Demand and supply of differentiated products

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References

- Reviews:
  - Gandhi and Nevo (2021)
  - Berry and Haile (2021)
  - Aguirregabiria (2021) chapter 2
  - Hortaçsu and Joo (2023) 2.1-2.2 and chapter 3
  - Ackerberg et al. (2007) section 1 (these slides use their notation)
  - Reiss and Wolak (2007) sections 1-7, especially 7
- Classic papers:
  - Berry (1994)
  - Berry, Levinsohn, and Pakes (1995)

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Section 1

Introduction

Introduction

- Typical market for consumer goods has many differentiated, but similar products, e.g.
  - Cars
  - Cereal
- Differentiated products are a source of market power
- Having many products can result in many parameters creating estimation difficulties and requiring departures from textbook demand and supply models

Motivation

- Counterfactuals that do not change production technology
  - Mergers
  - Tax changes
- Effects of new goods
- Cost-of-living indices
- Product differentiation and market power
  - Cross-price elasticities

Section 2

Demand in product space

Demand in product space 1

- J products, each treated as separate good
- Classical demand,

$$\begin{aligned} q_1 =& D_1(p_1, \ldots, p_J, z_1, \eta_1; \beta_1) \\ \vdots =& \vdots \\ q_J =& D_J(p_1, \ldots, p_J, z_J, \eta_J; \beta_J), \end{aligned}$$

and supply (firms' first-order conditions for prices):

$$\begin{aligned} p_1 =& g_1(q_1, \ldots, q_J, w_1, \nu_1; \theta_1) \\ \vdots =& \vdots \\ p_J^d =& g_J(q_1, \ldots, q_J, w_J, \nu_J; \theta_J), \end{aligned}$$

where

Demand in product space 2

- p_j = price
- q_j = quantity
- z_j = observed demand shifter
- η_j = unobserved demand shock
- β_j = demand parameters
- w_j = observed supply shifter
- ν_j = unobserved supply shock
- θ_j = supply parameters

- D_j typically parametrically specified, e.g.

$$\ln q_j = \beta_{j0} + \beta_{j1} p_1 + \cdots + \beta_{jJ} p_J + \beta_{jy} \ln y + Z_1 \gamma + \nu_j$$

Demand in product space

- Use reduced form to find instruments

$$
\begin{aligned}
q_1 =& \Pi_1^q(Z, W, \nu, \eta; \beta, \theta) \\
\vdots =& \vdots \\
q_J =& \Pi_J^q(Z, W, \nu, \eta; \beta, \theta) \\
p_1 =& \Pi_1^p(Z, W, \nu, \eta; \beta, \theta) \\
\vdots =& \vdots \\
p_J =& \Pi_J^p(Z, W, \nu, \eta; \beta, \theta)
\end{aligned}
$$

- Cost shifters of product j excluded from demand and supply of product k, but in reduced form
  - Cost data often not available
  - If available, unlikely to be product specific
- Attributes of other products
  - Hausman (1996) uses prices of other products
  - Hard to justify, especially with prices

Demand in product space 1

- Advantages of product space:
  - Flexible substitution patterns
  - Does not require detailed product attribute data
- Problems with product space:
  1. Representative agent and aggregation issues
     - With heterogeneous preferences, aggregate market demand need not meet restrictions on individual demand derived from economic theory
     - Cannot use restrictions easily to improve estimates
     - Can use simulation to aggregate (Pakes, 1986)
  2. Too many parameters, $O(J^2)$
     - Can limit by restricting cross-price elasticities, e.g. Pinkse, Slade, and Brett (2002)
  3. Too many instruments needed, J
  4. Cannot analyze new goods

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Demand in characteristic space

Demand in characteristic space

- Why do firms differentiate products?

Demand in characteristic space

- Why do firms differentiate products?
- Because consumers have heterogeneous tastes for product characteristics
    - E.g. cars: tastes for size, safety, fuel efficiency, etc

Demand in characteristic space

- Model consumer preferences for characteristics and treat products as bundles of characteristics

Demand in characteristic space

- Model consumer preferences for characteristics and treat products as bundles of characteristics
- Reduces number of parameters
- Predict demand for new goods
- Demand system consistent with utility maximization

Demand in characteristic space

- Model consumer preferences for characteristics and treat products as bundles of characteristics
- Reduces number of parameters
- Predict demand for new goods
- Demand system consistent with utility maximization
- Early work: Lancaster (1971), McFadden (1973)
- Key extension to early work: Berry, Levinsohn, and Pakes (1995)

Early work in characteristic space

- Consumer chooses one or none of J products
- Utility of consumer i from product j

$$u_{ij} = x_j\beta + \epsilon_{ij}$$

  with ϵ_{ij} iid across i and j (usually Type I extreme value)
- Implies aggregate demand (for Type I extreme value)

$$q_j = \frac{\exp(x_j\beta)}{1 + \sum_{k=1}^{J} \exp(x_k\beta)}$$

Downside of Logit

- Problem: restrictive substitution "independence of irrelevant alternatives"
  - Two goods with the same shares have the same cross price elasticities with any third good (think about a luxury and bargain good with equal shares)
  - Goods with same shares should have same markups
- Solution: add heterogeneity in β and/or allow correlation across j in ϵ_{ij}

Model 1

- Consumers i, goods j, markets t
- Utility: (include good $0 =$ buy nothing)

$$u_{ijt} = U\left(\overbrace{\underbrace{p_{jt}, \tilde{x}_{jt}}_{\text{observed}}, \underbrace{\xi_{jt}}_{\text{unobserved}}}^{\text{product characteristics}}, \overbrace{\underbrace{z_{it}}_{\text{observed}}, \underbrace{\nu_{it}}_{\text{unobserved}}}^{\text{consumer characteristics}} ; \theta \right)$$

  - $x_{jt} = (\tilde{x}_{jt}, p_{jt}) \in \mathbb{R}^K$, $z_{it} \in \mathbb{R}^R$, $\nu_{it} \in \mathbb{R}^L$
- Choose j if $u_{ijt} > u_{ikt}$ $\forall k \neq j$

Model 1

- Usually $U(\cdot)$ linear:

$$u_{ijt} = \overbrace{x_{jt}}^{1\times K} \overbrace{\underbrace{\theta_{it}}_{=\bar{\theta}+\theta^o z_{it}+\theta^u \nu_{it}}}^{K\times 1} + \overbrace{\xi_{jt}}^{1\times 1} + \epsilon_{ijt}$$

for $j = 1...J$ and normalize $u_{i0t} = 0$

- Assume ϵ_{ijt} i.i.d. double exponential
- Assume $\nu_{it} \sim f_\nu(\cdot;\theta)$, e.g. independent normal
- Write as product specific + observed interactions + unobserved interactions

$$u_{ijt} = \underbrace{\delta_j}_{=x_{jt}\bar{\theta}+\xi_{jt}} + x_{jt} \underbrace{\theta^o}_{K\times R} z_{it} + x_{jt} \underbrace{\theta^u}_{K\times L} \nu_{it} + \epsilon_{ijt}$$

Endogeneity

- Usually assume $\mathrm{E}[\nu_{it}|x_{jt}, z_{it}] = 0$ and $\mathrm{E}[\epsilon_{ijt}|x_{jt}, z_{it}] = 0$
  - Not interested in counterfactuals with respect to changes in z_{it}, so can treat as residual, i.e.

$$\nu_{it} = \theta_{it} - \mathrm{E}[\theta_{it}|z_{it}]$$

  - Market average ν_{it} or ϵ_{ijt} plausibly correlated with p_{jt} or other product characteristics, but this correlation absorbed into ξ_{jt} and/or market fixed effects

Endogeneity

- Problem is ξ_{jt}
  - Prices and other flexible product characteristics will be correlated with ξ_{jt}
  - If ξ_{jt} serially correlated, then possibly also correlated with inflexible product characteristics
  - Need instrument(s), w_{jt} such that $\mathrm{E}[\xi_{jt}|w_{jt}] = 0$
    - Cost shifters
    - Characteristics of other products

Aggregate data 1

- Often only have data on product characteristics and market shares
- Maybe also distribution of some individual characteristics for each market (e.g. income and education from CPS or census)
- Instrument w such that $\mathrm{E}[\xi_j | w] = 0$
- Distribution of $\nu \sim f_\nu(\cdot; \theta_\nu)$
    - Combination of estimated market level distribution of observed individual characteristics and parametric distributions of unobserved individual characteristics
    - e.g. $\nu_{it} = (educ_{it}, income_{it}, e_{it})$

$$F_{\nu,t}(s, y, e; \theta_\nu) = \underbrace{\hat{F}_t(s, y)}_{\text{empirical distribution}} \Phi\left(\frac{e - \theta_\nu^\mu}{\theta_\nu^\sigma}\right)$$

$\hat{F}_t(s, y)$ estimated from CPS or other similar data set

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Aggregate data 2

- Assume $\epsilon_{ijt} \sim$ double exponential (aka Gumbel or type I extreme value) as in logit
  - Computationally convenient, but other distributions feasible too

Estimation outline

- Estimate θ from moment condition

$$\mathrm{E}[\xi(\cdot;\theta)|w] = 0$$

- Where $\xi(\cdot;\theta)$ is such that model predicted market shares = observed market shares[1]
  1. Compute shares given θ, $\sigma(\cdot;\theta,\delta)$
  2. Find $\delta(\cdot;\theta) = x_{jt}\bar{\theta} + \xi(\cdot;\theta)$ such that observed shares, s_{jt} = model shares, $\sigma(\cdot;\theta,\delta)$, then

$$\xi(\cdot;\theta) = \delta(\cdot;\theta) - x_{jt}\bar{\theta}$$

[1] In this slide $\cdot$ means the data. I will leave the $\cdot$ out of the notation in subsequent slides. I will also leave out t subscripts.

Computing model shares

- Integrate over ν

$$\sigma_j(\theta, \delta) = \int \frac{\exp(\delta_j + x_j \theta^u \nu)}{1 + \sum_{k=1}^{j} \exp(\delta_k + x_k \theta^u \nu)} dF_\nu(\nu)$$

- Integral typically has no closed form, so compute numerically, usually by Monte Carlo integration

$$\sigma_j(\theta, \delta) = \sum_{r=1}^{N_s} \frac{\exp(\delta_j + x_j \theta^u \nu_r)}{1 + \sum_{k=1}^{j} \exp(\delta_k + x_k \theta^u \nu_r)}$$

where ν_r are N_s random draws from f_ν

  - Issues about how best to compute integral – simulation vs quadrature, type of simulation (Skrainka and Judd, 2011)
  - Simulation (more generally approximation) of integral affects distribution of estimator

Solving for δ and ξ

- Want δ s.t. $\sigma_j(\theta, \delta) = \hat{s}_j$
- Berry, Levinsohn, and Pakes (1995) show

$$T(\delta) = \delta + \log(\hat{s}_j) - \log(\sigma_j(\theta, \delta))$$

  is a contraction
  - Unique fixed point δ such that $\delta = \delta + \log(\hat{s}_j) - \log(\sigma_j(\theta, \delta))$, i.e. $\hat{s}_j = \sigma_j(\theta, \delta)$
  - Can compute $\delta(\theta)$ by repeatedly applying contraction (in theory and practice often faster to use other method)
- $\xi_j(\theta) = \delta_j(\theta) - x_j\bar{\theta}$
- Important identifying assumption: only θ s.t. $\xi_j(\theta) = \xi_j^0$ is true θ_0

Estimating θ

- Conditional moment restriction $\mathrm{E}[\xi_j(\theta)|w] = 0$
- Empirical unconditional moments:

$$G_{J,T,N,N_s} = \frac{1}{JT}\sum_{j=1}^{J}\sum_{t=1}^{T}\xi_{jt}(\theta)f(w_t)$$

where

  - $f(w)$ = vector of function of w
  - J = number of products
  - T = number of markets
  - N = number of observations in each market underlying $\hat{s}_j$
  - N_s = number of simulations
- Asymptotic properties (consistency, distribution), depend on which of J, T, N, and N_s are $\to\infty$, see Berry, Linton, and Pakes (2004)
- Reynaert and Verboven (2014): using optimal instruments greatly improves efficiency and stability

Pricing equation 1

- More moments give more precise estimates
- Assumption about form of equilibrium allows use of firm first order condition (pricing equation) as additional moment
- Nash equilibrium in prices
- Log linear marginal cost

$$\log mc_j = r_j \theta^k + \omega_j$$

  - r_j = observed product characteristics, input prices, maybe quantity, etc
  - ω_j = unobserved productivity, possibly endogenous
- Firm f producing set of product $\mathcal{J}_f$,

$$\max_{p_j : j \in \mathcal{J}_f} \sum_{j \in \mathcal{J}_f} \left(p_j - C_j(\cdot)\right) M s_j(\cdot, p)$$

Pricing equation 2

- First order condition:

$$\sigma_j(\cdot) + \sum_{l \in \mathcal{J}_f} (p_l - mc_l) \frac{\partial \sigma_l(\cdot)}{\partial p_j} = 0$$

- Collect as

$$s + (p - mc)\Delta = 0$$

- Rearrange and use log linear marginal cost

$$\log(p - \Delta^{-1}\sigma) - r\theta^c = \omega(\theta)$$

- Conditional moment restriction $\mathrm{E}[\omega(\theta)|w] = 0$
- Add empirical moments to G, $\frac{1}{JT}\sum_{jt} \omega_{jt}(\theta) f(w_t)$

Micro data

- Berry, Levinsohn, and Pakes (2004)
- Data on individual choices and characteristics

$$u_{ijt} = \underbrace{\delta_j}_{=x_{jt}\bar{\theta}+\xi_{jt}} + x_{jt}\underbrace{\theta^o}_{K\times R} z_{it} + x_{jt}\underbrace{\theta^u}_{K\times L} \nu_{it} + \epsilon_{ijt}$$

- Random coefficients discrete choice model, so can identify and estimate δ, θ^o, and θ^u without assumptions about ξ and x
  - Ichimura and Thompson (1998) give conditions for nonparametric identification of random coefficients binary choice models
  - Estimate by MLE or (usually) GMM
- Still need $\bar{\theta}$ for price elasticities, etc

$$\delta_j = x_{jt}\bar{\theta} + \xi_{jt}$$

  - Use IV
  - Use IV with a pricing equation

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References

Ackerberg, D., C. Lanier Benkard, S. Berry, and A. Pakes. 2007. "Econometric tools for analyzing market outcomes." *Handbook of econometrics* 6:4171–4276. URL http://www.sciencedirect.com/science/article/pii/S1573441207060631. Ungated URL http://people.stern.nyu.edu/acollard/Tools.pdf.

Aguirregabiria, Victor. 2021. "Empirical Industrial Organization: Models, Methods, and Applications." URL http://aguirregabiria.net/wpapers/book_dynamic_io.pdf.

Berry, S., J. Levinsohn, and A. Pakes. 2004. "Differentiated Products Demand Systems from a Combination of Micro and Macro Data: The New Car Market." *Journal of Political Economy* 112 (1):68–105. URL http://www.jstor.org/stable/10.1086/379939.

Berry, Steve, Oliver B. Linton, and Ariel Pakes. 2004. "Limit Theorems for Estimating the Parameters of Differentiated Product Demand Systems." *The Review of Economic Studies* 71 (3):613–654. URL http://restud.oxfordjournals.org/content/71/3/613.abstract.

Berry, Steven, James Levinsohn, and Ariel Pakes. 1995. "Automobile Prices in Market Equilibrium." *Econometrica* 63 (4):pp. 841–890. URL http://www.jstor.org/stable/2171802.

Berry, Steven T. 1994. "Estimating Discrete-Choice Models of Product Differentiation." *The RAND Journal of Economics* 25 (2):pp. 242–262. URL http://www.jstor.org/stable/2555829.

Berry, Steven T. and Philip A. Haile. 2021. "Chapter 1 - Foundations of demand estimation." In *Handbook of Industrial Organization, Volume 4*, *Handbook of Industrial Organization*, vol. 4, edited by Kate Ho, Ali Hortaçsu, and Alessandro Lizzeri. Elsevier, 1–62. URL https://www.sciencedirect.com/science/article/pii/S1573448X21000017.

Gandhi, Amit and Aviv Nevo. 2021. "Chapter 2 - Empirical models of demand and supply in differentiated products industries." In *Handbook of Industrial Organization, Volume 4*, *Handbook of Industrial Organization*, vol. 4, edited by Kate Ho, Ali Hortaçsu, and Alessandro Lizzeri. Elsevier, 63–139. URL https://www.sciencedirect.com/science/article/pii/S1573448X21000029.

Hausman, J.A. 1996. "Valuation of new goods under perfect and imperfect competition." In *The economics of new goods*. University of Chicago Press, 207–248. URL http://www.nber.org/chapters/c6068.pdf.

Hortaçsu, Ali and Joonhwi Joo. 2023. *Structural Econometric Modeling in Industrial Organization and Quantitative Marketing: Theory and Applications*. Princeton University Press.

Ichimura, Hidehiko and T.Scott Thompson. 1998. "Maximum likelihood estimation of a binary choice model with random coefficients of unknown distribution." *Journal of Econometrics* 86 (2):269 – 295. URL http://www.sciencedirect.com/science/article/pii/S0304407697001176.

Lancaster, K. 1971. *Consumer demand: A new approach*. Columbia University Press (New York).

McFadden, D. 1973. "Conditional logit analysis of qualitative choice behavior." *Frontiers in Econometrics* :105–142URL http://elsa.berkeley.edu/pub/reprints/mcfadden/zarembka.pdf.

Pakes, Ariel. 1986. "Patents as Options: Some Estimates of the Value of Holding European Patent Stocks." *Econometrica* 54 (4):pp. 755–784. URL http://www.jstor.org/stable/1912835.

Pinkse, J., M.E. Slade, and C. Brett. 2002. "Spatial price competition: a semiparametric approach." *Econometrica* 70 (3):1111–1153. URL http://onlinelibrary.wiley.com/doi/10.1111/1468-0262.00320/abstract.

Reiss, P.C. and F.A. Wolak. 2007. "Structural econometric modeling: Rationales and examples from industrial organization." *Handbook of econometrics* 6:4277–4415. URL http://www.sciencedirect.com.ezproxy.library.ubc.ca/science/article/pii/S1573441207060643.

Reynaert, Mathias and Frank Verboven. 2014. "Improving the performance of random coefficients demand models: The role of optimal instruments." *Journal of Econometrics* 179 (1):83 – 98. URL http://www.sciencedirect.com/science/article/pii/S0304407613002649.

Skrainka, B. and K. Judd. 2011. “High performance quadrature rules: How numerical integration affects a popular model of product differentiation.” Available at SSRN 1870703. URL http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1870703.