Estimating Production Functions

Methodology Extensions

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Mistakes have been made

Control function estimators of production functions have repeatedly been used without fully thinking through the underlying model and assumptions

1. Colinearity of flexible inputs with each other
   - Pointed out by Ackerberg, Caves, and Frazer (2015)
2. Lack of relevant instrument for flexible input
   - Pointed out by Gandhi, Navarro, and Rivers (2016)
3. Heterogeneous markups are incompatible with the monotonicity assumption
   - Mistake in De Loecker and Warzynski (2012) (1337 citations), repeated in De Loecker, Eeckhout, and Unger (2020) (1268 citations)
   - Pointed out by Doraszelski and Jaumandreu (2019), Doraszelski and Jaumandreu (2021)

Extensions

- Levinsohn and Petrin (2003): investment often zero, so use other inputs instead of investment to form control function
- Ackerberg, Caves, and Frazer (2015): control function often collinear with l_{it} – for it not to be must be firm specific unobervables affecting l_{it} (but not investment / other input or else demand not invertible and cannot form control function)
- Gandhi, Navarro, and Rivers (2016): relax scalar unobservable in investment / other input demand
- Wooldridge (2009): more efficient joint estimation
- Maican (2006) and Doraszelski and Jaumandreu (2013): endogenous productivity

Section 1

Ackerberg, Caves, and Frazer (2015)

Ackerberg, Caves, and Frazer (2015): contributions

- Document collinearity problem in OP and Levinsohn and Petrin (2003)
  - Need $l_{it}, f_{it}(k_{it}, i_{it})$ not collinear, i.e. something causes variation in l, but not k
- Propose alternative estimator
- Relates estimator to dynamic panel (Blundell and Bond, 2000) approach

[0]*These slides are based on the working paper version Ackerberg, Caves, and Frazer (2006).

Collinearity in OP 1

- OP assume $i_{it} = I_t(k_{it}, \omega_{it})$
- Symmetry, parsimony suggest $l_{it} = L_t(k_{it}, \omega_{it})$
- Then $l_{it} = L_t(k_{it}, I_t^{-1}(k_{it}, i_{it})) = g_t(k_{it}, i_{it})$

$$y_{it} = \beta_l l_{it} + f_t(k_{it}, i_{it}) + \epsilon_{it}$$

  l_{it} collinear with $f_t(k_{it}, i_{it})$
- Worse in Levinsohn and Petrin (2003)
  - Uses other input m_{it} to form control function

$$\begin{aligned} y_{it} = & \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + \omega_{it} + \epsilon_{it} \\ m_{it} = & M_t(k_{it}, \omega_{it}) \end{aligned}$$

  - Even less reason to treat labor demand differently than other input demand
- Collinearity still problem with parametric input demand

Collinearity in OP 2

- Plausible models that do not solve collinearity
  - Input price data
    - Must include in control function to preserve scalar unobservable
    - Same logic above implies m and l are functions of both prices, so still collinear
  - Adjustmest costs in labor
    - Need to add l_{it-1} to control function
  - Change in timing assumptions
  - Measurement error in l (but not m)
    - Solves collinearity, but makes $\hat{\beta}_l$ inconsistent
- Potential model change that removes collinearity
  - Optimization error in l (but not m)
  - m chosen, l specific shock revealed, l chosen
  - OP only: l_{it} chosen at $t - 1/2$, $l_{it} = L_t(\omega_{it-1/2}, k_{it})$, i_{it} chosen at t

ACF estimator

- Idea: like capital, labor is harder to adjust than other inputs
- Model: l_{it} chosen at time $t - 1/2$, m_{it} at time t
  - Implies $m_t = M_t(k_{it}, l_{it}, \omega_{it})$
- Estimation:
  1. $y_{it} = \underbrace{\beta_k k_{it} + \beta_l l_{it} + f_t(m_{it}, k_{it}, l_{it})}_{\equiv \Phi_t(m_{it}, k_{it}, l_{it})} + \epsilon_{it}$ gives

$$\hat{\omega}_{it}(\beta_k, \beta_l) = \hat{\Phi}_{it} - \beta_k k_{it} - \beta_l l_{it}$$

  2. Moments from timing and Markov process for ω_{it} assumptions:

$$\omega_{it} = \mathrm{E}[\omega_{it} | \omega_{it-1}] + \xi_{it}$$

     - $\mathrm{E}[\xi_{it} | k_{it}] = 0$ as in OP
     - $\mathrm{E}[\xi_{it} | l_{it-1}] = 0$ from new timing assumption
     - $\hat{\xi}_{it}(\beta_k, \beta_l)$ as residual from nonparametric regression of $\hat{\omega}_{it}$ on $\hat{\omega}_{it-1}$
     - Can add moments based on $\mathrm{E}[\epsilon_{it} | \mathcal{I}_{it}] = 0$

Relation to dynamic panel estimators

- Both derive moment conditions from assumptions about timing and information set of firm
- Dealing with ω
  - Dynamic panel: AR(1) assumption allows quasi-differencing
  - Control function: makes ω estimable function of observables
- Dynamic panel allows fixed effects, does not make assumptions about input demand
- Control function allows more flexible process for ω_{it}

Simulations

- DGPS:
  1. Consistent with their model, but not LP
  2. Consistent with both
  3. Combination that consistent with neither
- Add measurement error to materials

Simulation Results

TABLE I

MONTE CARLO RESULTS[a]

Meas. Error	ACF β_l Coef.	ACF β_l Std. Dev.	ACF β_k Coef.	ACF β_k Std. Dev.	LP β_l Coef.	LP β_l Std. Dev.	LP β_k Coef.	LP β_k Std. Dev.
DGP1—Serially Correlated Wages and Labor Set at Time $t-b$								
0.0	0.600	0.009	0.399	0.015	0.000	0.005	1.121	0.028
0.1	0.596	0.009	0.428	0.015	0.417	0.009	0.668	0.019
0.2	0.602	0.010	0.427	0.015	0.579	0.008	0.488	0.015
0.5	0.629	0.010	0.405	0.015	0.754	0.007	0.291	0.012
DGP2—Optimization Error in Labor								
0.0	0.600	0.009	0.400	0.016	0.600	0.003	0.399	0.013
0.1	0.604	0.010	0.408	0.016	0.677	0.003	0.332	0.011
0.2	0.608	0.011	0.410	0.015	0.725	0.003	0.289	0.010
0.5	0.620	0.013	0.405	0.017	0.797	0.003	0.220	0.010
DGP3—Optimization Error in Labor and Serially Correlated Wages and Labor Set at Time $t-b$ (DGP1 plus DGP2)								
0.0	0.596	0.006	0.406	0.014	0.473	0.003	0.588	0.016
0.1	0.598	0.006	0.422	0.013	0.543	0.004	0.522	0.014
0.2	0.601	0.006	0.428	0.012	0.592	0.004	0.473	0.012
0.5	0.609	0.007	0.431	0.013	0.677	0.005	0.386	0.012

[a] 1000 replications. True values of β_l and β_k are 0.6 and 0.4, respectively. Standard deviations reported are of parameter estimates across the 1000 replications.

Section 2

Gandhi, Navarro, and Rivers (2016)

Gandhi, Navarro, and Rivers (2016)

- Show that control function method is not nonparametrically identified when there are flexible inputs
- Propose alternate estimate that uses data on input shares and information from firm's first order condtiion
- Show that value-added and gross output production functions are incompatible
- Application to Colombia and Chile

Assumptions

1. Hicks neutral productivity $Y_{jt} = e^{\omega_{jt}+\epsilon_{jt}} F_t(L_{jt}, K_{jt}, M_{jt})$
2. ω_{jt} Markov, ϵ_{jt} i.i.d.
3. K_{jt} and L_{jt} determined at $t-1$, M_{jt} determined flexibly at t
    - K and L play same role in the model, so after this slide I will drop L
4. $M_{jt} = \mathbb{M}_t(L_{jt}, K_{jt}, \omega_{jt})$, monotone in ω_{jt}

Reduced form

- Let $h(\omega_{jt-1}) = \mathrm{E}[\omega_{jt}|\omega_{jt-1}]$, $\eta_{jt} = \omega_{jt} - h(\omega_{jt-1})$
- log output

$$\begin{aligned} y_{jt} =& f_t(k_{jt}, m_{jt}) + \omega_{jt} + \epsilon_{jt} \\ =& f_t(k_{jt}, m_{jt}) + \underbrace{h(\mathbb{M}_{t-1}^{-1}(k_{jt-1}, m_{jt-1}))}_{=h_{t-1}(k_{jt-1}, m_{jt-1})} + \eta_{jt} + \epsilon_{jt} \end{aligned}$$

- Assumptions imply

$$\mathrm{E}[\eta_{jt}|\underbrace{k_{jt}, k_{jt-1}, m_{jt-1}, \ldots k_{j1}, m_{j1}}_{=\Gamma_{jt}}] = 0$$

- Reduced form

$$\mathrm{E}[y_{jt}|\Gamma_{jt}] = \mathrm{E}[f_t(k_{jt}, m_{jt})|\Gamma_{jt}] + h_{t-1}(k_{jt-1}, m_{jt-1}) \qquad (1)$$

- Identification: given observed $\mathrm{E}[y_{jt}|\Gamma_{jt}]$ is there a unique f_t, h_{t-1} that satisfies (3)?

Example: Cobb-Douglas 1

- Let $f_t(k,m) = \beta_k k + \beta_m m$
- Assume firm is takes prices as given
- First order condition for m gives

$$m = constant + \frac{\beta_k}{1-\beta_m} k + \frac{1}{1-\beta_m}\omega$$

- Put into reduced form

$$\mathrm{E}[y_{jt}|\Gamma_{jt}] = C + \frac{\beta_k}{1-\beta_m} k_{jt} + \frac{\beta_m}{1-\beta_m}\mathrm{E}[\omega_{jt}|\Gamma_{jt}] + h_{t-1}(k_{jt-1}, m_{jt-} \tag{2}$$

- ω Markov and $\omega_{jt-1} = \mathbb{M}_{t-1}^{-1}(k_{jt-1}, m_{jt-1})$ implies

$$\mathrm{E}[\omega_{jt}|\Gamma_{jt}] = \mathrm{E}[\omega_{jt}|\omega_{jt-1} = \mathbb{M}_{t-1}^{-1}(k_{jt-1}, m_{jt-1})] = \\ = h_{t-1}(k_{jt-1}, m_{jt-1})$$

Example: Cobb-Douglas 2

- Which leaves

$$\mathrm{E}[y_{jt}|\Gamma_{jt}] = constant + \frac{\beta_k}{1-\beta_m} k_{jt} + \frac{1}{1-\beta_m} h_{t-1}(k_{jt-1}, m_{jt-1}) \tag{3}$$

  from which β_k, β_m are not identified
- Rank condition fails, $\mathrm{E}[m_{jt}|\Gamma_{jt}]$ is colinear with $h_{t-1}(k_{jt-1}, m_{jt-1})$
- After conditioning on $k_{jt}, k_{jt-1}, m_{jt-1}$, only variation in m_{jt} is from η_{jt}, but this is uncorrelated with the instruments

Identification from first order conditions 1

- Since m flexible, it satisfies a simple static first order condition,

$$\rho_t = p_t \frac{\partial F_t}{\partial M} \mathrm{E}[e^{\epsilon_{jt}}] e^{\omega_{jt}}$$

$$\log \rho_t = \log p_t + \log \frac{\partial F_t}{\partial M}(k_{jt}, m_{jt}) + \log \mathrm{E}[e^{\epsilon_{jt}}] + \omega_{jt}$$

- Problem: prices often unobserved, endogenous ω
- Solution: difference from output equation to eliminate ω, rearrange so that it involves only the value of materials and the value of output (which are often observed)

$$\underbrace{s_{jt}}_{\equiv \log \frac{\rho_t M_{jt}}{p_t Y_{jt}}} = \log \underbrace{G_t(k_{jt}, m_{jt})}_{\equiv \left(M_t \frac{\partial F_t}{\partial M}\right)/F_t} + \log \underbrace{\mathrm{E}[e^{\epsilon_{jt}}]}_{\mathcal{E}} - \epsilon_{jt}$$

Identification from first order conditions 2

- Identifies elasticity up to scale, $G_t\mathcal{E}$ and ϵ_{jt} which identifie $\mathcal{E}$
- Integrating,

$$\int_{m_0}^{m_{jt}} G_t(k_{jt}, m)/m = f_t(k_{jt}, m_{jt}) + c_t(k_{jt})$$

  identifies f up to location
- Output equation

$$y_{jt} = \int_{m_0}^{m_{jt}} \tilde{G}_t(k_{jt}, m)/m - c_t(k_{jt}) + \omega_{jt} + \epsilon_{jt}$$

$$-c_t(k_{jt}) + \omega_{jt} = \underbrace{y_{jt} - \int_{m_0}^{m_{jt}} \tilde{G}_t(k_{jt}, m)/m - \epsilon_{jt}}_{\equiv \mathcal{Y}_{jt}}$$

Identification from first order conditions 3

where the things on the right have already been identified

- Identify c_t from

$$\mathcal{Y}_{jt} = -c_t(k_{jt}) + \tilde{h}_t(\mathcal{Y}_{jt-1}, k_{jt-1}) + \eta_{jt}$$

Value added vs gross production

- Value added:

$$\begin{aligned} VA_{jt} = & p_t Y_{jt} - \rho_t M_{jt} \\ = & p_t F_t(K_{jt}, \mathbb{M}_t(K_{jt}, \omega_{jt})) e^{\omega_{jt}+\epsilon_{jt}} - \rho_t \mathbb{M}_t(K_{jt}, \omega_{jt}) \end{aligned}$$

- Envelope theorem implies $\text{elasticity}^Y_{e^\omega} \approx \text{elasticity}^{VA}_{e^\omega}(1 - \frac{\rho_t M_{jt}}{p_t Y_{jt}})$

Problems

- Production Hicks-neutral productivity does not imply value-added Hicks-neutral productivity
- Ex-post shocks ϵ_{jt} not accounted for in approximation

Empirical results

- Look at tables
- Value-added estimates imply much more productivity dispersion than gross (90-10) ratio of 4 vs 2

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