Demand and supply of differentiated products

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Part I

Implementation

Computational issues

- Non-convex optimization problems are almost always difficult to solve, this is no exception
- Nested iteration can be problematic
  - Solve for $\delta(theta)$:

    ```
    while norm(T(delta) - delta) > tolerance1 { delta
    ```

  - Minimize

    ```
    while norm(theta - thetaOld) > tolTheta && norm(f
      thetaOld = theta
      fold = f
      // update theta by e.g. newton's method, set f =
    }
    ```

  - Error in δ can lead to error in minimization
  - Error in δ is not a continuous with respect to θ (where changing θ changes number of iterations)

pyBLP

- Conlon and Gortmaker (2020), Conlon and Gortmaker (2023)
- Most used and feature complete implementation
- https://pyblp.readthedocs.io/en/stable/introduction.html

Nevo

- Popular code provided by Nevo (2000)
- Requires: Matlab, optimization toolbox
- Nevo's code does not run in current version of Matlab, but Rasmusen (2006) update does
- Code runs in Octave after changing `fminsearch` to another optimization routine
- Worked on by three people
- Used by at least six other papers (see Knittel and Metaxoglou (2014) footnote 5 for list)
- Fast for data provided

Nevo - issues

- Minimization difficult and not robust
  - Starting value
  - Algorithm
  - Tolerance for finding δ (Dubé, Fox, and Su (2012b) show loose tolerance affects estimates)
  - Knittel and Metaxoglou (2014) algorithms often stop at point where first and/or second order conditions fail
  - Knittel and Metaxoglou (2014) differences among convergence points economically significant

Dubé, Fox, and Su (2012b) 1

- Fixed point iteration to compute δ messes up GMM minimization; also is not best method for finding δ
  - Table 1: shows problem is too large a tolerance. NFP gives good estimates when tolerances are tight
- Can recast problem as constrained minimization

$$\min_{\theta,\delta} \sum_{\ell} \left(\frac{1}{JT} \sum_{j=1}^{J} \sum_{t=1}^{T} \xi_{jt}(\theta, \delta) f_{\ell}(w_t) \right)^2$$

subject to

$$\hat{s} = \sigma(\cdot; \theta, \delta)$$

- Su and Judd (2012): "mathematical programming with equilibrium constraints" (MPEC)
- Use state of the art algorithm to solve constrained minimization

Dubé, Fox, and Su (2012b) 2

- Solvers work best with accurate (i.e. not finite difference) derivatives—supplying 1st and 2nd order derivatives makes algorithm take approximately 1/3 as long as with just 1st order (Dubé, Fox, and Su, 2012a)
- Gains from exploiting sparsity of Jacobian of constraints and Hessian of objective function

Dubé, Fox, and Su (2012b) code

- Code requires: Matlab, KNITRO
- KNITRO proprietary, free version limited to 300 variables & constraints
- KNITRO can be replaced with other optimization algorithm, but others do not seem to work as well:
    - IPOPT uses similar algorithm, but I had trouble installing
    - NLOPT has no interior point algorithm, its algorithms do not seem to deal with nonlinear constraints very well
    - Skrainka (2012) uses SNOPT, which is similar algorithm to NLOPT's SLSQP
- Runs in Octave with KNITRO replaced by NLOPT

Observations

- High quality commercial solver appears necessary; my attempts with NLOPT fail and take longer
  - Skrainka (2012) uses SNOPT instead of KNITRO
- KNITRO and SNOPT not perfect
  - Still sensitive to starting values
  - MPEC replaces a contraction – a problem we know we can solve – with constraints that may make the optimization harder
    - Reynaerts, Varadhan, and Nash (2012) give method to improve accuracy and speed of computing δ
    - Dubé, Fox, and Su (2012a) using nested fixed point requires fewer solver iterations than MPEC, but takes as long or longer because of time spend solving for δ (can be much longer if contraction mapping is slow)
- Reynaert and Verboven (2014): using optimal instruments makes optimization more robust

References about implementation

- Overviews
  - Nevo (2000)
  - Dubé, Fox, and Su (2012b), Dubé, Fox, and Su (2012a)
  - Knittel and Metaxoglou (2014)
  - Skrainka (2012)
- Particular issues
  - Skrainka and Judd (2011): integration
  - Reynaerts, Varadhan, and Nash (2012): solving for δ
- Course on discrete choice models with simulation by Kenneth Train `http://elsa.berkeley.edu/users/train/distant.html`
- Bayesian: Jiang, Manchanda, and Rossi (2009), Brian Viard, Gron, and Polson (2014), Sun and Ishihara (2013)
- Overview of optimization methods and software Leyffer and Mahajan (2010)

Section 1

Fosgerau, Monardo, and de Palma (2024)

The Inverse Product Differentiation Logit Model

- Fosgerau, Monardo, and de Palma (2024)
- Flexible demand model with closed from inverse share
- Generalization of nested logit
- Faster, more stable computation

Setup

- Typical demand models (logit, nest logit, random coefficients logit) have:
  1. Linear index of average product desirability:

$$\underbrace{\delta_{jt}}_{\text{product, market}} = \underbrace{x_{jt}}_{\text{product characteristics}}\beta - \alpha \underbrace{p_{jt}}_{\text{price}} + \underbrace{\xi_{jt}}_{\text{demand shock}}$$

  2. Share equations:

$$s_{jt} = \underbrace{\sigma_j}_{\text{known share function}}(\delta_t; \underbrace{\theta_2}_{\text{additional parameters}})$$

Example Share Functions

- Logit:

$$s_{jt} = \frac{e^{\delta_{jt}}}{\sum_{k=0^J} e^{\delta_{kt}}}$$

- Random coefficients logit:

$$s_{jt} = \int \frac{e^{\delta_{jt} + x_{jt}\Sigma\nu}}{\sum_{k=0^J} e^{\delta_{kt} + x_{kt}\Sigma\nu} dF(\nu)}$$

- Nested logit:

$$s_{jt} = \frac{\exp\left(\mu \log\left(\sum_{k \in \mathcal{G}(j)} e^{\delta_{kt}}\right)\right)}{\sum_{\mathcal{G}} \exp\left(\mu \log\left(\sum_{k \in \mathcal{G}} e^{\delta_{kt}}\right)\right)} \frac{e^{\delta_{jt}}}{\sum_{k \in \mathcal{G}(j)} e^{\delta_{kt}}}$$

Nest(ed) Logit is Invertible

- Logit inverse share

$$\delta_{jt} = \log(s_{jt}) - \log(s_{0t})$$

- Mixed logit inverse share

$$\delta_{jt} = (1-\mu)\log(s_{jt}) + \mu \log\Big(\sum_{k \in \mathcal{G}(j)} s_{kt}\Big) - \log(s_{0t})$$

- Can estimate by linear IV

$$\log(s_{jt}/s_{0t}) = x_{jt}\beta - p_{jt}\alpha + \mu \log\Big(\sum_{k \in \mathcal{G}(j)} s_{kt}\Big) + \xi_{jt}$$

Inverse Product Differentiation Logit

- D discrete product characteristics define partitions / groups
- $d(j) \subseteq \{1, \dots, J\}$ = products in same group as j in partition d
- Specify inverse share

$$\sigma_j^{-1}(\mathbf{s}_t; \theta_2) = \left(1 - \sum_{d=1}^{D} \mu_d\right) \log(s_{jt}) + \sum_{d=1}^{D} \mu_d \log(\overbrace{s_{d(j)t}}^{\equiv \sum_{k \in d(j)} s_{kt}}) - \log(s_{0t})$$

IPDL Properties

- Inverse share has closed form, but share does not
- Can be micro founded from model of consumers choosing shares (not discrete choice)
    - Allows complements
- Simulations indicate it “accommodates similar consumer heterogeneity as the RCL model with independent normal random coefficients on dummies for groups defined by market segmentation”

IPDL Estimation

$$\log(s_{jt}/s_{0t}) = x_{jt}\beta - p_{jt}\alpha + \sum_{d=1}^{D} \mu_d \log\left(\frac{s_{jt}}{s_{d(j)t}}\right) + \xi_{jt}$$

- Need instruments for price and share terms (at least $1 + D$ instruments)
-

Simulations

- See paper

Section 2

References

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