0037	0.0006	9	58	4	1	-0.5179	0.3005	-0.3890	283	8
Clinical Data Inc.	Curagen	0.0037	0.0023	9	174	4	3	-0.5134	0.0108	-0.3450	283	9
EntreMed Inc.	AVI BioPharma Inc.	0.0004	0.0000	62	67	3	1	-0.5120	0.2734	-0.3213	283	10
WELFARE GAIN												
Isis Pharm. Inc.	Takeda Pharm. Co. Ltd.	0.0014	0.6445	4472	19460	4	7	0.8643	0.3406	-0.3517	283	1
Cell Genesys Inc.	Pfizer Inc.	0.0001	2.7679	236	78061	5	15	0.8636	0.6395	-0.3692	283	2
Exelixis	Pfizer Inc.	0.0057	2.7679	58	78061	7	15	0.8235	0.5397	-0.4127	283	3
Dyax Corp	Pfizer Inc.	0.0007	2.7679	227	78061	6	15	0.7717	0.5548	-0.4120	283	4
Bristol-Myers Squibb Co.	Novartis	1.0287	2.0691	22312	18924	6	15	0.7696	0.4889	-0.2978	283	5
Exelixis	Takeda Pharm. Co. Ltd.	0.0057	0.6445	58	19460	7	7	0.7661	0.5511	-0.3254	283	6
Exelixis	Novartis	0.0057	2.0691	58	18924	7	15	0.7637	0.5130	-0.3872	283	7
Genzyme Corp.	Pfizer Inc.	0.1830	2.7679	1116	78061	3	15	0.7441	0.4206	-0.3572	283	8
Medarex Inc.	Allergan Inc.	0.0028	0.1759	168	6154	9	3	0.7441	0.3586	-0.2983	283	9
Medarex Inc.	Amgen	0.0028	0.8193	168	6960	9	13	0.7411	0.7776	-0.2699	283	10

[a] Market share in the primary 3-digit sector in which the firm is operating.
[b] The relative welfare change due to a merger of firms i and j is computed as $\Delta W = \left(\mathbb{E}_{\mu^{\vartheta}}[W_{i\cup j}(G,\mathbf{q})] - W(\mathbf{q}^{\mathrm{obs}}, G^{\mathrm{obs}})\right)/W(\mathbf{q}^{\mathrm{obs}}, G^{\mathrm{obs}})$, where $\mathbf{q}^{\mathrm{obs}}$ and G^{obs} denote the observed R&D expenditures and network, respectively.
[c] ΔW_{F} denotes the relative welfare change due to a merger of firms assuming a fixed network of R&D collaborations.
[d] ΔW_{N} denotes the relative welfare change due to a merger of firms in the absence of a network of R&D collaborations.

Collaboration subsidies

Table 6: Subsidy ranking for firms in the chemicals and allied products sector (SIC-28).

Firm i	Firm j	Mkt. Sh. i [%][a]	Mkt. Sh. j [%]	Pat. i	Pat. j	d_i	d_j	ΔW [%][b]	ΔW_F [%][c]	SIC i	SIC j	Rank
Dynavax Technologies	Shionogi & Co. Ltd.	0.0003	0.0986	162	10156	0	0	0.7646	0.0509	283	283	1
Ar-Qule	Kemira Oy.	0.0004	0.3340	43	510	1	0	0.7622	0.0252	283	280	2
Indevus Pharm. Inc.	Solvay SA	0.0029	1.2445	37	22689	0	3	0.7603	0.0713	283	280	3
Nippon Kayaku Co. Ltd.	Koninklijke DSM NV	0.1342	1.1059	4398	4674	0	1	0.7543	0.0369	280	280	4
Encysive Pharm. Inc.	Johnson & Johnson Inc.	0.0011	3.0547	280	1212	0	7	0.7466	0.1111	283	283	5
Kaken Pharm. Co. Ltd.	Elancorp	0.0377	0.0322	821	462	0	3	0.7315	0.0986	283	283	6
Tsumura & Co.	Syngenta AG	0.0451	4.1430	23	5397	0	0	0.7215	-0.0188	283	287	7
NOF Corp.	Alkermes Inc.	0.1361	0.0138	431	31	0	0	0.7166	0.0132	280	283	8
Toagosei Co. Ltd.	Mitsubishi Tanabe Pharma Corp.	0.1412	0.0877	771	5296	0	1	0.7160	-0.0004	280	283	9
DOV Pharm. Inc.	Mochida Pharm. Co.	0.0015	0.0366	80	575	1	0	0.7158	0.0188	283	283	10
Geron	Elancorp	0.0002	0.0322	240	462	1	3	0.7146	0.0039	283	283	11
Tanox Inc.	PPG Industries Inc.	0.0032	7.5437	139	29784	0	0	0.7145	0.0283	283	285	12
Gedeon Richter	Dade Behring Inc.	0.0572	0.0999	11115	152	0	0	0.7103	0.0173	283	283	13
Nippon Kayaku Co. Ltd.	Valeant Pharm.	0.1342	0.0521	4398	312	0	0	0.7087	0.0695	280	283	14
Geron	Akzo Nobel NV	0.0002	11.7496	240	11366	1	2	0.7080	0.0114	283	285	15
Rigel Pharm. Inc.	Kyorin Holdings Inc.	0.0019	0.0381	259	2986	1	0	0.7074	0.0319	283	283	16
Indevus Pharm. Inc.	MannKind Corporation	0.0029	0.0000	37	32	0	0	0.7064	0.0144	283	283	17
Biosite Inc.	Toyama Chemical Co. Ltd.	0.0177	0.0083	182	2320	1	0	0.7062	-0.0179	283	283	18
Tsumura & Co	Alnylam Pharm. Inc.	0.0451	0.0015	23	114	0	3	0.7053	0.0222	283	283	19
Gen-Probe Inc.	Mitsubishi Tanabe Pharma Corp.	0.0201	0.0877	1179	5296	1	1	0.7046	0.0101	283	283	20

[a] Market share in the primary 3-digit sector in which the firm is operating.
[b] The relative welfare gain due to subsidizing the R&D collaboration costs between firms i and j is computed as $\Delta W = \left(\mathbb{E}_{\mu^\vartheta}[W(\mathbf{q}, G|\psi_{ij}=0)] - W(\mathbf{q}^{\text{obs}}, G^{\text{obs}})\right)/W(\mathbf{q}^{\text{obs}}, G^{\text{obs}})$, where $\mathbf{q}^{\text{obs}}$ and G^{obs} denote the observed R&D expenditures and network, respectively.
[c] ΔW_F denotes the relative welfare loss due to a merger of firms assuming a fixed network of R&D collaborations.

Section 2

Atalay et al. (2011)

Atalay et al. (2011): Network structure of production

- Model of buyer-supplier network of US firms
- Common features of observed social & economic networks: (see Jackson (2010))
    - Scale-free: degree distribution is Pareto: $P(d) = cd^{-\gamma}$ i.e. $\log P(d)$ is linear function of $\log d$.
    - Small worlds: the diameter & average path length tends to be small even for a large number of nodes (e.g. 6 degrees of Kevin Bacon; Erd os number)

Preferential attachment 1

- Growing random network model that is scale-free and has small worlds
- Model: nodes born over time and indexed by date of birth
  - Begin with m nodes fully connnected
  - Time t one node added and forms m connections with existing nodes, connects to node i with probability $\frac{d_i(t)}{\sum_j d_j(t)} = \frac{d_i(t)}{2tm}$

Mean-field approximation

- Solving for degree distribution: “mean-field approximation”
  - $\mathrm{P}(i \text{ gets new link}) = m\frac{d_i(t)}{2tm} = \frac{d_i(t)}{2t}$
  - Approximate time as continuous instead of discrete

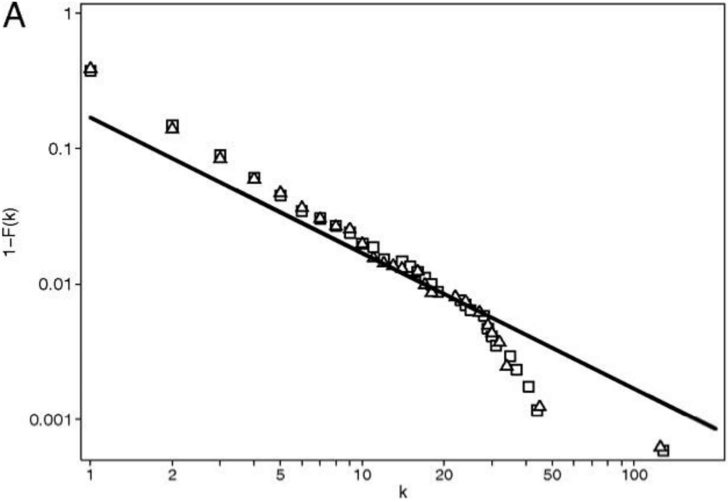
    $$\frac{d}{dt}d_i(t) = \frac{d_i(t)}{2t}$$

    and $d_i(1) = m$, implies

    $$d_i(t) = m\left(\frac{t}{i}\right)^{1/2}$$

  - Degree of older nodes > degree of younger nodes, at time t node born at time $i = t\left(\frac{m}{d}\right)^2$ has degree d, so $F_t(d) = 1 - m^2 d^{-2}$, $P_t(d) = m^2 d^{-3}$

Observed degree distribution



Model overview

- Directed network of buyers and suppliers
- Mix of preferential attachment and random attachment
- Adds node death & reattachment of survivors
- Better incorporate features of the actual firm network: firms often go out of business, and many suppliers actively prefer to work with less-connected downstream firms because of product specialization and long-term contracting issues

Model

- Notation:
    - $N(t)$ firms at time t
    - $n(k,t)$ firms with in-degree k at time t
    - $m(t) = \frac{\sum_k kn(k,t)}{N(t)}$ average in-degree
- Each period:
    1. Exit: each firm exists with probability q; destroys $q(2-q)N(t)m(t)$ edges, $q(1-q)N(t)m(t)$ of which have receiving vertex survive
    2. Reconnection: surviving firms whose connections were lost due to exit reconnect; $q(1-q)N(t)m(t)$ reconnections to make
        - r uniformly at random
        - $1-r$ by preferential attachment
    3. Entry:
- $(g+q)N(t)$ firms enter, each form $m(t)$ edges
    - $\delta(1-r)$ by preferential attachment to existing firms
    - $r\delta$ randomly to existing firms
    - $1-\delta$ randomly to other entrants

Mean-field approximation 1

$$\frac{\partial}{\partial t} n(k,t) + \frac{\partial}{\partial k}\left[n(k,t)\gamma(k,t)\right] = \beta(k,t)N(t)(q+g) - qn(k,t)$$

- $\gamma(k,t) =$ in-degree growth rate
  - $= \frac{dk}{dt} = qr(m(t)-k) + \frac{\delta(k+r(m(t)-k))(q+g)}{1-q}$
- $\beta(k,t) =$ in-degree distribution of entering vertices
  - $=$ binomial $\left((g+q)N(t)(1-\delta)m(t), \frac{1}{N(t)(g+q)}\right)$
  - $\approx \frac{1}{m(t)(1-\delta)} e^{-\frac{k}{m(t)(1-\delta)}}$ (exponential)
- Let $p(k,t) = \frac{n(k,t)}{N(t)}$,

$$\frac{\partial p(k,t)}{\partial t} + \frac{\partial}{\partial k}\left[p(k,t)\gamma(k,t)\right] = \beta(k,t)(q+g) - qp(k,t)$$

Mean-field approximation 2

- Solve for stead-state degree distribution, $p(k)$

$$\frac{\partial}{\partial k}\left[p(k)\gamma\right] = \beta(k)(q+g) - qp(k)$$

so

$$p(k) = \lambda(k+R)^{-1-S}\left(\Gamma[1+S, R/(m(1-\delta))] - \Gamma[1+S,(R+k)\right.$$

where R, S and λ are functions of δ, q, g, m, and r

Data

- Data yearly firm-level data from Compustat
- 1979-2007 publicly listed firms
- Link = major customer = firm that purchases ≥10% of seller's revenue

Table 1. Top 10 firms from 1979 to 1983 and from 2003 to 2007

	1979–1983		2003–2007	
Rank	Firm	*k*	Firm	*k*
1	GM	86.4	Wal-Mart	129.8
2	Sears	50.0	GM	42.0
3	Ford	48.2	Cardinal Health	37.4
4	IBM	33.4	Home Depot	33.0
5	JCPenney	26.4	Ford	31.2
6	Chrysler	20.2	Hewlett-Packard	30.8
7	GE	19.0	Daimler-AG	30.8
8	AT&T	18.2	AmerisourceBergen	30.6
9	Boeing	15.0	McKesson	28.8
10	McDonnell Douglas	12.8	Target	25.8

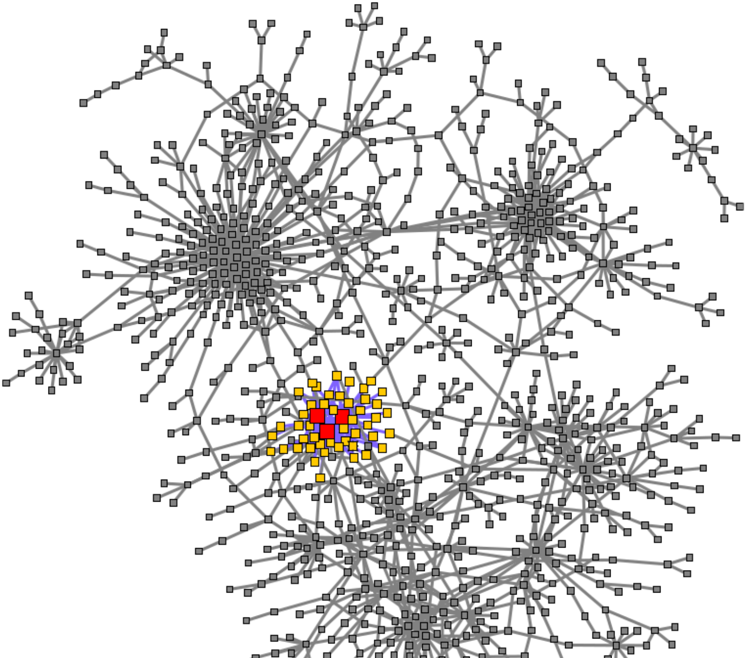
k, number of suppliers in the average year.

Atalay et al.

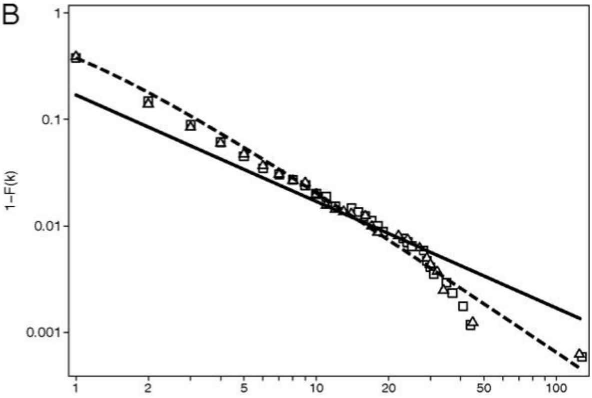
Estimation

- 5 parameters
  - q = exit rate = empirical average = 0.24
  - m = edges per vertex = 1.06
  - δ = portion of new vertices to existing firms = 0.75
  - g = growth rate of number of firms = 0.04
  - r = fraction of edges assigned randomly estimated by MLE for probability a new link among surviving vertices given in-degree = 0.18
- Not fitting CDF directly

Network



Fit



Section 3

Strategic network formation

- Christakis et al. (2010)
- Lee and Fong (2013)
- Chandrasekhar and Jackson (2013)
- Leung (2013)
- Sheng (2012)
- Graham (2014a), Graham (2014b)

Christakis et al. (2010)

- Tractable empirical model of network formation
- Estimable from data on a single network
- Bayesian estimation
- Applied to social network of high school students

Model

- Sequential: N nodes, T periods
- Begin with no links
- Each period two nodes meet and have opportunity to form a link
- Payoff of i from linking with j at time t

$$U_i(j| \underbrace{X}_{\text{Node characteristics}}, \underbrace{C}_{\text{link characteristics}}, G_{t-1}, t)$$

- Link formed if

$$g\left(U_i(j|X, C, G_{t-1}, t), U_j(i|X, C, G_{t-1}, t)\right) > 0$$

- Myopic behavior:

$$U_i(j|X, C, G_{t-1}, t) = U_i(j|X, C, G_{t-1})$$

  - Individuals do no have to take expectation over future links
  - Avoids multiple equilibria & computational difficulties

Empirical specification

- Preferences:

$$U_i(j|X, C, G_{t-1}) = \beta_0 + \beta_1' x_j - (x_i - x_j)'\Omega(x_i - x_j) + \\ + \alpha_1 d_{jt} + \alpha_2 d_{jt}^2 + \alpha_3 d(i, j; G_{t-1}) + \delta c_{ij} + \epsilon_{ij}$$

  Non-transferable:

$$g(u_i, u_j) = \mathbf{1}\{u_i \geq 0 \; \& \; u_j \geq 0\}$$

- $\epsilon_{ij} \sim$ logistic, independent
- Sequence of meetings, M: assume $T = N(N-1)/2$, each potential pair meets exactly once, all sequences equally likely
- Parameter meanings:
  - β individual characteristics
  - Ω captures homophily
  - α network characteristics
  - δ pair characteristics

Estimation

- Bayesian
- Likelihood

$$\mathcal{L}(\theta|G,X,C) = \mathrm{P}(G|X,C;\theta) = \sum_{M \in \mathbb{M}} \mathrm{P}(M|X,C;\theta)\mathrm{P}(G|M,X,C;\theta)$$

  - $\mathrm{P}(G|M,X,C;\theta)$ is product of logit probabilities
  - $|\mathbb{M}| = (N(N-1)/2)!$ is too large for MLE
- Compute posterior using MCMC — Metropolis-Hastings with data augmentation
  - Draw $\theta_k|M_k$ from $\mathrm{P}(\theta|M_k,G,X,C) \propto \mathrm{P}(G|M_k,X,C,\theta)\mathrm{P}(\theta)$
  - Draw $M_{k+1}|\theta_k$ from $\mathrm{P}(M|\theta_k,G,X,C) \propto \mathrm{P}(G|M_k,X,C,\theta)\mathrm{P}(M)$
- Data from a single large network
  - Properties of estimator as $N \rightarrow \infty$ unknown
  - Chandrasekhar and Jackson (2013), Leung (2013) also have data from a single network and show consistency of their estimators (but models differ)

Data

- Friendship network in single high school of 669 students, 1541 links
- From AddHealth data set

Summary statistics

Table 1: Summary Statistics of Student Characteristics (N=669)

Characteristic	Mean	Standard Deviation	median	Min	Max
Sex (0 Male, 1 Female)	0.48	(0.50)	0	0	1
Grade	10.7	(1.1)	11.0	8.0	13.0
Age	17.3	(1.3)	17.3	13.3	21.3
Sports Participation	0.49	(0.50)	0	0	1
Number of Friendships	4.6	(3.3)	4	0	18

Summary statistics

Table 2: Summary Statistics of Student Pair Characteristics (223,446 Pairs)

	All (223,446)		Friends (1,541)		Not Friends (221,905)	
Characteristic	Mean	SD	Mean	SD	Mean	SD
# Classes in Common	0.65	1.45	2.13	2.48	0.64	1.44
Abs Diff in Gender	0.50	0.50	0.41	0.49	0.50	0.50
Abs Dif in Grade	1.21	1.01	0.43	0.67	1.22	1.01
Abs Diff in Age	1.43	1.07	0.70	0.64	1.43	1.07
Abs Dif in Sports Participation	0.50	0.50	0.40	0.49	0.50	0.50

Estimates

		ML Estimates		Moments of Posterior Distribution			
		Model I		Model I		Model II	
		No Network Effects		No Network Effects		Network Effects	
Parameter	Description	est.	s.e.	mean	s.d.	mean	s.d.
α_1	# of friends of alter	0	–	0	–	-0.14	(0.03)
α_2	total # of friends of alter sq	0	–	0	–	0.004	(0.003)
α_3	degr of sep is two	0	–	0	–	2.66	(0.07)
α_4	degr of sep is three	0	–	0	–	1.22	(0.07)
β_0	intercept	-2.12	(0.05)	-2.11	(0.04)	-2.11	(0.06)
β_1	female	-0.06	(0.04)	-0.06	(0.04)	-0.04	(0.05)
β_2	alter grade	0.08	(0.03)	0.08	(0.03)	0.07	(0.03)
β_3	alter age	0.05	(0.03)	0.05	(0.03)	0.05	(0.03)
β_4	participates in sport	0.10	(0.04)	0.09	(0.04)	0.04	(0.05)
Ω_{11}	diff in sex	0.19	(0.03)	0.19	(0.03)	0.20	(0.03)
Ω_{22}	diff in grades squared	0.17	(0.02)	0.17	(0.01)	0.14	(0.01)
Ω_{33}	diff in age squared	0.10	(0.02)	0.10	(0.01)	0.09	(0.01)
Ω_{44}	diff in sports participation	0.21	(0.03)	0.22	(0.03)	0.19	(0.03)
δ	# of classes in common	0.14	(0.01)	0.14	(0.01)	0.12	(0.01)

Fit

Table 3: Triangle Census (total number of triples 49,679,494)

Triangle Type	Actual Count	Predicted Count Model I Covariates Only	Predicted Count Model II Network Effects
No Edges	48,660,171	48,660,484.8	48,697,654.4
Single Edge	1,011,455	1,010,674.3	974,304.9
Two Edges	7,212	8,294.5	7,075.2
Three Edges	656	40.3	459.6
Overall Clustering Coefficient	0.083	0.005	0.061

Fit

Figure 1a: Histogram Number of Friends

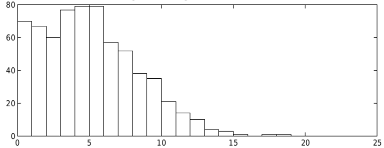


Figure 1b: Histogram Predicted Number of Friends (covariates only)

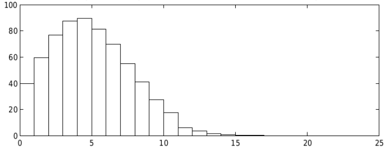


Figure 1c: Histogram Predicted Number of Friends (network effects)

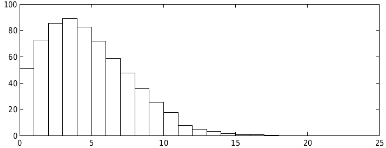


Figure 2a: Histogram Path Length

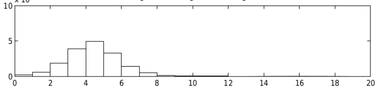


Figure 2b: Histogram Predicted Path Length (covariates only))

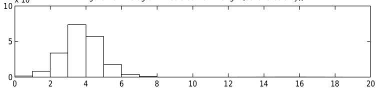


Figure 2c: Histogram Predicted Path Length (network effects)

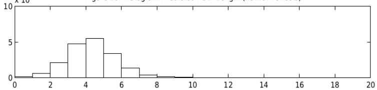


Table 7: Friendship Rates by Sex Composition

Friendship Type	Actual # of Pairs	Actual Frienship Rate	Predicted Rate Network Model Current Assignment (Mixed Sex Classrooms)	Predicted Rate Network Model Counterfactual (Single Sex Classrooms)
Boy-Boy	61,075	0.0087	0.0082	0.0079
Boy-Girl	111,650	0.0056	0.0055	0.0037
Girl-Girl	50,721	0.0076	0.0074	0.0071

Chandrasekhar and Jackson (2013)

- Consistent and tractable network formation model
- Setup nests variant of Christakis et al. (2010) model
- Starting point: exponential random graph (ERGM):
    - Network $g \in G$
    - Vector of statistics $S(g)$
    - Likelihood:

$$P_\theta(g) = \frac{e^{\theta S(g)}}{\sum_{g' \in G} e^{\theta S(g')}}$$

    - Broad class, can represent any random graph model
    - Used in many applications
- Challenges of ERGMs: set of networks, G very large, typically estimated by MCMC, but consistency unknown and mixing time exponential in number of nodes
- This paper: propose a related class of models, give conditions for consistent and asymptotically normal estimation, give examples of strategic network formation models that fit into setup

SERGM

- Statistical exponential random graph model
- Write model on space of statistic instead of network

$$\mathrm{P}_{\beta,K}(s) = \frac{K(s)e^{\beta s}}{\sum_{s' \in A} K(s')e^{\beta s'}}$$

- Estimate β by MLE or GMM
- Sum in denominator is over space of statistic instead of possible networks
- Sufficient conditions for consistent, asymptotically normal $\hat{\beta}$ (loosely):
    - Statistics are counts, e.g. of links, triangles, stars, etc
    - Graph is not too dense

SUGM

- Subgraph generation models
- List of subgraph types G^n_ℓ, $\ell = 1, ..., k$
- Probabilities p^n_ℓ of each type
- Formation:
    - Each subnetwork in G^n_1 formed with probability p^n_1
    - Repeat for $\ell = 2, ..., n$
- E.g. Erdos-Renyi: G^n_1 = all pairs of nodes
- $\hat{p}^n_\ell$ consistent and asymptotically normal if network is sparse

Strategic network formation as SUGM

- If payoff depends only on subgraph, then natural
- I.e. if $u_i(g)$ only depends on direct connection or direct connections + friends of friends etc
- E.g. in Christakis et al. (2010)

$$
\begin{aligned}
U_i(j|X,C,G) = & \beta_0 + \beta_1' x_j - (x_i - x_j)'\Omega(x_i - x_j) + \\
& + \alpha_1 d_j + \alpha_2 d_j^2 + \delta c_{ij} + \\
& + \alpha_3 1\{d(i,j;G) = 2\} + \alpha_4 1\{d(i,j;G) = 3\} + \epsilon_{ij}
\end{aligned}
$$

Lee and Fong (2013)

- Dynamic network formation model with transfers
- Applicable to bilateral contracting between firms, e.g.
  - Manufacturers & retailers
  - Health insurers & providers
  - Hardware & software

Model 1

- Infinite horizon, discrete time
- Network $g \in G$
- Contracts (payments) $t_g = \{t_{ij;g}\}_{ij \in g}$
- Per-period payoffs: $\pi_i(g, t_g)$

Model: each period

- Start with network $g^{\tau-1}$

1. Network formation:
    1. Simultaneously announce links a_i that want to negotiate, private payoff shock $\epsilon_{a_i,i}$ received
    2. Network of negotiations: $\tilde{g}(a)$
        - If i & j both announced link, $ij \in \tilde{g}(a)$,
        - Everyeone pays cost $c_i(\tilde{g}(a)|g^{\tau-1})$

Model: each period

- Start with network $g^{\tau-1}$

1. Network formation:
   1. Simultaneously announce links a_i that want to negotiate, private payoff shock $\epsilon_{a_i,i}$ received
   2. Network of negotiations: $\tilde{g}(a)$
      - If i & j both announced link, $ij \in \tilde{g}(a)$,
      - Everyeone pays cost $c_i(\tilde{g}(a)|g^{\tau-1})$
2. Bargaining:
   1. Additive payoff shocks η_{ij} observed
   2. Unstable links $ij \in \tilde{g}$ with no gains from trade (given rest of network) dissolves, repeat until no such pairs remain to get $g^\tau \subseteq \tilde{g}$
   3. Contracts t_g^τ determined by Nash bargaining, payoffs realized

$$\bar{\pi}_i(g^\tau, \eta, t_g^\tau) = \pi_i(g^\tau, t_g^\tau) + \sum_{ij \in g^\tau} \eta_{ij}$$

Model - dynamics

- Markov strategies $\sigma_i(g, \epsilon_i)$
- Conditional choice probabilities $P_i^\sigma(a|g) = \int \mathbf{1}\{\sigma_i(g,\epsilon_i) = a\} f(\epsilon_i) d\epsilon_i$
- $\Gamma(g; \eta, V^\sigma)$ = subnetwork $g' \subseteq g$ such that all pairs stable
- Negotiation network probabilities

$$q_i^\sigma(g'|a_i, g) = \sum_{a_{-i}} \prod_{j \neq i} P_j^\sigma(a_j|g) I\{\tilde{g}(a) = g'\}$$

- Choice-specific value function

$$v_i^\sigma(a, g) = \sum_{g'} q_i^\sigma(g'|a, g)\Big(c_i(g'|g) + \mathrm{E}_\eta\Big[\bar{\pi}_i(g'', \eta, t^\sigma_{g''}) + \beta V_i^\sigma(g'') : \\ : g'' = \Gamma(g; \eta, V^\sigma)\Big]\Big)$$

- Value function

$$V_i^\sigma(g) = \int \left(\max_a \epsilon_{a,i} + v_i^\sigma(a_i, g)\right) f(\epsilon_i) d\epsilon_i$$

Model - bargaining

- Nash bargaining:
  - Surplus of i from trading with j

$$\Delta S^{\sigma}_{i,j}(g;\eta,\{t,t^{\sigma}_{-ij:g}\}) = \left(\bar{\pi}_i(g,\eta,\{t,t_{-ij:g}\}) + V^{\sigma}_i(g)\right) - \\ - \left(\bar{\pi}_i(g-ij,\eta,t_{-ij:g}) + V^{\sigma}_i(g-ij)\right)$$

  - Assumes if ij do not link, other links unaffected today (but they could be in the future)

$$t_{ij;g}(\eta) \in \arg\max_{\tilde{t}} \Delta S^{\sigma}_{i,j}(g;\eta,\{\tilde{t},t^{\sigma}_{-ij:g}\})^{b_{ij}} \Delta S^{\sigma}_{j,i}(g;\eta,\{\tilde{t},t^{\sigma}_{-ij:g}\})^{b_{ji}}$$

- Equilibrium existence from Brouwer's fixed point theorem
- Equilibrium may not be unique

Example

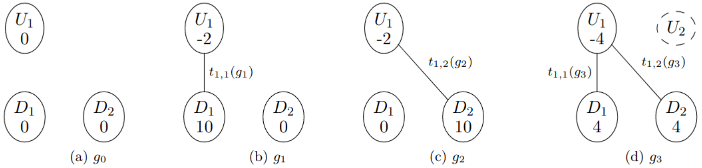


Figure 1: Potential Networks g_0, g_1, g_2, g_3 between firms U_1, D_1, D_2. Period payoffs contained within circles; $t_{ij}(g_k)$ represents payment between U_i and D_j under network g_k.

- Contracting externalities
- Static model (or equivalently $\beta = 0$) with equal bargaining power
  - $t_{1,j}(g_2) = 6$, $t_{1,j}(g_3) = 4$
- Dynamic model with $\beta = 0.9$, $c() = 1$, $var(\epsilon) = \pi^2/8$
  - $t_{1,j}(g_2) \approx 7.6$, $t_{1,j}(g_3) = 4.4$
    - Chance of downstream firms being unlinked for multiple periods lowers value of their outside option
  - Distribution of states $[g_0, g_1, g_2, g_3] \approx [.00, .43, .43, .14]$, $P(g_1|g_2) = P(g_2|g_2) \approx 0.8$

Estimation

- Much like dynamic games
- Approaches:
  - Constrained MLE: maximize pseudo-likelihood subject to equilibrium constraints
  - Two-step:
    1. Estimate policy functions: using Hotz-Miller inversion (e.g. with type I extreme value shocks)

$$\hat{\sigma}_i(g, \epsilon) = \arg\max_a \log(\hat{P}_i(a|g)) + \epsilon$$

    2. Let $\tilde{\sigma}_i(\cdot; \theta)$ be the best response of player i when payoff parameters are θ and other players play $\hat{\sigma}_{-i}$, estimate θ to minimize

$$\hat{\theta} = \arg\min \sum_{a,g,i} \left(P_i^{\tilde{\sigma}_i;\hat{\sigma}_{-i}}(a|g) - P_i^{\hat{\sigma}}(a|g) \right)^2$$

Identification 1

- "Intuitively, if there are gains from trade between two agents who form a link (given the actions of others), a static model would predict that the link should form regardless of which agent obtains a larger share. However, in a dynamic model, different values of Nash bargaining parameters will change each agent's respective outside options through their continuation values, and hence only certain parameter values will be consistent with a link forming in equilibrium."
- What data is observed?
    - Realized sequence of networks?
    - Sequence of networks $+$ actions $=$ announcements (i.e. we see potential links where negotiations failed)
    - 2-step estimator assumes the announcements observed, single step estimator allows only networks to be observed
    - Section 4.2 about estimation of bargaining parameter assumes (N, G, π, β, f, c) either observed, assumed, or can be separately estimated

Identification if π, c not known 1

- Assuming announcements observed, usual dynamic decision model identifies per-period payoff:

$$\tilde{\pi}(a|g) = \sum_{g'} q^P(g'|a,g)\left(c_i(g'|g) + \mathrm{E}_\eta[\pi_i(\Gamma(g',\eta), t^P_{\Gamma(g',\eta)}, \eta)]\right)$$

- $q^P(g'|a_i, g)$ is known, so variation in a_i identifies

$$c_i(g'|g) + \mathrm{E}_\eta[\pi_i(\Gamma(g',\eta), t^P_{\Gamma(g',\eta)}, \eta)]$$

- Need restriction to separate c_i and π_i, e.g. assume $c_i(g'|g) = 0$ if $g' = g$
- $\Gamma(g',\eta) =$ stable subnetwork of g'

$$\Gamma(g,\eta) = \begin{cases} g \\ \Gamma(g',\eta) \text{ otherwise where } g' = g \setminus \{ij \in g : \Delta S_{ij}(G, \eta, t \end{cases}$$

- Need to untangle Γ, η, and π from bargaining
- Estimator assumes η degenerate

Example: Insurer-Provider negotiations

- Simulate version of model designed to reflect features of HMO-hospital network
- Look at performance of estimator
- Ignoring dynamics biases estimates of payoffs (table 2)
- Estimates of bargaining power appear unbiased and precise (table 3)
- Simulate hospital mergers

Table 1: Simulated Equilibrium Network Distributions

	"B-Pow"	# Eq Net	Full Net	Eff. Net	Single (90%)	Single (50%)	Single & Full	Single & Eff	Active Hosp	Exp. Links
1 Hosp	Equal	1.03	0.01	0.88	0.97	1.00	0.01	0.88	1.00	1.00
2 HMOs	Hospitals	1.01	0.00	0.91	0.99	1.00	0.00	0.91	1.00	0.99
	HMOs	1.02	0.00	0.80	0.98	1.00	0.00	0.80	1.00	0.99
2 Hosp	Equal	3.36	0.39	0.90	0.01	0.17	0.04	0.14	2.00	2.65
2 HMOs	Hospitals	3.57	0.22	0.83	0.00	0.23	0.00	0.23	2.00	2.49
	HMOs	2.67	0.01	0.92	0.01	0.73	0.01	0.67	1.99	2.30
3 Hosp	Equal	1.92	0.00	0.72	0.01	0.05	0.00	0.01	2.99	2.88
2 HMOs	Hospitals	1.89	0.00	0.54	0.01	0.15	0.00	0.10	2.94	2.55
	HMOs	1.53	0.00	0.63	0.00	0.45	0.00	0.36	2.91	2.42

Summary statistics from 100 market draws for each specification. *"B-Pow"*: Equal - $b_{ij} = .5 \; \forall \, ij$; Hospitals - $b_i j = .8$ when i is a hospital, .2 otherwise; HMOs - $b_i j = .8$ when i is an HMO, .2 otherwise. *# Eq Net*: Average number of networks that occur more than 10% in the equilibrium network distribution (E.N.D.). *Full Net / Eff Net* : % of runs in which full / efficient network occurs more than 10% in E.N.D. *Single (x%)*: % of runs in which a single network occurs more than x% in E.N.D. *Single & Full / Eff*: % of runs in which a single network occurs more than 90% in E.N.D., and that network is full / efficient. *Active Hosp*: average number of hospitals that have contracts with at least one HMO more than 10% of the time in E.N.D. *Expected Links*: expected number of bilateral links in E.N.D.

Table 2: Regression of Hospital Margins on Observables / Characteristics

Timing:	Dynamic						Static					
	Equal		Hospital		HMO		Equal		Hospital		HMO	
	Coeff	s.e.	Coeff	s.e.	Coeff	s.e.	Coeff	s.e.	Coeff	s.e.	Coeff	s.e.
Const.	-2.40	1.33	0.72	1.43	1.96	1.48	21.77	0.73	23.94	0.63	18.31	0.69
Avg. Cost	-0.94	0.05	-0.96	0.05	-0.77	0.07	-0.65	0.06	-0.56	0.05	-0.70	0.05
Cost-AC	-0.23	0.07	-0.20	0.07	0.10	0.10	-0.23	0.08	-0.36	0.07	-0.16	0.07
# Patient	-0.01	0.08	0.05	0.06	0.18	0.10	0.41	0.05	0.38	0.05	0.31	0.06
Total # Patients	-0.04	0.04	-0.11	0.03	-0.12	0.05	-0.30	0.03	-0.27	0.02	-0.31	0.02
HMO Marg	12.03	0.52	11.58	0.49	8.67	0.68	2.04	0.33	1.66	0.27	3.86	0.37
R^2	0.77		0.79		0.50		0.57		0.62		0.65	

Projection of simulated equilibrium expected per-patient margins between hospital i and HMO j onto equilibrium market observables as bargaining power varies (Equal - $b_{ij} = .5\ \forall\ ij$; Hospitals - $b_i j = .8$ when i is a hospital, .2 otherwise; HMOs - $b_i j = .8$ when i is an HMO, .2 otherwise). Results pool across 2x2 and 3x2 settings. Av. Cost: average hospital marginal cost in the market; Cost-AC: difference between hospital's marginal cost and average cost in the market; # Patient (Total # Patients): expected number of patients of HMO j (from all HMOs) served by hospital i; HMO Marg: expected HMO margins (premiums minus marginal cost). Extra Hospital: indicator for whether there are 3 hospitals (instead of 2) in the market.

Table 3: Monte Carlo Estimates of b_H

	True b_H	1 Markets / Sample	5 Markets / Sample	10 Markets / Sample
Avg. Estimate:	0.50	0.48	0.47	0.51
95% C.I.:		(0.10,0.90)	(0.20,0.70)	(0.40,0.60)
Avg. Estimate:	0.80	0.60	0.76	0.77
95% C.I.:		(0.10,0.90)	(0.40,0.90)	(0.60,0.80)
Avg. Estimate:	0.20	0.20	0.24	0.23
95% C.I.:		(0.10,0.40)	(0.20,0.50)	(0.20,0.30)

Estimated values of hospital bargaining power b_H for 40 samples of either 1, 5, or 10 markets in 2x2 settings where a sequence of 20 networks were observed. Grid search conducted over b_H in increments of .05.

Merger simulation

	"B-Pow"	$+\Delta\pi^H$	$-\Delta\pi^H_{5\%}$	$+\Delta\pi^M$	$-\Delta\pi^M_{5\%}$	$+p^M$	$-p^M_{5\%}$	+Ins	$-\text{Ins}_{5\%}$
(i) Dynamic	Equal	0.72	0.28	0.73	0.25	0.81	0.14	0.19	0.76
	Hospitals	0.59	0.29	0.12	0.29	0.75	0.20	0.25	0.71
	HMOs	0.80	0.17	0.76	0.24	0.85	0.11	0.15	0.77
(ii) Dynamic,	Equal	-	-	0.97	0.01	0.99	0.00	0.01	0.99
$+\Delta\pi^H \geq 0$	Hospitals	-	-	0.15	0.07	1.00	0.00	0.00	0.95
	HMOs	-	-	0.89	0.11	0.99	0.00	0.01	0.90
(iii) Static	Equal	0.12	0.85	0.02	0.91	1.00	0.00	0.00	1.00
	Hospitals	0.04	0.87	0.01	0.98	1.00	0.00	0.00	1.00
	HMOs	0.25	0.71	0.02	0.87	1.00	0.00	0.00	1.00
(iv) Static,	Equal	-	-	0.17	0.25	1.00	0.00	0.00	1.00
$+\Delta\pi^H \geq 0$	Hospitals	-	-	0.25	0.50	1.00	0.00	0.00	1.00
	HMOs	-	-	0.08	0.52	1.00	0.00	0.00	1.00

Summary statistics from merger simulations, where: (i) and (ii) are from a dynamic model ($\beta = .9$), (iii) and (iv) from a static model, and (ii) and (iv) condition also on markets where hospitals find it profitable to merge. *"B-Pow"*: Equal - $b_{ij} = .5 \; \forall \; ij$; Hospitals - $b_ij = .8$ when i is a hospital, .2 otherwise; HMOs - $b_ij = .8$ when i is an HMO, .2 otherwise. $+\Delta\pi^H, -\Delta\pi^H_{5\%}$: percentage of markets in which total hospital profits increases at all or falls by 5%; $+\Delta\pi^M, -\Delta\pi^M_{5\%}$: percentage of markets in which total HMO profits increases at all or falls by 5%; $+p^M, -p^M_{5\%}$: percentage of markets in which both HMO premiums increase or fall by 5%; $+Ins, -Ins_{5\%}$: percentage of markets in which total patients insured increases at all or falls by 5%.

Atalay, Enghin, Ali Hortaçsu, James Roberts, and Chad Syverson. 2011. "Network structure of production." *Proceedings of the National Academy of Sciences* 108 (13):5199–5202. URL http://www.pnas.org/content/108/13/5199.abstract.

Chandrasekhar, Arun and Matthew O Jackson. 2013. "Tractable and consistent random graph models." *arXiv preprint arXiv:1210.7375* URL http://arxiv.org/abs/1210.7375.

Christakis, Nicholas A., James H. Fowler, Guido W. Imbens, and Karthik Kalyanaraman. 2010. "An Empirical Model for Strategic Network Formation." Working Paper 16039, National Bureau of Economic Research. URL http://www.nber.org/papers/w16039.

Graham, Bryan S. 2014a. "An Empirical Model of Network Formation: Detecting Homophily When Agents Are Heterogenous." Working Paper 20341, National Bureau of Economic Research. URL http://www.nber.org/papers/w20341.

———. 2014b. "Methods of Identification in Social Networks." Working Paper 20414, National Bureau of Economic Research. URL http://www.nber.org/papers/w20414.

Hsieh, Chih-Sheng, Michael D König, and Xiaodong Liu. 2017. "Network Formation with Local Complements and Global Substitutes: The Case of R&D Networks." URL http://www.econ.uzh.ch/static/wp/econwp217.pdf. Retitled "Endogenous Technology Spillovers in R&D Collaboration Networks" and now forthcoming in RAND.

———. 2024. "Endogenous Technology Spillovers in R&D Collaboration Networks." *The RAND Journal of Economics* Forthcoming.

Jackson, Matthew O. 2010. *Social and economic networks*. Princeton University Press.

Lee, Robin S and Kyna Fong. 2013. "Markov-Perfect Network Formation An Applied Framework for Bilateral Oligopoly and Bargaining in Buyer-Seller Networks." Tech. rep. URL http://www.people.fas.harvard.edu/~robinlee/papers/MPNENetworkFormation.pdf.

Leung, Michael. 2013. "Two-step estimation of network-formation models with incomplete information." *Available at SSRN* 2254145. URL http://ftp.zew.de/pub/zew-docs/veranstaltungen/SEEK2013/SocialNetwork_Workshop/SEEK2013_BEN_Leung.pdf.

Sheng, Shuyang. 2012. "Identification and estimation of network formation games." *Unpublished Manuscript* URL http://www-scf.usc.edu/~ssheng/network.pdf.